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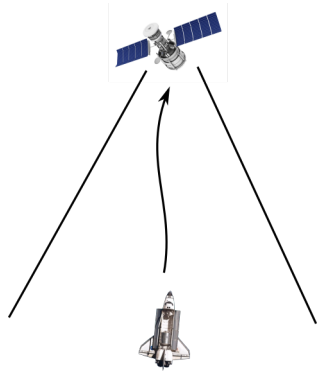
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Verification of satellite rendezvous problem via SReachSet This example will

```
%demonstrate the use of SReachTools in verification of stochastic
%continuous-state discrete-time linear time-invariant (LTI) systems.
%
% Specifically, we will discuss how SReachTools can use Fourier
% transforms
% (<http://www.math.wsu.edu/faculty/genz/software/matlab/qsimvnm.m
% Genz's
% algorithm> and MATLAB's patternsearch), convex chance constraints,
% and
% Lagrangian methods to construct underapproximative stochastic reach
% sets.
%
% Our approaches is grid-free and recursion-free resulting in highly
% scalable
% solutions, especially for Gaussian-perturbed LTI systems.
%
% This Live Script is part of the SReachTools toolbox. License for the
% use of
% this function is given in
% <https://github.com/unm-hscl/SReachTools/blob/master/LICENSE
% https://github.com/unm-hscl/SReachTools/blob/master/LICENSE>.
%
% Prescript running
close all;clc;clear;
srtinit
```

Problem formulation: Spacecraft motion via CWH dynamics

We consider both the spacecrafts, referred to as the deputy spacecraft and the chief spacecraft, to be in the same circular orbit. In this example, we will consider the problem of verification for the spacecraft rendezvous problem, i.e., identify all the initial states from which the deputy can rendezvous with the chief while staying within the line-of-sight cone with a likelihood above a user-specified threshold.



Dynamics model for the deputy relative to the chief spacecraft

The relative

```
%planar dynamics of the deputy with respect to the chief are described
% by the
%<https://doi.org/10.1109/CDC.2013.6760626 Clohessy-Wiltshire-Hill
% (CWH)
%equations,>
%
% 
$$\ddot{x} - 3\omega^2 x - 2\omega \dot{y} = \frac{F_x}{m_d}$$

%
% 
$$\ddot{y} + 2\omega \dot{x} = \frac{F_y}{m_d}$$

%
% where the position of the deputy relative to the chief is  $x, y$  in
%  $\mathbf{R}^2$ ,  $\omega = \sqrt{\frac{\mu}{R_0^3}}$  is the orbital
% frequency,
%  $\mu$  is the gravitational constant, and  $R_0$  is the orbital
% radius of the
% chief spacecraft. We define the state as  $\overline{x} = [x \ y \ \dot{x} \ \dot{y}]^T$ 
% which is the position and
% velocity of the
% deputy relative to the chief along  $x$ - and  $y$ -
% axes, and
% the input as  $\overline{u} = [F_x \ F_y]^T$  in
%  $\mathcal{U} \subset \mathbf{R}^2$ .
%
% We will discretize the CWH dynamics in time, via zero-order hold, to
% obtain
```

```

% the discrete-time linear time-invariant system and add a Gaussian
disturbance
% to account for the modeling uncertainties and the disturbance
forces,
%
%  $\overline{x}_{k+1} = A \overline{x}_k + B \overline{u}_k +$ 
%  $\overline{w}_k$ 
%
% with  $\overline{w}_k \in \mathbf{R}^4$  as an IID Gaussian zero-
mean
% random process with a known covariance matrix  $\Sigma_{\overline{w}}$ .
\Sigma_{\overline{w}}$.

```

System definition

```

umax=0.1;
mean_disturbance = zeros(4,1);
covariance_disturbance = diag([1e-4, 1e-4, 5e-8, 5e-8]);
% Define the CWH (planar) dynamics of the deputy spacecraft relative
to the
% chief spacecraft as a LtSystem object
sys = getCwhLtSystem(4, Polyhedron('lb', -umax*ones(2,1),...
                                     'ub', umax*ones(2,1)),...
                    RandomVector('Gaussian',
                                mean_disturbance,covariance_disturbance));

```

Methods to run

```

ft_run = 1;
cc_open_run = 1;

```

Target tube construction --- reach-avoid specification

```

time_horizon=5;           % Stay within a line of sight cone for 4 time
steps and
                           % reach the target at t=5% Safe Set --- LoS
cone
% Safe set definition --- LoS cone  $|x| \leq y$  and  $y \in [0, y_{max}]$  and  $|v_x| \leq v_{x_{max}}$  and
%  $|v_y| \leq v_{y_{max}}$ 
ymax=2;
vxmax=0.5;
vymax=0.5;
A_safe_set = [1, 1, 0, 0;
              -1, 1, 0, 0;
               0, -1, 0, 0;
               0, 0, 1, 0;
               0, 0, -1, 0;
               0, 0, 0, 1;
               0, 0, 0, -1];

```

```

b_safe_set = [0;
              0;
              ymax;
              vxmax;
              vxmax;
              vymax;
              vymax];
safe_set = Polyhedron(A_safe_set, b_safe_set);
% Target set --- Box [-0.1,0.1]x[-0.1,0]x[-0.01,0.01]x[-0.01,0.01]
target_set = Polyhedron('lb', [-0.1; -0.1; -0.01; -0.01],...
                        'ub', [0.1; 0; 0.01; 0.01]);
target_tube = Tube('reach-avoid', safe_set, target_set, time_horizon);
slice_at_vx_vy = zeros(2,1);

```

Preparation for set computation

```

prob_thresh = 0.8;

n_dir_vecs = 16;
theta_vec = linspace(0, 2*pi, n_dir_vecs);
set_of_dir_vecs_ft = [cos(theta_vec);
                      sin(theta_vec);
                      zeros(2,n_dir_vecs)];

n_dir_vecs = 40;
theta_vec = linspace(0, 2*pi, n_dir_vecs);
set_of_dir_vecs_cc_open = [cos(theta_vec);
                           sin(theta_vec);
                           zeros(2,n_dir_vecs)];

init_safe_set_affine = Polyhedron('He', [zeros(2,2) eye(2,2)
      slice_at_vx_vy]);

```

CC (Linear program approach)

```

if cc_open_run
    options = SReachSetOptions('term', 'chance-open',...
                              'set_of_dir_vecs', set_of_dir_vecs_cc_open,...
                              'init_safe_set_affine', init_safe_set_affine);
    timer_cc_open = tic;
    [polytope_cc_open, extra_info] = SReachSet('term', 'chance-open',
    sys,...
        prob_thresh, target_tube, options);
    elapsed_time_cc_open = toc(timer_cc_open);
end

```

Fourier transform (Genz's algorithm and MATLAB's patternsearch)

```

if ft_run
    options = SReachSetOptions('term', 'genzps-open',...
                              'set_of_dir_vecs', set_of_dir_vecs_ft,...
                              'init_safe_set_affine', init_safe_set_affine, 'verbose', 1);

```

```

    timer_ft = tic;
    polytope_ft = SReachSet('term','genzps-open', sys, prob_thresh,...
        target_tube, options);
    elapsed_time_ft = toc(timer_ft);
end

```

Polytopic underapproximation exists for $\alpha = 0.80$ since $W(x_{\max}) = 0.866$.

Analyzing direction : 1/16 | Upper bound of theta: 0.64

OptRAProb	OptTheta	LB_theta	UB_theta	OptInp^2	Exit reason
0.8660	0.00e+00	0.00e+00	6.45e-01	5.20e-03	Infeasible (0.001)
0.8050	1.61e-01	0.00e+00	3.22e-01	5.78e-03	Feasible
0.8050	2.42e-01	1.61e-01	3.22e-01	8.38e-03	Feasible
0.8050	2.82e-01	2.42e-01	3.22e-01	8.38e-03	Feasible
0.8050	3.02e-01	2.82e-01	3.22e-01	9.20e-03	Feasible
0.8050	3.12e-01	3.02e-01	3.22e-01	9.20e-03	Feasible
0.8050	3.17e-01	3.12e-01	3.22e-01	9.20e-03	Feasible

Analyzing direction : 2/16 | Upper bound of theta: 0.49

OptRAProb	OptTheta	LB_theta	UB_theta	OptInp^2	Exit reason
0.8660	0.00e+00	0.00e+00	4.88e-01	5.20e-03	Infeasible (0.001)
0.8050	1.22e-01	0.00e+00	2.44e-01	1.11e-02	Feasible
0.8050	1.83e-01	1.22e-01	2.44e-01	1.19e-02	Feasible
0.8050	2.14e-01	1.83e-01	2.44e-01	1.19e-02	Feasible
0.8050	2.29e-01	2.14e-01	2.44e-01	1.19e-02	Feasible
0.8050	2.36e-01	2.29e-01	2.44e-01	1.19e-02	Feasible

Analyzing direction : 3/16 | Upper bound of theta: 0.46

OptRAProb	OptTheta	LB_theta	UB_theta	OptInp^2	Exit reason
0.8050	2.28e-01	0.00e+00	4.56e-01	1.02e-02	Feasible
0.8050	3.42e-01	2.28e-01	4.56e-01	1.35e-02	Feasible
0.8050	3.99e-01	3.42e-01	4.56e-01	1.03e-02	Feasible
0.8050	4.28e-01	3.99e-01	4.56e-01	1.73e-02	Feasible
0.8050	4.42e-01	4.28e-01	4.56e-01	2.33e-02	Feasible

```

    0.8050 | 4.49e-01 | 4.42e-01 | 4.56e-01 | 2.76e-02 |
Feasible
Analyzing direction : 4/16 | Upper bound of theta: 0.51
OptRAProb | OptTheta | LB_theta | UB_theta | OptInp^2 | Exit
reason
    0.8050 | 2.56e-01 | 0.00e+00 | 5.12e-01 | 1.45e-02 |
Feasible
    0.8050 | 3.84e-01 | 2.56e-01 | 5.12e-01 | 1.30e-02 |
Feasible
    0.8050 | 4.48e-01 | 3.84e-01 | 5.12e-01 | 2.44e-02 |
Feasible
    0.8050 | 4.80e-01 | 4.48e-01 | 5.12e-01 | 1.83e-02 |
Feasible
    0.8050 | 4.96e-01 | 4.80e-01 | 5.12e-01 | 2.01e-02 |
Feasible
    0.8050 | 5.04e-01 | 4.96e-01 | 5.12e-01 | 2.44e-02 |
Feasible
Analyzing direction : 5/16 | Upper bound of theta: 0.59
OptRAProb | OptTheta | LB_theta | UB_theta | OptInp^2 | Exit
reason
    0.8050 | 2.93e-01 | 0.00e+00 | 5.87e-01 | 1.48e-02 |
Feasible
    0.8050 | 4.40e-01 | 2.93e-01 | 5.87e-01 | 1.17e-02 |
Feasible
    0.8050 | 5.13e-01 | 4.40e-01 | 5.87e-01 | 1.30e-02 |
Feasible
    0.8050 | 5.50e-01 | 5.13e-01 | 5.87e-01 | 1.29e-02 |
Feasible
    0.8050 | 5.68e-01 | 5.50e-01 | 5.87e-01 | 1.29e-02 |
Feasible
    0.8050 | 5.77e-01 | 5.68e-01 | 5.87e-01 | 1.29e-02 |
Feasible
Analyzing direction : 6/16 | Upper bound of theta: 0.47
OptRAProb | OptTheta | LB_theta | UB_theta | OptInp^2 | Exit
reason
    0.8050 | 2.36e-01 | 0.00e+00 | 4.72e-01 | 9.77e-03 |
Feasible
    0.8050 | 3.54e-01 | 2.36e-01 | 4.72e-01 | 1.34e-02 |
Feasible
    0.8050 | 4.13e-01 | 3.54e-01 | 4.72e-01 | 1.33e-02 |
Feasible
    0.8050 | 4.42e-01 | 4.13e-01 | 4.72e-01 | 1.66e-02 |
Feasible
    0.8050 | 4.57e-01 | 4.42e-01 | 4.72e-01 | 1.50e-02 |
Feasible
    0.8050 | 4.65e-01 | 4.57e-01 | 4.72e-01 | 1.67e-02 |
Feasible
Analyzing direction : 7/16 | Upper bound of theta: 0.46
OptRAProb | OptTheta | LB_theta | UB_theta | OptInp^2 | Exit
reason
    0.8660 | 0.00e+00 | 0.00e+00 | 4.61e-01 | 5.20e-03 |
Infeasible (0.001)
    0.8050 | 1.15e-01 | 0.00e+00 | 2.31e-01 | 1.26e-02 |
Feasible

```

0.8050	/	1.73e-01	/	1.15e-01	/	2.31e-01	/	2.09e-02	/
Feasible									
0.8050	/	2.02e-01	/	1.73e-01	/	2.31e-01	/	2.12e-02	/
Feasible									
0.8050	/	2.16e-01	/	2.02e-01	/	2.31e-01	/	2.12e-02	/
Feasible									
0.8050	/	2.24e-01	/	2.16e-01	/	2.31e-01	/	2.12e-02	/
Feasible									
Analyzing direction : 8/16 / Upper bound of theta: 0.54									
OptRAProb	/	OptTheta	/	LB_theta	/	UB_theta	/	OptInp^2	/ Exit
reason									
0.8660	/	0.00e+00	/	0.00e+00	/	5.43e-01	/	5.20e-03	/
Infeasible (0.001)									
0.8050	/	1.36e-01	/	0.00e+00	/	2.72e-01	/	5.50e-03	/
Feasible									
0.8050	/	2.04e-01	/	1.36e-01	/	2.72e-01	/	1.14e-02	/
Feasible									
0.8050	/	2.38e-01	/	2.04e-01	/	2.72e-01	/	1.14e-02	/
Feasible									
0.8050	/	2.55e-01	/	2.38e-01	/	2.72e-01	/	1.14e-02	/
Feasible									
0.8050	/	2.63e-01	/	2.55e-01	/	2.72e-01	/	1.14e-02	/
Feasible									
Analyzing direction : 9/16 / Upper bound of theta: 0.84									
OptRAProb	/	OptTheta	/	LB_theta	/	UB_theta	/	OptInp^2	/ Exit
reason									
0.8660	/	0.00e+00	/	0.00e+00	/	8.37e-01	/	5.20e-03	/
Infeasible (0.001)									
0.8050	/	2.09e-01	/	0.00e+00	/	4.18e-01	/	1.24e-02	/
Feasible									
0.8050	/	3.14e-01	/	2.09e-01	/	4.18e-01	/	2.21e-02	/
Feasible									
0.8050	/	3.66e-01	/	3.14e-01	/	4.18e-01	/	2.18e-02	/
Feasible									
0.8050	/	3.92e-01	/	3.66e-01	/	4.18e-01	/	2.19e-02	/
Feasible									
0.8050	/	4.05e-01	/	3.92e-01	/	4.18e-01	/	2.25e-02	/
Feasible									
0.8050	/	4.12e-01	/	4.05e-01	/	4.18e-01	/	2.25e-02	/
Feasible									
Analyzing direction :10/16 / Upper bound of theta: 2.31									
OptRAProb	/	OptTheta	/	LB_theta	/	UB_theta	/	OptInp^2	/ Exit
reason									
0.8660	/	0.00e+00	/	0.00e+00	/	2.31e+00	/	5.20e-03	/
Infeasible (0.001)									
0.8660	/	0.00e+00	/	0.00e+00	/	1.15e+00	/	5.20e-03	/
Infeasible (0.001)									
0.8660	/	0.00e+00	/	0.00e+00	/	5.76e-01	/	5.20e-03	/
Infeasible (0.001)									
0.8050	/	1.44e-01	/	0.00e+00	/	2.88e-01	/	1.63e-02	/
Feasible									
0.8050	/	2.16e-01	/	1.44e-01	/	2.88e-01	/	1.72e-02	/
Feasible									

```

    0.8050 / 2.52e-01 / 2.16e-01 / 2.88e-01 / 1.72e-02 /
Feasible
    0.8050 / 2.70e-01 / 2.52e-01 / 2.88e-01 / 2.04e-02 /
Feasible
    0.8050 / 2.79e-01 / 2.70e-01 / 2.88e-01 / 2.04e-02 /
Feasible
Analyzing direction :11/16 / Upper bound of theta: 1.57
OptRAProb / OptTheta / LB_theta / UB_theta / OptInp^2 / Exit
reason
    0.8660 / 0.00e+00 / 0.00e+00 / 1.57e+00 / 5.20e-03 /
Infeasible (0.001)
    0.8660 / 0.00e+00 / 0.00e+00 / 7.83e-01 / 5.20e-03 /
Infeasible (0.001)
    0.8050 / 1.96e-01 / 0.00e+00 / 3.91e-01 / 2.13e-02 /
Feasible
    0.8050 / 2.93e-01 / 1.96e-01 / 3.91e-01 / 2.23e-02 /
Feasible
    0.8050 / 3.42e-01 / 2.93e-01 / 3.91e-01 / 2.76e-02 /
Feasible
    0.8050 / 3.67e-01 / 3.42e-01 / 3.91e-01 / 3.20e-02 /
Feasible
    0.8050 / 3.79e-01 / 3.67e-01 / 3.91e-01 / 3.17e-02 /
Feasible
    0.8050 / 3.85e-01 / 3.79e-01 / 3.91e-01 / 3.22e-02 /
Feasible
Analyzing direction :12/16 / Upper bound of theta: 1.36
OptRAProb / OptTheta / LB_theta / UB_theta / OptInp^2 / Exit
reason
    0.8660 / 0.00e+00 / 0.00e+00 / 1.36e+00 / 5.20e-03 /
Infeasible (0.001)
    0.8660 / 0.00e+00 / 0.00e+00 / 6.81e-01 / 5.20e-03 /
Infeasible (0.001)
    0.8050 / 1.70e-01 / 0.00e+00 / 3.41e-01 / 1.16e-02 /
Feasible
    0.8050 / 2.56e-01 / 1.70e-01 / 3.41e-01 / 2.66e-02 /
Feasible
    0.8050 / 2.98e-01 / 2.56e-01 / 3.41e-01 / 2.74e-02 /
Feasible
    0.8050 / 3.19e-01 / 2.98e-01 / 3.41e-01 / 3.25e-02 /
Feasible
    0.8050 / 3.30e-01 / 3.19e-01 / 3.41e-01 / 3.58e-02 /
Feasible
    0.8050 / 3.35e-01 / 3.30e-01 / 3.41e-01 / 3.58e-02 /
Feasible
Analyzing direction :13/16 / Upper bound of theta: 1.43
OptRAProb / OptTheta / LB_theta / UB_theta / OptInp^2 / Exit
reason
    0.8660 / 0.00e+00 / 0.00e+00 / 1.43e+00 / 5.20e-03 /
Infeasible (0.001)
    0.8660 / 0.00e+00 / 0.00e+00 / 7.13e-01 / 5.20e-03 /
Infeasible (0.001)
    0.8050 / 1.78e-01 / 0.00e+00 / 3.56e-01 / 2.13e-02 /
Feasible

```

```

    0.8050 | 2.67e-01 | 1.78e-01 | 3.56e-01 | 2.35e-02 |
Feasible
    0.8050 | 3.12e-01 | 2.67e-01 | 3.56e-01 | 2.88e-02 |
Feasible
    0.8050 | 3.34e-01 | 3.12e-01 | 3.56e-01 | 3.67e-02 |
Feasible
    0.8050 | 3.45e-01 | 3.34e-01 | 3.56e-01 | 3.71e-02 |
Feasible
    0.8050 | 3.51e-01 | 3.45e-01 | 3.56e-01 | 3.71e-02 |
Feasible
Analyzing direction :14/16 | Upper bound of theta: 1.82
OptRAProb | OptTheta | LB_theta | UB_theta | OptInp^2 | Exit
reason
    0.8660 | 0.00e+00 | 0.00e+00 | 1.82e+00 | 5.20e-03 |
Infeasible (0.001)
    0.8660 | 0.00e+00 | 0.00e+00 | 9.12e-01 | 5.20e-03 |
Infeasible (0.001)
    0.8050 | 2.28e-01 | 0.00e+00 | 4.56e-01 | 1.74e-02 |
Feasible
    0.8050 | 3.42e-01 | 2.28e-01 | 4.56e-01 | 3.17e-02 |
Feasible
    0.8050 | 3.99e-01 | 3.42e-01 | 4.56e-01 | 2.87e-02 |
Feasible
    0.8050 | 4.27e-01 | 3.99e-01 | 4.56e-01 | 3.13e-02 |
Feasible
    0.8050 | 4.42e-01 | 4.27e-01 | 4.56e-01 | 3.23e-02 |
Feasible
    0.8050 | 4.49e-01 | 4.42e-01 | 4.56e-01 | 3.23e-02 |
Feasible
Analyzing direction :15/16 | Upper bound of theta: 1.27
OptRAProb | OptTheta | LB_theta | UB_theta | OptInp^2 | Exit
reason
    0.8660 | 0.00e+00 | 0.00e+00 | 1.27e+00 | 5.20e-03 |
Infeasible (0.001)
    0.8660 | 0.00e+00 | 0.00e+00 | 6.36e-01 | 5.20e-03 |
Infeasible (0.001)
    0.8050 | 1.59e-01 | 0.00e+00 | 3.18e-01 | 7.72e-03 |
Feasible
    0.8050 | 2.38e-01 | 1.59e-01 | 3.18e-01 | 1.34e-02 |
Feasible
    0.8050 | 2.78e-01 | 2.38e-01 | 3.18e-01 | 2.05e-02 |
Feasible
    0.8050 | 2.98e-01 | 2.78e-01 | 3.18e-01 | 2.05e-02 |
Feasible
    0.8050 | 3.08e-01 | 2.98e-01 | 3.18e-01 | 2.44e-02 |
Feasible
Analyzing direction :16/16 | Upper bound of theta: 0.64
OptRAProb | OptTheta | LB_theta | UB_theta | OptInp^2 | Exit
reason
    0.8660 | 0.00e+00 | 0.00e+00 | 6.45e-01 | 5.20e-03 |
Infeasible (0.001)
    0.8050 | 1.61e-01 | 0.00e+00 | 3.22e-01 | 5.78e-03 |
Feasible

```

```

0.8050 / 2.42e-01 / 1.61e-01 / 3.22e-01 / 8.38e-03 /
Feasible
0.8050 / 2.82e-01 / 2.42e-01 / 3.22e-01 / 8.38e-03 /
Feasible
0.8050 / 3.02e-01 / 2.82e-01 / 3.22e-01 / 9.20e-03 /
Feasible
0.8050 / 3.12e-01 / 3.02e-01 / 3.22e-01 / 9.20e-03 /
Feasible
0.8050 / 3.17e-01 / 3.12e-01 / 3.22e-01 / 9.20e-03 /
Feasible

```

Preparation for Monte-Carlo simulations of the optimal controllers

Monte-Carlo simulation parameters

```

n_mcarlo_sims = 1e5;
n_sims_to_plot = 5;

```

Plotting and Monte-Carlo simulation-based validation

```

figure(1);
clf
box on;
hold on;
plot(safe_set.slice([3,4], slice_at_vx_vy), 'color', 'y');
plot(target_set.slice([3,4], slice_at_vx_vy), 'color', 'g');
legend_cell = {'Safe set', 'Target set'};
if exist('polytope_cc_open', 'var')
    plot(polytope_cc_open.slice([3,4],
    slice_at_vx_vy), 'color', 'm', 'alpha', 1);
    legend_cell{end+1} = 'Underapprox. polytope (chance-open)';
else
    polytope_cc_open = Polyhedron();
    elapsed_time_cc_open = NaN;
end
if exist('polytope_ft', 'var')
    plot(polytope_ft.slice([3,4],
    slice_at_vx_vy), 'color', 'b', 'alpha', 1);
    legend_cell{end+1} = 'Underapprox. polytope (genzps-open)';
else
    polytope_ft = Polyhedron();
    elapsed_time_ft = NaN;
end
direction_index_to_plot = 30;
if ~isEmptySet(polytope_cc_open)
    init_state =
    extra_info(2).vertices_underapprox_polytope(:, direction_index_to_plot);
    input_vec =
    extra_info(2).opt_input_vec_at_vertices(:, direction_index_to_plot);

```

```

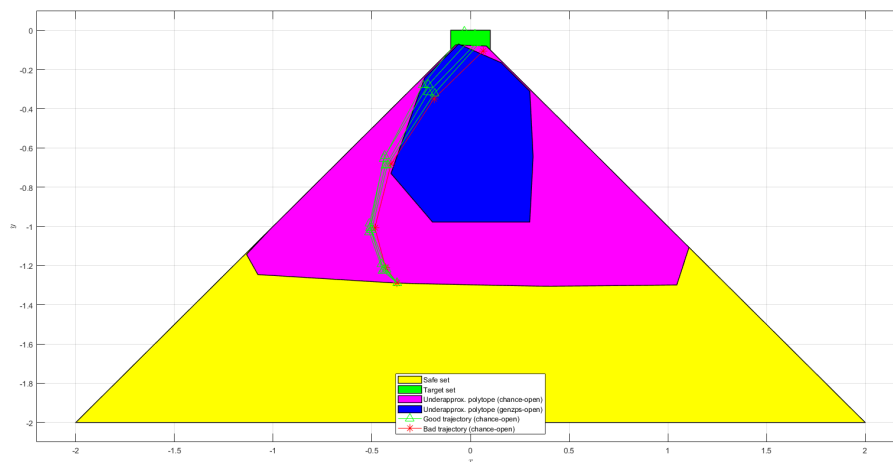
    opt_reach_avoid =
    extra_info(2).opt_reach_prob_i(direction_index_to_plot);

    concat_state_realization = generateMonteCarloSims(...
        n_mcarlo_sims,...
        sys,...
        init_state,...
        time_horizon,...
        input_vec);

    % Check if the location is within the target_set or not
    mcarlo_result =
    target_tube.contains([repmat(init_state,1,n_mcarlo_sims);
        concat_state_realization]);

    [legend_cell] = plotMonteCarlo(' (chance-open)', mcarlo_result,...
        concat_state_realization, n_mcarlo_sims, n_sims_to_plot,...
        sys.state_dim,init_state, legend_cell);
end
legend(legend_cell, 'Location','South');
xlabel('$x$','interpreter','latex');
ylabel('$y$','interpreter','latex');

```



Reporting solution times

```

if any(isnan([elapsed_time_ft, elapsed_time_cc_open]))
    disp('Skipped items would show up as NaN');
end
fprintf('Elapsed time: (genzps-open) %1.3f | (chance-open) %1.3f\n',...
    seconds\seconds\n',...
    elapsed_time_ft, elapsed_time_cc_open);

```

Elapsed time: (genzps-open) 1288.347 | (chance-open) 24.172 seconds

Helper functions

Plotting function

```
function [legend_cell] = plotMonteCarlo(method_str, mcarlo_result,...
    concat_state_realization, n_mcarlo_sims, n_sims_to_plot,
    state_dim,...
    initial_state, legend_cell)
% Plots a selection of Monte-Carlo simulations on top of the plot

green_legend_updated = 0;
red_legend_updated = 0;
traj_indices = floor(n_mcarlo_sims*rand(1,n_sims_to_plot));
for realization_index = traj_indices
    % Check if the trajectory satisfies the reach-avoid objective
    if mcarlo_result(realization_index)
        % Assign green triangle as the marker
        markerString = 'g^-' ;
    else
        % Assign red asterisk as the marker
        markerString = 'r*-' ;
    end
    % Create [x(t_1) x(t_2)... x(t_N)]
    reshaped_X_vector = reshape(...
        concat_state_realization(:,realization_index), state_dim,
[]);
    % This realization is to be plotted
    h = plot([initial_state(1), reshaped_X_vector(1,:)], ...
        [initial_state(2), reshaped_X_vector(2,:)], ...
        markerString, 'MarkerSize',10);
    % Update the legends if the first else, disable
    if strcmp(markerString,'g^-' )
        if green_legend_updated
            h.Annotation.LegendInformation.IconDisplayStyle
= 'off';
        else
            green_legend_updated = 1;
            legend_cell{end+1} = strcat('Good trajectory ',
method_str);
        end
    elseif strcmp(markerString,'r*-' )
        if red_legend_updated
            h.Annotation.LegendInformation.IconDisplayStyle
= 'off';
        else
            red_legend_updated = 1;
            legend_cell{end+1} = strcat('Bad trajectory ',
method_str);
        end
    end
end
end
end
```

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