The cost of strategic adaptation in a simple conceptual model of climate change

Antony Millner
Climate research group, Centre for the Environment, Oxford University, UK[*]

Giovanni Ciampaglia
Modelling and Applications of Complex Systems Lab, University of Lugano, Switzerland

Nish Aravamudan
Department of Computer Science, Portland State University, USA

Jacob Foster
Complexity Science Group, Department of Physics and Astronomy, University of Calgary, Canada
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A simple theoretical model of the process of strategic adaptation to climate change is proposed. Climate change is represented by a non-stationary Markov process on the space of climate states, and strategic adaptation by a simple resource allocation task in which agents incur costs when moving resources from one activity to another. A stationary analysis allows diagnostics that quantify the net costs of climate change, and the long-run benefits to adaptation, to be defined. A full dynamic analysis of the model allows for the computation of the costs of negotiating the transition between two stationary climate regimes. We analyze the dependence of these adaptation costs on the behavioural parameters of the model, and the costs of adjusting resources from one activity to another. We find that adaptation costs have a complex and counterintuitive dependence on adjustment costs, and can be more sensitive to the details of the climate change process than adaptation benefits are. This has important implications for adaptation planning, and understanding the linkages between adaptation and climate change mitigation.

I. INTRODUCTION

Over the past few years the issue of anthropogenic global climate change has been largely transformed in the public eye from one of considerable controversy and uncertainty to an established scientific fact. Even those who still dissent from the present weight of scientific and lay opinion are forced to accept that climate change is a major political issue, both domestically (as evinced in the recent Australian elections) and internationally. With this change in the political status of the science underlying climate change, attention has begun to shift away from whether it is happening to what can be done about it. A rapidly growing literature, spurred on by high profile government commissioned studies [24], has begun to investigate policy options in greater detail than ever before.

Policy responses are often differentiated into two broad categories – mitigation and adaptation. The literature on mitigation is rich and varied, drawing on a long intellectual history in the economics of public goods and externalities [14], game theoretic models of the efficacy and enforceability of multilateral environmental treaties [5, 11], and a wealth of real-world case studies [6]. The field is by no means without its major open questions - not least amongst them ethical concerns relating to inter-temporal welfare, and the equitable distribution of costs and damages - but these are well within the remit of traditional economics, and thus receive justly deserved attention.

In contrast, the literature on adaptation [see e.g. 1–3, 8, 21, 24], lacks an analytical framework comparable to that which informs the mitigation debate. In part this may be due to the inherent difficulties of the problem, which we will discuss more fully below. However, it must also be said that the incentive to develop a rigorous understanding of the adaptive process is not strong in most of the developed world. Setting aside possible catastrophic changes such as the melting of the West Antarctic ice sheet [19], or the shutting down of the thermohaline circulation [26], developed countries are reasonably well prepared to cope with moderate climate shifts, owing to their large and diversified economies, sophisticated risk distribution mechanisms, and a pernicious twist of geographical fate [10]. Indeed one of the largest costs they may incur due to climate change, at least in the short to medium term, may be the cost of complying with mitigation measures imposed on them by a more exacting international climate treaty than the Kyoto protocol. Hence their interest in mitigation policy. The same cannot be said for many developing countries,

[*]Electronic address: antony.millner@magd.ox.ac.uk

who are expected to bare the brunt of climate impacts [20], some of which the world is already committed to, due to the long residence time of atmospheric carbon dioxide [23]. It is these countries that stand to benefit the most from an understanding of the adaptive process.

Perhaps the central question the literature in climate adaptation seeks to answer is this: Under what conditions can a society successfully transit from a set of institutions, economic strategies, and technologies adapted to one climate regime to those adapted to another, and which policy interventions can aid this transition? By 'adapted' we mean a set of strategies and practices that successfully hedges against climate risks, and distributes both positive and negative outcomes relatively equitably amongst members of society.

Analytical approaches to this question are virtually nonexistent [see 12, 16, 28, for rare attempts], for a number of good reasons. Not least amongst these is the fact that the problem is inherently dynamic and nonstationary. Most economic analyses concerning behaviour in the face of uncertainty are concerned with decisions made under stationary risk distributions [13]. The best we can do with these techniques is appeal to a kind of comparative statics – comparing decision making under different stationary risks – however this approach cannot contribute to answering the dynamic question of how, or indeed if, behaviour shifts from one basin of attraction to another, and what the costs associated with this shift are. In order to understand these dynamic issues in all their detail, a theory which is capable of making statements about societies which are experiencing continual shocks drawn from nonstationary distributions is needed. Such a theory would need to provide some insight into how society attempts to adaptively cope with the cumulative effects of these shocks by reorganising itself and the strategies of production and consumption it is invested in. At a minimum, this theoretical juggernaut would need to confront questions relating to agent learning and innovation, institutional change, technology adoption, the role of prior beliefs and forecasts, and all the collective phenomena that arise when many agents engage in information exchange. Yohe & Tol [28] list an even more forbidding set of factors upon which adaptation is contingent. It will be no surprise to the reader that such a theory is not readily available.

It may well be unrealistic to expect any model to provide insight into such a complex question, whose answer, if attainable at all, must clearly be geographically localized, and ultimately, deeply empirical. Most of the integrated assessment models often used for policy evaluation neglect dynamic effects (by focusing on a few static snapshots of arbitrarily chosen scenarios), are heavily dependent on model assumptions and parameter choices, and are arguably too specialized to particular climate impacts to draw any conclusions of general applicability. Yet despite the manifest difficulty of the problem it is our belief that a simple and reasonably parsimonious representation of climate change processes, combined with a decision task so general as to have many relevant interpretations, can shed light on some of the key dependencies of the adaptive process, and in particular on when and for whom it can be expected to be costly. The requirements of tractability necessitate a dramatic paring down of the problem. We leave it to our readers to judge whether our abstract representation of the adaptive process retains sufficient verisimilitude to be useful in their own area of interest.

This paper attempts to contribute to the theoretical basis of adaptation policy. We will be particularly concerned with developing some intuition for which factors are likely to affect the costs of adaptation for agents who have different attitudes to risk and discounting behaviour, and face different impediments to strategic adjustment. While the literature has tended to focus on static measures of the benefits to adaptation we will also develop dynamic measures of the loss in productivity an agent sustains in the transient period between two stable climate regimes. We will see that, while static measures give us valuable information about long run costs and benefits of an environmental change, they are not capable of reflecting the losses an agent sustains in attempting to negotiate such a change. Such information is only captured by a fully dynamic treatment of the problem. When such dynamics are included our model suggests that the costs of adaptation may be much more sensitive to the parameters of the climate change process than the benefits. We illustrate this by means of a simple example. If this holds true in the real world, it has significant implications for informing adaptive planning, and for clarifying the linkages between mitigation and adaptation.

The structure of the paper is as follows: Section II describes our model of the adaptive task in detail. The model is comprised of an idealized specification of the climate an agent faces, in the form of a Markov process, and a very general resource allocation decision problem. Section III illustrates how the model may be solved in the case in which the climate is stationary. We define stationary measures of the cost of a climatic change, and the benefits to adaptation. These measures provide useful diagnostics of the long-run effects of an environmental change. In section IV we extend the model to the dynamic case, in which the climate evolves in time. We restrict our attention to a simple class of climate change processes, and show how the model may be solved in this case. The solution allows us to define a dynamic measure of the costs of adapting from one climate regime to another. By considering a simple example, we examine the dependence of these adaptation costs on the behavioural and economic parameters of the model, and explore their relationship to the stationary measures of section III. Section V discusses the findings of the modeling exercise, considers several possible extensions to the results, and concludes.

II. THE MODEL

In order to model strategic adaptation we consider an agent who must apportion a discrete finite set of resources (which could be labour, money, or physical or human capital) amongst a set of activities or technologies with climatically contingent returns. Our agent could represent a small-holder farmer choosing which crops to plant in his fields, a rural community assigning activities to its members, a multinational energy company evaluating its portfolio of renewable energy power stations, or a water resource manager attempting to provide water to a growing city. At each time step the agent can choose to move any of his resources from one activity to another. We assume a constant cost c for moving any unit of resource. In the examples mentioned above this could represent the cost of reconditioning a field, retraining a worker, building a new plant or moving physical capital, and changing catchment and demand management strategies. The agent's strategy is a set of rules which tell him if and where to move his resource units as a function of the history of previous climate states and his previous resource allocations. In this simple model we neglect all interactions between agents and focus on an individual decision maker. We formalize this setup below.

A given climate is partially specified by defining a finite set of possible environmental states E, whose total number $N_{env} := |E|$. The specification is completed by assuming that the time series of realized states is a Markov process $\varepsilon(t)$ with transition probability matrix $\mathbf{p}(t)$. Thus,

$$\Pr(\varepsilon(t+1) = e' | \varepsilon(t) = e) = p_{ee'}(t) \tag{1}$$

Such a simplified model of climate has been used elsewhere [e.g. 17] as a parsimonious and tractable representation of stochastic correlated environmental dynamics. It has the advantage of being able to represent changes in both mean conditions and the variability of a given climatic regime, as well as having a well specified correlation structure. Indeed, Markov processes were amongst the first statistical models of weather [27].

Now suppose that the agent has access to N_{tech} technologies or activities, indexed by k. Technology k is assumed to yield a return of $f_k^{(e)}$ in climate state e. Thus the $N_{tech} \times N_{env}$ matrix $\mathbf{f}$ contains all the information about the performance of the various technologies in each of the climate states. We assume that the agent has N_{res} units of resource to allocate dynamically amongst the technologies. The set of possible actions he can take is just the set of ways of allocating these resources amongst the N_{tech} technologies. We will refer to this set of possible allocations, the action set, as A. One can show that $N_{act} := |A|$, the total number of such allocations is

$$N_{act} = \binom{N_{res} + N_{tech} - 1}{N_{res}} \tag{2}$$

where the term on the right is a binomial coefficient. A generic allocation $a \in A$ is an N_{tech}-vector, where the components a_k of a are the number of resources allocated to technology k, and $\sum_{k=1}^{N_{tech}} a_k = N_{res}$.

The model's state at any time t can be represented as the ordered pair $s(t) = (e(t), a(t))$, where $e(t) \in E$ and $a(t) \in A$. The set of all such pairs form the state space $S = E \times A$ of the model. Using this state-space notation, the probability $P^b_{ss'}$ of transitioning from state $s = (e, a)$ to state $s' = (e', a')$ when action b is taken is

$$P^b_{ss'}(t) = \begin{cases} p_{ee'}(t) & \text{if } a' = b \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The reward structure of the model is as follows: Suppose that when the agent is in state $s = (e, a)$, he takes action a'. Assume that the next climate state is e', implying that the next state of the model is $s' = (e', a')$. Then we define the rewards $R^{a'}_{ss'}$ associated with taking action a' when in state s, and when the next state is s' as:

$$R^{a'}_{ss'} = \sum_{k=1}^{N_{tech}} \left(f_k^{(e')} a'_k - \frac{c}{2} |a_k - a'_k| \right). \tag{4}$$

The rewards are comprised of two parts: the benefits realized from allocation a' in the climate state e', and the costs of moving resources from allocation a to allocation a'. The magnitude of these costs is controlled by c, the adjustment cost per unit resource moved. For reasons that will become clear later, we assume that $c < \min_{e,k} f_k^{(e)}$, i.e. the adjustment costs are less than the minimum possible returns.

The agent's behaviour is encoded by a policy function $\pi : S \to A$, which maps states to actions [29]. Thus $\pi(s) = \pi((e, a_{old})) = a_{new}$ tells the agent to shift his resources from allocation a_{old} to allocation a_{new} if he is using allocation a_{old} when the environmental state is e.

III. SOLVING THE MODEL IN THE STATIONARY CASE

In the case where the climate is stationary, implying that the transition matrix $\mathbf{p}(t)$ is fixed for all time, an optimal policy can be computed by the method of dynamic programming. This solution method is especially elegant when we assume there is no time-horizon, i.e. the climatic regime is infinitely long.

We define the expected discounted utility in state s at time t when policy π is employed as

$$V^{\pi}(s) = E_t\Big[U_\eta(r_t) + \delta U_\eta(r_{t+1}) + \delta^2 U_\eta(r_{t+2}) + \ldots | s(t) = s, a(t) = \pi(s)\Big]. \tag{5}$$

Here E_t denotes the expected value of the quantity in brackets at time t, $\delta \in (0,1)$ is the discount rate, r_t is the reward received at time t, π is a policy, and U_η is the utility function, which encodes the agent's risk preferences [see e.g. 13]. We will assume that U_η is of the constant relative risk aversion form:

$$U_\eta(r) = \begin{cases} \ln r & \text{if } \eta = 1 \\ \frac{r^{1-\eta}}{1-\eta} & \text{otherwise} \end{cases} \tag{6}$$

The non-negative parameter η is the coefficient of relative risk aversion[30].

We are interested in computing the optimal policy, i.e. a policy $\pi^*(s)$ for which $V^{\pi^*} \geq V^{\pi}$ for all other policies π, and the inequality is strict for at least one of the components of V^{π^*}. In order to achieve this, we exploit the recursive structure of the value function V^{π^*} in order to write down the Bellman optimality condition [see 18]. Define the value function $V^*(s) := V^{\pi^*}(s)$, then

$$V^*(s) = \max_{a \in A} \sum_{s' \in S} P^a_{ss'} \left(R^a_{ss'} + \delta V^*(s')\right) \tag{7}$$

If the value function V^* is known, the optimal policy π^* is

$$\pi^*(s) = \text{argmax} \sum_{s' \in S} P^a_{ss'} \left(R^a_{ss'} + \delta V^*(s')\right) \tag{8}$$

Equation (7) can be solved iteratively by a variety of methods, to an arbitrary degree of accuracy [18]. It is then trivial to extract the optimal policy π^* using equation (8).

The behaviour of an agent who follows policy $\pi(s)$ can be thought of as a Markov chain on the state space S. The transition matrix $\mathbf{T}^{\pi}$ for this process can be calculated from the policy and equation (3) as follows:

$$T^{\pi}_{ss'} = P^{\pi(s)}_{ss'} \tag{9}$$

A quantity of particular interest that can be calculated from this matrix is the stationary distribution q_s, i.e. the long-run probability of the system being in state s. This is just the left (row) eigenvector of the matrix $\mathbf{T}^{\pi}$ that corresponds to the eigenvalue 1[31]. Thus, the vector q satisfies

$$q = q\mathbf{T}^{\pi} \tag{10}$$

where q is normalized so that $\sum_{s \in S} q_s = 1$. Using this quantity, one can calculate the expected rewards (ER) that the agent achieves from using policy π when the climate's transition matrix is $\mathbf{p}$:

$$ER(\mathbf{p}, \pi) = \sum_{s \in S} q_s \sum_{s' \in S} T^{\pi}_{ss'} R^{\pi(s)}_{ss'} \tag{11}$$

The preceding analysis holds for stationary processes in the case of an infinite time horizon. If we think of a climate change scenario as a path between two stationary, infinitely long, climates parameterized by the transition matrices $\mathbf{p}$ and $\mathbf{p}'$, then the static analysis yields the optimal policies $\pi^*_{\mathbf{p}}$ and $\pi^*_{\mathbf{p}'}$ that the agent should hold in each of these bounding cases. Moreover, we can use the expected reward calculation given in equation (11) to define static measures of the absolute cost of the climate change, and the cost of not adapting to the new climate:

1. Define the environmental change cost (ECC) of a shift from climatic transition matrix $\mathbf{p}$ to $\mathbf{p}'$ as:

$$ECC(\mathbf{p} \rightarrow \mathbf{p}') = ER(\mathbf{p}, \pi^*_{\mathbf{p}}) - ER(\mathbf{p}', \pi^*_{\mathbf{p}'}) \tag{12}$$

2. Define the maladaptation cost (MAC) of using policy $\pi^*_{\mathbf{p}}$ when the climate's transition matrix is $\mathbf{p}'$ as:

$$MAC(\mathbf{p} \rightarrow \mathbf{p}') = ER(\mathbf{p}', \pi^*_{\mathbf{p}'}) - ER(\mathbf{p}', \pi^*_{\mathbf{p}}) \tag{13}$$

These quantities are analogous to as the 'climate change damage' and 'net benefits of adaptation' respectively in chapter 18 of Stern [24].

TABLE I: Optimal policy in the initial environment, with $p_{11} = p_{22} = 0.1$.

Environmental state	Old Allocation	New Allocation	Stationary probability
1	(4,0)	(0,4)	0.45
1	(3,1)	(0,4)	0
1	(2,2)	(0,4)	0
1	(1,3)	(0,4)	0
1	(0,4)	(0,4)	0.05
2	(4,0)	(4,0)	0.05
2	(3,1)	(4,0)	0
2	(2,2)	(4,0)	0
2	(1,3)	(4,0)	0
2	(0,4)	(4,0)	0.45

A. A simple example

In order to develop some intuition for the model and the diagnostics defined above, we will consider a simple example. Suppose that there are only two technologies, and two possible climate states. We will assume, at random, that the matrix of climatically contingent returns is

$$\mathbf{f} = \begin{pmatrix} 1.12 & 0.53 \\ 0.86 & 1.51 \end{pmatrix} \tag{14}$$

where the rows of $\mathbf{f}$ correspond to the climate states, and the columns to each of the technologies. The climate's transition matrix is given by:

$$\mathbf{p} = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix} \tag{15}$$

In the following analysis, we pick $p_{11} = 0.1$ and $p_{22} = 0.1$ and use these to define an initial stationary climate $\mathbf{p}$. Setting $c = 0.2$, $\delta = 0.9$, $\eta = 0.5$, and $N_{res} = 4$, we can compute the optimal policy and stationary distribution on state space using equations (7), (8), and (10). The result is displayed in table I. Note that in the long run not all the elements of state space are visited with non-zero probability. This indicates that some of the policy rules are superfluous, and the policy may be pruned to only those rules associated with positive stationary probabilities. Even in more complex examples with many more environmental states, technologies, and resource units, the optimal policy can often be reduced to a small set of rules in this way.

In order to get some intuition for the quantities ECC and MAC we plot these quantities for the transition $\mathbf{p} \to \mathbf{p}'$, over all possible values of $\mathbf{p}'$, keeping $\mathbf{p}$ fixed at $p_{11} = p_{22} = 0.1$. In Figure 1, the environmental change costs (ECC) are plotted as a function of the two parameters p'_{11} and p'_{22} of the new environment. Clearly, $ECC = 0$ when $p'_{11} = p'_{22} = 0.1$, the initial environment. Note that the the sign of ECC can be either positive of negative - some transitions are beneficial, whereas others are deleterious. In general, there is no simple relationship between initial and final transition matrices that determines the effect of a given environmental change, as this is heavily dependent on the matrix of returns $\mathbf{f}$.

The left frame of figure 2 depicts the maladaptation costs as a function of the parameters of the alternative environment $\mathbf{p}'$. Note that this graph seems to have a simpler structure than that of the ECC. MAC seems to increase monotonically with 'distance' from the initial environment. This qualitative result can be further illustrated by appealing to a quantitative measure of the 'distance' between environmental transition matrices. One such measure is the Kullback-Leibler divergence, an information-theoretic measure of the amount of information that one probability distribution contains about another [9]. Following Kussell & Leibler [15], we define the Kullback-Leibler divergence (KLD) between transition matrices $\mathbf{p}$ and $\mathbf{p}'$ as:

$$KLD(\mathbf{p}||\mathbf{p}') = \sum_{\alpha \in E} \sigma_\alpha \sum_{\beta \in E} p_{\alpha\beta} \log\left(\frac{p_{\alpha\beta}}{p'_{\alpha\beta}}\right) \tag{16}$$

where the vector σ is the stationary distribution of the Markov process with transition matrix $\mathbf{p}$. The maladaptation costs are plotted against the Kullback-Leibler divergence between our initial environment and a random sampling of

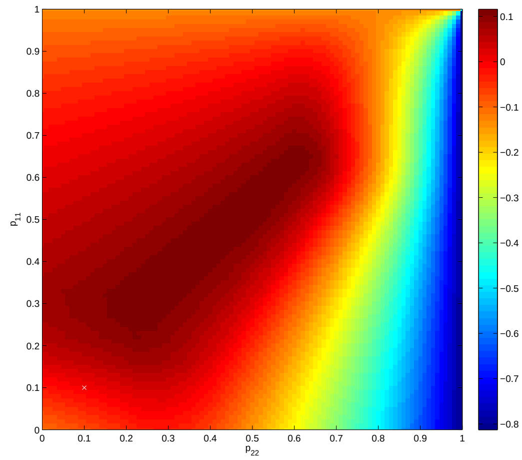

FIG. 1: Environmental change costs (ECC) for the example of section II.A

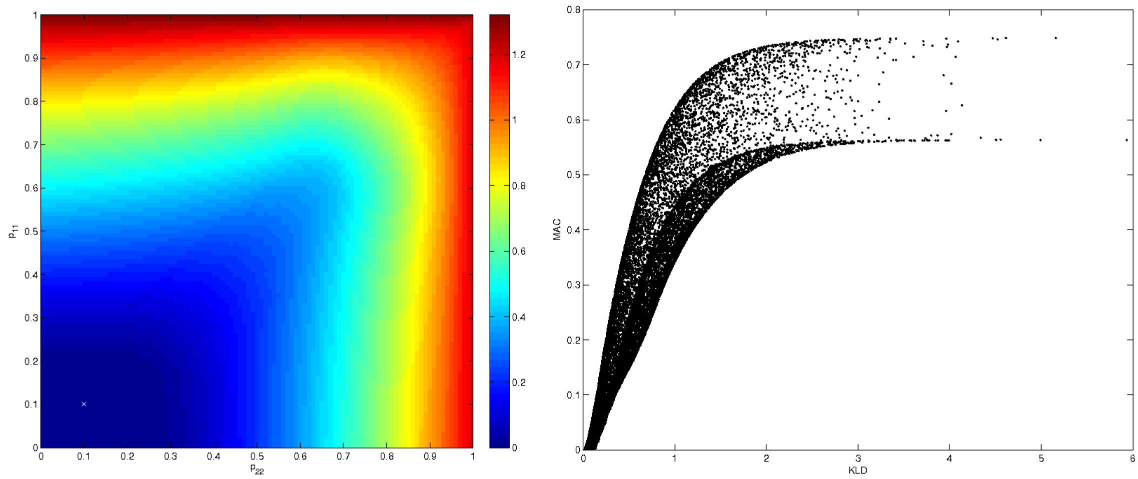

FIG. 2: Maladaptation costs (MAC) for the example of section II.A. The left frame shows maladaptation costs as a function of the parameters of the new environment $\mathbf{p}'$, while the right frame shows the relationship between the MAC and the Kullback-Leibler divergence between the two environmental transition matrices $\mathbf{p}$ and $\mathbf{p}'$.

new environments in the right frame of figure 2. Notice that the spray of points in this graph is bounded by a set of monotonically increasing envelope curves, and that these envelopes are approximately linear for values of KLD that are not too large. Numerical experiments suggest that the qualitative shape of these curves is independent of the initial value of the environmental transition matrix, and indeed all other parameters. All such plots feature monotonically increasing envelope curves which are approximately linear for small KLD, and exhibit a saturation effect due to the fact that the MAC is bounded above for any given $\mathbf{f}$ matrix. This feature is the analogue in our model of a result of Kussell & Leibler [15]. They consider a population of bacteria that stochastically switch between exhibiting alternative phenotypes with environmentally contingent growth rates. It is shown that for bacteria whose switching rates are optimal, the growth rate cost of having poor information about the transitions between alternative environmental states is linearly related to the Kullback-Leibler divergence between environmental transition matrices. This result is reproduced in spirit in our model. It is remarkable that such a simple measure, which has no access to the any of the details of the agent's decision problem, captures such a large amount of information about the potential benefits of adaptation.

IV. DYNAMIC ANALYSIS

The preceding section developed the machinery necessary for computing optimal policies in climates with stationary transition matrices that have infinite time horizons. It is a reasonable working assumption that agents faced with such stable conditions will have learned the optimal policies simply by trial and error. We also developed a set of diagnostics that enabled us to make comparative static statements about the differences between two climate regimes, as encoded by their transition matrices. The ECC is a measure of the absolute costs of a climate change, assuming that the agent has learned the optimal policy in the new climate, and the MAC is a measure of the costs of not changing one's strategy, or put another way, the benefits from adaptation.

While the comparative static measures developed above are useful for understanding the long-run consequences of an environmental change, they do not give an indication of how an agent can be expected to deal with such a change, since they contain no dynamic information. In this section we extend the stationary analysis presented above to the case where the climate process is dynamic, i.e. the transition matrix is a function of time. In this case, we are specifically interested in agent behaviour during the transient phase, in which the climate is in the process of shifting from one stable regime to another. It is in this phase that the agent must learn to adapt his strategy to the new climate. There are two related trade-offs involved in this adaptive process. In the case where the agent does not know what the environment will be like in the future, he must decide how to strike a balance between exploration of new allocation strategies and exploitation of current conditions. In the case where the future environment is known he must decide where to move his resources so that they are profitable now, and also have a low chance of needing to be moved later. The later consideration becomes increasingly important as c, the adjustment costs, increase.

We specify the particular climate change processes we will deal with in the next subsection. We then describe how the model may be explicitly solved for these processes if we assume that the climate transition matrix is known to the agent for all time. The solution proceeds by the method of backwards induction, and allows us to obtain a lower bound on the dynamic costs of adapting from one stationary climate regime to another. By considering a simple example related to that in section II.A, we explore the effect of the model parameters on these dynamic adaptation costs. We then demonstrate that adaptation costs are more sensitive to the parameters of the final climate than the benefits to adaptation are. This suggests that even small alterations in the path of a climate change can have a dramatic effect on the loss in productivity agents may experience while they adapt to new climatic conditions.

A. Climate change processes

We will consider a restricted class of climate change processes, in which the climate shifts from one infinitely long stationary regime to another in finite time. This is formalized below.

Let $\mathbf{\Pi}(t)$ be a discrete path ($t \in \mathbb{Z}$) in the space of possible transition matrices with the following properties:

$$\mathbf{\Pi}(t) = \begin{cases} \mathbf{p} & t \leq 0 \\ \mathbf{p}(t) & 0 < t < \tau \\ \mathbf{p}' & t \geq \tau \end{cases} \tag{17}$$

We refer to $\mathbf{\Pi}$ as a climate change process between the stationary climate regimes $\mathbf{p}$ and $\mathbf{p}'$. We will be particularly concerned with a special case of such processes, which we will refer to as linear change processes. In this case, we assume that

$$\mathbf{p}(t) = (1 - \frac{t}{\tau})\mathbf{p} + \frac{t}{\tau}\mathbf{p}' \quad . \tag{18}$$

For linear change processes, the path $\mathbf{\Pi}(t)$ is completely specified by the initial and final climate transition matrices, and the length of the transition period τ. We thus write $\mathbf{\Pi}$ as $\mathbf{\Pi}(\mathbf{p} \to \mathbf{p}'; \tau)$ when referring to a specific linear change process.

B. Dynamic solution assuming perfect information

Suppose for a moment that the agent has complete knowledge of the climate change process $\mathbf{\Pi}$ for all time. This is clearly a grossly unrealistic assumption. Changes in the climate, a highly nonlinear systems operating on a wide range of time-scales with many uncertain forcing terms, interactions, and parameters, are notoriously difficult to predict [22]. However, although this assumption is extremely optimistic, it does allow the model to be solved explicitly, and will allow us to calculate a lower bound on the costs of adapting to a change in climatic conditions in our model.

The method of solution in the dynamic, perfect information case, is similar to the infinite-time stationary case in flavor. However, instead of exploiting the recursive structure of the value function when there is no time horizon, the optimal policy is obtained by backward induction.

We will consider climate change processes of the type presented in equation (17). Let the infinite time value function associated with the final climate $\mathbf{p}'$ be the vector V_f^*. We now define a time-dependent value function u_t^*, and set $u_\tau^* = V_f^*$. The optimality conditions that define u_t^* are:

$$u_t^*(s) = \max_{a \in A} \sum_{s' \in S} P_{ss'}^a(t) \left(R_{ss'}^a + \delta u_{t+1}^*(s') \right), \tag{19}$$

where $P_{ss'}^a(t)$ is defined by equation (3). The time-dependent optimal policy $\pi_t(s)$ is naturally given by

$$\pi_t(s) = \text{argmax} \sum_{s' \in S} P_{ss'}^a(t) \left(R_{ss'}^a + \delta u_{t+1}^*(s') \right). \tag{20}$$

Thus using the terminal value $u_\tau^* = V_f^*$, we can iterate backwards and compute the optimal policy for all time. Now assuming that in the infinite period $t < 0$ the agent uses an optimal policy, we can calculate the stationary probability of his being in a given initial state when the climate transition begins. This is given by the stationary distribution in equation (10), which we call $q(0)$. In analogy with equation (9), we also define a time-dependent state-space transition matrix associated with the optimal policy π_t:

$$T_{ss'}(t) = P_{ss'}^{\pi_t(s)}. \tag{21}$$

Then the distribution $q(t)$ of the agent's position in state space for times $t > 0$ is:

$$q(t) = q(0) \prod_{i=1}^{t} \mathbf{T}(i) \tag{22}$$

Using this measure of the probability of the agent being in a given state at time t, we can calculate a dynamic version of the expected rewards given in equation (11):

$$ER^{\mathbf{\Pi}}(t) = \sum_{s \in S} q_s(t) \sum_{s' \in S} T_{ss'}(t) R_{ss'}^{\pi_t(s)} \tag{23}$$

C. Quantifying the costs of adaptation

The final definition we will make in this section is designed to quantify the adaptation costs of the climate change process $\mathbf{\Pi}$. It measures the total loss in productivity the agent sustains over the transition period relative to a hypothetical counterfactual in which the climate either did not change at all, or changes instantly from $\mathbf{p}$ to $\mathbf{p}'$. We will make this definition precise below, and then unpack it to get at the intuition behind it.

We define the adaptation costs $AC(\mathbf{\Pi})$ associated with the climate change process $\mathbf{\Pi}$ as follows: Let ER_i and ER_f be the expected rewards (given by equation (11)) associated with the initial stationary climate $\mathbf{p}$ and final stationary climate $\mathbf{p}'$ respectively, and let $ER_{\min} = \min(ER_i, ER_f)$. Suppose that $ER^{\mathbf{\Pi}}(t) < ER_{\min}$ for $t \in G$, where G is some subset of the time interval $[0, \tau]$. Then the adaptation costs are given by:

$$AC(\mathbf{\Pi}) = ER_{\min}|G| - \sum_{t \in G} ER^{\mathbf{\Pi}}(t) \tag{24}$$

The motivation for this definition is as follows: We wish the adaptation costs to be differentiated from the environmental change costs (ECC). While the later encode information about the long-run costs of a climate change, we wish the former to give us an indication of what it costs the agent to shift from one set of strategies to another in the transient period in which the climate is changing. Whenever the agent's expected rewards are between ER_i and ER_f, his returns are essentially interpolating between those he would achieve in the two bounding climate regimes. Thus in the time period for which this is the case, expected returns are simply adjusting to their new level, and we say that no adaptation costs are incurred. However, when $ER(t) < ER_{\min}$, the agent suffers a genuine loss due to the fact that his resource allocations are not perfectly suited to the current environment. This does not imply that the optimal policy the agent follows in this region is somehow defective, but rather that he is forced to accept low returns now, so as to facilitate his strategic adjustment to future conditions. This is driven partly by the fact that his policy at

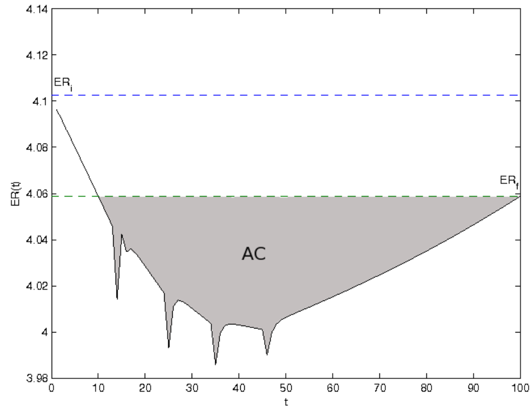

FIG. 3: Time series of expected rewards for the linear transition process $\mathbf{\Pi}(\mathbf{p_i} \to \mathbf{p_f}; 100)$. The adaptation costs $AC = 3.17$ in this example.

time t may be sacrificing immediate returns now so that returns are likely to be high later (perhaps be moving a large number of resources and incurring high adjustment costs), and partly by the fact that his probability $q(t)$ of being in a state with high expected rewards now may be low due to the inertial effect of his initial state-space distribution $q(0)$. Since these productivity losses are due to the process of strategic adaptation, and not a readjustment in the level of expected rewards as captured by the ECC, we use them to define the adaptation costs. Recall that, since we have assumed perfect information, AC is really a lower bound on these costs.

D. Dynamic example

In order to illustrate the solution process and the definition of adaptation costs outlined above, we calculate the time-dependent optimal policy for a representative linear transition processes $\mathbf{\Pi}(\mathbf{p_i} \to \mathbf{p_f}, \tau)$ with initial climate $\mathbf{p_i}$ and final climate $\mathbf{p_f}$ given by

$$\mathbf{p_i} = \begin{pmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{pmatrix} \tag{25}$$

$$\mathbf{p_f} = \begin{pmatrix} 0.6 & 0.4 \\ 0.8 & 0.2 \end{pmatrix}. \tag{26}$$

Note that $\mathbf{p_i}$ is the same as that used in the example of section II.A. The stationary distributions of these two transition matrices are $q_i = (1/2, 1/2)$ and $q_f = (2/3, 1/3)$ respectively. Thus the climate shifts from a highly oscillatory phase, in which each state occurs equally often, and seldom persists for more than a single time step, to a phase in which the first climate state is favored, occurring twice as often as the second, and often persisting for several time steps. Again we assume that $\mathbf{f}$ is given by equation (14). One possible interpretation of this example is as a representation of a shift in climatic conditions, where the two climate states are 'wet' and 'dry' years, and $\mathbf{f}$ represents returns to investment in two crop varieties with different water requirements. Resource units correspond to the number of fields the farmer plants with each crop, and adjustment costs are incurred when the farmer must recondition a field used for one of the crops so that it is suitable for the other. In figure 3, we plot $ER(t)$ for $c = 0.2, \delta = 0.9, \eta = 0.5, N_{res} = 4$, and $\tau = 100$.

It is interesting to ask what the effects of the risk aversion parameter η, discount rate δ, and the length of the environmental transition τ are on the adaptation costs. We plot AC as a function of these parameters in figure 4. The dependence of AC on η is consistent with what one might expect intuitively. The more risk averse the agent is, the greater his preference for income streams with low volatility, and thus the greater his preference for policies with well diversified allocations. Such diverse allocations will tend to perform tolerably well regardless of the current climate regime, and will also incur fewer adjustment costs, thus leading to lower adaptation costs when the climate changes.

Similarly, the dependence of AC on δ conforms to intuition. The lower the discount rate, the less the agent weights the future in choosing his current actions, and thus one would expect him to have higher adaptation costs, since his

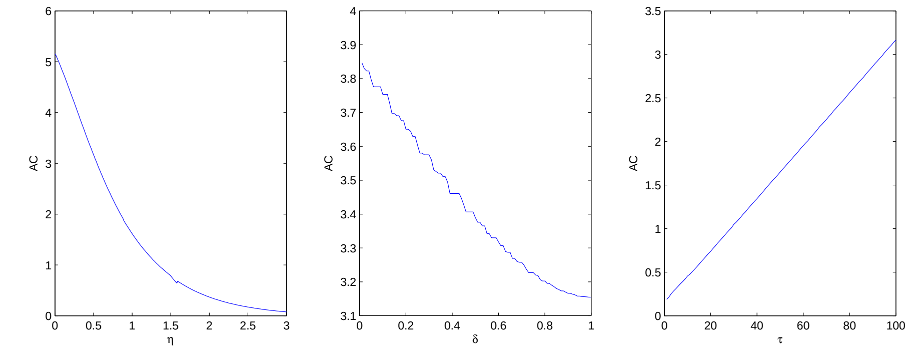

FIG. 4: From left to right, the figure shows the adaptation costs (AC) as a function of relative risk aversion (η), discount rate (δ), and the length of the environmental transition (τ) for the environmental change process the transition $\mathbf{\Pi}(\mathbf{p_i} \to \mathbf{p_f}, \tau)$. In all the graphs $\eta = 0.5, c = 0.2, \delta = 0.9, \tau = 100$, except for the variable that is varied.

planning horizon is short. Finally, the linear dependence of AC on the length τ of the transient period indicates that losses in per period productivity due to adaptation do not depend on the rapidity of the transition. This result is a consequence of our perfect information assumption and the linearity of the climate change process, and would doubtless be altered for a less prescient agent who responds to events as they unfold.

The adaptation costs have a complicated dependence on the adjustment costs c which is worth investigating. Changing c changes the reward structure of the model, and not just behavioural or environmental parameters. In order to get a handle on the effect of this parameter it is worth computing the static measures ECC and MAC as a function of c for the transition $\mathbf{p_i} \to \mathbf{p_f}$, as well as the adaptation costs. These are displayed in figure 5. One might

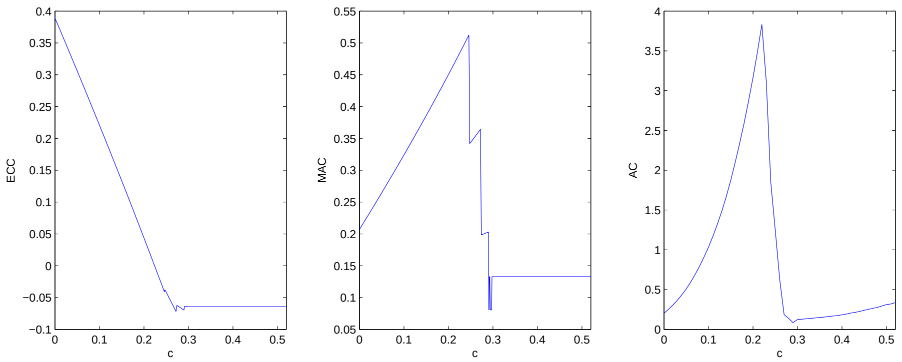

FIG. 5: From left to right, the figure shows the ECC, MAC and AC as functions of the adjustment costs c for the transition $\mathbf{\Pi}(\mathbf{p_i} \to \mathbf{p_f}, 100)$. In all the graphs $\eta = 0.5, \delta = 0.9, \tau = 100$.

naively think that the adaptation costs should increase monotonically with adjustment costs, however the figure shows this simple intuition to be flawed. The right most frame of figure 5 shows that, while AC increases initially, there is a threshold value of c at which it drops dramatically, and remains small for all values above this threshold. Some light is shed on this curious behaviour by considering the two frames on the left. The ECC shows a linear decrease up until a critical value of c above which it is constant, while the MAC exhibits a threshold similar to that in the AC. The downward slope of the ECC is due to the fact that the optimal policy in the initial phase is identical for values of c below the threshold at $c \approx 0.24$, so that linear changes in c lead to linear changes in the expected returns for this phase. For all c the optimal policy in the final phase is to have all resources on technology 1, so changing c has no effect on expected returns in the final phase. Thus the effect on the ECC of changing c is a linear decrease, as the expected returns in the initial phase drop as c is increased. However, when c hits its threshold value, the optimal policy in the initial phase changes in such a way that additional increases in the value of c affect expected returns in the initial and final phases in the same way, thus leading to a leveling off of the ECC. The existence of a threshold value of c at which the optimal policy in the initial phase shifts is also responsible for the thresholds in MAC and AC. In the case of the MAC, the explanation follows similar lines to that of the ECC. To understand the behaviour of the AC, recall that this quantity is a measure of losses relative to a baseline set by the minimum of the expected rewards in the initial and final climate phases. Thus, since these baselines exhibit threshold behavior, it is reasonable to expect the AC to as well. It is important to emphasize though, that these baselines capture vital information about the conditions that a given user is facing. The cost of adaptation is not an absolute quantity, and must be measured differently for different agents, decision problems, and climatic changes.

The main lesson of this rather involved discussion is that agents who face impediments to strategic adaptation (represented by c in our model) are engaging in adaptive behaviour even when the climate is not changing. Climate is not simply the mean of the distribution of a set of atmospheric variables, but the distribution itself. Even without climate change people's strategies must be adapted to stationary climate variability, and to a first approximation they face the same adjustment costs when adapting to this variability as they do when adapting to new climate regimes [2]. As such, understanding the effect of adjustment costs on adaptation costs necessitates an understanding of the effect of adjustment costs on stationary strategies.

While the preceding analysis shows that the dependence of dynamic adaptation costs on c can be understood by investigating the stationary bounding regimes, we now show that there are interesting effects in the adaptation costs that are not represented by the stationary measures MAC or ECC. In order to demonstrate this, we plot AC as a function of the parameters of all possible final climate regimes $\mathbf{p_f}$ in figure 6. Comparing figure 6 to figure 2, it is

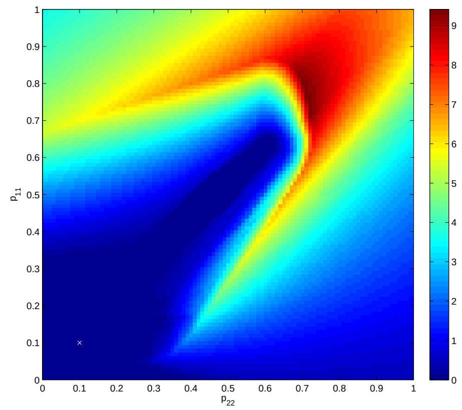

FIG. 6: Adaptation costs (AC) for transitions from $\mathbf{p_i}$ to all other possible environmental states. ($c = 0.2, \eta = 0.5, \delta = 0.9, \tau = 100$)

evident that AC captures information that MAC does not. The correlation between the two measures is only 0.63 in this example [32]. Moreover, AC exhibits much sharper gradients in the parameter space of the final climate $\mathbf{p_f}$ than MAC does. This later result is particularly important. It shows that while small changes in the statistics of the final climate have comparably small effects on the benefits to adaptation, they can have much larger effects on the costs of adaptation. This feeds back into the mitigation debate. Small changes in the final level of greenhouse gas stabilization, which will play a role in determining new stationary distributions for regional climates, can dramatically alter the costs of adaptation, with only marginal changes in benefits.

V. CONCLUSIONS AND EXTENSIONS

The model presented above was designed to develop our intuition for the factors that can affect the costs of strategic adaptation to climate change. It is undoubtedly a stylized (some might say simplistic) representation of a deeply complex issue. Yet therein, we hope, lies its strength. Our model is tractable, and, in its abstract terms, allows for a detailed analysis of the dynamic costs of negotiating a climate transition. This analysis yields several important insights. First, there is an interesting and counter-intuitive relationship between the total costs of adapting to a given climate change, and the size of the impediment to strategic adjustment that an agent faces. Large adjustment costs do not necessarily imply large adaptation costs, and a detailed understanding of the adaptive strategies already employed by agents in stationary environments is necessary in order to understand this dependence. Moreover, the example used to illustrate this in the text is not generic – the form of the dependence of adaptation costs on adjustment costs depends on the specific climate change process being examined. Thus stationary information is not necessarily sufficient for a full understanding of this dependence. The insufficiency of stationary information is further illustrated by examining the dependence of adaptation costs on the parameters that define the climate change process. We showed that simply knowing the long-run benefits of adaptation does not in general provide good information about the costs of achieving those benefits. Specifically, it is possible for the costs of adaptation to be more sensitive to the details of the climate change process than the benefits are.

Possible extensions to the analysis we have presented abound. Perhaps most obviously, it would be interesting to explore the effect of different initial climates $\mathbf{p_i}$, and return matrices $\mathbf{f}$, on the costs of adaptation. One could investigate several scenarios, such as changes in the frequency of extreme events, shifts in variability, and temporary climate changes that revert back to their original stationary regime. It would be useful to quantify the effect of the strength of the correlation between the performance of the various technologies on adaptation costs. One could also consider relaxing our assumption of constant adjustment costs, and making these dependent on where resources are transfered from and to.

At a more fundamental level, it would be interesting to ask what the effect of increasing or decreasing returns to investment in the technologies would be. We have tacitly assumed constant returns to scale, as increasing or decreasing returns makes it necessary to keep track of cumulative investment in the technologies, thus violating the Markov property that allowed us to employ dynamic programming in its simplest form. Other economies of scale will doubtless complicate the analysis, very likely introducing path dependence in the case of increasing returns [4]. This may be a more accurate representation of the barriers to adaptation that are faced in the real world.

A further pertinent extension would be to try to understand the effect of imperfect information on adaptation costs. One could conceive of a process in which the agent receives imperfect forecasts periodically over time, and updates his beliefs based on the past performance of these forecasts. It would be interesting to try to relate the adaptation costs sustained with these imperfect forecasts to a measure of forecast performance. Behavioural effects could also be included in order to understand how boundedly rational agents with imperfect information might learn about changes in climate as they unfold [e.g. 7, 25].

With shifts in climate already beginning to take their toll on natural and economic systems, it is vital that we take all reasonable measures to curtail greenhouse gas emissions, and adapt to those impacts we can now no longer avoid. It is our hope that the analysis presented here will help to inform this later task, and hopefully contribute to the thinking of the policy makers and decision takers who must carry it out.

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[29] We restrict our attention to Markov policies as these turn out to be sufficient to ensure optimality in the case where the rewards $R^a_{ss'}$ depend only on the current state, and not the entire history [18].
[30] Note that U is real for all positive values of η only when the rewards r are positive. This is ensured by our earlier assumption that $c < \min_{e,k} f_k^{(e)}$.
[31] Such an eigenvector is guaranteed to exist from the fact that the sum of each of the rows of $\mathbf{T}$ is 1, and the Perron-Frobenius theorem.
[32] Correlation of AC with ECC is -0.06. This makes sense, we would not expect any relationship between the net effects of the climate change and the cost of negotiating it