# Ignorance is Strength: Improving Performance of Decentralized Matching Markets by Limiting Information

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**Abstract**

I develop a model of a decentralized matching market in which buyers arrive over time and pursue workers by proposing jobs. Workers have heterogeneous preferences over jobs and independently decide what jobs to accept. First, I establish that the equilibrium outcome is inefficient because of scheduling externality: by rejecting a job a worker makes himself available and decreases the other workers' chances of getting subsequent jobs. Additionally, workers fail to internalize buyers' search efforts. Second, I show that the platform's policy of coarse revelation of buyer information to workers improves acceptance rates by decreasing the ability of workers to "cherry-pick" desirable buyers. This degrades worker match quality but improves buyer welfare. Notably, this information coarsening can be Pareto improving because it offsets the scheduling externality. Finally, I characterize the disclosure policy that maximizes the number of matches in the linear payoff environment and show that, compared to the static case, there are additional availability and patience effects that qualitatively change the form of the optimal disclosure policy.

## Contents

# 1 Introduction

New platforms, such as Airbnb and Upwork, were created with the aspiration of bringing to the market the idle capacities possessed by dispersed owners. These platforms are matching marketplaces in the sense that both buyer and worker preferences are potentially highly heterogenous (Dinerstein *et al.* (2014); Fradkin (2015); Cullen and Farronato (2015); **?**). Hence a platform's goal is to facilitate search and transactions between buyers and workers. In markets with pronounced heterogeneity, the information provision is the key because it improves matching efficiency (e.g. Hoppe *et al.* (2009)). Yet information availability also gives service providers more control over the matching process, grants them more market power that may lead to inferior market outcomes. This creates a tradeoff between matching efficiency and rent-seeking. The precise characterization and various consequences of this tradeoff constitute the theme of the present paper.

I develop a model of a *dynamic peer-to-peer market* in which buyers arrive stochastically and request services from workers; a participating worker has heterogenous matching value across buyers (e.g., heterogenous costs but posted price), his capacity is limited, and he is free to accept or reject buyer requests. Coarse segmentation of buyer tasks degrades matching quality for workers but also expands the set of equilibrium outcomes, namely it eliminates undesirable cream-skimming competition among workers. As a result, capacity utilization may increase. A combination of these two effects is that generically there exists a coarse segmentation of tasks on which the worker profits increase. Furthermore, buyers are also better off because higher capacity utilization implies shorter delays for buyers.

Dynamic peer-to-peer markets are increasingly relevant: there has been a 10-fold growth in worker participation in online platforms over the last 3 years (JP Morgan Chase, 2016). Nowadays 19% of the total US adult population has engaged in a sharing economy transaction (2015, PWC); Airbnb now lists more rooms than the three largest hotel chains combined (Fradkin); Uber has more drivers than taxi drivers in US. By 2025, online talent platforms could boost global GDP by $2.7 trillion (McKinsey, 2015).

Many existing platforms have the problem of failed matches. For example, Fradkin (2015) finds that hosts on Airbnb reject proposals to transact by searchers 49% of the time. Using TaskRabbit data (from June 2010 to May 2014), Cullen and Farronato (2015) find that only 49% of posted tasks were successfully completed. Horton (2015) provides empirical evidence from online labor market UpWork that information asymmetry about workers capacity impacts the formation of matches in important ways. Part of the failed matches occurs due to workers' picking out buyers[1]. In this paper I study how the platform can use segmentation of buyer tasks to improve the utilization rates, where utilization is defined as the fraction of the aggregate service providers' capacity being used on average.

Uber addressed the cream-skimming problem by limiting drivers rejection rate, and additionally hiding some passenger information from drivers, such as ride destination. Airbnb is also aware of the problem of worker low acceptance rate, and as a solution, introduced Instant Book feature. In my framework, a host who adopted Instant Book faces a single segment of guests so that he cannot discriminate among them. Airbnb is now working on fine tuning the Instant Book feature that would allow hosts to use a somewhat finer con-

[1] Fradkin (2015) reports that on Airbnb 20% of requests fail due to hosts' cream-skimming.

sumer segments. The natural question is how fine these segments should be allowed to be? TaskRabbit design defaults workers into accepting incoming tasks, so that rejecting tasks is harder because it requires additional actions. Moreover, TaskRabbit does not collect task details from customers before directing them to the search results. Offline markets which possess the same features as peer-to-peer platform are housing market, where the real-estate agency is the intermediary.

In a peer-to-peer market, coarsening the segmentation of consumer tasks can improve utilization rates. Workers can discriminate only across segments, and thus coarser segments limit the extent of discrimination. For intuition, consider a simple static example: there is one worker, his hourly rate $p$ is posted in advance but the costs across potential tasks are heterogenous. The worker's optimal strategy is to accept all tasks with cost below $p$ and reject the tasks with cost above $p$. The intermediary can increase the worker's acceptance rate by grouping tasks right above $p$ with the tasks below $p$ into one segment. The worker infers his expected cost for the segment. If the segment contains sufficiently many low-cost tasks, the average cost of a task from the segment is low enough, so that the worker is willing to accept a random task from the segment. Consequently, for any fixed price, the platform can increase the utilization rate by coarsening the segments of buyer tasks. This simple example, however, does not explain how segmentation influences the worker dynamic optimization problem and competition between workers, therefore we can't evaluate the equilibrium effect of segmentation and can't study implications for the welfare. Also in the example I assumed that the platform has complete information about the worker's cost, which would be a strong assumption in many applications.

Coarse segmentation may improve worker profits because information availability creates undesirable competition among workers for better buyers. Workers choose to keep their schedule open by ignoring low-return jobs in order to increase their individual chances of receiving high-return jobs. I call this behavior *dynamic cream-skimming.* Collectively, this behavior is sub-optimal for workers because all profitable jobs should be completed to maximize the joint profits. Dynamic cream-skimming results in wasted capacity and lowered worker profits.

Finally, I characterize the optimal segmentation for the intermediary that wants to maximize total matches, as a function of worker cost heterogeneity and the distribution of workers' productivity.

I proceed as follows. ...

## 1.1 Related Work

My paper is adjacent to several literatures.

First, vis-a-vis literature on pricing in two-sided platform (Rochet and Tirole (2006); Weyl (2010)), I make a point that in peer-to-peer markets, not only prices matter but also the matching mechanism. Second, vis-a-vis DAA literature which studies the centralized matching mechanism (kidney exchange, National Resident Matching Program), the markets I am interested in feature less control by the intermediary. The distinguishing feature of a peer-to-peer market is that a participant can typically decide whether to participate in a transaction or not after learning his counter-party. Intermediation in peer-to-peer platforms is therefore akin to regulation and anti-trust policies in industries. Third, peer-to-peer market is not a bargaining network often found in health (cite Robin Lee). The intermediary imposes some basic structure on the marketplace, such as posted prices.

Fourth, labor literature often assumes the black-box matching mechanism – matching function (Becker (1973); Shimer and Smith (2000)). This does not allow to evaluate the policies that affect the matching mechanism, such as information intermediation and minimal acceptance rate.

Fifth, on the search side of the platform, the interaction of product differentiation and search costs have been studied by Anderson and Renault (1999). They showed that when product differentiation increases, buyers search more. However, the total effect on price elasticity of demand is ambiguous. In the present paper the workers do not search but reject or accept incoming requests. Information coarsening decreases buyer task differentiation. On the one hand, workers are worse off because due to lack of information (Blackwell (1953)). On the other hand, information coarsening softens cream-skimming competition among workers.

There have been studies on dynamic matching markets and two-sided platforms in the literatures of economics and operations research. To the best of my knowledge, no prior work examined information disclosure in a dynamic mechanism design setting.

Our main contribution is to the market design literature. A related paper that also studies dynamic matching is Ashlagi *et al.* (2013). They show that waiting to thicken improves matching in kidney exchange because individual agents do not fully incorporate the benefits of waiting. I show there is a "reversed" effect: individual workers wait too long because they do not fully incorporate the lost value of rejection for buyers (or buyer waiting costs). Another related paper is Akbarpour *et al.* (2016).

Effects of information disclosure in matching markets are multi-dimensional. Hoppe *et al.* (2009) shows that disclosing too much information can induce excessive costly signaling which overcomes the gains from improved matching.

Milgrom (2010) shows that conflation increases prices of auctioned items if the items are identical and buyers have bidding costs. In his model, supply is fixed because the number of goods is pre-determined. Moreover, there is no cost of mismatch. In a closely related literature on targeting in advertising, Bergemann and Bonatti (2011) and Athey and Gans (2010) evaluate the economic implications of improved targeting on market for advertising. In both Bergemann and Bonatti (2011) and Athey and Gans (2010), better targeting increases demand for advertising (efficiency effect). Higher demand pushes prices up. In both papers, supply effect of better targeting gives the same effect on prices but the underlying mechanisms are distinct. In Bergemann and Bonatti (2011), better targeting leads to fewer advertisers in each outlet which lowers competition between advertisers and pushes prices down. In Athey and Gans (2010), better targeting allows each outlet to better allocate scarce advertising space, so that more advertisers can be accommodated. This pushes prices down.

Dinerstein *et al.* (2014) uses data from eBay to study ranking algorithms for differentiated products. In the framework of this paper, their Best Match algorithm is de-conflation of vertically differentiated goods. By making price a more important feature in search results, the goods are effectively grouped into price categories, so that it is now easier for buyers to find lower-priced goods.

Another closely related couple of papers is Hagiu and Wright (2015) and Hagiu and Wright (2016) that study the question of the intermediary's span of control in the platform it's running. There, the focus is on who, the platform or sellers, engages in complimentary activities (marketing, advertising). My explanation focuses on the matching mechanics and underscores the importance of buyer delays and supply heterogeneity in supply-demand fit.

My paper relies on communication games literature (Aumann *et al.* (1995); Grossman and Hart (1980); Milgrom (1981); Kamenica and Gentzkow (2011); Rayo and Segal (2012); Kolotilin *et al.* (2015); Bergemann *et al.* (2015)). Johnson and Myatt (2006) connect demand rotations to the company's information disclosure policy about its product. The contribution to this literature is that I study the effects of information coarsening on the market with network effects where one of the sides is potentially competitive. Bergemann and Morris (2013, 2016) examine the general question, in strategic many-player settings, of what behavior could arise in an incomplete information game if players observe additional information, private or public among themselves, that is not known to the analyst. Bergemann and Morris (2016) showed that a joint distribution over actions and states is Bayes correlated equilibrium if and only if it forms a Bayes Nash equilibrium distribution under some information structure. Bergemann and Morris (2013) illustrate how to find all BCE in linear best-response games with Gaussian uncertainty. Compared to them, the game the workers play in my model is asynchronous, there is no hidden common state but there are private types which are not observed by the intermediary.

There is extensive operations research literature on staffing and queuing problems for platforms. Gurvich *et al.* (2015); Banerjee *et al.* (2015) study price incentives schemes for staffing problem under uncertain demand but in both papers the supply side does not cream-skim. Cachon *et al.* (2015) shows that surge-price practices of ride-sharing platforms is nearly revenue-maximizing but also generates higher welfare than in the fixed-price or fixed-wage contract. Taylor (2016) and Arnosti *et al.* (2014) study how congestion affect the platform performance. Taylor (2016) show that congestion may have a counter-intuitive effect on optimal pricing in two-sided markets. Namely, in the presence of the uncertainty congestion may increase the optimal customer price and decrease the optimal wage. Arnosti *et al.* (2014) study the matching market when applicants can send multiple applications and employer's screening is costly. They show that if the screening cost is large enough, restricting the number of applications is strict Pareto improvement for both applicants and employers.

# 2 Dynamic Model of Peer-to-peer Market

In this section I lay out a dynamic model of peer-to-peer market, in which buyers arrive gradually over time and contact one worker to complete their task; workers have heterogenous costs across buyer tasks, worker capacities are limited, and workers have control over what matches to accept. The model will allow me to study the role of heterogeneous preferences and capacity utilization in two-sided markets. I make my first important observation, that in peer-to-peer markets, from the perspective of service providers, there is a tradeoff between match rate and match quality (average cost of performing a task).

## 2.1 Setup

**Dynamic matching.** There are three parties involved in the search and matching process: workers, buyers and the platform itself.

There is mass 1 of workers, who always stay on the platform, never leave or arrive. The workers are passive searchers; they do not actively look for jobs, but instead are presented with a sequence of job offers at Poisson rate, and decide whether to accept or reject them to maximize discounted profit flow. An accepted job takes time $\tau$ to complete, during which time the worker cannot receive new jobs. This is the search friction. At each moment in time, there are two pools of workers: available and busy. Available workers wait for jobs, busy workers work on their job.

There is a continuum of potential buyers who gradually arrive over time at flow rate $\beta$. Namely, within an time interval $dt$, mass $\beta dt$ of buyers arrive to the platform. Buyers are short lived. Buyer search is passive: they put up a job and the platform assigns them to an available worker. If their job is accepted, they stay until the job is completed, otherwise they leave.[2] We make the following assumption on the matching process.

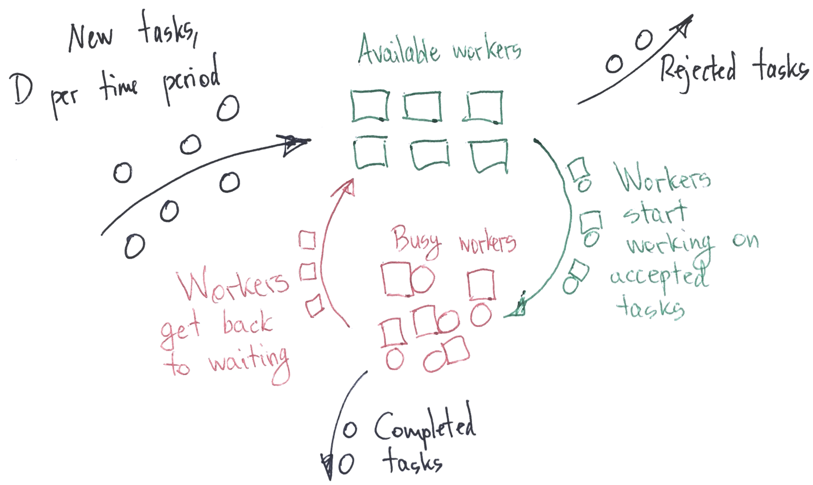
**Assumption 1** (Random Assignment)**.** *Buyers are assigned uniformly at random to the available workers.*

This assumption has two implications. First, worker of any type faces the same intensity of buyer traffic. Second, each worker faces the same distribution of buyer tasks. This is a simplifying assumption that helps me to focus on the mechanics on the supply side of the market. In the extension subsection 4.1 we show how to deal with a more general demand system where demand for each worker type can be different.

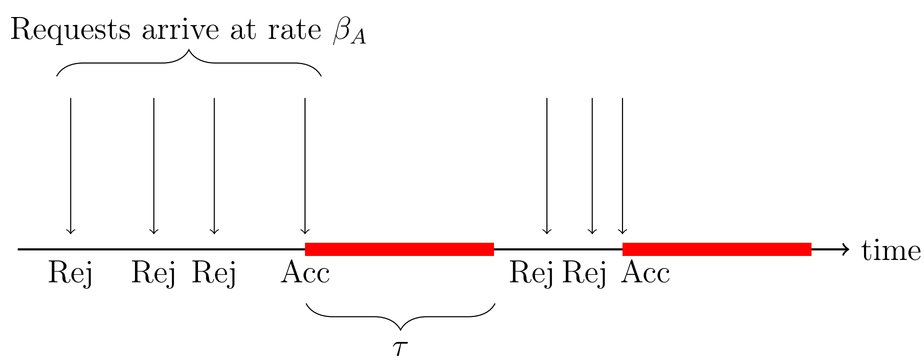
The platform opts out of the centralized matching process, instead it makes segmentation of buyer tasks part of its design, as described below. See Figure 1a for the illustration of the matching process.

There are two dimensions of heterogeneity in this matching market. First, buyer tasks are heterogenous. Second, workers are diverse. The former implies that the workers have

[2] Rejection-intolerant buyers is a simplifying assumption that makes the model of agent flows tractable. It captures the real aspect of matching markets that rejections are costly to buyers, e.g. wasted time, wasted search effort, bidding costs, etc. Moreover, buyers often do take rejections badly. For example, Fradkin (2015) reports that on Airbnb, an initial rejection decreases the probability that the guest eventually books any listing by 50%.




(a) Matching dynamics. Buyers arrive at exogenous rate $\beta$, and make one attempt to match with available workers. If rejected, a buyer leaves the platform. If accepted, the buyer forms a match which lasts for time $\tau$. After the time elapses, the buyer leaves the platform, and the workers return to the pool of available workers.




(b) Worker dynamic optimization problem with repeated search and waiting. An available worker receives requests at Poisson rate $\beta_A$. If a request is accepted, the worker becomes busy for time $\tau$ when he does not receive new requests.

Figure 1: Search and matching process.

heterogenous costs over tasks. The latter implies that different workers have different preferences over tasks. Worker type is $\theta \in \Theta$, where $\Theta$ is a compact subset of a Euclidean space. The cdf of $\theta$ is $G$, which admits probability density function $g$. Buyer task type is $\omega \in \Omega$, distribution of task types is $H$. $\Omega$ is a compact subset of a Euclidean space. Worker costs are heterogenous across tasks, namely, a worker of type $\theta$ has cost of completing task $\omega$ equal to

$$C(\theta, \omega) \geqslant 0.$$

For example, $\omega$ can be the difficulty of the task, destination of a ride, number of boxes to carry, etc. $\theta$ can indicate a worker's skill level, location, type of preferred task, etc. In the base model, I assume fixed prices, worker $\theta$'s price is $p_\theta \geqslant 0$.

Buyer gross payoff from matching is $u(\omega, \theta)$. Assume that all incoming buyers' net payoff from matching is positive:

$$u(\omega, \theta) - p_\theta \geqslant 0, \quad \forall \theta, \omega. \tag{1}$$

**Segmentation.** The platform observes buyer type $\omega$ but not worker type $\theta$.[3] A *segmentation* is a devision of tasks into different segments.[4] The segmentation framework I use below is adopted from Bergemann *et al.* (2015). Let $S = \Delta(\Omega)$ be the set of all possible segments. Thus, a segmentation $\mu \in \Delta(S)$ is a probability distribution on $S$,[5] with the interpretation that $\mu(S')$ is the proportion of tasks in segments $S' \subset S$. A segmentation can be seen as a two-stage lottery on $\Omega$ whose reduced lottery is prior $h(\omega)$. The set of possible segmentations is then:

$$\left\{ \mu \in \Delta(S) \colon \int s\mu(ds) = h \right\}.$$

When a buyer arrives and puts up a task of type $\omega$, the platform assigns it to one of the segments according to $\mu$, and then the corresponding worker only observes the segment the task belongs to. The perfect segmentation is the one where type $\omega$ is perfectly revealed to the workers. The perfect segmentation is denoted by $\mu_0$; we have $\mu_0(\delta_\omega) = h(\omega)$ for all $\omega \in \Omega$.

**Steady state.** A state of the dynamic matching system described above is characterized by the mass of available workers of each type and the acceptance rates of each worker type. Formally, let $\alpha(\theta) \geqslant 0$ be the acceptance rate of type-$\theta$ available workers, and let $\rho(\theta)$ be the type-$\theta$ *capacity utilization rate* – the fraction of type-$\theta$ workers who are busy. Denote the total mass of available workers by $A := \int_\Theta (1 - \rho(\theta)) dG(\theta)$.

To define steady state, I first need to define the worker inflow and outflow. The outflow of type-$\theta$ workers is the mass of workers that flows from "available" pool to "busy" pool in a unit of time. By the Random Assignment assumption, the buyer flow to type-$\theta$ workers is $\beta \frac{(1-\rho(\theta))g(\theta)}{A}$. The outflow is thus equal to buyer flow $\beta \frac{(1-\rho(\theta))g(\theta)}{A}$ times the acceptance

[3] $\omega$ captures all information the platform elicits from the buyer about the task, passively from the buyer's cookies and queries or actively by asking questions.

[4] I focus on the "public" segmentation in the base model, i.e. I am not studying the mech. design problem where the platform tries to elicit or learn the worker's information and tailor the segments to a worker type.

[5] When $\Omega$ is a compact subset of a Euclidean space, $\Delta(S)$ is the set of Borel-measurable probability distributions with the weak-* topology on $\Delta(\Omega)$.

rate $\alpha(\theta)$ . The inflow of type-$\theta$ workers is the mass of workers that flows from "busy" pool to "available" pool in unit of time. The size of the "busy" pool is $g(\theta)\rho(\theta)$, the tasks are completed in time $\tau$, therefore the inflow is equal to $g(\theta)\rho(\theta)/\tau$.

In a steady state, the inflow equals the outflow for each worker type:

$$\beta\frac{(1-\rho(\theta))g(\theta)}{A}\alpha(\theta) = \frac{g(\theta)\rho(\theta)}{\tau}, \quad \forall\theta\in\Theta. \tag{2}$$

**Supply: worker repeated search problem.** Buyers arrive at flow rate $\beta$, and by the Random Assignment assumption, get distributed uniformly at random across all available workers. Therefore, each individual available worker experiences a Poisson arrival process of buyers, and the rate of the arrival is equal to the buyer-to-seller ratio:

$$\beta_A := \frac{\beta}{A} \tag{3}$$

(Myerson (2000)). There is thus a contrast between individual arrival process and aggregate arrival process in that the individual arrival process is stochastic while the aggregate arrival process is deterministic (a consequence of law of large numbers). More concretely, an individual worker faces a stochastic arrival process, such that over an infinitesimal time interval $dt$ the probability that one buyer arrives is $\beta_A dt + o(dt)$. Jointly, available workers face the deterministic buyer arrival process, such that over time interval $dt$ the continuum mass $\beta_A dt$ of buyers arrive. As we show in the rest of this section, stochasticity of individual arrival process combined with independent decision-making creates matching inefficiency.

Worker $\theta$ takes $\beta_A$ as given[6] and solves the dynamic optimization problem where for each incoming task a worker has to decide to accept it or reject it. See Figure 1b for the illustration of the worker dynamic optimization problem.

If type-$\theta$ worker accepts task of type $\omega$, he immediately earns

$$p_\theta - C(\theta,\omega).$$

Workers are risk neutral, and so for the decision-making, it is sufficient to know only the distribution of posterior means. The expected cost on segment $s$ for type-$\theta$ worker is $\mathbb{E}[C(\theta,\omega)|s] = \int_\Omega C(\theta,\omega)s(d\omega)$.

Workers solve a dynamic optimization problem with repeated search and waiting. There is no discounting. Denote by $V(\theta)$ the average profit flow when worker of type $\theta$ acts optimally (value function)[7]. Denote the value of a new request from segment $s$ by $\pi(s,\theta)$. Value of the task is zero if the worker rejects it, and $p_\theta - \mathbb{E}[C(\theta,\omega)|s] - \tau V(\theta)$ if he accepts it, where $\tau V(\theta)$ is the opportunity cost of lost value for working on the task for time $\tau$. Worker optimization problem is then the system of the following two equations:

$$V(\theta) = \beta_A \int \pi(s,\theta)\mu(ds) \tag{4}$$

$$\pi(s,\theta) = \max\{0, p_\theta - \mathbb{E}[C(\theta,\omega)|s] - \tau V(\theta)\} \tag{5}$$

[6] This follows from the fact that there is a continuum of workers on the platform, and so any individual worker's actions does not affect $\beta_A$.

[7] For example, if a worker earns \$1 on each task, and time interval between starting consequent tasks is 2, then $V(\theta) = 1/2$.

For each $\theta$, worker strategy is function $\sigma(\cdot,\theta)\colon S \to [0,1]$ that maps a task's segment to the probability of accepting it. For each $\theta$ the worker acceptance rate is the ex ante probability of accepting a task:

$$\alpha(\theta) = \int \sigma(s,\theta)\mu(ds). \tag{6}$$

## 2.2 Equilibrium

We are interested in a steady state equilibrium of the market when workers act independently and take buyer arrival rate as given, and worker inflow and outflow balance out.

Before stating the definition of the equilibrium, recall that the exogenous objects in my model are the following: buyer flow rate $\beta$, prices $p_\theta$, cost function $C(\omega,\theta)$, distribution of types $G(\theta)$ and $H(\omega)$, and completion time $\tau$.

Based on the exogenous objects, a tuple of endogenous objects $(\alpha(\theta), \beta_A)_{\theta\in\Theta}$ constitute a *steady-state equilibrium* if

1. Acceptance rates $\alpha(\cdot)$ are optimal given buyer arrival rate $\beta_A$, as shown in (6).
2. Arrival rate $\beta_A$ arises in a steady state given workers' acceptance rates $\alpha(\cdot)$, as shown in $\{$(2), (3)$\}$.

We make the following assumption to make the exposition easier.

**Assumption 2** (No Excess Demand). *Collectively, it is physically possible for workers to complete every buyer task: $\beta\tau < 1$.*

Relaxing it will require dealing with demand overload and queues, which does not add more intuition. I relax the assumption in Appendix C and show that qualitatively results do not change.

The next lemma shows how to find arrival rate for available workers $\beta_A$ as a function of acceptance rates.

**Lemma 1.** *For an arbitrary vector of acceptance rates $\alpha(\theta)$, $\alpha(\theta) \in [0,1]$, $\beta_A$ is the unique solution to*

$$\int \frac{G(d\theta)}{\tau\alpha(\theta) + 1/\beta_A} = \beta. \tag{7}$$

In the proof I show that the steady state utilization rate is $\rho(\theta) = 1 - \frac{1}{\tau\alpha(\theta)\beta_A + 1}$, which yields (7). Then, the left hand side of (7) is non-decreasing in $\beta_A$, growing from zero to some value greater than $\beta$. This gives the existence and uniqueness.

**Proposition 1.** *Steady-state equilibrium exists and is unique.*

The uniqueness obtains by monotonicity of reaction curves in (6) and (7). Namely, in (6), as buyer traffic intensity $\beta_A$ increases, workers become more picky and acceptance rate $\alpha(\theta)$ decreases. In (7), as $\alpha(\theta)$ increases, buyers get directed to a smaller set of workers, hence $\beta_A$ increases. For details see the proof in the Appendix on page 29.

Proposition 1 demonstrates the role of capacity utilization in two-sided markets. Both buyers and workers are price-takers. The result shows that for any price $p > 0$ and any segmentation $\mu$, there is a steady-state equilibrium characterized by the vector of acceptance rates $(\alpha(\theta))$ where the market clears. This happens because there are two mechanisms to clear the market: through adjustments in price and through adjustments in capacity utilization. The former is the most familiar mechanism, but in this paper I emphasize the importance of the latter.

### 2.3 Examples of Marketplaces

In this section I discuss how the model maps to the reality in the case of Uber, Airbnb and labor platform such as TaskRabbit. Recall that $\theta$ denotes worker type, i.e. the worker's private component of his costs; $\omega$ denotes buyer type, i.e. the information about buyer available to the platform.

**Uber.** Drivers receive requests from passenger when they are idle. $\theta$ is driver's location, $\omega$ is passenger's destination and star rating. Price per mile and minute is fixed at $p$ (ignore surge pricing). Drivers do not like short rides or rides to the neighborhoods far removed from busy areas. Proposition 3 shows that if drivers have full discretion over accepting or rejecting, the driver utilization rates are inefficiently low, and the demand-supply fit is suboptimal.

**Airbnb.** Hosts are capacity constrained in rooms. Once a room is booked for a specific date, the host cannot accept a better guest. $\theta$ - room quality, location. $\omega$ - guest quality, e.g. college student vs. elderly couple. Every host sets his own price $p_\theta$ that applies to all guests but he may prefer to reject certain guests who he expects can be a pain in the neck. Proposition 3 shows that if hosts have full discretion over accepting or rejecting guest requests, the apartment utilization rates are inefficiently low, and the marketplace is removed from its full potential.

**TaskRabbit.** The service providers are capacity constrained because they can do only that many tasks per week. Once a service provider agreed to do some task, he is constrained in picking new tasks. $\theta$ - worker skill, specialization. $\omega$ - task category, task difficulty, professional/annoying client. Service provider sets hourly rate $p_\theta$ which applies to all tasks in the category. The worker may prefer to reject the tasks which he does not find to be a good fit, or too far, or bad timing of the lead. Proposition 3 shows that if service providers have full discretion over accepting or rejecting client leads, the utilization of participating labor is inefficiently low, and the matching efficiency is suboptimal.

# 3 Market Design: Efficient Information Disclosure

This section demonstrates that a dynamic peer-to-peer market with full disclosure generates suboptimal market outcomes and then shows how the platform can use information coarsening as a Pareto improving policy. A key insight is that information disclosure influences capacity utilization, and therefore can be used to balance the two sides of the market to achieve higher efficiency.

Market outcome $O = (\{V(\theta)\}, CS)$ is a combination of worker profits and consumer surplus that arises in a steady-state equilibrium. I say that a market outcome is *feasible* if there are acceptance strategies for workers that generate it, and $V(\theta) \geqslant 0$ for all $\theta$. A feasible outcome $O$ is Pareto-optimal if there is no other feasible $O'$ such that $V(\theta)' > V(\theta)$ for all $\theta$, and $CS' > CS$. The Pareto frontier is the set of all Pareto-optimal outcomes. We say that market outcome $O$ is *implementable* if there is a segmentation of $\Omega$ such that the equilibrium outcome is $O$.

## 3.1 Implementable outcomes with identical workers

We start by considering the case of identical workers, that is when $\Theta$ is a singleton. My first main result characterizes the set of combinations (worker profits, buyer surplus) implementable by some segmentation.

**Proposition 2.** *Let the workers be identical. Then for any point on a Pareto frontier there is a segmentation that implements it.*

The proof is in the Appendix on page 30 but here we discuss what market outcomes the Pareto frontier consists of and what segmentations implement them.

Figure Figure 2 shows the Pareto frontier in the axes of consumer surplus and joint worker profits. Moving along the Pareto frontier, the acceptance rate varies from worker-optimal acceptance $\alpha^{coord}$ to the maximal feasible acceptance $\alpha^{max}$. In lieu of (1), $\alpha^{max}$ is also the buyer-optimal outcome. Two things to notice here. First, $\alpha^{full-disc}$ — the acceptance rate when workers act independently under full disclosure — is not on the Pareto frontier. We will discuss this important observation in detail in the next section and show that it is a robust feature in dynamic matching environment. Second, the maximal feasible acceptance $\alpha^{max}$ is less than one when the price is below the average cost. Indeed, the proof shows that $\alpha^{max}$ is the maximal $\alpha \in [0, 1]$ such that $F^{\mu}(p) = \alpha$ for some coarsening $\mu$ of $\mu_0$. If $p < \int z dF^{\mu_0}(z)$, then $\alpha^{max} < 1$. To recap, normally[8] we have:

$$0 < \alpha^{full-disc} < \alpha^{coord} < \alpha^{max} < 1,$$

and the Pareto optimal outcomes lie in the range $[\alpha^{coord}, \alpha^{max}]$.

Segmentation $\hat{\mu}$ that implements acceptance rate $\alpha$ on the Pareto frontier has two seg-

[8] Normal for the applications I have in mind: $F^{\mu_0}$ has full support on $[0, p]$ and $p < \int z dF^{\mu_0}(z)$.

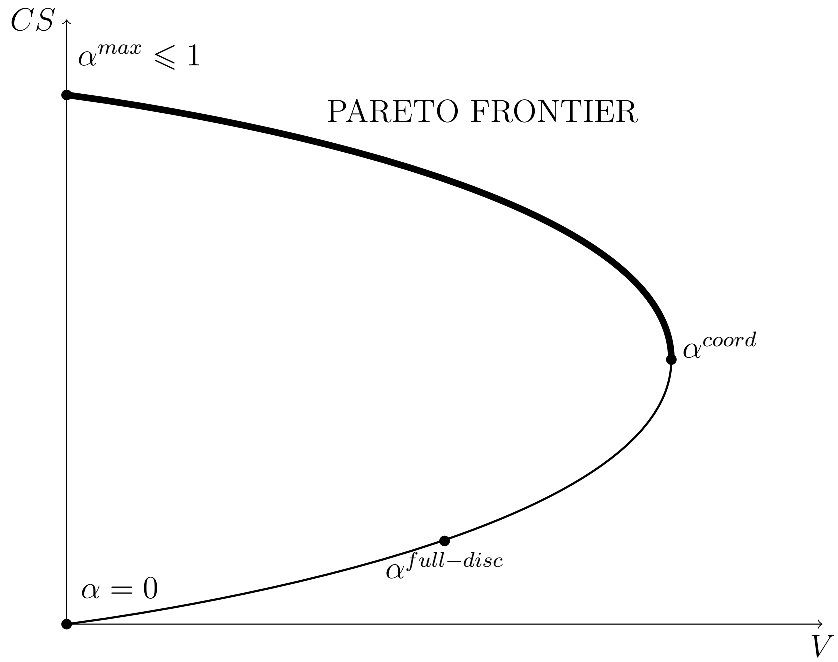



Figure 2: Utility possibility set for aggregate buyers and aggregate workers. Segmentation can implement any point on the Pareto frontier (thick solid line). Acceptance rate $\alpha^{full-disc}$ is the acceptance rate when workers act independently under full disclosure; $\alpha^{coord}$ is the worker-optimal acceptance rate; $\alpha^{max}$ is the maximal feasible acceptance rate (also buyer optimal). The Pareto frontier is the set of border points between $\alpha^{coord}$ and $\alpha^{max}$.

ments, $\Omega_1$ and $\Omega\backslash\Omega_1$, that satisfy:

$$\int_{\Omega_1} H(d\omega) = \alpha,$$
$$\mathbb{E}[C(\omega)|\omega \in \Omega_1] \leqslant p,$$
$$\mathbb{E}[C(\omega)|\omega \notin \Omega_1] > p.$$

Coarse segmentation improves acceptance by decreasing the ability of workers to "cherry-pick" desirable tasks. However it degrades their match quality. As a total effect, the increase in acceptance may outweigh the quality decrease, and the service providers benefit from moderate information coarsening. Buyers benefit from imperfect segmentation compared to perfect segmentation because they that increases their match rate.

The optimal solution is non-trivial: in a typical case the optimal segmentation involves only partial revelation of buyer type $\omega$. The two-segment form of the optimal segmentation $\hat{\mu}$ follows by the revelation principle from the fact that workers have two actions, accept or reject.

What is interesting is that $\hat{\mu}$ does not depend on the intensity of buyer traffic $\beta$. This implies that $\hat{\mu}$ is the same segmentation as would be optimal in a static case, when $\tau = 0$. This happens because the arrival rate to available workers ($\beta_A$) matters only to the extent that it creates option value of rejection for workers. When the workers have no private information, segmentation $\hat{\mu}$ induces the posterior mean distribution $F^{\hat{\mu}}$ that has only one acceptable value. Therefore, the option value is zero, and so $\beta$ does not matter. In the next section we solve for the optimal segmentation when workers do have private information.

Proposition 2 is related to the result of Bergemann *et al.* (2015) who show that segmentation of a monopolistic market can achieve every feasible combination of consumer and producer surplus. Their segmentation problem is static with single receiver (monopolist) while my model is dynamic and has an equilibrium effect. Neither dynamics nor equilibrium effect are present in the prior literature on information disclosure. Non-linearity of utility function does not allow me to use techniques used in Bergemann and Morris (2013) to find the set of all outcomes achievable under some information structure.

## 3.2 Worker coordination problem and scheduling externality

The next proposition shows that in un-mediated dynamic peer-to-peer markets (all available information about buyer tasks is revealed to workers), the market outcome is removed from the Pareto frontier. Reason — workers' coordination problem; due to scheduling externality they resolve match rate/match quality tradeoff collectively sub-optimally. The inefficiency arises due to dynamic nature of the matching.

In this subsection we return to the most general setting to emphasize that the inefficiency is robust to the specifications of the agent's preferences and arises due to the dynamic nature of the matching.

To show the result, I compare the market outcome under perfect segmentation with the coordination benchmark in which workers coordinate to maximize the joint profits. Obviously, workers cannot do worse under coordination. Proposition 3 shows that first, generically they do strictly better, and additionally, the buyers also do better.

The result demonstrates that when the platform relies on independent service providers who dynamically screen buyers and have discretion over accepting/declining tasks, the service providers accept too few tasks, and matching on the platform is not efficient.

Given segmentation $\mu$, denote by $F_\theta^\mu$ the distribution of cost posterior means for worker of type $\theta$:

$$F_\theta^\mu(y) = \int_S I\{\mathbb{E}[C(\theta,\omega)|s] \leqslant y\}\mu(ds), \tag{8}$$

Denote the acceptance rates under worker coordination by $\alpha^{coord}(\theta)$, profits by $V^{coord}(\theta)$ and buyer surplus by $CS^{coord}$.

**Proposition 3.** *Suppose $F_\theta^\mu$ has full support on $[0, p_\theta]$. Then:*

$$\begin{aligned} V^{coord}(\theta) &> V(\theta), \\ \alpha^{coord}(\theta) &> \alpha(\theta), \\ CS^{coord} &\geqslant CS. \end{aligned}$$

The result has three parts: worker profits, acceptance rates, and buyer surplus. The first part shows that coordination strictly improves worker surplus. This happens because individual workers have positive option value of rejecting tasks while coordinated workers have zero option option value of rejecting. Indeed, type-$\theta$ worker has option value of rejecting equal to $\tau V(\theta)$ (see Eq. (5)). As a result, type-$\theta$ worker rejects profitable tasks that have expected cost lying in the interval $c \in (p_\theta - \tau V(\theta), p_\theta]$. However, the aggregate buyer arrival process is deterministic, and by the No Excess Demand assumption, all tasks can be accepted. Therefore, the coordinated workers have no option value from rejecting a task, and they accept all tasks with $c \leqslant p_\theta$. The fundamental reason behind this distinction in option values for individual and coordinated workers is that collectively, the workers are not capacity constrained (in time) while individually, the workers *are* capacity constrained.

Perhaps more surprisingly, buyers *also* benefit from worker coordination. This happens because worker coordination results in higher acceptance rates, and due to (1), the buyer surplus increases.

Worker profits in the equilibrium are lower because workers impose negative *scheduling externality* on each other: by rejecting a task a worker decreases the other workers' chances of getting consequent tasks. Coordination benefits workers because it makes them internalize the scheduling externality.

Proposition 3 shows that scheduling externality leads to a market failure. Proposition 2 shows that when workers are identical, the platform can improve the welfare of both buyers and workers by the right choice of segmentation. However, in the case when workers are not identical and have private information about their costs, things are getting more tricky.

## 3.3 Optimal information disclosure when workers are differentiated by skill

Typically, the platform does not observe worker costs, and there is a private cost component that is only known to the worker (skill level, willingness to do long-haul rides, etc). If there is privately known match value held by workers, the intermediary's ability to segment effectively is limited. Depending on the heterogeneity of service provider costs, there may or may not exist a coarse segmentation that improves acceptance rates. In this section I provide sufficient conditions for this.

To see the intuition behind additional problems the platform faces when workers have private information, imagine there are two types of workers: professionals and amateurs. Professionals can profitably complete a larger set of tasks than amateurs. When the platform designs task segments it should take into consideration different effects the segmentation will have on the two groups of workers. Amateurs can only do a small subset of tasks therefore the segmentation has to be fine to keep them on the platform. On the other hand, fine segmentation allows the professionals to be picky and decreases their acceptance rate. The resolution of this tradeoff depends on the relative sizes of amateur and professional populations. If there are more professionals than amateurs, then coarser segmentation increases the total acceptance rates even though amateurs stop working. If there are more amateurs, coarse segmentation decreases the total acceptance rate. In the rest of this section we rigorously study this problem.

Formally, consider a type space where tasks are vertically differentiated, and workers differ in their cost efficiency. Namely, let the space of buyer types be $\Omega = [0, 1]$, cdf $H(\omega)$ admits pdf $h(\omega)$. Let the space of worker skill level be $\Theta = [0, 1]$; cdf $G(\theta)$ admits pdf $g(\theta)$. The cost function is $C(\theta, \omega) = \omega - \theta$. Prevailing market price $p$ is used by all workers. I call this *linear cost* setting. In this formulation, "professionals" are workers with high $\theta$, and "amateurs" are workers with low $\theta$.

Since workers are risk-neutral, the sufficient statistic for a segment is the expected cost on it. Denote the expected value of $\omega$ on segment $s$ by $m(s) := \int_\Omega \omega s(d\omega)$. I will use $m$ as a typical name for posterior value of $\omega$. Denote by $H^\mu$ the distribution of posterior means of $\omega$ when the platform uses segmentation $\mu$. Define the *option value function* $W^\mu\colon [0, 1] \to \mathbb{R}_+$:

$$W^\mu(m) := \int_0^m (m - y) dH^\mu(y), \tag{9}$$

For any $\mu$, $W^\mu(m)$ is non-negative, weakly increasing from 0 to $1 - \mathbb{E}[\omega]$, and is convex. As we will see from the next lemma, $W^\mu(m)$ is equal to the option value of rejecting a task with expected cost $m$.

The next lemma shows that a worker's optimization problem depends on $\mu$ only through $W^\mu$.

**Lemma 2.** *For any segmentation $\mu$, worker's optimal strategy has a cutoff form with cutoff $\hat{m}^\mu(\theta)$ such that $\theta$-worker accepts all tasks with expected cost $m < \hat{m}^\mu(\theta)$ and rejects all tasks with $m > \hat{m}^\mu(\theta)$. Furthermore, for any $\beta_A$, worker payoff $V(\theta)$ and cutoff $\hat{m}^\mu(\theta)$ are solutions to the following system of equations:*

$$V^\mu(\theta) = \frac{p + \theta - \hat{m}^\mu(\theta)}{\tau} = \beta_A W^\mu(\hat{m}^\mu(\theta)). \tag{10}$$

Let $\overline{W}$ be the option value function under perfect segmentation (full disclosure of $\omega$), $\overline{W}(z) := \int_0^z (z - y) dH^{\mu_0}(y)$. Similarly, let $\underline{W}$ be the option value function under no information, $\underline{W}(z) := \max\{0, z - \mathbb{E}[\omega]\}$. Denote by $\mathcal{W}$ the set of all option value functions $W$ implementable by some $\mu$. The next lemma characterizes $\mathcal{W}$.

**Lemma 3.** *Option value function $W$ is implementable by some segmentation if and only if $W$ is a convex function point-wise between $\overline{W}$ and $\underline{W}$ .*

The result is shown in e.g. Gentzkow and Kamenica (2016). The power of Lemma 3 is that it shows that any segmentation can be represented as some non-negative, non-decreasing and convex function $W$. Therefore, in any optimization problem that requires to find an optimal signaling structure, it is equivalent to find an optimal $W \in \mathcal{W}$. It is easier to optimize with respect to $W \in \mathcal{W}$ because the platform's optimization problem is more tractable when the set of optimizers is bounded convex functions.

Denote the flow of matches consummated on the platform by $M$, i.e. over time period $dt$, mass $Mdt$ of matches is consummated. Consider the platform's problem of maximizing the utilization rate. Since we hold worker participation fixed, the problem is equivalent to maximizing $M$. In light of Lemma 3, the platform's problem can be expressed as

$$\max_{W \in \mathcal{W}} M.$$

The problem is not trivial because: 1) workers are heterogenous; 2) besides the direct effect of segmentation on worker's dynamic optimization problem in (10), there is an equilibrium effect when arrival rate $\beta_A$ changes in response to the workers' change in acceptance rate, as shown in Eq. (7). We approach this problem using calculus of variations. Denote by $\nu(\theta) := x(\theta)/g(\theta)$ the fraction of available type-$\theta$ workers relative to all type-$\theta$ workers.

**Theorem 1.** *The first variation of $M$ with respect to $W$ exists and is proportional to:*

$$\frac{\delta M}{\delta W} \propto -(g(\theta)\nu^2(\theta))' + g(\theta)\nu'(\theta). \tag{11}$$

A way to see the contribution of theorem Theorem 1 is to compare it with the static case which has been studied in the prior literature. The worker's problem becomes static when $\tau = 0$. A variation of the following result has been shown in Kolotilin *et al.* (2015), and the implied concavification reasoning goes back to Aumann *et al.* (1995) and Kamenica and Gentzkow (2011).

**Corollary 1.** *Suppose $\tau = 0$. Then*

$$\frac{\delta M}{\delta W} \propto -g'(\theta). \tag{12}$$

*If $G$ is concave, then perfect segmentation is optimal. If $G$ is convex, no segmentation is optimal.*

The result easily follows from theorem Theorem 1 after noting that $\nu(\theta) = (1+\tau\beta_A\alpha(\theta))^{-1}$.

When matching is dynamic, the original formula (12) has to be adjusted for the dynamic effects, as shown in (11). I now explain the additional dynamic effects in more detail.

To better understand the additional effects, consider uniform distribution of worker types, $G = U[0, 1]$. Under uniform distribution in the static case, the segmentation does not matter: $\frac{\delta M}{\delta W} = 0$ for any $W$. In my model, however, we have that

$$\frac{\delta M}{\delta W} \propto -(\nu^2(\theta) - \nu(\theta))'.$$

There are two additional forces that the intermediary has to consider when the matching problem becomes dynamic: *availability effect* and *patience effect*. The availability effect arises because in equilibrium, high types are less available than low types. This makes the effective distribution of workers concave, and creates a motive for the intermediary to use finer segmentation. Term $\nu^2(\theta)$ in the expression above is responsible for the availability effect.

The patience effect arises because workers do not act myopically and may reject tasks due to temporal optimization. High types have greater option value of rejecting a task, and so all held equal they are more picky than low types. Note that this effect is dynamic and is distinct from the pure static effect of high types having higher match value. Relatively higher pickiness of high types creates a motive for the intermediary to coarsen segmentation for high types. Term $-\nu(\theta)$ in the expression above is responsible for the patience effect.

The next four results give necessary and sufficient condition for optimality of no segmentation (one segment, no signal about buyer task $\omega$) and of perfect segmentation ($\omega$ is perfectly revealed).

**Proposition 4** (Necessary condition for optimality of no segmentation)**.** *If*

$$g'(\theta) < g(\mathbb{E}\omega)\tau\beta(1-\beta\tau)^2, \quad \forall\theta,$$

*then no segmentation cannot be optimal.*

For example, if worker types are distributed uniformly, $g \equiv const$, then no segmentation cannot be optimal. In the static case $g'(\theta) > 0$ implies optimality of no segmentation (1). In dynamic case, it is not longer the case: there are slightly convex $G$ such that no segmentation is not optimal. Proposition 4 gives the corresponding bound on the convexity of $G$.

**Proposition 5** (Sufficient condition for local optimality of perfect segmentation)**.** *If $G$ is concave, and $\beta\tau < 1/2$, then it's impossible to improve upon perfect segmentation by local coarsening.*

**Proposition 6** (Necessary condition for optimality of perfect segmentation)**.** *Assume $G = U[0,1]$. If $\beta\tau > \Pi^* := \int \frac{dG(\theta)}{1+H(\theta-\tau\beta(1-\beta\tau)^{-1}\overline{W}(\theta))}$, then perfect segmentation cannot be optimal.*

# 4 Extensions

## 4.1 General demand: workers are imperfect substitutes

Worker type space $\Theta$ is finite. There is a continuum of potential buyers who arrive continuously over time. Let $\beta(\theta)$ be the flow rate of buyers who arrive to request services of type-$\theta$ workers. In other words, within an infinitesimal time interval $dt$, a continuum mass $\beta(\theta)dt$ of buyers arrive to the platform and requests one of type-$\theta$ workers. I refer to the collection of functions $\{\beta(\theta)\}_{\theta\in\Theta}$ as "demand" although it is different from the textbook notion of demand in that my demand arrives continuously. We make the following assumptions on the demand.

**Assumption 3** (Frictional Consumer Search)**.** *Let the fraction of available type-$\theta$ workers relative to the mass of all available workers be $\nu(\theta)= x(\theta)/\sum x(\theta')$. Demand for type-$\theta$ workers is*

$$\beta(\theta) = \beta(\theta, \alpha(\theta), p_\theta, \nu(\theta)).$$

*$\beta$ is differentiable in $\alpha$, $p$ and $\nu$. Most importantly,*

$$\frac{\partial \beta}{\partial \nu} \geqslant 0. \tag{13}$$

The assumption posits that the demand for type-$\theta$ workers depends not only on the characteristics of type-$\theta$ workers but also on their number. Critically, the larger the fraction of available type-$\theta$ workers, the more demand they receive.

The steady state condition becomes:

$$\beta(\theta)\alpha(\theta) = \frac{g(\theta) - x(\theta)}{\tau}, \quad \forall \theta \in \Theta. \tag{14}$$

Each buyer type now had individual buyer arrival rate:

$$\beta_A(\theta) := \frac{\beta(\theta)}{x(\theta)} \tag{15}$$

A tuple of endogenous objects $(\alpha(\theta), x(\theta), \beta_A(\theta))_{\theta\in\Theta}$ constitute a *steady-state equilibrium* if

1. Workers act optimally taking buyer arrival rate $\beta_A(\theta)$ as given, so that the acceptance rates $\alpha(\theta)$ are determined by (6).

2. Given $\alpha(\cdot)$, worker flows balance out, as shown in (14). Steady state flows determine $\beta_A(\theta)$ according to (15).

**Assumption 4** (No Excess Demand-2)**.** *Collectively, it is physically possible for workers to complete every buyer task: $\beta(\theta, 1, p(\theta), 1)\tau < g(\theta)$ for all $\theta \in \Theta$.*

**Proposition 7.** *Steady-state equilibrium exists and is unique.*

First I establish that for any vector of acceptance rates $\alpha(\theta)$, there exists a unique steady state masses of available workers $x(\theta)$, as a solution to (14) (lemma Lemma 7). The existence of $\{x(\theta)\}$ crucially relies on the No Excess Demand assumption, and the uniquess follows from the fact that demand depends on $\{x(\theta)\}$ through the share $\nu(\theta)= x(\theta)/\sum x(\theta')$. Then treating $x$ as a funciton of $\alpha$, I show that there is a unique solution $(\alpha(\theta), \beta_A(\theta))_{\theta\in\Theta}$ to $\{$(6), (15)$\}$. For details see the proof in the Appendix on page 29.

**Proposition 8.** *Assume $F^\mu_\theta$ are absolutely continuous on $\mathbb{R}_+$, and $F^\mu_\theta(p_\theta) > 0$. Assume buyers are delay-sensitive. Then*

$$\begin{aligned} V^{coord}(\theta) &> V(\theta), \quad \theta\in\Theta \\ \alpha^{coord}_\theta &> \alpha_\theta \\ CS^{coord} &\geqslant CS. \end{aligned}$$

## 4.2 Endogenous price

In our base model we assumed that the price is set by the larger outside market. In this section we consider other scenarios of price setting: competitive price and price set by the intermediary. Main results of this section are: 1) there is a continuum of competitive price equilibria; 2) characterization of #matches-maximizing pricing.

We adjust our notation a little bit to account for the fact that demand depends on price. Namely, buyer arrival rate is given by

$$\beta(\alpha, p),$$

$\beta(0, p) = 0$, $\beta_\alpha > 0$, $\beta_p < 0$. The assumption I am making here is that buyer arrival rate responds to the changes in platform policies only through the average acceptance rate $\alpha$ and price $p$.[9] Denote demand elasticities wrt $\alpha$ and wrt $p$ by $\varepsilon_\alpha = \frac{\beta_\alpha \alpha}{\beta} > 0$ and $\varepsilon_p = -\frac{\beta_p p}{\beta} > 0$, respectively.

## 4.3 Price set by the intermediary

In this section we study a situation when the platform controls the price on the platform (e.g. in case of Uber and Instacart). This brings us closer to the literature on pricing in two-sided markets (Rochet and Tirole (2006); Weyl (2010)). However, unlike these papers, we are more specific about where the network effect is coming from and also workers have a decision margin not related to participation – acceptance rate.

[9] Let buyer $i$'s expected acceptance rate for his task be $\alpha_i$, the expected match value conditional on being matched be $v_i$. Additionally, buyer $i$ has hassle cost $h_i$ from using the platform. If the buyer does not use the platform he gets payoff of zero. If he uses the platform he experiences a disutility of $h_i$. After requesting a service, if he is accepted, he pays price $p$ and gets payoff of $v_i$. Therefore, buyer $i$ joins iff

$$\alpha_i(v_i - p) - h_i \geqslant 0.$$

As defined above, the average acceptance rate across buyers is $\alpha$. The assumption is that [#potential buyers]$\cdot$ $\Pr(\alpha_i(v_i - p) - h_i \geqslant 0) = \beta(\alpha, p)$.

**Match-maximizing platform.** From the previous section we know that whatever price a Walrasian auctioneer picks, it can be sustained in a competitive equilibrium. We thus pose a question: if the platform can set the price, what price and disclosure policy should it use to maximize the number of matches? The platform solves

$$\max_{\mu,p} \alpha\beta(\alpha,p),$$

subject to {(4), (5)}.

We know from Proposition 2 the form of the optimal disclosure policy for given price. How should the price be set? To induce more demand, the platform would like to choose lower price. But the lower the price, the lower worker utilization rate can be implemented. The next result quantifies this tradeoff.

**Proposition 9.** *Assume workers have no private information. The #matches-maximizing disclosure policy has the form*

$$\hat{\mu}(\omega) = \begin{cases} \mathbb{E}[\omega|\omega \leqslant mc], & \alpha \\ \mathbb{E}[\omega|\omega > mc], & 1-\alpha \end{cases}$$

*and $\alpha$, $p$ and $mc$ solve the system of equations*

$$\begin{aligned} \mathbb{E}[\omega|\omega \leqslant mc] &= p \\ F_0(mc) &= \alpha \\ \frac{\varepsilon_\alpha + 1}{\varepsilon_p} &= \frac{mc-p}{p} \end{aligned} \tag{16}$$

The proof is the Appendix on page ??. From (16) note that $mc > p$, which is indication of the platform taking into account network effects, namely the demand expansion for higher $\alpha$.

**Revenue-maximizing platform.** When the platform in the stage of growth, it wants to be bigger. Later on, the platform turn to monetization. We are interested in answering the following question: How does the platform policies change when it is profit-maximizing?

To get a cut from workers revenue, the platform uses two prices: price $p$ for buyers and wage $w$ for workers. Additionally, it has disclosure policy as an instrument. The platform's objective is

$$\begin{aligned} &\max_{\mu,p,w} \beta(\alpha,p)\alpha(p-w) \\ \text{s.t.}\quad & AC(\alpha) \leqslant w \end{aligned}$$

and subject to {(4), (5)}.

**Proposition 10.** *Assume workers have no private information. The profit-maximizing disclosure policy has the form*

$$\hat{\mu}(\omega) = \begin{cases} \mathbb{E}[\omega|\omega \leqslant mc], & \alpha \\ \mathbb{E}[\omega|\omega > mc], & 1-\alpha \end{cases}$$

*and $\alpha$, $p$, $w$ and $mc$ solve the system of equations*

$$\begin{aligned}
\mathbb{E}[\omega|\omega \leqslant mc] &= w \\
F_0(mc) &= \alpha \\
\frac{1}{\varepsilon_p} &= \frac{p-w}{p} \\
\frac{\varepsilon_\alpha + 1}{\varepsilon_p} &= \frac{mc-w}{p}
\end{aligned}$$

The next results states how the marketplace variables change after the platform transitions from the match-maximizing mode to the revenue-maximizing mode. Imagine the platform starts with match-maximizing as in Proposition 9. It makes zero revenue there because it takes zero fee on transactions. There are two ways to increase the intermediary's revenue. One is increase buy price, the other one is to decrease the wage. However, decreasing $w$ decreases acceptance rate $\alpha$, so it might be profitable to increase both $p$ and $w$. Depending on the properties of the demand, the platform may find it optimal either to increase both $p$ and $w$, or decrease both $p$ and $w$, or increase $p$ and decrease $w$. The next result provides sufficient conditions for when the acceptance rate is higher (lower) under profit-maximization than under match-maximization.

**Conjecture 1.** *If $\beta_p + \alpha\beta_{\alpha p} > 0$, $\beta_{pp} > 0$, then $\alpha^{PM} < \alpha^{MM}$ and $w^{PM} < p^{MM}$. If $\beta_{pp} < 0$, $\beta_{\alpha p} < 0$, then $\alpha^{PM} > \alpha^{MM}$ and $w^{PM} > p^{MM}$.*

# 5 Conclusion

# Appendix

# A Lemmas

The next lemma solves for optimal cutoff in worker problem (4)-(5) and $V$, which we call worker profit.

**Lemma 4.** *The optimal cutoff $c^*$ is found from*

$$p - c^* = \tau\beta_A \int_0^{c^*} (c^* - c)dF(c). \tag{17}$$

*worker accepts all $c \leqslant c^*$ and rejects all $c > c^*$.*[10]

*worker profit is*

$$V = \frac{p - c^*}{\tau}. \tag{18}$$

Before going into formal proof, let me give intuition behind (17): When faced with a task with cost $c^*$, over the next time interval $\tau$, the worker can: either accept the task and earn $p - c^*$; or skip the task and receive on average $\beta_A \tau$ new requests, each of them having option value $\int_0^{c^*}(c^* - c)dF(c)$. At $c^*$, the worker is indifferent between taking and skipping.

*Proof.* From (5), there is cutoff $c^*$ such that $V_1(c) = p - c - \tau V$ if $c < c^*$ and $V_1(c) = 0$ if $c > c^*$. Note that $c^*$need not be in the support of $F$, this is just a cutoff value. Therefore,

$$p - c^* = V\tau. \tag{19}$$

We find

$$\mathbb{E}[V_1(c)] = \int_0^{c^*} (c^* - c)dF(c).$$

Plug in into (4) to obtain:

$$V = \beta_A \int_0^{c^*} (c^* - c)dF(c). \tag{20}$$

Combine (19) and (20) to obtain the desired results. □

*Note.* I consider time average payoff rather than discounted sum because discount rate is not essential for my tradeoffs. However, the results generalize to the case when the worker has discount rate $\rho$ by changing $\tau$ to $\tau_\rho = \frac{1 - e^{-\rho\tau}}{\rho}$.

The next two results provide comparative statics on parameters. We find that the worker's utilization rate $\alpha$ is higher when each job is more profitable (higher price $p$) and takes less time to complete (smaller $\tau$). Importantly, the utilization rate is lower when buyer arrival rate is higher (high $\beta_A$). The result also shows that as the platform conflates more, the workers become worse off. However, the effect of information coarsening on $\alpha$ is ambiguous (see **??**).

[10] When $F$ is discrete and takes values $c_1 < c_2 < ... < c_n$ with probabilities $f(c_i)$, you should read formula (17) as $p - c^* = \tau\beta_A \sum_{c_j \leqslant c^*}(c^* - c_j)f(c_j)$.

**Lemma 5.** *$c^*$ is increasing in $p$, and is decreasing in $\beta_A$ and $\tau$. Worker's payoff is increasing in $p$ and $\beta_A$, and is decreasing in $\tau$. Furthermore, let $F_1$ be a mean-preserving spread of $F_2$. Then $c^*(F_1) \leqslant c^*(F_2)$, and worker's payoff is weakly greater under $F_1$ than under $F_2$.*

*Proof of lemma Lemma 5.* Comparative statics with respect to $p, \beta_A$ and $\tau$ is straightforward.

Denote $c_1 := c^*(F_1)$ and $c_2 := c^*(F_2)$. Suppose towards contradiction that $c_1 > c_2$. We have that

$$p - c_2 > p - c_1 = \tau\beta_A \int_0^\infty (c_1 - c)^+ dF_1(c) \geqslant \tau\beta_A \int_0^\infty (c_2 - c)^+ dF_1(c) \geqslant$$
$$\geqslant \tau\beta_A \int_0^\infty (c_2 - c)^+ dF_2(c) = p - c_2.$$

A contradiction. The last inequality follows from the fact that $(c_2 - c)^+ \equiv \max\{c_2 - c, 0\}$ is a convex function of $c$ and $F_2$ second-order stochastically dominates $F_1$. □

Denote by $n_j$ the average number of matches worker $j$ completes per unit of time. Denote by $N$ the measure of matches all workers complete per unit of time.

**Lemma 6.** *$N$ is increasing in $\beta$ if*

1. $$\tau \frac{n_j}{F_j(c')} \frac{f_j(c')}{F_j(c')} W_j(c') \leqslant 1 \quad \forall c', \forall j,$$
   *where $W_j(c') = \int_0^{c'} F_j(c)dc$.*
2. *$G$ is not too dispersed, namely $var_G \left\{ \frac{dn_j}{d\beta} \right\} var_G \{\lambda_j\} \leqslant 1$.*

*Moreover, if all workers are identical, $N$ is increasing in $\beta$ if and only if $\tau\beta \frac{f(c^*)}{F(c^*)} W(c^*) \leqslant 1$, where $c^*$ is the optimal cutoff from (17).*

An interesting case comes out from the case of identical workers when the condition is violated. If the condition is violated, then the platform can increase the flow of matches by <u>contracting</u> the demand.

*Proof.* First, worker $j$'s profit flow $V_j$ can be expressed as

$$V_j = (p_j - AC_j(\alpha_j))n_j,$$

where $n_j$ is the average number of matches, and $p_j - AC_j(\alpha_j)$ is the average markup. Using (19), find that

$$\begin{aligned} \frac{p_j - c_j^*}{\tau} &= (p_j - AC_j(\alpha_j))n_j, \\ \frac{p - c^*}{\tau(p - \mathbb{E}[c|c < c^*])} &= n_j. \end{aligned}$$

Second, I find the derivative of $n_j$ with respect to $\alpha_j$. Start by finding the following derivative:

$$\frac{d}{dc_j^*}\left(\mathbb{E}[c|c<c_j^*]\right) = \frac{d}{dc_j^*}\left(\frac{\int_0^{c_j^*} c\,dF_j(c)}{F_j(c_j^*)}\right) = \frac{f_j(c_j^*)}{F_j(c_j^*)^2}\int_0^{c_j^*}(c_j^*-c)dF_j(c).$$

Now differentiate $n_j$ wrt $c_j^*$:

$$\begin{aligned}
\frac{dn_j}{dc_j^*} &\propto -1(p-\mathbb{E}[c|c<c_j^*]) + (p-c_j^*)\frac{f_j(c_j^*)}{F_j(c_j^*)^2}\int_0^{c_j^*}(c_j^*-c)dF_j(c) \\
&\propto -1+\tau n_j\frac{f_j(c_j^*)}{F_j(c_j^*)^2}\int_0^{c_j^*}(c_j^*-c)dF_j(c) = -1+\frac{\tau n_j}{F_j(c_j^*)}\frac{f_j(c_j^*)}{F_j(c_j^*)}W_j(c_j^*).
\end{aligned}$$

$$\frac{dn_j}{dc^*} = \frac{-1+\frac{\tau n_j}{F_j(c_j^*)}\frac{f_j(c_j^*)}{F_j(c_j^*)}W_j(c_j^*)}{\tau(p-\mathbb{E}[c|c<c^*])}$$

$$\frac{dn_j}{d\alpha_j} = \frac{dn_j}{f(c_j^*)dc_j^*} = \frac{-1+\frac{\tau n_j}{F_j(c_j^*)}\frac{f_j(c_j^*)}{F_j(c_j^*)}W_j(c_j^*)}{f(c^*)\tau(p-\mathbb{E}[c|c<c^*])}.$$

Third, the flow balance condition (7) can be rewritten as

$$\int \frac{n_j}{\alpha_j}dG(j) = \beta.$$

Differentiate wrt $\beta$:

$$\begin{aligned}
\int\left[\frac{dn_j}{d\beta}\frac{1}{\alpha_j} - \frac{d\alpha_j}{d\beta}\frac{n_j}{\alpha_j}\right]dG(j) &= 1 \\
\int\frac{dn_j}{d\beta}\left[\frac{1}{\alpha_j} - \frac{d\alpha_j}{dn_j}\frac{n_j}{\alpha_j}\right]dG(j) &= 1
\end{aligned}$$

I will show now that for any $j$,

$$\frac{1}{\alpha_j} - \frac{d\alpha_j}{dn_j}\frac{n_j}{\alpha_j} \geqslant 0$$

$$\frac{1}{\alpha_j} - \frac{n_j}{\alpha_j^2}\frac{f(c^*)\tau(p-\mathbb{E}[c|c<c^*])}{-1+\frac{\tau n_j}{F_j(c_j^*)}\frac{f_j(c_j^*)}{F_j(c_j^*)}W_j(c_j^*)} \geqslant 0$$

$$\frac{-\alpha_j+\tau n_j\frac{f_j(c_j^*)}{F_j(c_j^*)}W_j(c_j^*) - n_j f(c^*)\tau(p-\mathbb{E}[c|c<c^*])}{-1+\frac{\tau n_j}{F_j(c_j^*)}\frac{f_j(c_j^*)}{F_j(c_j^*)}W_j(c_j^*)} \geqslant 0$$

$$\frac{-\alpha_j - n_j f(c_j^*)\tau(p-c_j^*)}{-1+\frac{\tau n_j}{F_j(c_j^*)}\frac{f_j(c_j^*)}{F_j(c_j^*)}W_j(c_j^*)} \geqslant 0$$

The numerator is non-positive because price is above the marginal cost. The denominator is negative by the assumption. Therefore, the inequality holds. Finally, this implies that

$$\frac{dN}{d\beta} = \int \frac{dn_j}{d\beta} dG(j) \geqslant 0.$$

$\square$

**Lemma 7.** *For any vector $\alpha(\theta)$, there exists a unique vector solution $x(\theta)$ to (2).*

*Proof.* Denote $x = \sum_{\theta'} x(\theta')$ the mass of all available workers. Rewrite (2) as

$$\tau\beta(\theta, \alpha(\theta), p_\theta, x(\theta)/x)\alpha(\theta) + x(\theta) = g(\theta). \tag{21}$$

Denote the left-hand side of the equation by $\mathbf{A}(\mathbf{x})$, where $\mathbf{x} = (x(\theta))_{\theta\in\Theta}$ and $\mathbf{A}$ is a correspondence that maps $\mathbf{x}$ into $\mathbb{R}^{dim\Theta}$. First we show that the solution to $\mathbf{A}(\mathbf{x}) = \mathbf{g}$ exists, then we show that it is unique.

For existence, consider a mapping $T$ defined as

$$T_i(\mathbf{x}) = g_i - \tau\beta(\theta_i, \alpha(\theta_i), p_{\theta_i}, x(\theta_i)/x)\alpha(\theta_i).$$

Define $X = \{(x_1, ...x_{\dim\Theta})\colon 0 \leqslant x_i \leqslant g_i\}$. We will show that mapping $T$ satisfies the conditions of Brouwer's fixed point theorem. First, $T$ maps $X$ to itself. Indeed, $T_i(\mathbf{x}) \leqslant g_i$. But also

$$T_i(\mathbf{x}) \geqslant g_i - \tau\beta(\theta_i, 1, p_{\theta_i}, 1) > 0.$$

The first inequality uses Assumption 3 and demand monotonicity in $\alpha$. The second inequality uses Assumption 2. Second, $X$ is compact. Finally, $T$ is continuous. Therefore, Brouwer's theorem ensures the existence of a fixed point, which is a solution to (21).

In the rest of the proof we prove uniqueness. We will show that the principal minors of Jacobian matrix of $\mathbf{A}$ are non-negative everywhere. Then by Gale-Nikaido (1965), the mapping is one-to-one and hence $\mathbf{A}(\mathbf{x}) = \mathbf{g}$ has a unique solution.

$$J = \begin{bmatrix} \frac{\partial A_1}{\partial x_1} & \frac{\partial A_1}{\partial x_2} & \cdots \\ \frac{\partial A_2}{\partial x_1} & \frac{\partial A_2}{\partial x_2} & \\ \vdots & & \ddots \end{bmatrix},$$

where we use a simplified notation $x_1 := x(\theta_1)$. Use notation $\nu_1 = x_1/x$.

$$\begin{aligned}\frac{\partial A_1}{\partial x_1} &= \tau\frac{d\beta(\theta_1, \alpha(\theta_1), p_{\theta_1}, x(\theta_1)/x)\alpha(\theta_1)}{d\nu_1}\frac{d(x(\theta_1)/x)}{dx_1} + 1 = \\ &= \tau\frac{d\beta(\theta_1, \alpha(\theta_1), p_{\theta_1}, x(\theta_1)/x)\alpha(\theta_1)}{d\nu_1}\frac{x - x_1}{x^2} + 1.\end{aligned}$$

Denote for brevity $\beta_1 = \frac{d\beta(\theta_1,\alpha(\theta_1),p_{\theta_1},x(\theta_1)/x)\alpha(\theta_1)}{d\nu_1}$. Then

$$\frac{\partial A_1}{\partial x_1} = \frac{\tau}{x}\beta_1(1 - \nu_1) + 1.$$

Similary find

$$\frac{\partial A_1}{\partial x_2} = -\frac{\tau}{x}\beta_1\nu_1.$$

We need to show that all principle minors of matrix $J$ are non-negative. Without loss of generality, consider the leading principal minor of size $K = 1, ..., \dim \Theta$, denote it by $J_K$. It can be rewritten as follows

$$J_K = \begin{bmatrix} 1+\frac{\tau}{x}\beta_1 & ... & 0 \\ \vdots & \ddots & \\ 0 & & 1+\frac{\tau}{x}\beta_K \end{bmatrix} + \frac{\tau}{x}\begin{bmatrix} -\beta_1\nu_1 \\ \vdots \\ -\beta_2\nu_2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.$$

Now I will use the following formula for the determinants:

$$\det(X + AB) = \det X \det(I + BX^{-1}A).$$

In case of matrix $J_K$, we obtain:

$$\det J_K = \det \begin{bmatrix} 1+\frac{\tau}{x}\beta_1 & ... & 0 \\ \vdots & \ddots & \\ 0 & & 1+\frac{\tau}{x}\beta_K \end{bmatrix} \det(1 + \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{\tau}{x}\beta_1 & ... & 0 \\ \vdots & \ddots & \\ 0 & & 1+\frac{\tau}{x}\beta_K \end{bmatrix}^{-1} \frac{\tau}{x}\begin{bmatrix} -\beta_1\nu_1 \\ \vdots \\ -\beta_2\nu_2 \end{bmatrix}).$$

We need to show that $\det J_K \geqslant 0$ for all $\mathbf{x}$. It is sufficient to the second determinant on the right hand side of the formula above. We need to show

$$1 + \frac{\tau}{x}\sum_{i=1}^{K}\frac{-\beta_i\nu_i}{1+\frac{\tau}{x}\beta_i} \geqslant 0.$$

First observe that $\frac{\beta_i\tau/x}{\beta_i\tau/x+1} \leqslant 1$, and then

$$1 + \frac{\tau}{x}\sum_{i=1}^{K}\frac{-\beta_i\nu_i}{1+\frac{\tau}{x}\beta_i} \geqslant 1 - \sum_{i=1}^{K}\nu_i \geqslant 0,$$

because $\sum_{i=1}^{\dim\Theta}\nu_i = 1$. This proves that all principal minors of $J$ are non-negative, and by Gale-Nikaido (1965), $\mathbf{A}(\mathbf{x}) = \mathbf{g}$ has a unique solutions. □

# B Proofs omitted from the main text

*Proof of Proposition 1.* We need to show that there is a unique solution $(\alpha(\theta), \beta_A(\theta))_{\theta\in\Theta}$ to $\{(6), (\textbf{??})\}$

Use Kakutani theorem. □

*Proof of Proposition 7.* We need to show that there is a unique solution $(\alpha(\theta), \beta_A(\theta))_{\theta\in\Theta}$ to $\{(6), (15)\}$, where we treat $x$ as a function of $\alpha$, as has been determined in lemma Lemma 7.

Use Kakutani theorem. □

*Proof of theorem Theorem 1.* Step 1. $M$ and $\beta_A$ are positively related. Indeed, when the mass of available workers is $X$, the flow of matches is

$$M = \frac{1-X}{\tau}.$$

The buyer arrival rate on available workers is $\beta_A = \beta/X$. Therefore,

$$M = \frac{1-\beta/\beta_A}{\tau}.$$

We are interested in the sign of $\delta M/\delta W$, therefore we will find $\delta\beta_A/\delta W$.

Step 2. The equilibrium values of $\alpha(\theta)$ and $\beta_A$ are found from the system of equations (10) and (7), which we reproduce here:

$$p + \theta - \hat{m}(\theta) = \tau\beta_A W(\hat{m}(\theta)), \quad \forall\theta \in \Theta; \tag{22}$$

$$\int \frac{dG(\theta)}{\tau\alpha(\theta) + 1/\beta_A} = \beta. \tag{23}$$

Take the differential of (23):

$$-\int \frac{dG(\theta)\left(\tau\delta\alpha(\theta) - \delta\beta_A/\beta_A^2\right)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2} = 0.$$

$$\frac{\delta\beta_A}{\beta_A^2}\int \frac{dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2} - \tau\int \frac{\delta\alpha(\theta)dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2} = 0.$$

$$\delta\beta_A = \frac{\tau\int \frac{\delta\alpha(\theta)dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2}}{\frac{1}{\beta_A^2}\int \frac{dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2}}. \tag{24}$$

Now use the fact that

$$\alpha(\theta) = W'(\hat{m}(\theta)+) \tag{25}$$

to find the differential

$$\delta\alpha(\theta) = \delta W'(\hat{m}(\theta)) + W''(\hat{m}(\theta))\delta\hat{m}(\theta). \tag{26}$$

Take the differential of (22):

$$-\delta\hat{m}(\theta) = \tau\delta\beta_A W(\hat{m}(\theta)) + \tau\beta_A W'(\hat{m}(\theta))\delta\hat{m}(\theta) + \tau\beta_A\delta W(\hat{m}(\theta))$$

$$-\delta\hat{m}(\theta)(1 + \tau\beta_A\alpha(\theta)) = \tau\delta\beta_A W(\hat{m}(\theta)) + \tau\beta_A\delta W(\hat{m}(\theta)). \tag{27}$$

Plug in (27) into (26) and then into (24):

$$\begin{aligned}
\frac{\delta\beta_A}{\beta_A^2}\int \frac{dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2} &= \tau\int \frac{\delta\alpha(\theta)dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2} = \\
&= \tau\int \frac{\left(\delta W'(\hat{m}(\theta)) + W''(\hat{m}(\theta))\delta\hat{m}(\theta)\right)dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2} = \\
= \tau\int \frac{\left(\delta W'(\hat{m}(\theta))\right)dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^2} &- \tau\int \frac{W''(\hat{m}(\theta))\left(\tau\delta\beta_A W(\hat{m}(\theta)) + \tau\beta_A\delta W(\hat{m}(\theta))\right)dG(\theta)}{\left(\tau\alpha(\theta)+1/\beta_A\right)^3\beta_A}
\end{aligned}$$

$$\delta\beta_A\left[\int \frac{dG(\theta)}{(\tau\alpha(\theta)\beta_A+1)^2}+\tau^2\int\frac{W''(\hat m(\theta))\beta_A^2 W(\hat m(\theta))dG(\theta)}{(\tau\alpha(\theta)\beta_A+1)^3}\right]=$$
$$=\tau\int\frac{\delta W'(\hat m(\theta))dG(\theta)}{(\tau\alpha(\theta)+1/\beta_A)^2}-\tau^2\int\frac{W''(\hat m(\theta))\delta W(\hat m(\theta))dG(\theta)}{(\tau\alpha(\theta)+1/\beta_A)^3}$$

We have both $\delta W$ and $(\delta W)'$ on the right hand side of the expression above, and we need to have only $\delta W$ . To get it, we will use integration by parts. Consider separately the first term in the previous line:

$$Z_1 := \tau\int\frac{\delta W'(\hat m(\theta))dG(\theta)}{(\tau\alpha(\theta)+1/\beta_A)^2}=\tau\beta_A^2\int\frac{(\delta W(\hat m(\theta)))'\,g(\theta)d\theta}{(\tau\alpha(\theta)\beta_A+1)^2}=\tau\beta_A^2\int\frac{g(\theta)d\,(\delta W(\hat m(\theta)))}{(\tau\alpha(\theta)\beta_A+1)^2}=$$
$$=\tau\beta_A^2\frac{g(\theta)\delta W(\hat m(\theta))}{(\tau\alpha(\theta)\beta_A+1)^2}\Big|_{\underline{\theta}}^{\overline{\theta}}-\tau\beta_A^2\int\delta W(\hat m(\theta))d\left(\frac{g(\theta)}{(\tau\alpha(\theta)\beta_A+1)^2}\right)=$$
$$=-\tau\beta_A^2\int\delta W(\hat m(\theta))d\left(\frac{g(\theta)}{(\tau\alpha(\theta)\beta_A+1)^2}\right)=$$
$$=-\tau\beta_A^2\int\delta W(\hat m(\theta))\frac{g'(\theta)\left(\tau\alpha(\theta)\beta_A+1\right)^2-2g(\theta)\left(\tau\alpha(\theta)\beta_A+1\right)\tau\beta_A\alpha'(\theta)}{(\tau\alpha(\theta)\beta_A+1)^4}d\theta=$$
$$=\tau\beta_A^2\int\delta W(\hat m(\theta))\left[-\frac{g'(\theta)}{(\tau\alpha(\theta)\beta_A+1)^2}+\frac{2g(\theta)\tau\beta_A\alpha'(\theta)}{(\tau\alpha(\theta)\beta_A+1)^3}\right]d\theta$$

Using (25),

$$\alpha'(\theta)=W''(\hat m(\theta))\hat m'(\theta).$$

Differentiate (22) wrt $\theta$,

$$1-\hat m'(\theta)=\tau\beta_A\alpha(\theta)\hat m'(\theta)$$
$$\hat m'(\theta)=\frac{1}{1+\tau\beta_A\alpha(\theta)}.$$

For brevity, adopt the notation

$$\nu(\theta):=\frac{1}{1+\tau\beta_A\alpha(\theta)}.$$

The notation is not coincidental, $\nu(\theta)$ is equal to the fraction of type-$\theta$ workers who are available in equilibrium. Return to compute

$$Z_1=\tau\beta_A^2\int\delta W(\hat m(\theta))\left[-g'(\theta)\nu(\theta)^2+\nu(\theta)^3 2g(\theta)\tau\beta_A W''\nu\right]d\theta.$$

Now we finish computing the differential $\delta\beta_A$:

$$\frac{\delta\beta_A}{\beta_A^2}\int\frac{dG(\theta)}{(\tau\alpha(\theta)+1/\beta_A)^2}=$$
$$=\tau\beta_A^2\int\delta W(\hat m(\theta))\left[-g'(\theta)\nu(\theta)^2+\nu(\theta)^3 2g(\theta)\tau\beta_A W''\nu\right]d\theta-\tau^2\int W''(\hat m(\theta))\delta W(\hat m(\theta))\nu^3\beta_A^3 dG(\theta)=$$
$$=\tau\int\delta W(\hat m(\theta))\left[-g'(\theta)\nu(\theta)^2\beta_A^2+\nu(\theta)^4 2g(\theta)\tau\beta_A W''\beta_A^2-\tau W''(\hat m(\theta))\nu^3 g(\theta)\beta_A^3\right]d\theta.$$

We are interested in the expression in the square brackets in the formula above. Therefore, for some $K > 0$ that is constant in $\theta$ we have:

$$\begin{aligned}\frac{\delta\beta_A}{\delta W} &= K\left(-g'(\theta)\nu(\theta)^2 + \nu(\theta)^4 2g(\theta)\tau\beta_A W'' - \tau W''(\hat{m}(\theta))\nu^3 g(\theta)\beta_A\right) = \\ &= K\nu(\theta)^2\left(-g'(\theta) + g(\theta)\nu(\theta)\tau\beta_A W''(\hat{m}(\theta))(2\nu(\theta)-1)\right).\end{aligned}$$

□

# C Technical Extensions

## C.1 No Excess Demand Assumption relaxed

Allow for the case when $\beta(1)\tau \geqslant 1$. If $\beta(1)\tau > 1$, then workers get overwhelmed by the buyer requests and can't respond to all of them to the extent that they can't even reject them. To cover this situation we assume that if there are no available workers to reject a pending buyer request, the platform rejects it automatically.

Since some requests can be rejected by the platform, the acceptance rate as perceived by buyers does not coincide with the acceptance rate $\alpha$ generated by workers. Denote by $\alpha^e$ the effective acceptance rate that buyers face. Let at some moment of time there is $x \in [0,1]$ mass of available workers, and let buyers arrive to the platform at rate $\beta$. Then within the next time interval $dt$, there are $\beta dt$ new request, and $x + \left(\frac{1-x}{\tau}dt\right)$ available workers. What is $\alpha^e$ when workers use acceptance rate $\alpha$? Consider three cases.

1. $x > 0$. There is plenty of available workers, $x + \left(\frac{1-x}{\tau}dt\right) > \beta dt$. Fraction $\alpha$ of buyers are accepted, therefore $\alpha^e = \alpha$.

2. $x = 0$ and $\alpha\beta < \frac{1}{\tau}$. There are few workers that just became available but in sufficient number to process all buyers. In the same fashion as in case 1, $\alpha^e = \alpha$.

3. $x = 0$ but $\alpha\beta \geqslant \frac{1}{\tau}$. Not sufficient workers to process all buyers, some buyers are rejected by the platform. The number of accepted tasks is $\frac{1}{\tau}dt$. The acceptance rate is therefore $\alpha^e = \frac{1/\tau}{\beta}$.

Combining all there cases, we have that

$$\alpha^e = \min\{\alpha, \frac{1}{\tau\beta}\}.$$

The adjusted definition of equilibrium is then the following.

1. $$\alpha \in [F(c^*(\beta_A)-), F(c^*(\beta_A)+)].$$

2. $$\beta_A = \frac{\beta(\alpha^e)}{1-\beta(\alpha^e)\alpha^e\tau} = \begin{cases} \frac{\beta(\alpha)}{1-\beta(\alpha)\alpha\tau}, & \alpha\beta(\alpha)\tau < 1 \\ +\infty, & \alpha\beta(\alpha)\tau \geqslant 1 \end{cases}$$

$\beta_A = +\infty$ reflects the fact that when the demand is overwhelming, buyers line up for workers so workers start a new task immediately after they finish the previous one. The No Excess Demand assumption makes sure that this never happens, $\alpha\beta(\alpha)\tau < 1$ for all $\alpha \in [0, 1]$.

The next result shows that in the equilibrium there are no lines. By this reason for the clarity of exposition we decided to restrict the analysis to the case of no lines to begin with.

*Claim.* In equilibrium, $\beta_A < \infty$.

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