Ignorance is Strength: Improving Performance of Matching Markets by Limiting Information*

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Abstract

This paper develops a model for studying the problem of information disclosure faced by a platform matching buyers and sellers. Buyers search for sellers and are time-sensitive, while sellers have limited capacity for serving buyers and derive heterogeneous payoffs from being matched with different buyers. The platform controls the information the sellers observe about the buyers before forming a match. I show that full information disclosure is inefficient because of excessive rejections by the sellers. When the platform observes the sellers' payoff function, it can restore the full efficiency using a coarse disclosure policy, which recommends to each seller an action. When seller preferences are unknown to the platform, I characterize the disclosure policy that maximizes the total welfare. Tighter capacity constraints or higher buyer-to-seller ratio require coarser disclosure. In a linear payoff environment with uniform distribution of seller attributes, the efficient disclosure is *upper-coarsening*. For general distribution of seller attributes, I develop an approach to solving the disclosure problem with heterogeneous and forward-looking sellers.

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1 Introduction

Information provision about goods or trading partners is important for the efficiency of platforms that match buyers and sellers of products or services, such as platforms for house rental, labor contracting and transportation. Providing more information allows participants to identify and pursue the most valuable matches, and online platforms spend significant resources eliciting match-relevant information from users. However, sometimes this information is not fully revealed to the users. For example, Uber currently does not show drivers the passenger's destination until after the driver has accepted the ride, even once the passenger entered it into the application. Airbnb hosts who turn on the "Instant Book" feature commit to accepting all requests without knowing the details of prospective guests. Why do such platforms choose not to disclose all the relevant information fully? What is the optimal information intermediation policy for platforms that care about both sides of the market? What does it depend on?

I study the information intermediation problem in the context of the following model. Buyers seek sellers' services to complete their jobs. Buyers contact sellers, who then review buyer attributes and choose whether to accept or reject the buyer job. Buyers are short-lived, must match quickly, but are indifferent about which seller completes their task. Sellers are long-lived, are indifferent about the match speed, but have heterogeneous preferences over buyers. In particular, not all jobs are profitable. Sellers have limited capacity for serving buyers. If a seller accepts a job, he becomes unavailable for a fixed period of time and cannot accept new jobs. The platform designs an information disclosure policy that governs which buyer attributes are disclosed to sellers before deciding on buyer requests. I consider the general platform's objective of maximizing the weighted average of buyer surplus and seller profits. Note that in this model, buyers are sellers are asymmetric, which focuses the information disclosure problem on the seller side of the market. If buyers have preferences about a match, the disclosure problem would be two-sided.

In this model, a key tradeoff arises between match quality and match rate. Full information disclosure need not be optimal because information disclosure can negatively affect the match rate overpassing the positive effect on the match quality. Seller profit is a product of match quality and match rate, and buyer surplus is an increasing function of the match rate. Seller profit function does not align with the platform objective. Specifically, profit-maximizing sellers will *cream-skim* and reject inefficiently many jobs. The platform, unlike sellers, cares about welfare on both sides of the market, and so efficient disclosure is coarse if the negative effects of information disclosure on the match rate are strong. This paper considers how the optimal disclosure policy depends on the details of the seller side of the market.

Efficiency requires that both match quality and match rate be high. Seller profit is larger when both are high, whereas buyer surplus is large when the match rate is high. Yet, match quality and match rate are in conflict. Enforcing higher match rates compromises quality because sellers are forced to accept more inferior jobs. Similarly, allowing sellers to cherrypick the most valuable buyers leads to higher rejection and lower match rates. For example, if Uber drivers reject passenger requests more often, passengers will have longer wait times. On Airbnb, if hosts reject guest inquiries more frequently, guests must spend more time searching.

Optimal disclosure policy must balance three effects: the positive effect on the seller match quality and the negative static and dynamic effects on the match rate. First, the effect of disclosure on the seller match quality is straightforward. From a seller's point of view, more information increases his set of attainable payoffs. Holding the match rate fixed, he benefits from more information about buyers. Second, information disclosure reduces the platform's ability to induce sellers to accept jobs. A key observation is that the platform can increase a seller's expected marginal profit by limiting the information revealed to him. To see why, note that pooling marginally rejected jobs (rejected because the seller would prefer to wait for a better job) with inframarginal (accepted) jobs alters a seller's expected marginal profit.. Ignoring dynamic effects, higher marginal profit induces sellers to accept more jobs, which leads to higher match rates. Third, making more information available to a seller increases returns to his search. Indeed, since acceptance precludes further search, the opportunity cost of accepting any offer is higher. As a result, sellers reject more often, and the match rate goes down. This dynamic effect on match rate exists only when sellers have limited capacity and are forward-looking. The match quality effect gives a motive for the platform to disclose information, while the match rate effects give the motive to limit information.

If sellers are identical, then some information coarsening is always optimal. In fact, such coarsening is necessary whether the platform maximizes total welfare, buyer surplus, or the joint seller surplus (see Figure 2). The total welfare maximizing disclosure policy is coarse but has a simple form. It sends one of two recommendations to sellers, "accept" or "reject", which are chosen such that the sellers have incentives to follow them. The welfare maximizing policy is the intermediate case between the buyer-optimal disclosure and seller-optimal disclosure. To maximize the buyer surplus, coarsening is necessary because sellers underweight the match rate relative to buyers in their payoff functions. The higher are buyer search costs (and thus the costlier are rejections), the coarser is optimal disclosure. Perhaps more surprisingly, when maximizing the joint seller profits, the optimal disclosure is also coarse.

The adverse effect of disclosure on joint seller profits is a form of seller coordination failure. In a marketplace where sellers act independently, each seller keeps his schedule open by rejecting low-value jobs to increase his own chances of getting high-value jobs. As a result, sellers spend significant time waiting for high-value jobs. Collectively, this behavior is suboptimal because all profitable jobs have to be completed to maximize the joint profits. I attribute the source of the coordination failure to what I call the *cream-skimming externality*: By rejecting a job, a seller remains available on the marketplace and attracts a fraction of subsequent buyers, who otherwise would move to other sellers. As a result, the other sellers face fewer profitable jobs and obtain lower profits. The cream-skimming externality arises only when sellers have limited capacity and are forward-looking. When this externality is present, sellers underestimate the effect of their own acceptance rate on the platform-wide match rate and collectively resolve the match quality-match rate tradeoff sub-optimally. Coarsening information decreases opportunity cost of rejecting and can increase the match rate to the seller-optimal level.

When sellers are unobservably heterogeneous, the optimal disclosure is finer than in the case of identical sellers. With heterogeneous sellers, the match quality effect of information disclosure is stronger, but there are important subtleties. Coarse disclosure tailored to increase one seller's acceptance rate can drive another seller's average profit below zero and violate his individual rationality constraint. If any coarsening is optimal, the degree is now not obvious. The optimal policy should accommodate the possibly opposite reactions of sellers to disclosure and will depend on the shape of the seller type distribution.

To understand how seller heterogeneity affects optimal disclosure policy, I study a linear payoff environment, with vertically differentiated buyers and sellers. In this case, the optimal disclosure policy depends on the shape of seller type distribution, the intensity of buyer traffic, and the tightness of seller capacity constraints. In this case, the seller match payoff is linear in buyer and seller characteristics, and buyer match payoff is constant. I first consider the case of uniform distribution of seller types, where I can fully characterize the optimal disclosure policy. A key result with a uniform distribution of sellers, is that the match-rate maximizing disclosure policy is *upper-coarsening*: high buyer types are pooled and low buyer types are revealed truthfully. This is in stark contrast with the case of unconstrained sellers, in which information disclosure does not affect the match rate (cf. Kolotilin *et al.* (2015)). When buyer-to-seller ratio is high or when the sellers are more capacity constrained, the efficient disclosure is also upper-coarsening. In the converse case, the full disclosure is efficient.

Turning to a general (non-uniform) distribution of sellers, I find that optimal disclosures can have a variety of qualitatively different shapes depending on the distribution. The heuristic in the case of unconstrained sellers is to pool buyer type on the increasing part of buyer probability density function g, and reveal buyer type on the decreasing part of g. With capacity constrained sellers, this heuristic should be further qualified with seller utilization rates. Despite the complexity I find the first-order condition for the general case in Lemma 3.

The model is mainly motivated by the matching problems of digital marketplaces. To use Airbnb as an example again, guests (buyers) are differentiated by age, gender, race, personality, etc. Hosts (sellers) have preferences over the number of guests, their gender, race, lifestyle, etc. While guests prefer to minimize time spent searching and book a listing instantly, hosts want to avoid offensive or inconvenient guests. Airbnb introduced Instant-Book feature to satisfy the guests' demand for convenience. In my model, it corresponds to the no disclosure policy. However, one could imagine a finer tool that allowed a host to specify guest types that were permitted to use the InstantBook. The problem of optimal guest segments is equivalent to the problem of the optimal information disclosure, and, as argued above, has important tradeoffs. Uber's matching system is another notable example. Uber directs passenger (buyer) requests to drivers (sellers), and the requests include information about the passenger. In the current version of UberX, the passenger's destination, though relevant to the driver payoff, is not shown. One final example is on-demand labor platforms, such as TaskRabbit. On this platform, freelancers (sellers) commit to an hourly rate over a broad category of tasks, such as Moving. The problem of the optimal category breadth is equivalent to the problem of the optimal disclosure policy of client task characteristics, and can also be analyzed with the framework outlined.

There are two primary contributions of this paper relative to the existing literature. First, the paper considers the role of information intermediation in matching markets. It shows that strategic information disclosure can be an effective tool to balance the match quality and the match speed. When the sides of the market are not symmetric in their preference for match quality and match rate, the more patient and more selective side of the market (sellers in my model) tends to cream-skim. Strategically limiting information can be used to decrease cream-skimming and improve total welfare. Additionally, limiting information can alleviate the seller coordination failure by offsetting the cream-skimming externality.

The paper also contributes to the literature on information design. The paper extends the model of signaling game with heterogeneous audience to the case with endogenously available and dynamically optimizing receivers. In this case, the design of the optimal information disclosure is a non-trivial problem. With forward-looking receivers, information disclosure policy determines not only the receiver's stage payoff but also the distribution of his potential future payoffs. As a result, receiver's decision to accept depends not only on the posterior mean of the state but also on the entire signaling structure. This makes the concavification approach of Kamenica and Gentzkow (2011), as well as the linear programming approach of Kolotilin (2015) unsuitable for the analysis of my model. I approach it by representing signaling structures as a particular class of convex functions and then using the calculus of variations to find the first-order necessary conditions. Section 3.5 sketches the main steps of the approach.

The rest of the paper is organized as follows. The next subsection relates this paper to the existing literature and highlights my contributions. Section 2 introduces the model of matching market, and establishes the existence and uniqueness of equilibrium. Section 2.4 contains the discussion of the key assumptions of the model. Section 3 sets up the platform's information disclosure problem, solves and explains it in different settings. First, Section 3.1 explains the seller coordination failure. Then, Section 3.2 studies the setting with identical sellers, and Section 3.3 discusses the competing effects of information disclosure . Section 3.4 studies the setting with heterogeneous sellers and presents the main characterization result of the paper. Section 3.5 describes the technique for proving the main theoretical result.

2 The Model of Matching Market

In this section, I lay out a model of a matching market that will allow me to evaluate how information disclosure policy affects the equilibrium market outcome. The model has two main components: the matching process between buyers and sellers, and the seller optimization problem. At the end of this section, I define the steady state equilibrium, in which seller actions are individually optimal, and the dynamic matching system is in steady state.

2.1 Setup

Spot matching process. There are three parties involved in the search and matching process: sellers, buyers and the platform itself. Time is continuous.

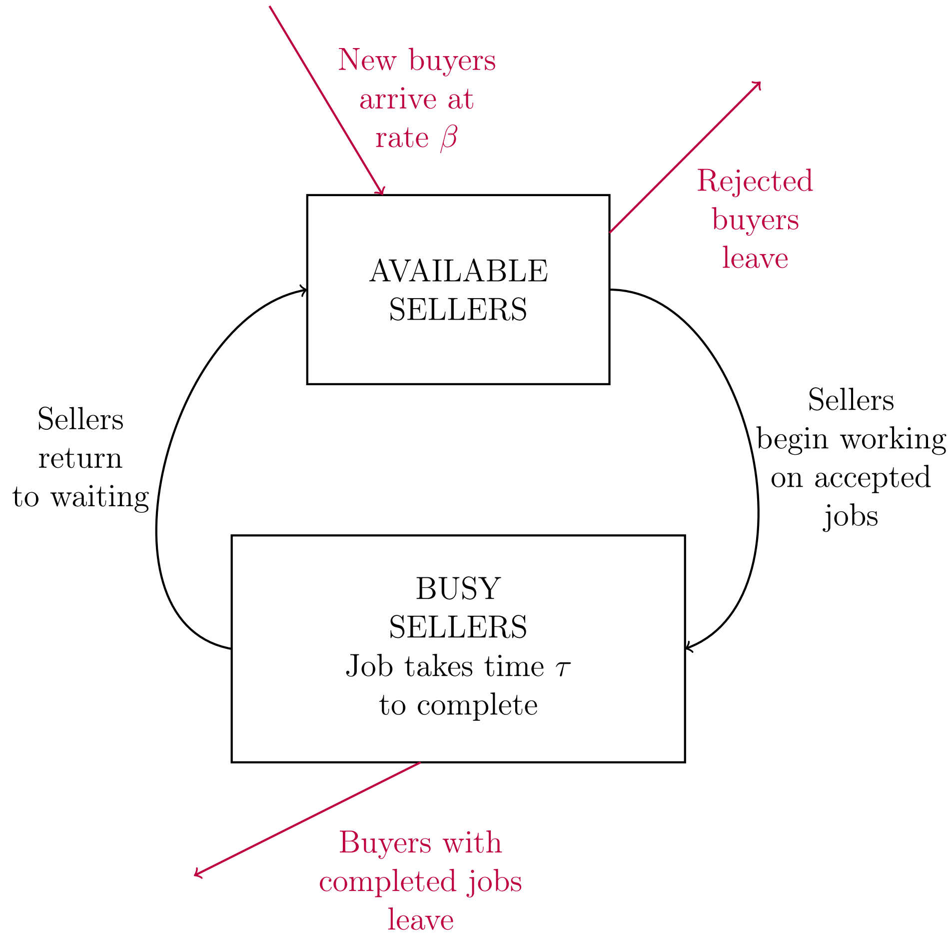
There is mass 1 of sellers, who always stay on the platform, never leave or arrive. The sellers do not actively look for jobs, but instead screen the buyer requests: each worker is presented with a sequence of job offers at Poisson rate, and decides whether to accept or reject them to maximize discounted profit flow. At each moment of time a seller is either available and waits for new jobs, or busy working on a job. An accepted job takes time τ to complete, during which time the seller cannot receive new jobs. This is the main source of matching friction. In what follows, I also refer to τ as the seller capacity constraint because higher τ implies the seller can complete fewer jobs over the same time interval.

There is a continuum of potential buyers who gradually arrive over time at flow rate β: within time interval dt, mass βdt of buyers arrive at the platform. Each new buyer arrives with a single job that he proposes to one of the available sellers. The seller is chosen uniformly at random from the pool of available workers. If the buyer's job is accepted, the buyer stays until his job is completed; otherwise, he leaves the platform. Assume $\beta\tau < 1$ which implies that collectively, it is physically possible for sellers to complete every buyer job. See the detailed discussion of the assumptions on buyer search and buyer arrival rate in Section 2.4. See figure 1a for the illustration of the matching process.

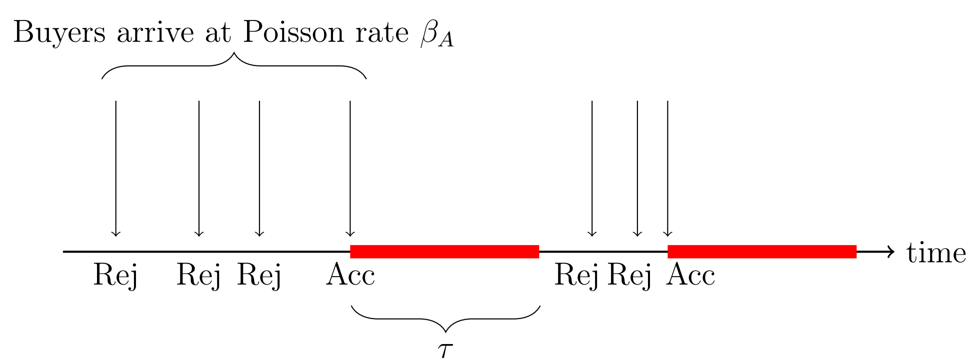
Buyer and seller preference heterogeneity. There are two dimensions of heterogeneity in the market. First, each seller has heterogeneous match payoff across buyers. Second, different sellers have different payoff functions over jobs. Concerning the platform's information disclosure problem, I need the following pieces of notation. Let x be buyer type, with the interpretation that x is buyer characteristics observed by the platform.[1] The space of buyer types X is a compact subset of a Euclidean space. The distribution of x is F with full support. Let y be seller type, with the interpretation that y is seller characteristics unobserved by the platform.[2] The space of seller types Y is a compact subset of a Euclidean

[1]Buyer type x captures the payoff-relevant information the platform elicits from the buyer about the job, passively from the buyer's cookies and queries or actively by asking questions. For example, on Uber, x would include rider's destination; on Airbnb, x would include guest's race, age and gender.

[2]Seller type y captures the payoff-relevant information the platform did not elicit from sellers by whatever reason – costly, unethical, etc. For example, on Uber, y would include the driver's preference for long rides and traffic; on Airbnb, y would include the host's preference for his guest's age, gender, socio-economic status, race, etc.



(a) Spot matching process. Buyers arrive at exogenous rate β, and contact available sellers. If rejected, a buyer leaves the platform. If accepted, the buyer forms a match which lasts for time τ. After the time elapses, the buyer leaves the platform, and the seller returns to waiting.



(b) Seller dynamic optimization problem with screening and waiting. An available seller receives requests at Poisson rate β_A. If a request is accepted, the seller becomes busy for time τ during which he does not receive new requests.

Figure 1: The model of matching market has two main components: the spot matching process, and the seller dynamic optimization problem.

space. The distribution of y is G with full support that admits density g, g is differentiable on Y. Seller profit for one match is $\pi(x, y)$. Assume π is continuous and for any y there is x such that $\pi(x, y) > 0$. Buyer net match payoff is $u(x, y)$. Assume that all incoming buyers have non-negative match payoff:

$$u(x, y) \geqslant 0 \quad \forall x, y. \tag{1}$$

Platform: Information intermediation. Before the matching process starts running, the platform designs and commits to a disclosure policy that governs which buyer characteristics are disclosed to sellers. The platform observes buyer type x and sends a signal about x to the seller. The seller does not receive any additional information about x beside the platform's signal. Let $S = \Delta(X)$ be the set of all posterior distributions over X. *Information disclosure policy* $\lambda \in \Delta(S)$ is a probability distribution of posteriors.[3] The interpretation is that $s \in S$ is the platform's signal to the seller, and so $\lambda(S')$ is the fraction of buyers with signals $S' \subset S$.[4] Note that a disclosure policy can be seen as a two-stage lottery on X whose reduced lottery is the prior F. The set of possible disclosure policies is then:

$$\left\{ \lambda \in \Delta(S) \colon \int s \, d\lambda(s) \sim F \right\}.$$

When a buyer of type x arrives, the platform draws a signal according to λ and shows it to the seller. The seller knows the platform's choice of λ, and so his interpretation of a signal as a posterior is correct. The full disclosure policy, denoted by λ^{FD}, perfectly reveals buyer type x to the sellers. No disclosure policy fully conceals x. Disclosure policy λ' is *coarser* than λ'' if λ' is a Blackwell garbling of λ''. That is, the platform can obtain λ' from λ'' by taking λ'' and pooling some x's.

Steady state distribution of sellers. The matching process is the dynamic system in which sellers become repeatedly busy and available. A steady state of the matching process is characterized by the fraction of available sellers of every type and their acceptance rates. Formally, let $\alpha(y) \geqslant 0$ be the *acceptance rate* – a fraction of buyers accepted by type-y sellers. Let $\rho(y)$ be the *utilization rate* – the fraction of type-y sellers who are busy. Denote the average utilization rate by $\bar{\rho} := \int_Y \rho(y) dG(y)$. Since the total mass of sellers is 1, $\bar{\rho}$ is also the mass of busy sellers.

In a steady state, the flow of sellers who begin working is equal to the flow of sellers who finished the job and return to waiting. The flow of beginning sellers is equal to the buyer flow to type-y sellers times type-y sellers' acceptance rate. Since buyers distribute uniformly across the available sellers, the buyer flow to type-y sellers is $\beta \frac{(1-\rho(y))g(y)}{1-\bar{\rho}}$. The acceptance rate is $\alpha(y)$. Thus, the beginning flow is $\beta \frac{(1-\rho(y))g(y)}{1-\bar{\rho}} \alpha(y)$. The flow of returning sellers is

[3] When X is a compact subset of a Euclidean space, $\Delta(S)$ is the set of Borel probability distributions with the weak-* topology on $\Delta(X)$.

[4] I focus on the "public" signaling when the same λ applies to all seller. I am not studying the mechanism design problem where the platform tries to elicit or learn the seller's type y and tailor the disclosure policy to seller type. Kolotilin *et al.* (2015) find in the one-shot signaling game with linear payoffs that public signaling is equivalent to private signaling.

$g(y)\rho(y)/\tau$ because the mass of busy y-sellers is $g(y)\rho(y)$, and jobs are completed in time τ. In a steady state, the flow of beginning sellers is equal to the flow of returning sellers:

$$\beta\frac{(1-\rho(y))g(y)}{1-\bar{\rho}}\alpha(y)=\frac{g(y)\rho(y)}{\tau},\quad \forall y\in Y. \tag{2}$$

Seller dynamic screening problem. Denote by β_A the Poisson rate at which buyers request an available seller. Since buyers contact only available sellers, β_A depends on the mass of available sellers. In a steady state, the mass of available sellers is $1-\bar{\rho}$, and so,

$$\beta_A := \frac{\beta}{1-\bar{\rho}}. \tag{3}$$

Note that β is the flow rate at which buyers arrive at the platform, while β_A is the Poisson rate at which buyers arrive to available sellers.[5] The particularly simple form of the relation between the two in Eq. (3) follows from the uniform distribution of buyers across available sellers. Sellers take β_A as given because there is a continuum of sellers on the platform, and any individual seller's actions do not affect β_A.

A risk-neutral seller solves the dynamic optimization problem to maximize the average profit flow. The seller faces the sequence of jobs arriving at Poisson rate β_A, for each job observes the platform's signal s and chooses to accept or reject it. See figure 1b for the illustration. Denote by $\pi(s,y)=\int_X \pi(x,y)ds(x)$ the seller y's expected profit if he accepts a job with signal s. Denote by $V(y)$ be the average profit flow when the seller of type y acts optimally (the value function)[6]. Let $v(s,y)$ be the value of a new job with signal s, where v includes the option value of rejecting the job and the opportunity cost of accepting. The value of a new job is zero if the seller rejects it, and $\pi(s,y)-\tau V(y)$ if he accepts it, where $\tau V(y)$ is the opportunity cost of accepting due to being unavailable for time τ. Therefore, $v(s,y)=\max\{0,\pi(s,y)-\tau V(y)\}$. The average profit per unit of time equals the expected value from one job times the expected number of new jobs: $V(y)=\beta_A\mathbb{E}\left[v(s,y)\right]$.[7] Put together, the seller optimization problem is given by:

$$V(y) \;=\; \beta_A\int \max\{0,\pi(s,y)-\tau V(y)\}d\lambda(s). \tag{4}$$

The seller strategy is function $\sigma(\cdot,y)\colon S\to[0,1]$ that for every seller type y maps signal to the probability of accepting it. The seller acceptance rate is the ex ante probability of accepting a job:

$$\alpha(y)=\int\sigma(s,y)d\lambda(s). \tag{5}$$

[5]In more detail, on the one hand, an individual available seller faces a stochastic arrival process, such that the probability of arrival of a new buyer over time interval dt is $\beta_A dt+o(dt)$. On the other hand, available sellers jointly face the deterministic arrival process of buyers, such that over time interval dt the mass $\beta_A dt$ of buyers arrive.

[6]For example, if a seller earns \$1 on each job, and the time interval between starting consequent jobs is 2, then $V(y)=1/2$.

[7]I consider time average payoff rather than discounted sum because discount rate is not essential for my argument. However, the results immediately generalize to the case when the seller has discount rate r by replacing τ with $\tau_r=\frac{1-e^{-r\tau}}{r}$.

2.2 Examples of marketplaces

In this section, I explain how the model fits the marketplaces of Uber, Airbnb and labor platforms, such as TaskRabbit. I will return to these applications in the discussion section after I state my main results. Recall that y captures the seller heterogeneity unobserved by the platform, and x captures buyer heterogeneity observed by the platform.

Uber. When idle, drivers receive requests from passengers. Type y includes driver home location, preference for long rides, tolerance to congestion; x includes passenger's destination and his ride history. Price per mile and minute is fixed (conditional on aggregate multipliers, such as surge pricing). Drivers do not like very short rides or the rides to the remote neighborhoods. Passengers do not like waiting. Concealing passenger destination from drivers is information coarsening.

Airbnb. Hosts are capacity constrained in rooms: once a room is booked for a given date, the host cannot accept a better guest. Type y includes host's preference for age, race, personality, daily schedule. Type x includes guests' gender, age, socio-economic status, lifestyle. Every host sets a price that applies to all guests, but he may prefer to reject guests who he expects will be a bad fit. The InstantBook feature, if adopted by a host, is effectively the no disclosure policy because the host commits to accepting all guests.[8]

TaskRabbit. The service providers are capacity constrained in the number of tasks they can do per week. Once a service provider agrees to do one task, he is limited in picking new tasks. Type y includes service provider's skill and work ethics. Type x includes job category, job difficulty, client's professionality and location. Service providers set hourly rate that apply to all tasks in the same category. Making service providers to commit and price to broad categories is a form of information coarsening.

2.3 Equilibrium definition and existence

A steady state equilibrium is a market outcome in which the sellers take buyer arrival rate β_A as given and optimize independently, and the seller busy-available flows balance out. Formally, a tuple $(\sigma, \bar{\rho})$ constitutes a *steady-state equilibrium* if

1. [Optimality] For every type-y seller for all y, $\sigma(\cdot, y)$ is an optimal strategy given buyer Poisson arrival rate $\beta_A = \beta/(1-\bar{\rho})$.

2. [Steady state] Average utilization rate $\bar{\rho}$ arises in a steady state when sellers play σ, as shown in $\{(2), (5)\}$.

Proposition 1. *A steady-state equilibrium exists. It is unique up to the acceptance of marginal jobs in the following sense. If $(\sigma^i, \bar{\rho}^i)$, $i = 1, 2$ are two steady-state equilibria, then (1) $\bar{\rho}^1 = \bar{\rho}^2$, and (2) for any $y \in Y$, $\sigma^1(\cdot, y)$ and $\sigma^2(\cdot, y)$ coincide except on $\{s \colon \pi(s, y) = \tau V(y)\}$.*

[8]InstantBook is interesting because instead of imposing coarsening, Airbnb offers the feature as an option. This way, InstantBook also serves as a screening device for the platform.

To prove the result, first, I show that for an arbitrary vector of acceptance rates $\alpha(y)$, there is a unique steady state value of $\bar{\rho}$ (Lemma 5). Then, average utilization $\bar{\rho}$ is increasing and continuous in $\alpha(y)$ for any $y \in Y$. The uniqueness of equilibrium follows from monotonicity of reaction curves of α in $\bar{\rho}$ and $\bar{\rho}$ in α. Namely, if average utilization $\bar{\rho}$ increases, then buyer traffic to each available worker increases; sellers become pickier and acceptance rate $\alpha(y)$ decreases. As $\alpha(y)$ increases, sellers become less available, and$\bar{\rho}$ goes down. For details see the proof in the Appendix on page 26.

2.4 Discussion of the modeling assumptions

In this subsection, I discuss in detail the critical assumptions of my model. The assumptions are motivated by the stylized facts about the online platforms.

Assumption 1. *Buyers make a single search attempt.*

Rejection-intolerant buyers is a simplifying assumption but captures a real aspect of matching markets that rejections are costly to buyers, e.g. wasted time, wasted search effort, bidding costs. Moreover, buyers often do not continue searching after a rejection. For example, Fradkin (2015) reports that on Airbnb, an initial rejection decreases the probability that the guest eventually books any listing by 51%.[9]

Assumption 2. *Buyers contact available sellers only.*

The goal of the paper is to explore the matching friction that pertains to preference heterogeneity and screening. Therefore, I assume away the coordination friction owing to simultaneity, when several buyers request the same seller at the same time. Also, I assume away the coordination friction due to unavailability, which arises when buyers request unavailable sellers who did not update their status or do not have the means to do so. The coordination frictions in matching markets have been extensively studied in the theoretical literature (Burdett *et al.* (2001); Kircher (2009); Halaburda *et al.* (2015); Arnosti *et al.* (2014)), and digital platforms usually have good technological means of resolving the simultaneity driven friction[10] . I assume away the simultaneity and unavailability driven frictions to focus on screening.

Assumption 3. *Buyers contact an available seller chosen uniformly at random.*

This assumption implies that buyers do not search for better sellers and holds, for instance, in the situation when buyers are indifferent between what seller completes their job. The assumptions allows me to analyze the match quality - match rate tradeoff in a cleaner model, in which one side of the market (buyers) has homogenous match quality. If buyers have heterogeneous match quality, then the platform will face a similar disclosure problem on the buyer side of the market.

The assumption has two implications to the seller optimization problem. First, the seller of any type faces the same intensity of buyer traffic β_A. Second, each seller faces the same distribution of buyer jobs F.

[9] The results will generalize if buyers return for consequent search attempts. If buyer makes several search attempts, then buyer arrival rate is effectively increased. Sellers still cream-skim but information coarsening is even more effective because higher buyer arrival rate means higher seller option value of rejection.

[10] E.g. Fradkin (2015) finds that on Airbnb the coordination friction explains only 6% of failed matches.

Assumption 4 (No Excess Demand)**.** *Collectively, it is physically possible for sellers to complete every buyer job:* $\beta\tau < 1$

The assumption makes the exposition cleaner and does not add more insights. Relaxing it requires more notation to deal with either automatic rejections or queues. I do this in Appendix D and show that qualitatively results do not change.

3 Market Design: Information Disclosure Policy

This section studies how the platform's information disclosure policy affects the equilibrium market outcome. I consider the general platform's objective of maximizing the weighted average of buyer and seller surplus, which includes as special cases the welfare maximization, seller profits maximization and match rate maximization. I show the full disclosure produces the sub-optimal market outcome because information disclosure aggravates sellers' cream-skimming behavior. I start with the benchmark case of identical sellers, and then move on to a more general case of vertically differentiated sellers. In the latter, the optimal disclosure policy depends non-trivially on the shape of seller distribution, buyer arrival rate, and seller capacity constraints.

3.1 Full disclosure: Seller coordination failure

This subsection establishes that the full disclosure leads to a Pareto sub-optimal market outcome. The reason behind it is the sellers' coordination failure. The inefficiency arises due to sellers' decentralized decision-making and capacity constraints.

To state the results of this and the next sections, I need to define Pareto-optimality in my setting. The market outcome $O = (V(\cdot), U)$ is a combination of seller profits and buyer surplus. I say that a market outcome is *feasible* if there is a seller strategy profile that generates it, and $V(y) \geqslant 0$ for all y. A feasible outcome O is Pareto-optimal if there is no other feasible O' such that $V(y)' \geqslant V(y)$ for all y, and $U' \geqslant U$, and at least one seller type or buyers are strictly better off. The Pareto frontier is the set of all Pareto-optimal outcomes. Market outcome O is *implementable* if there is a disclosure policy such that the equilibrium outcome is O. The welfare is the sum of consumer surplus and joint seller profits. A market outcome is *efficient* if there is no other feasible outcome with higher welfare.

Write $V^\sigma(y)$, $\alpha^\sigma(y)$, U^σ for steady-state seller profits, acceptance rates, and consumer surplus when strategy profile σ is played. Imagine the platform starts with the full disclosure as its default disclosure policy. The next proposition shows that there is a strategy profile under which sellers are strictly better off than in the full disclosure equilibrium, they complete more jobs, and additionally, the buyers are also better off. The result implies that in matching markets, sellers face a coordination problem.

Proposition 2. *Let σ^{FD} be the equilibrium strategy profile under full disclosure. Then there exists $\tilde{\sigma}$ such that for all y:*

$$\begin{aligned} \tilde{V}(y) &> V^{FD}(y), \\ \widetilde{U} &> U^{FD}, \\ \tilde{\alpha}(y) &> \alpha^{FD}(y). \end{aligned}$$

The full proof is in the Appendix on page 27. A very high-level intuition for the coordination problem is the following. A seller keeps his schedule open by rejecting low-value jobs to increase his own chances of getting high-value jobs. As a result, in equilibrium, sellers spend a lot of time waiting for the high-value jobs. Collectively, this behavior is suboptimal because all profitable jobs have to be completed. I call this behavior *cream-skimming.*

Here I give the proof sketch for the case of identical sellers, which gives further insights into the nature of the seller coordination problem. Under the full disclosure, seller profit V^{FD} is strictly positive because sellers accept only profitable jobs. The opportunity cost of accepting equals τV^{FD}, and so it is also strictly positive. In equilibrium, the jobs that are accepted have profits $\pi(x) \geqslant \tau V^{FD}$ (see Eq. (4)). However, the profitable jobs are those with $\pi(x) \geqslant 0$. Hence, some profitable jobs are rejected. Consider strategy $\tilde{\sigma}$ that maximizes the joint seller profits. It prescribes every seller to accept jobs with $\pi(x) \geqslant 0$. . Naturally, $\tilde{\sigma}$ yields $\tilde{V} > V^{FD}$. Acceptance rate is also higher, $\tilde{\alpha} > \alpha^{FD}$. Consumers are better off because consumer surplus is increasing in the acceptance rate.

Interestingly, unlike price competition that benefits buyers and improves market efficiency, seller competition for better buyers hurts buyers and decreases market efficiency. Thus, cream-skimming is a market failure. I attribute the source of the failure to what I call the *cream-skimming externality*. By rejecting a job, a seller remains available on the marketplace and attracts a fraction of subsequent buyers, who otherwise would go the other sellers. As a result, the other sellers face fewer profitable jobs and obtain lower profits. The necessary conditions for the cream-skimming externality to arise is forward-looking and capacity constrained sellers.

The fundamental reason behind the coordination failure is that collectively, the sellers are not capacity constrained (in time) while individually, the sellers *are* capacity constrained. When an individual seller accepts a job, he is off the market for time τ and cannot accept new jobs. Therefore, when he is available, he is afraid to miss high-value future jobs and therefore rejects low-value jobs. However, there are always some available sellers in the market, and it is feasible to accept all profitable jobs. Mathematically, the distinction between individual and collective capacity constraints is captured by having a continuum of sellers, so that while buyer traffic to each individual seller is stochastic, the aggregate buyer traffic is deterministic.

Proposition 2 shows that the full disclosure equilibrium is Pareto dominated by some strategy profile $\tilde{\sigma}$. Is there an information disclosure policy that induces $\tilde{\sigma}$? In the next section, I give the affirmative answer to this question in the case of identical workers. The situation with heterogeneous sellers is more tricky, and I study it in Section 3.4.

3.2 Benchmark: Optimal information disclosure with identical sellers

I start by considering the case of identical sellers, i.e. Y is a singleton. The next proposition establishes that any Pareto optimal outcome (in seller profits-buyer surplus space) is implementable by a disclosure policy.

Proposition 3. *Suppose the sellers are identical. Then for any Pareto optimal outcome (V, U) there is a disclosure policy that implements it. Furthermore, an optimal disclosure policy has binary structure.*

The proof relies on the Revelation Principle. Since there is one seller type and two actions, it is sufficient to consider only disclosure policies that send two signals, where a signal is "action recommendation". With such a binary signaling structure, the seller dynamic optimization problem reduces to the static optimization problem. Indeed, since there is only

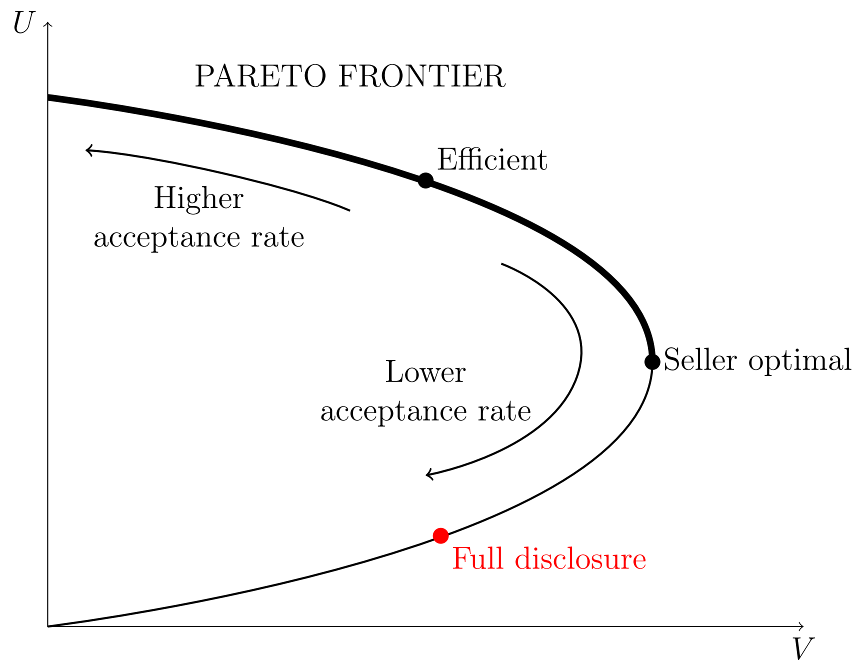


Figure 2: Graphical illustration of the set of feasible and implementable market outcomes in the case of identical workers. U stands for buyer surplus, V stands for seller surplus. Disclosure policy can implement any point on the Pareto frontier (thick solid line). The full disclosure outcome is suboptimal.

one signal with the recommendation to accept, there is only one type of acceptable jobs. All profitable jobs are the same, and there is no return to cream-skimming them. Since a Pareto optimal outcome is feasible with $V \geqslant 0$, sellers have incentives to follow the platform's recommendations to accept. For the details of the proof, see the Appendix.

Proposition 3 characterizes at once the range of possible objective functions. Indeed, any point on the Pareto frontier maximizes $\gamma U + (1-\gamma)V$ for some $\gamma \in [0,1]$. The welfare maximization policy corresponds to $\gamma = 1/2$, and so the first-best efficient outcome is implementable by a disclosure policy. Figure 2 illustrates the result. Note that by Proposition 2 the full disclosure equilibrium outcome lies below the Pareto frontier.

Proposition 3 is related to the result of Bergemann *et al.* (2015) who show that segmentation of a monopolistic market can achieve every feasible combination of consumer and producer surplus. Their segmentation problem is a static signaling game with a single receiver (monopolist), who also sets a price, while my model is a signaling game with dynamically optimizing receivers, whose prices are fixed.

3.3 Three effects of information disclosure

Disclosure of buyer characteristics to sellers has three competing effects on welfare. The stronger effect determines the form and coarseness of the optimal disclosure.

The welfare is the sum of buyer surplus and seller profits,

$$W = U + V.$$

In the case of identical sellers, I am going to represent the welfare as a function of seller match quality and match rate, and explain how information disclosure affects both of them.

Denote the match rate by M, that is M is the number of matches that form on the platform over a unit time interval. Higher M implies that more buyers are served, and so the buyer surplus is the increasing function of the match rate, $U = U(M)$.

Denote the seller (average) match quality by v. We have:

$$V = Mv.$$

The welfare can be represented as

$$W = U(M) + Mv.$$

Obviously, W increases in both match rate M and seller match quality v.

First, information disclosure has the positive effect on the seller match quality. From a seller's point of view, more information increases his set of attainable payoffs. Therefore, holding fixed the match rate, he individually benefits from more information about buyers. Formally,

Claim 1 (Match quality effect). Let λ' be coarser than λ''. Holding M fixed, the match quality under λ' is lower than under λ'', or $v' \leqslant v''$.

Second, information disclosure reduces the platform's ability to induce sellers to accept more jobs. Namely, the platform can coarsen information to increase the seller expected marginal profit. Ignoring dynamic effects, higher marginal profit induces sellers to accept more jobs, which leads to higher match rates. Formally,

Claim 2 (Static match rate effect). Suppose $\tau = 0$. Let λ'' be such that there is $s \in \operatorname{supp} \lambda''$ with $\pi(s) < 0$. Then there is λ' coarser than λ'' such that $M' > M''$.

Third, more information available to sellers increases their return to search. Therefore, the opportunity cost of accepting is higher, because acceptance precludes further search. As a result, sellers reject more often, and the match rate goes down. In equilibrium, it hurts seller profits because it is not collectively optimal for sellers to reject profitable jobs. Formally,

Claim 3 (Dynamic match rate effect). Let λ' be coarser than λ''. Holding M fixed, the option value of rejecting a marginal job under λ' is lower than under λ''.

The match quality effect gives a motive for the platform to disclose information, while the effects on match rate give the motive to coarsen information. What effect is stronger depends on the primitives of the economic environment.

With identical sellers, too many efficient jobs are rejected under the full disclosure. Denote the efficient disclosure policy by $\hat{\lambda}$ and the set of efficient matches by $\hat{X}_a := \{x \in X : \pi(x) + u(x) \geqslant 0\}$. Disclosure $\hat{\lambda}$ sends the recommendation "accept" for any jobs in $\hat{X}_a$ and recommendation "reject" for jobs in $X \backslash \hat{X}_a$. Under the full disclosure, the accepted jobs are $X_a^{FD} := \{x \in X : \pi(x) \geqslant \tau V^{FD}\}$. Clearly, X_a^{FD} is a proper subset of $\hat{X}_a$.

When sellers are identical, the match rate effects win over the match quality effect. Indeed, jobs $\{x : -u(x) \leqslant \pi(x) \leqslant 0\}$ are rejected because sellers fail to internalize the effect of their acceptance decisions on buyers. This is the static match rate effect. Jobs

$\{x\colon 0 \leqslant \pi(x) \leqslant \tau V\}$ are rejected because sellers fail to internalize the cream-skimming externality they impose on each other. This is the dynamic match rate effect. The match quality effect is weak because the platform observes seller preferences and can finely control their surplus. Therefore, $\hat{\lambda}$ is coarse.

The next result gives comparative statics of the efficient disclosure policy with respect to buyer traffic β, seller capacity constraint τ and buyer cost of rejection.

Corollary 1. *Suppose the sellers are identical. Efficient information disclosure policy $\hat{\lambda}$ is independent of the intensity of buyer traffic β and seller capacity constraints τ. When buyer cost of rejection is higher, $\hat{\lambda}$ prescribes pooling more of marginal unprofitable jobs with inframarginal profitable jobs.*

Interestingly, $\hat{\lambda}$ does not depend on β or τ. This implies that the same $\hat{\lambda}$ would be optimal if sellers were able to complete jobs instantaneously ($\tau = 0$). Independence of $\hat{\lambda}$ from β and τ obtains due to the binary structure of $\hat{\lambda}$. Arrival rate to available sellers β_A matters to sellers only to the extent that higher arrival rate magnifies the opportunity cost of accepting. With binary $\hat{\lambda}$, the opportunity cost of accepting a profitable job is zero because all profitable jobs are the same.

The next section shows that with heterogeneous sellers, the match quality effect can be the strongest. Moreover, the optimal disclosure policy depends on β and τ.

3.4 Optimal information disclosure with heterogeneous sellers

This section studies the information disclosure problem in the linear payoff environment when sellers have payoff heterogeneity unobserved by the platform. It is a leading case of my analysis because in most marketplaces, the intermediary observes seller preferences imperfectly. I characterize the optimal disclosure policy and show that it is qualitatively different from the statically optimal disclosure found in the prior literature. Moreover, unlike with identical sellers, the Pareto frontier is generally not attainable. Finally, under certain conditions, which I provide, the full disclosure is the only optimal policy.

With heterogeneous sellers, the match quality effect is stronger. To see why, imagine there are two types of sellers: professionals and amateurs. Professionals can profitably complete a larger set of jobs than amateurs. The platform does not observe seller type. When the platform designs the disclosure policy, it should take into account that the same disclosure policy has different impacts on professionals and on amateurs. Amateurs can profitably complete only a small subset of jobs, and so they need finer disclosure to tell apart profitable jobs from unprofitable ones. If the disclosure is too coarse, amateurs reject all jobs. Professionals have a large set of profitable jobs, and so their average profit per job is high. Pooling more unprofitable marginal jobs with profitable inframarginal jobs keeps professionals' average profits positive but induces higher acceptance rate. Whether it is optimal to coarsen the information disclosure or not thus depends on the relative population sizes of amateurs and professionals. If there are more professionals than amateurs, then coarser disclosure is optimal: it increases the total acceptance rates even though amateurs stop working. If there are more amateurs, finer disclosure is optimal. In the rest of this section I rigorously study this problem for the general continuous distribution of seller types.

Consider the setting that I call the *linear payoff* environment. The space of buyer types is $X = [0, 1]$, with the interpretation that x is the difficulty of the job. The space of seller types is $Y = [0, \overline{y}]$, $\overline{y} \geqslant 1$, with the interpretation that y is the seller skill level. The seller profit function is $\pi(x, y) = y - x$.[11] High-y sellers are "professionals", and the low-y sellers are "amateurs". Buyer match value is $u(x, y) = u$. Impose the regularity condition necessary for the results below: $f(0) > 0$.

The platform's objective is maximizing the weighted average of buyer surplus and joint seller profits.

$$\mathcal{J}(\gamma) = \gamma U + (1 - \gamma)V, \tag{6}$$

where $V = \int_Y V(y)dG(y)$ is the joint seller profits, and $U = u \cdot M$ is buyer surplus, where M is the total number of matches formed over unit of time. The general objective $\mathcal{J}(\gamma)$ includes as special cases welfare maximization ($\gamma = 1/2$), seller profits maximization ($\gamma = 0$) and maximization of the number of matches ($\gamma = 1$).

The next proposition is the second main result of the paper. It characterizes the disclosure policy that maximizes $\mathcal{J}(\gamma)$ for the case of uniform seller type distribution.

Definition. Disclosure λ is x^**-upper-coarsening* for some $x^* \in [0, 1]$ if λ fully reveals $x < x^*$ and pools all $x > x^*$.[12]

Proposition 4. *Suppose $G = U[0, \overline{y}]$, $\overline{y} \geqslant 1$. Then for any $\gamma \in [0, 1]$, there is unique $x^*_\gamma \in [0, 1]$ such that x^*_γ-upper-coarsening maximizes $\mathcal{J}(\gamma)$. Furthermore, x^*_γ is decreasing in γ.*

- *For any $\gamma \in [0, 1]$, there exist $\beta\tau$ and $\overline{y}$ large enough such that $x^*_\gamma < 1$ (some coarsening is strictly optimal).*
- *If $0 < \beta\tau < 1/2$, then $x^*_\gamma = 1$ for any γ (full disclosure is strictly optimal).*

I reserve notation x^*_γ to denote the cutoff in the upper-coarsening disclosure policy that maximizes $\mathcal{J}(\gamma)$. This way, x^*_0, x^*_1 and $x^*_{1/2}$ are the highest truthfully revealed buyer types under seller profits maximizing disclosure, match rate maximizing disclosure and the efficient disclosure, respectively.

First, Proposition 4 shows that the optimal disclosure involves pooling hard tasks and truthfully revealing easy tasks. As we will see in the discussion below, the reason behind this form of disclosure is to reduce of the professionals' opportunity cost of accepting.

Second, when the platform puts more weight on buyer surplus, the optimal disclosure policy is coarser. In particular the match-maximizing disclosure is coarser than the efficient disclosure that in turn, is coarser than the seller profits maximizing disclosure ($x^*_0 \geqslant x^*_{1/2} \geqslant x^*_1$).

Third, Proposition 4 shows that the optimal disclosure policy in the case of the uniform distribution of seller types depends on the intensity of buyer traffic β, tightness of seller capacity constraints τ and the spread of seller types $\overline{y}$. If $\beta\tau$ is large enough and there are

[11] The linear payoff environment was used in the prior literature on disclosure, e.g. in Kolotilin *et al.* (2015).

[12] The terminology is borrowed from Kolotilin *et al.* (2015).

sufficiently high seller types, then upper-coarsening is optimal. Conversely, if buyer traffic is low or capacity constraints are loose, full disclosure is strictly optimal.

I am going to contrast the optimal disclosure policy found in Proposition 4 to the solution of the static disclosure problem. The *static disclosure problem* is the one shot interaction between the platform and the sellers. My model reduces to the static disclosure problem when $\tau = 0$. Indeed, when $\tau = 0$, sellers are always available, has zero opportunity cost of accepting, and so act myopically.

The upper-coarsening form of the optimal policy is qualitatively different from the solution to the static disclosure policy in the same environment.

Fact 1. *Suppose $\tau = 0$ and consider the platform's objective of maximizing the number of matches ($\gamma = 1$).*

- *If g is decreasing, then the full disclosure is optimal;*
- *If g is increasing, then the no disclosure is optimal;*
- *If g is constant, then any disclosure is optimal.*

The static disclosure problem has been extensively studied in the prior literature, and the above result appears in e.g. Kolotilin *et al.* (2015). The implied concavification reasoning goes back to Aumann *et al.* (1995) and Kamenica and Gentzkow (2011). The proof of Fact 1 follows from Corollary 4 in the next section.

In the static disclosure problem, if the distribution of seller types is uniform, the information disclosure does not affect the number of matches. Any coarsening of the full disclosure policy decreases the acceptance rate of lower-type sellers and increases the acceptance rate of high-type sellers. When distribution of seller types is uniform, these two forces cancel out and the total number of matches is unchanged.

In contrast, in dynamic setting, Proposition 4 shows that information disclosure does affect the number of matches. In particular, the upper-coarsening is strictly optimal. The additional affects that arise when $\tau > 0$ are the dynamic match quality effect and the availability effect.

To better see what forces make the upper-coarsening optimal, consider the following two simplifications of the model. Each simplification alters a part of the original model. Recall that the model has two main components: the matching system, call it $\mathcal{M}$, and the seller optimization problem, call it $\mathcal{O}$. The first simplification keeps $\mathcal{O}$ in its exact form as in (4) but has the alternative matching system $\mathcal{M}'$ that replenishes any available seller immediately after he becomes busy with a new seller of the same type. In $(\mathcal{O}, \mathcal{M}')$, the mass of available sellers is always 1, and the distribution is always G. The second simplification keeps $\mathcal{M}$ but has the alternative $\mathcal{O}'$ in which sellers act myopically. In $(\mathcal{O}', \mathcal{M})$, the seller accept all profitable jobs but it sill takes time τ to complete them.

Corollary 2. *Suppose the distribution of available seller types is exogenous $G = U[0, 1]$ (simplification $(\mathcal{O}, \mathcal{M}')$). Then only the no disclosure policy maximizes the number of matches.*

The result demonstrates the workings of the dynamic match quality effect in the case of heterogeneous sellers. Under the premises of Corollary 2, the dynamic match quality effect completely dominates the match quality effect. Compare the no disclosure to the full disclosure. If seller were myopic, then by Fact 1 coarsening does not affect the number of matches. However, with forward-looking sellers, there is a difference. The coarsening of information increases the acceptance rate of professionals unproportionally because it decreases their option value of rejection. As a result, the no disclosure is optimal.

Corollary 3. *Suppose $G = U[0, 1]$. If sellers are myopic (simplification $(\mathcal{O}', \mathcal{M})$), then full disclosure is strictly optimal to maximize the matching rate.*

In this result, I shut down the dynamic match quality effect and instead focus on the effect of endogenous distribution of seller types. In the equilibrium, the pdf of available seller types is decreasing because high seller types have higher acceptance rates and so are less available. Results from the static disclosure (Fact 1) suggest that in this case the full disclosure must be optimal. Corollary 3 confirms that this is indeed the case.

In the original model $(\mathcal{O}, \mathcal{M})$ both effects from Corollary 2 and Corollary 3 are present. The resulting optimal policy is a partial coarsening. In the case of uniform G, as Proposition 4 shows, it is upper-coarsening.

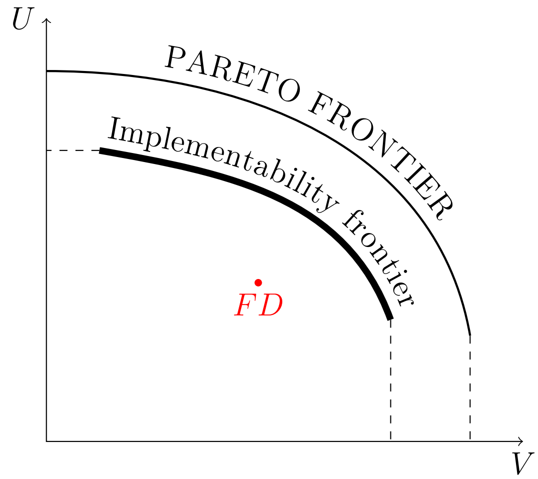
For general G, the shape of the optimal disclosure can be quite complex even in the static case. The heuristic is to pool x on the increasing part of pdf g, and reveal x on the decreasing part of g. With forward-looking agents, it gets even less tractable. Nevertheless, I give the first order condition in Lemma 3 that can be used to analyze the general case.

Maximizing seller profit may also require coarse disclosure policy. Proposition 4 shows that when $\gamma = 0$, the non-trivial upper-coarsening can be optimal ($x_0^* < 1$). The coarsening is necessary to alleviate the seller coordination problem when $\beta\tau$ is large. Large $\beta\tau$ implies large option value of rejecting, and the option value is the largest for professionals. Large x are the professionals' marginal buyers. Therefore coarsening at the right end of X decreases the professionals' return to rejecting and increases the fraction of accepted profitable jobs. This improves seller profits.

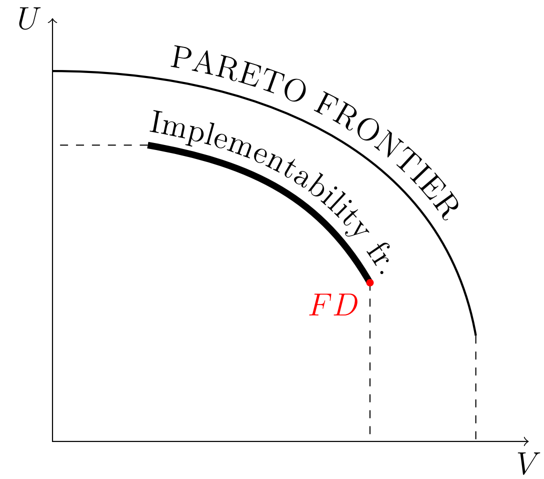
Figure 3 illustrates Proposition 4 and contrasts it with the case of identical sellers. From Proposition 2 we know that the full disclosure outcome is suboptimal. In the case of identical sellers, it was possible to implement any point on the Pareto frontier by information disclosure (Proposition 3), and therefore coarsening was necessary for optimality. In the case of heterogeneous sellers, the Pareto frontier is not implementable in the generic case because sellers have private information. Therefore, the "implementability frontier" in Figure 3 is below the Pareto frontier. Further, if $\beta\tau$ and $\overline{y}$ are large enough then the full disclosure outcome is below the implementability frontier, and efficiency requires some coarsening. The optimal disclosures in this case are described in Proposition 4. If $\beta\tau$ is small, the implementability frontier consists of only the full disclosure outcome. That is, the full disclosure is optimal for welfare maximization, match-rate maximization or joint seller profits maximization.

3.5 Main lemma and proof outline of Proposition 4

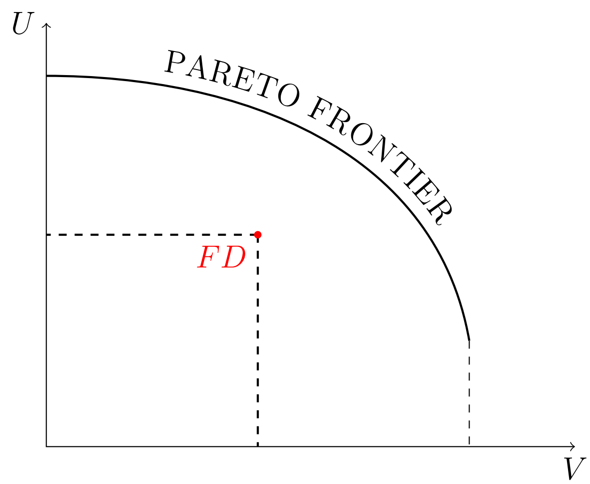
In this subsection I sketch the proof of Proposition 4. The main idea behind the proof is to represent information disclosure λ as some bounded convex function $\Lambda(\cdot)$, and then use the calculus of variations to find the optimal Λ. The main technical result of the paper is Lemma 3.



(a) $\beta\tau$ is large enough and $\overline{y}$ is large enough. The full disclosure is neither efficient, nor seller profits maximizing, nor match rate maximizing.



(b) The full disclosure is seller profits maximizing but not efficient.



(c) $\beta\tau \in (0, 1/2)$. The implementability frontier is degenerate and consists of only the full-disclosure outcome.

Figure 3: Graphical illustration of the set of feasible market outcomes and the limits of implementability using information design in the case of heterogeneous workers in the linear payoff environment with the uniform seller type distribution. U stands for buyer surplus, V stands for seller surplus. FD stands for the full-disclosure outcome.

The proof of Proposition 4 relies on a sequence of four lemmas. Lemma 1 establishes the one-to-one correspondence between information disclosure policies and convex functions from some set. Lemma 2 finds the convenient representation of the seller dynamic optimization problem. Lemma 3 finds a variational derivative of the platform's objective $\mathcal{J}$. Lemma 4 provides a necessary condition for optimality of a disclosure policy.

Denote the posterior mean of x conditional on signal s by $z(s) := \int_X x\, ds(x)$. I reserve notation z for a typical posterior mean of x. Denote by F^λ the distribution of $z(s)$ when the platform uses disclosure policy λ. We have $F^\lambda(\zeta) = \lambda\{z(s) \leqslant \zeta\}$. Define the *option value function* $\Lambda\colon [0, \infty) \to \mathbb{R}_+$:

$$\Lambda(z; \lambda) := \int_0^z F^\lambda(\zeta)\, d\zeta. \tag{7}$$

As we will see from the Lemma 2, $\Lambda(z)$ is proportional to the option value of rejecting a job with expected difficulty z. Let $\overline{\Lambda}$ be the option value function under full disclosure, $\overline{\Lambda}(z) := \int_0^z F(\zeta)\, d\zeta$. Similarly, let $\underline{\Lambda}$ be the option value function under no disclosure, $\underline{\Lambda}(z) := \max\{0, z - \mathbb{E}[x]\}$. Let

$$\mathcal{L} := \left\{\Lambda(z)\colon \Lambda(z) \text{ is increasing, convex and pointwise between } \overline{\Lambda}(z) \text{ and } \underline{\Lambda}(z)\right\}.$$

The next lemma establishes a one-to-one correspondence between functions from $\mathcal{L}$ and option value functions defined in (7).

Lemma 1. *$\ell \in \mathcal{L}$ if and only if there is $\lambda \in \Delta(S)$ such that $\Lambda(\cdot, \lambda) = \ell$.*

The power of Lemma 1 is that it shows that any disclosure policy can be represented as some non-negative, non-decreasing and convex function from $\mathcal{L}$. See Figure 4 for illustration. I am going to use notation Λ for a typical element of $\mathcal{L}$. The optimization of $\mathcal{J}$ with respect to $\lambda \in \Delta(S)$ is equivalent to the optimization with respect to $\Lambda \in \mathcal{L}$. The latter allows me to use the calculus of variations to find the optimality condition below.

The next lemma characterizes the optimal seller strategy and demonstrates that the seller optimization problem depends on λ only through Λ.

Lemma 2. *For any disclosure policy λ, seller's optimal strategy has a cutoff form with cutoff $\hat{z}(y)$ such that y-seller accepts all jobs with expected difficulty $z < \hat{z}(y)$ and rejects all jobs with $z > \hat{z}(y)$. Furthermore, for any β_A, seller payoff $V(y)$ and cutoff $\hat{z}(y)$ are solutions to the following system of equations:*

$$V(y) = \frac{y - \hat{z}(y)}{\tau} = \beta_A \Lambda(\hat{z}(y)). \tag{8}$$

The lemma implies that the platform's objective $\mathcal{J}$ depends on λ only through Λ. It is used in the next lemma, which is the main technical result of the paper.

Consider functional $\mathcal{I}(\Lambda)\colon \mathcal{L} \to \mathbb{R}$ and consider variation $\delta\Lambda(y)$. The first variation of $\mathcal{I}$ is $\delta\mathcal{I} = \mathcal{I}(\Lambda + \delta\Lambda) - \mathcal{I}(\Lambda)$. The variational derivative of $\mathcal{I}$ with respect to Λ is function $\phi(y)$ such that $\delta\mathcal{I} = \int \phi(y)\delta\Lambda(y)dy + o(\delta\Lambda)$ as $\delta\Lambda \rightrightarrows 0$. If the variational derivative exists, it is denoted by $\delta\mathcal{I}/\delta\Lambda$.

Denote by $\nu(y) := 1 - \rho(y)$ the fraction of type-y sellers who are available.

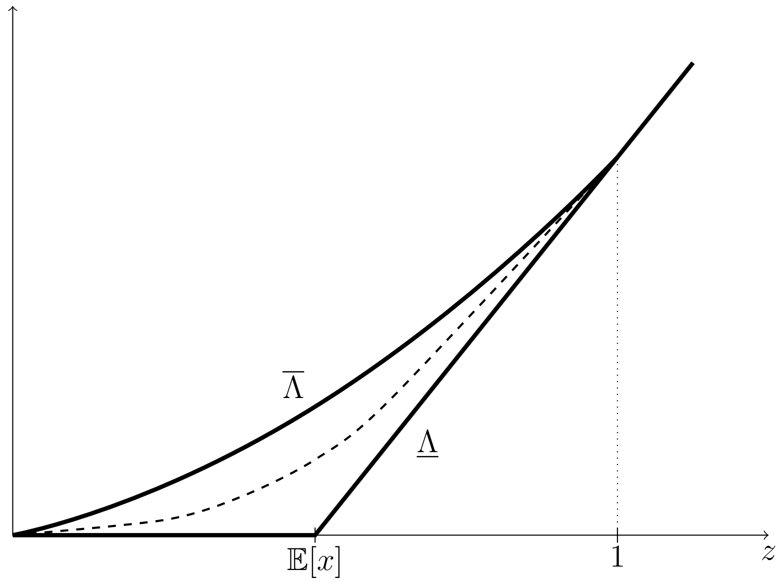


Figure 4: Any disclosure policy λ can be represented as an increasing, convex function Λ point-wise between $\underline{\Lambda}$ and $\overline{\Lambda}$.

Lemma 3 (Main Lemma)**.** *For any initial $\Lambda \in \mathcal{L}$, the variational derivative of the flow of matches M with respect to Λ exists and equals:*

$$\frac{\delta M}{\delta \Lambda} = K_1 \cdot \left[g(y)\nu'(y) - (g(y)\nu^2(y))' \right], \tag{9}$$

where $K_1 = \beta / \int [\nu^2(y) - \tau V(y)\nu'(y)]\, dG(y) > 0$. Similarly, the variational derivative of the joint seller profits V with respect to Λ exists and equals:

$$\frac{\delta V}{\delta \Lambda} = \frac{\delta M}{\delta \Lambda} \cdot K_2 + \beta_A \nu(y) g(y),$$

where $K_2 = \tau \int \nu(y)V(y)dG(y)/\bar{\nu} > 0$.

To see the contribution of Lemma 3, compare it to the static disclosure problem.

Corollary 4. *Suppose $\tau = 0$. Then*

$$\frac{\delta M}{\delta \Lambda} = -\beta g'(y). \tag{10}$$

If g is decreasing, then the full disclosure is optimal. If g is increasing, then the no disclosure is optimal.

The result easily follows from Lemma 3 using the fact that when $\tau = 0$, $\nu(y) = (1 + \tau\beta_A\alpha(y))^{-1} = 1$. When sellers are capacity constrained ($\tau = 0$), the original formula (10) has to be adjusted, as shown in (9).

I now explain the additional terms in (9) in more detail. Consider uniform distribution of seller types, $G = U[0, 1]$. By (9),

$$\frac{\delta M}{\delta \Lambda} \propto -(\nu^2(y) - \nu(y))'. \tag{11}$$

When $\tau = 0$, it reduces to

$$\frac{\delta M}{\delta \Lambda} = 0.$$

Therefore, the disclosure has no effect on the number of matches in the static case but does have effect in the dynamic case.

Term $-\nu(y)$ in (11) corresponds to the match quality effect. The match quality effect arises because sellers do not act myopically and reject low-value jobs due to dynamic optimization. High types have greater option value of rejecting a job (see Eq. (4)), and so coarsening information has additional effect on acceptance by decreasing the sellers' option value of rejection. By Corollary 4, $-\nu(y)$ acts as a density function, and since $\nu(y)$ is decreasing, this creates a motive for the intermediary to use coarser disclosure policy.

Term $\nu^2(y)$ in (11) corresponds to the availability effect. The availability effect arises because in equilibrium, high types are less available than low types. Therefore, the pdf of available seller types is decreasing. By Corollary 4, $\nu^2(y)$ acts as a density function, and since $\nu(y)$ is decreasing, this creates a motive for the intermediary to use finer disclosure policy.

The next lemma provides a necessary condition for optimality of a disclosure policy.

Lemma 4. *If λ_0 maximizes $\mathcal{J}$, and $\delta\mathcal{J}/\delta\Lambda$ evaluated at λ_0 crosses zero from above at most once, then λ_0 is upper-coarsening.*

Now I sketch the key steps of the proof of the main result for the case of heterogeneous seller Proposition 4. The complete proof relies on more technical lemmas and is deferred to the Appendix on page 31.

Proof sketch of Proposition 4. By Lemma 3,

$$\begin{aligned}\frac{\delta \mathcal{J}}{\delta \Lambda} &= \gamma u \frac{\delta M}{\delta \Lambda} + (1-\gamma)\left(\frac{\delta M}{\delta \Lambda} K_2 + \nu(y) g(y) \beta_A\right) = \\ &= (\gamma u + K_2(1-\gamma))\frac{\delta M}{\delta \Lambda} + (1-\gamma)\beta_A \nu(y),\end{aligned} \tag{12}$$

where $K_2 > 0$. Evaluating with $G = U[0, \overline{y}]$,

$$\frac{\delta M}{\delta \Lambda} = K_1 \overline{y}^{-1}(\nu(y) - \nu^2(y))' = K_1 \overline{y}^{-1}(1 - 2\nu(y))\nu'(y), \tag{13}$$

where $K_1 > 0$. Since $\nu(0) = 1$ and $\nu(y)$ is non-negative and decreasing, $\delta\mathcal{J}/\delta\Lambda$ is either positive for all $y \in [0, \overline{y}]$ or crosses zero once from above. Denote by λ^*_γ the disclosure policy that maximizes $\mathcal{J}(\gamma)$. By Lemma 4, λ^*_γ is upper-coarsening.

To see that the cutoff x^*_γ is decreasing in γ, note that larger γ puts more weight on the positive term $\beta_A \nu(y)$ in (12). Therefore the region of Y with negative $\delta\mathcal{J}/\delta\Lambda$ is smaller.

Whether $x^*_\gamma = 1$ or strictly less than 1 depends on the existence of $y \leqslant \overline{y}$ with $\nu(y) < 1/2$. Indeed, from (13), if $\nu(y) > 1/2$ for all $y \leqslant \overline{y}$, then $\delta\mathcal{J}/\delta\Lambda \geqslant 0$ for all y. For the details, refer to the proof in the Appendix on page 31. □

4 Appendix

A Lemmas

Lemma 5. *Fix the arbitrary increasing $\alpha(y) \in [0,1]$, $y \in Y$.*

- *Average utilization rate $\bar{\rho} \in [0,1]$ is a solution to*

$$1 = \int \frac{dG(y)}{1 - \bar{\rho} + \beta\tau\alpha(y)}. \tag{14}$$

- The solution $\bar{\rho}$ exists, is unique, *increases in $\alpha(y)$ for any $y \in Y$, in β and in τ.*

- *Utilization rate of type y $\rho(y)$ increases in $\alpha(y')$ for any $y' \in Y$.*

Proof. Find from (2) that

$$1 - \rho(y) = \frac{1}{1 + \tau\beta\alpha(y)/(1-\bar{\rho})}. \tag{15}$$

Take the integral using cdf $G(y)$ to obtain $1-\bar{\rho} = \int dG(y)/(1+\tau\beta\alpha(y)/(1-\bar{\rho}))$. Rearrange and get (14). The right-hand side of (14) is increasing in $\bar{\rho}$. Evaluated with $\bar{\rho} = 0$, it equals $\int \frac{dG(y)}{1+\beta\tau\alpha(y)} \leqslant 1$. Evaluated with $\bar{\rho} = 1$, it equals $\int \frac{dG(y)}{\beta\tau\alpha(y)} \geqslant 1$, using Assumption Assumption 4. Therefore, the solutions exists and is unique. Monotonicity of $\bar{\rho}$ is straightforward.

Finally, if $\alpha(y')$ increases (at the neighborhood of y'), $\bar{\rho}$ increases. Therefore, the right hand side of (15) decreases. Therefore, $\rho(y)$ increases. □

Lemma 6. *In the steady state,*

$$\beta \leqslant \beta_A \leqslant \frac{\beta}{1-\tau\beta}.$$

Proof. From (3), $\beta_A = \beta/(1-\bar{\rho})$. Therefore, $\beta_A \geqslant \beta$.

Find from (2) that $\tau\beta\frac{1-\rho(y)}{1-\bar{\rho}}\alpha(y) = \rho(y)$. Take expectation wrt y and obtain $\bar{\rho} = \tau\beta\int\frac{1-\rho(y)}{1-\bar{\rho}}\alpha(y)dG(y) \leqslant \tau\beta\int\frac{1-\rho(y)}{1-\bar{\rho}}dG(y) = \tau\beta$. Therefore, $\beta_A \leqslant \frac{\beta}{1-\tau\beta}$. □

Lemma 7. *Assume the linear payoff environment. In the steady state equilibrium with disclosure policy λ, $\alpha(y) = \Lambda'(\hat{z}(y))$ for sellers of mass 1. Furthermore,*

- *$\alpha(y)$ is increasing in y.*

- *For any $\Lambda \in \mathcal{L}$, there is $\breve{y} > 0$ such that $\alpha(y) = 1$ for $y \geqslant \breve{y}$.*

- *Considering exogenous variations of $\beta_A\tau$ in the seller optimization problem (8), $\alpha(y)$ is decreasing in $\tau\beta_A$ for any $y \in Y$.*

Proof. By Lemma 2, the the accepted jobs are those with $z < \hat{z}(y)$. Using definition of Λ from (7), probability of accepting is

$$\lambda\{z(s) < \hat{z}(y)\} \leqslant \alpha(y) \leqslant \lambda\{z(s) \leqslant \hat{z}(y)\}$$
$$F^{\lambda}(\hat{z}(y)-) \leqslant \alpha(y) \leqslant F^{\lambda}(\hat{z}(y)+)$$
$$\Lambda'(\hat{z}(y)) \leqslant \alpha(y) \leqslant \Lambda'(\hat{z}(y))$$

Points of discontinuity of F^{λ} make a set of measure zero. Since seller types have full support on $[0, \overline{y}]$, $\alpha(y) = \Lambda'(\hat{z}(y))$ for sellers of mass 1.

From (8), $\hat{z}(y)$ is increasing in y. Since Λ is convex, $\alpha(y)$ is increasing in y.

Since $X = [0, 1]$, $\Lambda'(z) = 1$ for $z \geqslant 1$. And for y large enough, $\hat{z}(y) > 1$.

From (8), $\hat{z}(y)$ is decreasing in $\tau\beta_A$ for any $y \in Y$. Since Λ is convex, $\alpha(y)$ is decreasing in $\tau\beta_A$. □

B Proofs of Propositions 1-3

Proof of Proposition 1. Step 1. Restricting set of strategies to cutoff strategies. Fix $y \in Y$. By (4) the optimal seller strategy is such that all $S_a(y) = \{s \colon \pi(s, y) > \tau V(y)\}$ are accepted, all $S_r(y) = \{s \colon \pi(s, y) < \tau V(y)\}$ are rejected, and the seller is indifferent between accepting and rejecting signals from $S_m(y) := \{s \colon \pi(s, y) = \tau V(y)\}$. Therefore any optimal strategy has the cutoff form:

$$\sigma(s, y) = \begin{cases} 1, & s \in S_a(y) \\ \in [0, 1], & S_m(y) \\ 0, & s \in S_r(y) \end{cases}$$

Denote the cutoff for type-y seller by $\hat{\pi}(y) := \tau V(y)$. Using (4),

$$V(y) = \beta_A \int (\pi(s, y) - \tau V(y))\sigma(s, y)\lambda(ds)$$
$$\hat{\pi}(y) = \tau\beta_A \int (\pi(s, y) - \hat{\pi}(y))\sigma(s, y)\lambda(ds)$$
$$\hat{\pi}(y) = \tau\beta_A \int (\pi(s, y) - \hat{\pi}(y))I\{\pi(s, y) > \hat{\pi}(y))\}\lambda(ds) \tag{16}$$

Given λ and β_A, the cutoff of the optimal strategy is a solution to (16). The solution is unique because the left-hand side of (16) is strictly increasing in $\hat{\pi}(y)$ while the left hand side is decreasing in $\hat{\pi}(y)$.

Step 2. Existence. Consider a correspondence $\psi \colon [0, 1] \rightrightarrows [0, 1]$, which maps $\bar{\rho}$ to a set of "reaction" $\bar{\rho}$'s by the following procedure. To find $\psi(\bar{\rho})$, first find the unique cutoff function $\hat{\pi}$ from (16) using $\beta_A = \beta/(1-\bar{\rho})$. Cutoff $\hat{\pi}$ does not pin down the acceptance rates α uniquely because marginal signals from S_m can have positive probability under λ. The acceptance rates congruent with $\hat{\pi}(y)$ are integrable $\alpha(y)$ such that

$$\lambda(S_a(y)) \leqslant \alpha(y) \leqslant \lambda(S_a(y)) + \lambda(S_m(y)), \quad \forall y \in Y.$$

Take all α that are congruent with $\hat{\pi}$, call this set $\mathcal{A}$. For any $\alpha \in \mathcal{A}$, find $\bar{\rho}$ as shown in Lemma 5. Going over all $\mathcal{A}$ will produce the set of $\bar{\rho}$. This will be $\psi(\bar{\rho})$.

Clearly, $\mathcal{A}$ is convex. Thus, $\psi(\bar{\rho})$ is also convex. Since $\psi(\bar{\rho})$ is an interval subset of $[0, 1]$, it is easy to see that $\psi(\bar{\rho})$ is also closed. ψ is upper hemi-continuous because $\hat{\pi}$ is continuous in β_A according to (16). Therefore, by Kakutani's theorem, ψ has a fixed point.

Step 3. Uniqueness. Suppose $\bar{\rho}^1$ and $\bar{\rho}^2$ are two distinct fixed points of ψ. Suppose $\bar{\rho}^1 > \bar{\rho}^2$. Then $\beta_A^1 > \beta_A^2$. By (16) $\hat{\pi}^1(y) > \hat{\pi}^2(y)$ for any $y \in Y$. If so, then whenever $\pi(s, y) \geqslant \hat{\pi}^1(y)$ we also have $\pi(s, y) > \hat{\pi}^2(y)$. But this means that $S_a^2(y) \supseteq S_a^1(y) \cup S_m^1(y)$. Therefore $\alpha^2(y) \geqslant \lambda(S_a^2) \geqslant \lambda(S_a^1 \cup S_m^1) \geqslant \alpha^1(y)$ for all y. By Lemma 5, this implies $\bar{\rho}^2 \geqslant \bar{\rho}^1$. A contradiction.

We showed that there is a unique $\bar{\rho}$ that can arise in a steady-state equilibrium. By the argument below (16), the strategy cutoffs $\hat{\pi}(\cdot)$ are also pinned down uniquely in the steady-state equilibrium. □

Proof of Proposition 2. By (4),

$$\begin{aligned} V(y) &= \beta_A \int (\pi(s, y) - \tau V(y))\sigma(s, y) d\lambda(s) \\ V(y) &= \beta_A \nu(y) \int \pi(s, y)\sigma(s, y) d\lambda(s), \end{aligned}$$

where $\nu(y) = (1 + \tau\beta_A \alpha(y))^{-1}$. In a steady state, by (2), $\beta_A \nu(y) = \rho(y)/(\tau\alpha(y))$. Therefore,

$$\begin{aligned} V(y) = \frac{\int \pi(s, y)\sigma(s, y) d\lambda(s)}{\alpha(y)} \frac{\rho(y)}{\tau} = \\ = \mathbb{E}[\pi(s, y) \mid \text{acc.}] \frac{\rho(y)}{\tau}. \end{aligned}$$

Now σ^{FD} prescribes that jobs $\{x \colon \pi(x, y) > \tau V(y)\}$ are accepted with probability 1. Consider an alternative strategy profile

$$\tilde{\sigma}(x, y) = \begin{cases} 1, & \pi(x, y) > 0 \\ 0, & \pi(x, y) \leqslant 0 \end{cases}$$

Since $V(y) > 0$, we have $\mathbb{E}[\pi(s, y) \mid \text{acc. with } \sigma^{FD}] < \mathbb{E}[\pi(s, y) \mid \text{acc. with } \tilde{\sigma}]$. Also, $\alpha^{FD}(y) < \tilde{\alpha}(y)$. Finally, by Lemma 5, $\rho^{FD}(y) < \tilde{\rho}(y)$. Therefore, $V^{FD}(y) < \tilde{V}(y)$. □

Proof of Proposition 3. Take any Pareto optimal pair $O = (V, CS)$. Since O is feasible, there is seller strategy profile σ that induces O. Since there is one seller type and two actions, it is sufficient to consider only the binary signaling structures (Revelation principle). A binary signaling structure has two signals, where a signal is "action recommendation". Let s_a be the recommendation to accept, and s_r be the recommendation to reject. Denote this signaling structure by $\hat{\lambda}$. We need to check the incentive constraints, that is, to make sure that the sellers would follow the recommendations of $\hat{\lambda}$.

From (4) we have that $v(s_a) = \pi(s_a) - \tau V$, $v(s_r) = 0$, and $V = \beta_A(\pi(s_a) - \tau V)\hat{\lambda}(s_a)$. The incentive constraints require that $\pi(s_a) \geqslant \tau V$ and $\pi(s_r) \leqslant 0$. For the former,

$$\tau V = \frac{\tau\beta_A \hat{\lambda}(s_a)}{1 + \tau\beta_A \hat{\lambda}(s_a)} \pi(s_a) < \pi(s_a).$$

For the latter, recall that O is Pareto optimal, hence σ accepts all profitable jobs. This implies that

$$\pi(s_r) \leqslant 0 < \tau V.$$

$\square$

C Proof of Proposition 4

The proof of Proposition 4 relies on a sequence of lemmas.

Lemma 8. *Suppose the linear payoff environment with $G = U[0, \overline{y}]$. For any (large) $L > 0$ there is $\beta\tau < 1$ large enough and $\breve{y}$ (large enough) such that if $\overline{y} \geqslant \breve{y}$, we have $\tau\beta_A \geqslant L$.*

Proof. Since $\beta_A = \beta/\bar{\nu}$, rewrite (14) as

$$\beta\tau = \int_0^{\overline{y}} \frac{dy/\overline{y}}{\frac{1}{\tau\beta_A} + \alpha(y; \tau\beta_A)}, \tag{17}$$

where I made explicit that α depends on $\tau\beta_A$ in equilibrium. Treat the right hand side of (17) as a function of $\tau\beta_A$ and $\overline{y}$ and denote it by $\psi(\tau\beta_A, \overline{y})$. First, ψ is strictly increasing in $\tau\beta_A$ because by Lemma 7 α is decreasing in $\tau\beta_A$. Second, ψ is decreasing in $\overline{y}$ because by Lemma 7 α is increasing in y. Third, using the second part of Lemma 7 and uniformity of G, $\lim_{\overline{y}\to\infty} \psi = ((\tau\beta_A)^{-1} + 1)^{-1} < 1$. Pick $\breve{y}$ such that $\psi(L, \breve{y}) < 1$. Let $(\beta\tau)^* := \psi(L, \breve{y})$. We have that $\beta_A\tau > L$ whenever $\beta\tau > (\beta\tau)^*$ or $y > \breve{y}$. $\square$

Lemma 9. *Suppose the linear payoff environment with $G = U[0, \overline{y}]$. For any (small) $\varepsilon > 0$ there is $\beta\tau < 1$ large enough and $\overline{y}$ large enough, such that $\nu(\overline{y}) = \varepsilon$.*

Proof. Note that $\nu(y) = \frac{1}{1+\tau\beta_A\alpha(y)}$. Pick $L > \varepsilon^{-1} - 1$. By Lemma 8 for any $L > 0$, we can find $\beta\tau$ and $\breve{y}$ such that if $\overline{y} \geqslant \breve{y}$, $\tau\beta_A \geqslant L$. Since $\operatorname{supp} F = [0, 1]$, we have from (8) that $\alpha(\overline{y}) = 1$ for $\overline{y}$ large enough. Therefore, $\nu(\overline{y}) = \frac{1}{1+\tau\beta_A\alpha(\overline{y})} < \frac{1}{1+L} < \varepsilon$. $\square$

Lemma 10. *$\lambda \in \Delta(S)$ is x^*-upper-coarsening if and only if the corresponding Λ has the following form:*

$$\Lambda(z) = \begin{cases} \overline{\Lambda}(z), & z \in [0, x^*] \\ \overline{\Lambda}(x^*) + F(x^*)(z - x^*), & z \in (x^*, \mathbb{E}[x|x > x^*]) \\ \underline{\Lambda}(z), & z \in [\mathbb{E}[x|x > x^*], 1] \end{cases} \tag{18}$$

Proof of Lemma 3. Step 1. The equilibrium values of $\alpha(y)$ and $\bar{\rho}$ are found using the system of equations (8) and (14). Since $\beta_A = \beta/(1 - \bar{\rho})$, rewrite them as:

$$y - \hat{z}(y) = \tau\beta_A\Lambda(\hat{z}(y)), \quad \forall y \in Y; \tag{19}$$

$$\beta_A \int \frac{dG(y)}{\tau\beta_A\alpha(y) + 1} = \beta. \tag{20}$$

Denote the fraction of available sellers by

$$\nu(y) := \frac{1}{\tau\beta_A\alpha(y)+1} = 1-\rho(y).$$

Differentiating (19) wrt y, find that

$$\hat{z}'(y) = \nu(y).$$

Step 2. Consider a feasible variation $\delta\Lambda(y)$, that is the one such that $\Lambda + \delta\Lambda \in \mathcal{L}$. We are going to track variation in all endogeneous variables to eventually solve for δM and δV. Differentiate (19):

$$-\delta\hat{z}(y) = \tau\delta\beta_A\Lambda(\hat{z}(y)) + \tau\beta_A\delta\Lambda(\hat{z}(y)) + \tau\beta_A\alpha(y)\delta\hat{z}(y),$$

from where

$$\delta\hat{z}(y) = -\nu(y)\left[\tau\delta\beta_A\Lambda(\hat{z}(y)) + \tau\beta_A\delta\Lambda(\hat{z}(y))\right]. \tag{21}$$

Differentiate with respect to y:

$$\begin{aligned}
\delta\hat{z}'(y) =& -\nu'(y)\left[\tau\delta\beta_A\Lambda(\hat{z}(y)) + \tau\beta_A\delta\Lambda(\hat{z}(y))\right] \\
& -\nu(y)\left[\tau\delta\beta_A\alpha(y)\hat{z}'(y) + \tau\beta_A\delta\Lambda'(\hat{z}(y))\hat{z}'(y)\right] \\
\delta\hat{z}'(y) =& -\tau\nu'(y)\left[\delta\beta_A\Lambda(\hat{z}(y)) + \beta_A\delta\Lambda(\hat{z}(y))\right] \\
& -\tau\nu^2(y)\left[\delta\beta_A\alpha(y) + \beta_A\delta\Lambda'(\hat{z}(y))\right]
\end{aligned}$$

From (24),

$$\begin{aligned}
\delta\beta_A\bar{\nu} = -\beta_A\delta\bar{\nu} = -\beta_A\int\delta\nu(y)dG(y) &= -\beta_A\int\delta\hat{z}'(y)dG(y) \\
= \tau\beta_A\int\delta\beta_A\left[\Lambda(\hat{z}(y))\nu'(y) + \alpha(y)\nu^2(y)\right]dG(y) \\
+\tau\beta_A\int\left[\nu'(y)\beta_A\delta\Lambda(\hat{z}(y)) + \nu^2(y)\beta_A\delta\Lambda'(\hat{z}(y))\right]dG(y).
\end{aligned} \tag{22}$$

Manipulate separately the integral

$$\begin{aligned}
&\int\nu^2(y)\delta\Lambda'(\hat{z}(y))dG(y) = \\
&\qquad = \int\nu^2(y)g(y)d\left(\delta\Lambda(\hat{z}(y))\right) = \\
&\qquad = \nu^2(y)g(y)\delta\Lambda(\hat{z}(y))|_0^\infty - \int\delta\Lambda(\hat{z}(y))\left(\nu^2(y)g(y)\right)'dy,
\end{aligned}$$

where we used the integration by parts in the third line. Since feasible variations have $\delta\Lambda|_0^\infty = 0$, the first term in the expression above is zero, and so,

$$\int\nu^2(y)\delta\Lambda'(\hat{z}(y))dG(y) = -\int\delta\Lambda(\hat{z}(y))\left(\nu^2(y)g(y)\right)'dy.$$

Put it back into (22), collect terms with $\delta\beta_A$ in the left hand side, and obtain the equality

$$\delta\beta_A \int \left[\nu(y) - \tau\beta_A\Lambda(\hat{z}(y))\nu'(y) + \tau\beta_A\alpha(y)\nu^2(y)\right] dG(y) = \\ \tau\beta_A^2 \int \delta\Lambda(\hat{z}(y)) \left[\nu'(y)g(y) - \left(\nu^2(y)g(y)\right)'\right] dy.$$

Use the fact that $\tau\beta_A\alpha(y) = 1/\nu(y) - 1$ to simplify the expression.

$$\delta\beta_A \int \left[\nu^2(y) - \tau\beta_A\Lambda(\hat{z}(y))\nu'(y)\right] dG(y) = \\ \tau\beta_A^2 \int \delta\Lambda(\hat{z}(y)) \left[\nu'(y)g(y) - \left(\nu^2(y)g(y)\right)'\right] dy.$$

By the definition of variational derivative,

$$\frac{\delta\beta_A}{\delta\Lambda} = \frac{\tau\beta_A^2}{\int \left[\nu^2(y) - \tau V(y)\nu'(y)\right]} \left[\nu'(y)g(y) - \left(\nu^2(y)g(y)\right)'\right]. \tag{23}$$

Step 3. The flow of type-y sellers who become available has rate $\rho(y)g(y)/\tau$. In a steady state, each vacancing seller is equivalent to one completed match. Hence the number of matches per unit of time is

$$M = \int \frac{\rho(y)g(y)}{\tau} dy = \int \frac{1-\nu(y)}{\tau} dG(y) = \frac{1-\bar{\nu}}{\tau}.$$

Since $\beta_A = \beta/(1-\bar{\rho}) = \beta/\bar{\nu}$,

$$\beta_A\bar{\nu} = \beta.$$

Take the differential,

$$\delta\beta_A\bar{\nu} + \beta_A\delta\bar{\nu} = 0. \tag{24}$$

Combine (23) and (24):

$$\begin{aligned}
\frac{\delta M}{\delta\Lambda} &= \frac{\bar{\nu}}{\tau\beta_A}\frac{\delta\beta_A}{\delta\Lambda} = \\
&= \frac{\bar{\nu}\beta_A}{\int \left[\nu^2(y) - \tau V(y)\nu'(y)\right]} \left[\nu'(y)g(y) - \left(\nu^2(y)g(y)\right)'\right] = \\
&= \frac{\beta}{\int \left[\nu^2(y) - \tau V(y)\nu'(y)\right]} \left[\nu'(y)g(y) - \left(\nu^2(y)g(y)\right)'\right].
\end{aligned}$$

Step 4. By Lemma 2,

$$V(y) = \frac{y - \hat{z}(y)}{\tau},$$

the differential of joint profits is

$$\delta V = \frac{1}{\tau}\int (-\delta\hat{z}(y)) dG(y).$$

Use (21) to find:

$$
\begin{aligned}
\delta V &= \frac{1}{\tau} \int \nu(y) \left[\tau \delta\beta_A \Lambda(\hat{z}(y)) + \tau\beta_A \delta\Lambda(\hat{z}(y))\right] dG(y) = \\
&= \int \nu(y) \left[\frac{\tau\beta_A}{\bar{\nu}} \delta M \Lambda(\hat{z}(y)) + \beta_A \delta\Lambda(\hat{z}(y))\right] dG(y) = \\
&= \delta M \frac{\tau\beta_A}{\bar{\nu}} \int \nu(y)\Lambda(\hat{z}(y)) dG(y) + \int \nu(y)\beta_A \delta\Lambda(\hat{z}(y)) dG(y) = \\
&= \delta M \frac{\tau}{\bar{\nu}} \int \nu(y) V(y) dG(y) + \int \nu(y)\beta_A \delta\Lambda(\hat{z}(y)) dG(y)
\end{aligned}
$$

By the definition of variational derivative,

$$
\frac{\delta V}{\delta \Lambda} = \frac{\delta M}{\delta \Lambda} \cdot \frac{\tau}{\bar{\nu}} \int \nu(y) V(y) dG(y) + \nu(y)\beta_A g(y).
$$

□

Proof of Lemma 4. Suppose the contrary, λ_0 is not upper-coarsening. I will show that there is a deviation from λ_0 that increases $\mathcal{J}$.

Let $\Lambda_0 \in \mathcal{L}$ which corresponds to λ_0. Let y^* be the zero of $\delta\mathcal{J}/\delta\Lambda$, and the corresponding cutoff in the seller optimization problem be $z^* = \hat{z}(y^*; \Lambda_0)$. I am now going to construct a feasible variation Λ^ε from Λ_0 that increases $\mathcal{J}$.

Let $\tilde{\Lambda}$ be the upper-coarsening (of the form (18)) that passes through point $(z^*, \Lambda_0(z^*))$. There is only one such function because there is only one line passing through $(z^*, \Lambda_0(z^*))$ and is tangential to $\overline{\Lambda}$. We have that $\Lambda_0(z) \geqslant \tilde{\Lambda}(z)$ on $z > z^*$, and $\Lambda_0(z) \leqslant \tilde{\Lambda}(z)$ on $z < z^*$. Moreover, since Λ_0 is not upper-coarsening, for some $z \in [0, 1]$ one of these inequalities is strict. Consider variation

$$
\Lambda^\varepsilon(z) = (\tilde{\Lambda}(z) - \Lambda_0(z))\varepsilon.
$$

Since $\delta\mathcal{J}/\delta\Lambda < 0$ on $z > z^*$ and $\delta\mathcal{J}/\delta\Lambda > 0$ on $z < z^*$, Λ^ε increases $\mathcal{J}$. □

Proof of Proposition 4. Here I provide the entire complete proof. Step 1. By Lemma 3,

$$
\begin{aligned}
\frac{\delta \mathcal{J}}{\delta \Lambda} &= \gamma u \frac{\delta M}{\delta \Lambda} + (1-\gamma)\left(\frac{\delta M}{\delta \Lambda} K_2 + \nu(y) g(y) \beta_A\right) = \\
&= (\gamma u + K_2(1-\gamma)) \frac{\delta M}{\delta \Lambda} + (1-\gamma)\beta_A \nu(y), \qquad (25)
\end{aligned}
$$

where $K_2 > 0$. Evaluating with $G = U[0, \overline{y}]$,

$$
\frac{\delta M}{\delta \Lambda} = K_1 \overline{y}^{-1} (\nu(y) - \nu^2(y))' = K_1 \overline{y}^{-1} (1 - 2\nu(y)) \nu'(y), \qquad (26)
$$

where $K_1 > 0$. Since $\nu(0) = 1$ and $\nu(y)$ is non-negative and decreasing, $\delta\mathcal{J}/\delta\Lambda$ is either positive for all $y \in [0, \overline{y}]$ or crosses zero from above once. Let λ^*_γ denote the disclosure policy that maximizes $\mathcal{J}(\gamma)$. By Lemma 4, λ^*_γ is upper-coarsening.

Step 2. To see that the cutoff x^*_γ is decreasing in γ, note that larger γ puts more weight on the positive term in (25). Therefore the region of Y with negative $\delta\mathcal{J}/\delta\Lambda$ is smaller.

Step 3. Suppose $\beta\tau < 1/2$. Using Lemma 6, $\beta_A\tau \leqslant \beta\tau/(1-\beta\tau) < 1$. Next, $\nu(y) = (1+\beta_A\tau\alpha(y))^{-1} > (1+1\cdot 1)^{-1} = 1/2$ for any y. Using (26), $\frac{\delta M}{\delta\Lambda} \geqslant 0$ for any y, and so it $\delta\mathcal{J}/\delta\Lambda$. Therefore, full disclosure is optimal for any γ.

□

D Technical Extensions

D.1 No Excess Demand Assumption relaxed

Allow for the case when $\beta(1)\tau \geqslant 1$. If $\beta(1)\tau > 1$, then sellers get overwhelmed by the buyer requests and can't respond to all of them to the extent that they can't even reject them. To cover this situation we assume that if there are no available sellers to reject a pending buyer request, the platform rejects it automatically.

Since some requests can be rejected by the platform, the acceptance rate as perceived by buyers does not coincide with the acceptance rate α generated by sellers. Denote by α^e the effective acceptance rate that buyers face. Let at some moment of time there is $x \in [0,1]$ mass of available sellers, and let buyers arrive to the platform at rate β. Then within the next time interval dt, there are βdt new request, and $x + \left(\frac{1-x}{\tau}dt\right)$ available sellers. What is α^e when sellers use acceptance rate α? Consider three cases.

1. $x > 0$. There is plenty of available sellers, $x + \left(\frac{1-x}{\tau}dt\right) > \beta dt$. Fraction α of buyers are accepted, therefore $\alpha^e = \alpha$.

2. $x = 0$ and $\alpha\beta < \frac{1}{\tau}$. There are few sellers that just became available but in sufficient number to process all buyers. In the same fashion as in case 1, $\alpha^e = \alpha$.

3. $x = 0$ but $\alpha\beta \geqslant \frac{1}{\tau}$. Not sufficient sellers to process all buyers, some buyers are rejected by the platform. The number of accepted jobs is $\frac{1}{\tau}dt$. The acceptance rate is therefore $\alpha^e = \frac{1/\tau}{\beta}$.

Combining all there cases, we have that

$$\alpha^e = \min\{\alpha, \frac{1}{\tau\beta}\}.$$

The adjusted definition of equilibrium is then the following.

1.
$$\alpha \in [F(c^*(\beta_A)-), F(c^*(\beta_A)+)].$$

2.
$$\beta_A = \frac{\beta(\alpha^e)}{1-\beta(\alpha^e)\alpha^e\tau} = \begin{cases} \frac{\beta(\alpha)}{1-\beta(\alpha)\alpha\tau}, & \alpha\beta(\alpha)\tau < 1 \\ +\infty, & \alpha\beta(\alpha)\tau \geqslant 1 \end{cases}$$

$\beta_A = +\infty$ reflects the fact that when the demand is overwhelming, buyers line up for sellers so sellers start a new job immediately after they finish the previous one. The No Excess Demand assumption makes sure that this never happens, $\alpha\beta(\alpha)\tau < 1$ for all $\alpha \in [0,1]$.

The next result shows that in the equilibrium there are no lines. By this reason for the clarity of exposition we decided to restrict the analysis to the case of no lines to begin with.

Claim. In equilibrium, $\beta_A < \infty$.

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