Double Integrator Reach-Avoid Via Dynamic Programming

This example demonstrates how to use the SReach toolbox to solve a terminal-hitting time reach-avoid problem using dynamic programming.

In this example, we analyze the following problems via dynamic programming for a stochastic system with known dynamics:

1. **the terminal-hitting time reach-avoid problem** posed as the stochastic reachability of a target tube problem
2. **stochastic reachability of a moving target tube**

The terminal-hitting time reach-avoid problem computes a controller that maximizes the probability of reaching a target, `target_set`, at a time horizon, `N`, while maintaining the system in a set of safe states, `safe_set`. This problem is generalized as the problem of stochastic reachability of a target tube --- maximize the probability of staying within a target tube. For reach-avoid problems, the target tube has a specific structure. Finally, we implement a dynamic programming solution to the problem of stochastic reachability of a general target tube.

Double Integrator

In this example we use a discretized double integrator dynamics given by:

$$x_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} u_k + w_k$$

where T is the discretization time-step, and w_k is the stochastic disturbance.

Notes about this Live Script:

1. **MATLAB dependencies**: This Live Script uses MATLAB's Statistics and Machine Learning Toolbox.
2. **External dependencies**: This Live Script uses Multi-Parameteric Toolbox (MPT).
3. Make sure that `sreachinit` is run before running this script.

Setup the system

```
% discretization parameter
T = 0.25;

% define the system
sys = LtiSystem('StateMatrix', [1, T; 0, 1], ...
    'InputMatrix', [T^2/2; T], ...
    'InputSpace', Polyhedron('lb', -0.1, 'ub', 0.1), ...
    'DisturbanceMatrix', eye(2), ...
    'Disturbance', StochasticDisturbance('Gaussian', zeros(2,1), 0.01*eye(2)));
```

Setup the target and safe sets

```
% safe set definition
safe_set = Polyhedron('lb', [-1, -1], 'ub', [1, 1]);
target_set = Polyhedron('lb', [-0.5, -0.5], 'ub', [0.5, 0.5]);
```

Setup the target tube

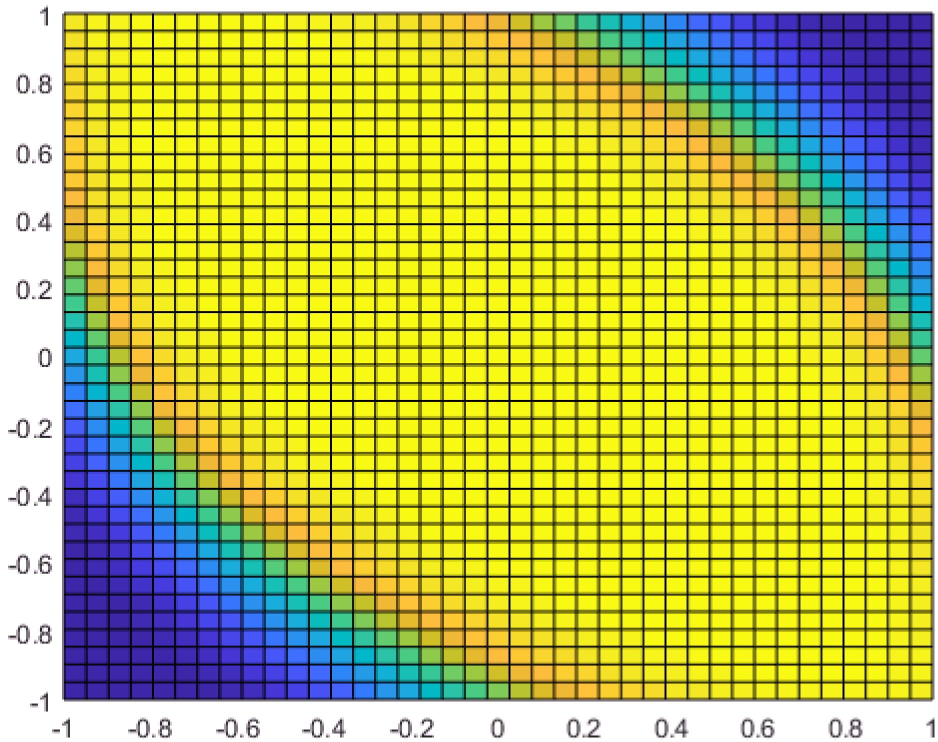
Target tube is a generalization of the reach problem. The reach avoid target-tube is created by setting the first $N-1$ sets in the tube as the `safe_set` and the final set as the `target_set`.

```
% in target tube for the viability problem is equivalent to a tube of repeating
% safe sets
target_tube = { safe_set, ...
                safe_set, ...
                safe_set, ...
                safe_set, ...
                safe_set, ...
                safe_set, ...
                safe_set, ...
                safe_set, ...
                safe_set, ...
                safe_set, ...
                target_set};

N = length(target_tube);
```

Dynamic programming recursion via gridding

For dynamic programming we need to create a grid over which we will perform the recursion

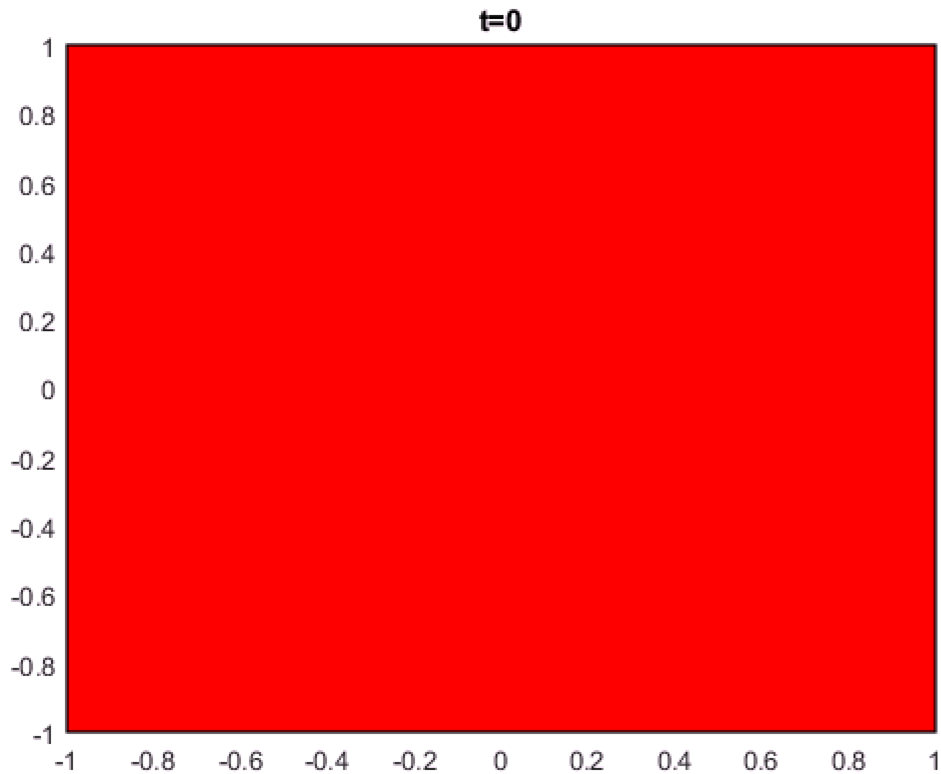
```
% need to create a state space grid and input space grid
ss_grid = SpaceGrid([-1, -1], [1, 1], 40);
in_grid = InputGrid(-1, 1, 20);

grid_probability = getDynProgSolForTargetTube(sys, ...
    ss_grid, in_grid, target_tube);


figure(1);
ss_grid.plotGridProbability(grid_probability);
view(0, 90)
```

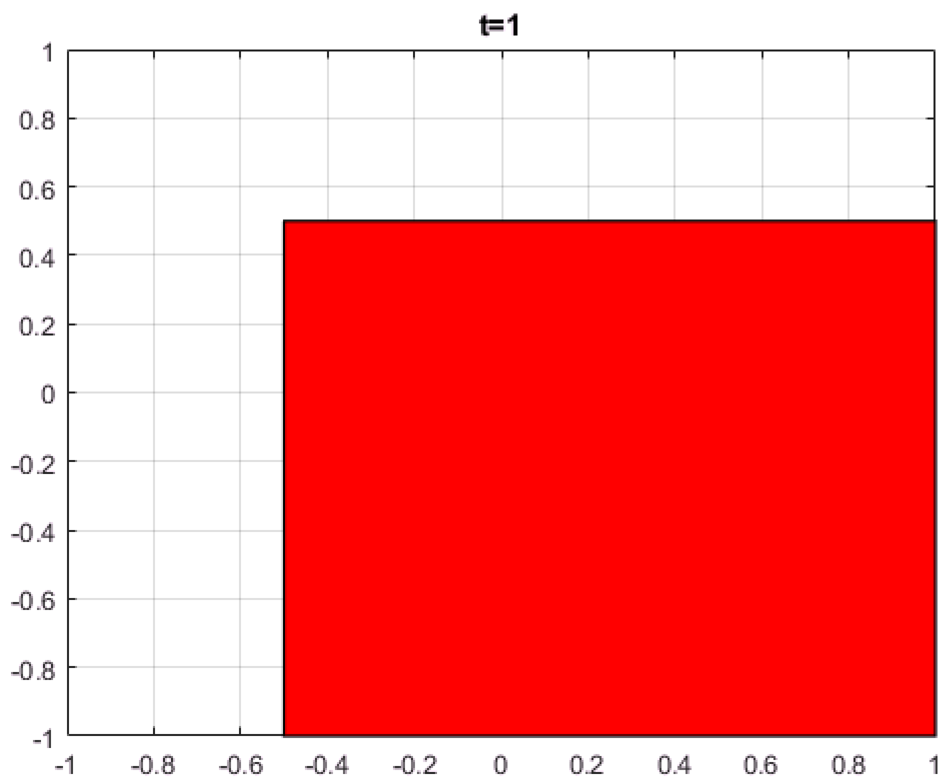


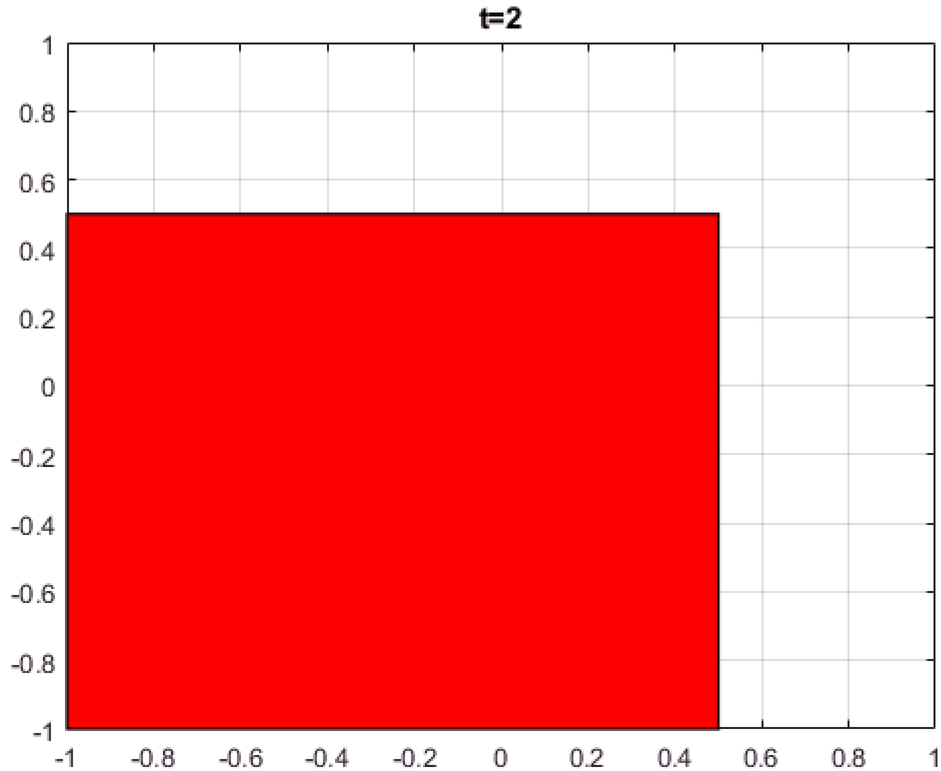
Moving target problem

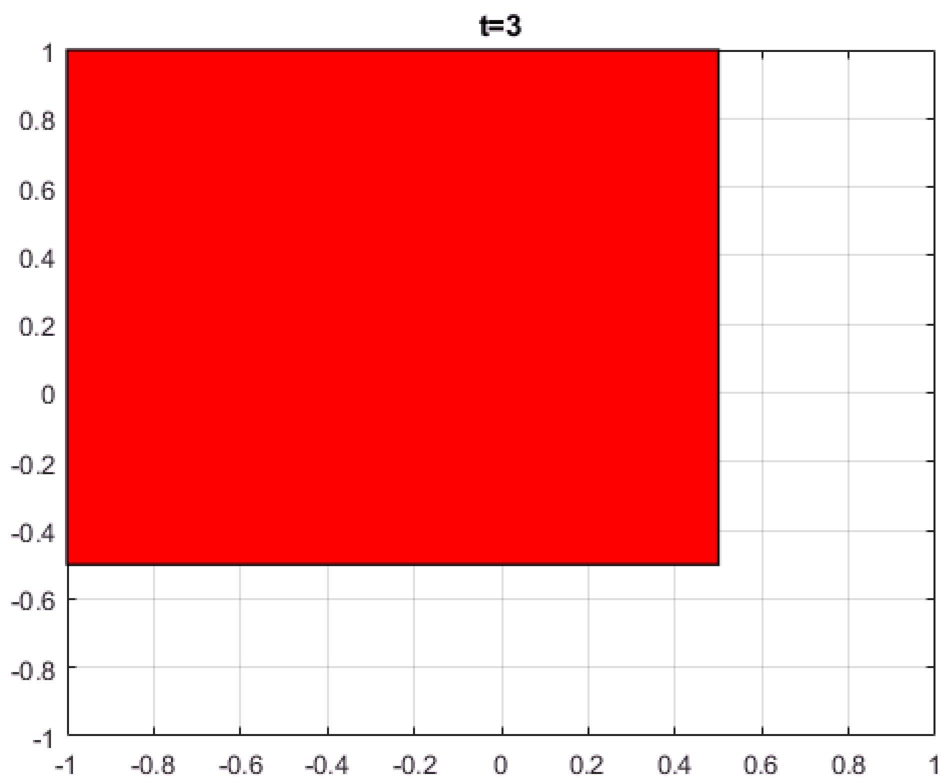
The advantage of target-tubes is that it allows for abstractions in which we would like to reach a moving target.

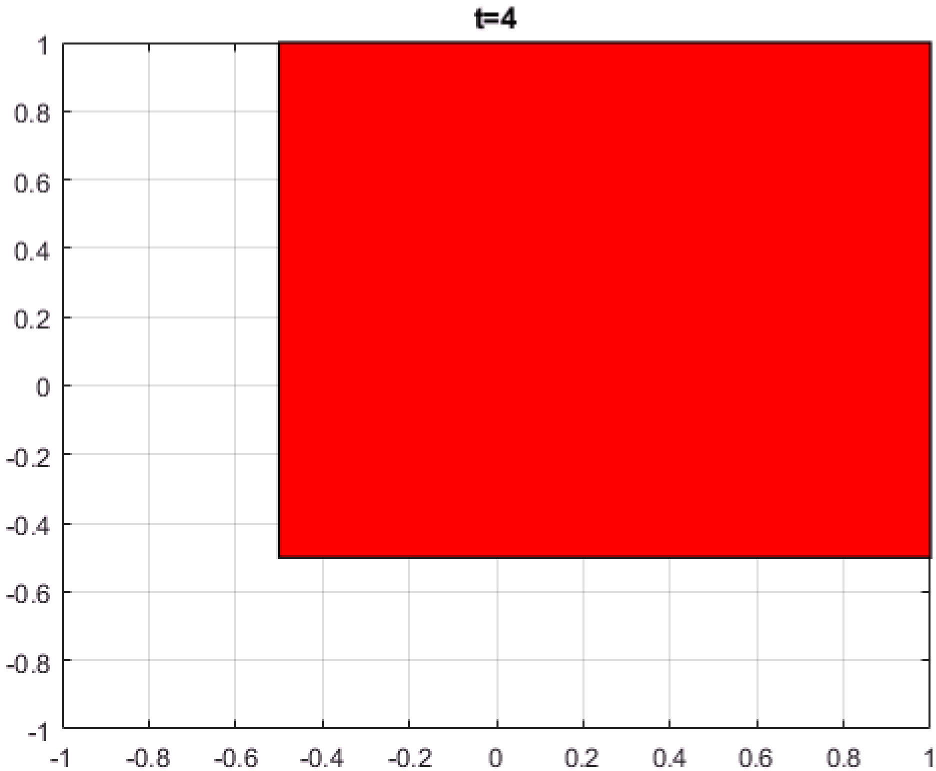
```
target_tube = {Polyhedron('lb', [-1, -1], 'ub', [1, 1]), ...
    Polyhedron('lb', [-0.5, -1], 'ub', [1, 0.5]),...
    Polyhedron('lb', [-1, -1], 'ub', [0.5, 0.5]), ...
    Polyhedron('lb', [-1, -0.5], 'ub', [0.5, 1]), ...
    Polyhedron('lb', [-0.5, -0.5], 'ub', [1, 1])};
% Plotting of target tube
for time_indx=1:5
    figure()
    plot(target_tube{time_indx});
    axis([-1 1 -1 1]);
    box on;
    grid on;
    title(sprintf('t=%d',time_indx-1));
end
```


t=0
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


t=1
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

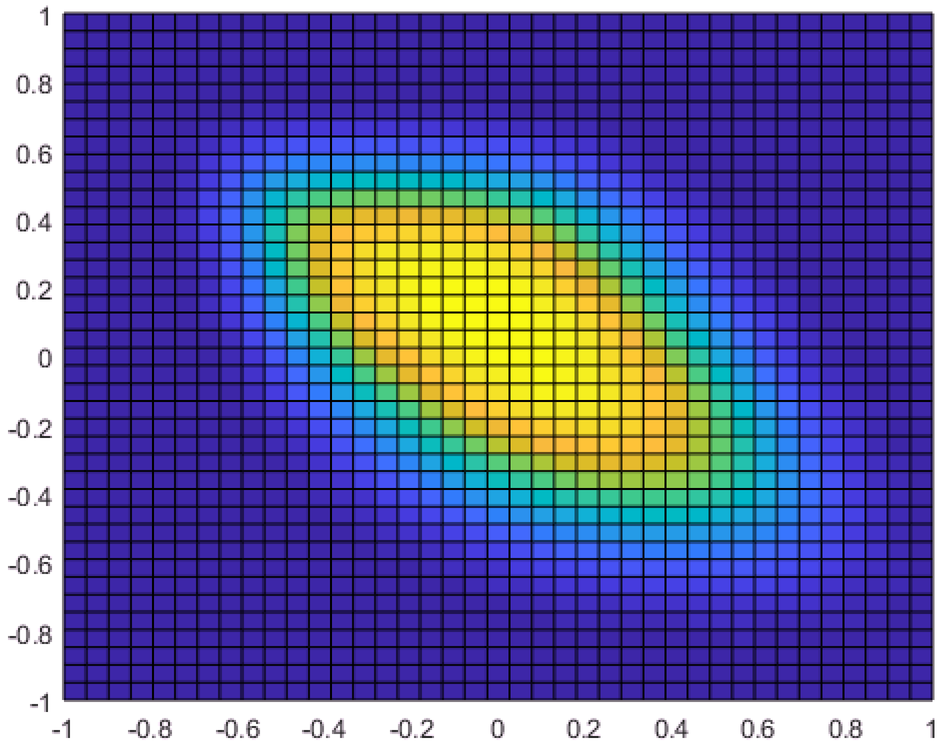

t=2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


t=3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1



Dynamic programming solution on target tube

```
ss_grid = SpaceGrid([-1, -1], [1, 1], 40);
in_grid = InputGrid(-0.1, 0.1, 3);

grid_probability = getDynProgSolForTargetTube(sys, ...
    ss_grid, in_grid, target_tube);

figure(2);
ss_grid.plotGridProbability(grid_probability);
view(0, 90)
```


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1