Computer Assignment 2

Solutions should be submitted on Blackboard by 11pm on Wednesday, 1 November

1. [15 marks]
   Consider the system of linear equations

$$\begin{bmatrix} 5 & 3 & 36 & 7 \\ 15 & 9 & 28 & -1 \\ -23 & -4 & -13 & 7 \\ 9 & 3 & 40 & -2 \end{bmatrix} x = \begin{bmatrix} 67 \\ 63 \\ -52 \\ 88 \end{bmatrix}.$$

a) Try to solve this system using Matlab function `gaussel` on pp. 30-31 of Matlab Beginner's Guide and explain why it does not work.

b) Modify the function `gaussel` to implement Gaussian elimination with *scaled partial pivoting*, as described in Section 3.2.1 of Pav's lecture notes. To do this, define vector l with elements l_i, which are set to $l_i = i,\ i = 1, \dots, N$, at the start. Then calculate the vector s of row scale factors: $s_i = \max_{1\le j\le N} |a_{ij}|$. Then, inside the j loop of the Gaussian elimination part of the code, determine index k such that $|a_{l_k,j-1}|/s_{l_k} \ge |a_{l_i,j-1}|/s_{l_i}$ for $j-1 \le i \le N$. If $k \ne j-1$, then swap the values of l_{j-1} and l_k. Finally, in both the Gaussian elimination and back substitution parts of the code replace the row index of a_{ij} and the index of b_i with the corresponding variable index l_i. Your modified function should have the header line
`function [x, l] = gaussel_spp(A, b)`, so that it outputs both x and l.

c) Test your function from part b) by solving the above system, presenting the results for both x and l. Given that the exact x has integer elements, determine the norm of the error of your solution. Compare this to the error of the solution obtained with Matlab's backslash '\' operator. Which solution is more accurate?
Note. If you did not manage to get your code in b) to work correctly, just determine the norm of the error of the solution with the backslash operator.

2. [15 marks]

a) Write a Matlab function `run_newton.m` which implements Newton's method according to the algorithm on p.52 of Pav's lecture notes (see Exercise 4.12 on p.60).

b) Use your function to find the real root of the polynomial $x^5 - x^4 - 4x^3 + 4x^2 + 5x - 5$.

c) Modify your code in a) to output all iterates of Newton's method, x_k, and calculate $e_k = x_k - x^*$, where $x^* = 1$ is the exact root. From the definition of the convergence rate (see Heath, Ch. 5, slide 13), we should observe a linear relationship between $\log|e_{k+1}|$ and $\log|e_k|$, that is

$$\log|e_{k+1}| \approx \log C + r \log|e_k|.$$

Plot $\log|e_{k+1}|$ as a function of $\log|e_k|$. Do the results follow a straight line? Explain possible deviations from the straight line at large and small $|e_k|$. Estimate the values of r and C from the linear part of the plot. Compare these values to those you would expect from the convergence analysis for Newton's method.

d) Repeat part c) for the polynomial $(x-1)^5$. Estimate the values of r and C and explain these values. *Hint:* See Heath, Ch. 5, slide 28.