Ignorance is Strength: Improving Performance of Decentralized Matching Markets by Limiting Information

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Job Market Paper

Abstract

I develop a model of a decentralized matching market in which buyers pursue sellers by proposing jobs. Buyer requests are perfectly coordinated. Sellers have non-trivial preferences over jobs and independently choose what jobs to accept. Preference heterogeneity induces same-side and cross-side externalities that generate sellers' excessive screening and leads to welfare loss. Platform's policy of coarse revelation of buyer information improves acceptance rates by decreasing the ability of sellers to "cherry-pick" desirable buyers. Implication for welfare-maximizing platform is to limit information to the sellers when they are more patient and more substitutable side of the market. However, if sellers has unobserved heterogeneity, coarsening is effective only if sellers are tightly capacity constrained or buyer-to-seller ratio is large.

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1 Introduction

In decentralized matching markets, such as market for lodging, labor market, dating market and others, information availability is the key to the well-functioning market. Information allows the participants to identify the most valuable options and facilitates trade (Akerlof 70, Roth (2008)). However, information availability exacerbates the problem of excessive screening, which is an important source of market failure in matching markets. A distinguishing feature of a matching market relative to a consumer market is that buyers not only choose but must also be chosen by sellers (Roth 08). In fast-moving marketplaces seller rejections are consequential. As a stark example, on Airbnb, conditional on being rejected from their first request, guests are 51% less likely to eventually book; however 20% of inquiries are screened by hosts (Fradkin 2016). On Uber, passengers are highly sensitive to wait times (Hall *et al.* (2015)). Buyer disutility from delays or longer search decreases welfare and participation and undermines the marketplace performance. Can the platform design an information intermediation policy to alleviate the excessive screening problem and improve efficiency (revenue, number of matches)? What does the optimal disclosure policy depend on?

Optimal information disclosure must balance two opposing forces—letting the participants find their preferred matches, and preventing the cream-skimming behavior that leads to slow matching. On the one hand, information availability reveals the heterogeneity of buyers and allows the sellers to identify those they value the most. This increases seller surplus. On the other hand, the screening can take long. If the sellers' preferences for matches are not aligned with the buyers', or if buyers have higher cost of rejection (e.g. due to higher time-sensitivity), then meticulous screening hurts buyer surplus. When little information is available, the sellers settle for less and screen faster. Information coarsening is thus a welfare-improving intervention. It has been recognized in the information economics literature that the latter, "strategic", effect of information disclosure can outweigh the former, "individual choice" effect (Bergemann and Morris (2016)). Despite the fact that information intermediation is potentially powerful design tool, the implications of this general observations to the context of decentralized matching markets have not been well explored.

In this paper I develop a model of information intermediation in decentralized matching markets in order to formalize this information disclosure tradeoffs and to characterize the optimal disclosure policy for a class of decentralized matching environments. section 2 sets up the model in which short-lived buyers gradually arrive over time and pursue long-lived sellers by proposing jobs. Sellers have heterogeneous preferences over jobs and independently decide what jobs to accept. Sellers have limited capacity: when a seller accepts a job, he becomes busy and cannot accept new jobs. Buyer and seller preferences over matches are not aligned: jobs unprofitable for sellers may be valuable for buyers. In contrast to many search-and-matching models that have a coordination friction as the main matching friction (Burdett *et al.* (2001); Kircher (2009); Arnosti *et al.* (2014)), I assume no coordination friction: only available sellers receive buyer requests. The assumption is motivated by the observation that most digital platforms have good technological means of solving the coordination problem.[1] The matching friction of the model therefore pertains to preference heterogeneity.

[1] E.g. Fradkin (2015) finds that on Airbnb the coordination friction explains only 6% of failed matches.

The platform uses information disclosure policy as a design tool. Before the matching process starts running, the platform commits to a disclosure policy that governs what buyer information is revealed to sellers each time they have to make the acceptance decision. The policy is modeled as a signaling game between the platform and heterogeneous sellers, built on Kolotilin *et al.* (2015). The new element is the seller dynamic optimization and endogenous availability of sellers. The platform's objective is to maximize the weighted sum of buyer surplus and seller profits.

If sellers are identical, then some information coarsening is always optimal. In this case, as shown in subsection 3.2, the efficient coarsening involves pooling inframarginal profitable jobs with marginal unprofitable but efficient jobs. This way, sellers' actions get aligned with the platform's objective. Higher buyer search costs imply a larger set of efficient matches, and so the efficient disclosure exhibits more coarsening. However, the sellers may have payoff heterogeneity unobserved by the platform. In this case, information coarsening tailored to increase the acceptance rate of one seller can overburden another seller and violate his individual rationality constraint. What coarsening is optimal, if any, is now not obvious, as the optimal policy depends on the distribution of seller types and should accommodate the diverse reactions to information.

In the linear payoff environment, with vertically differentiated buyers and vertically differentiated sellers, the optimal disclosure policy depends on the intensity of buyer traffic and the tightness of seller capacity constraints. Here, seller match payoff is $y-x$, with seller type $y \sim U[0,\overline{y}]$ and buyer type $x \in [0,1]$, and buyer payoff is constant. In this case, as shown in subsection 3.4, the optimal match-maximizing disclosure policy is *upper-coarsening*: high buyer types are pooled, and low buyer types are revealed truthfully. This is in stark contrast with the static case (cf. Kolotilin *et al.* (2015)), in which information disclosure does not affect the number of matches. When buyer-to-seller ration is high or the seller capacity constraints are tighter, the efficient disclosure is also upper-coarsening. In a converse case, the full disclosure is efficient. For the general distribution of seller types, the optimal disclosure depends on whether pdf of seller types is decreasing or increasing and the seller utilization rates. If give the first-order condition for the general case in Lemma 3.

Disclosure to sellers has three competing effects on welfare, with the optimal disclosure determined by what effect is dominant. The first is the standard *Individual Choice effect* operating on the seller side of the market. From an individual seller's point of view, more information increases his set of attainable payoffs (Blackwell (1953)). Holding the other sellers' behavior fixed, he individually benefits from more information about buyers. The second effect of information disclosure is the *Buyer-side effect*. More information available to sellers reduces the platform's ability to induce sellers to accept buyer-valuable jobs. The third effect is the *Seller Option Value effect* which arises only with capacity constrained sellers. More information available to sellers increases their ability to "cream-skim" the stream of jobs and increases the option value of rejection. In equilibrium, the Option Value effect leads to a range profitable jobs being rejected, which hurts the joint seller surplus. The Buyer-side and the Option Value effect are the instances of a general principle that revealing more information to sellers imposes more constraints on the designer and reduces the set of outcomes he can induce (Bergemann and Morris (2016)). Both the Buyer-side and the Option Value effects lead to sellers' excessive screening — rejection rates are inefficiently high. Therefore, the Buyer-side and the Option Value effects give a motive to the platform to coarsen information. On the contrary, the Individual Choice effect gives a motive to disclose information.

Negative effects of disclosure on sellers (the Option Value effect) is a form of seller coordination failure. In a marketplace where the sellers act independently, each seller keeps his schedule open by rejecting low-value jobs to increase his individual chances of getting high-value jobs. As a result, sellers spend a lot of time waiting for the high-value jobs. Collectively, this behavior is suboptimal because all profitable jobs have to be completed. The source of the coordination failure is what I call the *cream-skimming externality*: By rejecting a job, a seller remains available on the marketplace and attracts a fraction of subsequent buyers, who otherwise would go the other sellers. As a result, the other sellers face fewer profitable jobs and obtain lower profits.

The contribution of the present paper is two-fold. First, the paper explains the role of information intermediation in decentralized matching markets. It shows that strategic information disclosure can be an effective tool to balance the seller's demand for more transparency and buyer's demand for less hassle and more speed. Simultaneously, it can alleviate the seller coordination failure due to cream-skimming externality.

On the technical side, the paper contributes to the information design literature by extending the signaling game with heterogeneous audience to the case with endogenously available and dynamically optimizing receivers. In this case, the design of information disclosure is a non-trivial problem. Information disclosure policy determines not only the stage payoff of a receiver but also the distribution of future potential payoffs. As a result, a patient receiver's decision to accept depends not only on the posterior mean of the state but also on the entire signaling structure. This makes the concavification approach of Kamenica and Gentzkow (2011), as well as the linear programming approach of Kolotilin (2015) unsuitable for the analysis. I approach this information design problem by representing signaling structure as a convex function from a certain set and using calculus of variations to find the first-order necessary conditions. subsection 3.5 sketches the main steps of the approach.

Related Literature. The paper contributes to matching markets and information design strands of economic literature. There have been studies on matching markets in the literatures of economics and operations research. To the best of my knowledge, no prior work examined information disclosure as a design tool in a search-and-matching setting.

Search and matching also plays a large role in the economics of labor, housing, and dating markets. Theoretical results in this literature such as Burdett, Shi and Wright (2001), Albrecht, Gautier and Vroman (2006), and Kircher (2009) show that markets where sellers have limited capacity are inherently more frictional than markets with large firms. However, the main cause of inefficiency in these papers is mis-coordination due to simultaneity and unavailability. Halaburda (2010); Arnosti *et al.* (2014) propose to limit the intensity of applications to allay this friction. On the contrary, I focus on the frictions owing to preference heterogeneity and screening. In a market with pronounced heterogeneity, information intermediation is an important design tool. Although the first-order effect of information is positive (Lewis (2011), Tadelis and Zettelmeyer (2015)), the problem of failed matches can be severe (Fradkin (2015); Horton (2015); Cullen and Farronato (2015)). Flexible pricing and innovative matching mechanisms such as auctions (Einav et al. (Forthcoming), employer initiated search (Horton (Forthcoming)), and surge pricing (Hall, Kendrick and Nosko (2016)) are frequently used to clear the market. Milgrom (2010) shows that conflation increases prices of auctioned items if the items are identical and buyers have bidding costs. In his model, supply is fixed because the number of goods is pre-determined.

On the methodological side, my work relies and extends the literature of communication games (Aumann *et al.* (1995); Grossman and Hart (1980); Milgrom (1981); Kamenica and Gentzkow (2011); Rayo and Segal (2012); Kolotilin *et al.* (2015); Bergemann and Morris (2016)). The most closely related paper is Kolotilin *et al.* (2015) from where I adopt the framework of the signaling game with heterogeneous audience by extend it to the case with patient and dynamically available receivers. The main setting with heterogeneous sellers assumes that buyer surplus is linear in the number of matches and implies that buyer-seller complementarities are not realized . The information disclosure with buyer-seller complementarities is the topic of Rayo and Segal (2012), who show in the static setting that the optimal disclosure policy is based on "non-ordered prospects". My work focuses on the role of time in information disclosure and I assume all prospects are "ordered" in the language of Rayo and Segal (2012).

The paper is related to the broader literature of information disclosure in markets. The seminal papers of Akerlof and Hirschleifer drew attention to the role of information and its possible opposing effects on welfare (asymmetric information and risk-sharing, respectively). The advertising literature finds that effects of better targeting is ambiguous because it affects both demand and supply. On the one hand, better targeting increases demand for advertising. On the other hand, it decreases competition on the supply side (Bergemann and Bonatti (2011)) and improves ad space allocation (Athey and Gans (2010)).

A related paper of Hoppe *et al.* (2009) shows that disclosing too much information can induce excessive costly signaling which overcomes the gains from improved matching. Hoppe *et al.* (2009) as well as some other papers in labor literature assume the black-box matching function (Becker (1973); Shimer and Smith (2000)) that does not allow to evaluate the policies affecting the matching mechanism.

Bergemann and Morris (2016) organize the different effects of information disclosure scattered in the literature into the Individual Choice effect, that benefits each individual due to the "experiment" value of information (Blackwell (1953)), and the Strategic effect, that restricts the set of outcomes attainable in equilibrium.

My work differs from the theory of centralized matching in that it generally assumes that agents know their true preferences over potential partners prior to engaging in a match — For a survey, see Roth and Sotomayor (1990). Critically, the participants of decentralized matching markets have to inspect potential matches to identify the valuable ones. Related papers that also study dynamic matching are Ashlagi *et al.* (2013); Akbarpour *et al.* (2016). They show that waiting to thicken improves matching in kidney exchange because individual agents do not fully incorporate the benefits of waiting. I show there is a "reversed" effect: individual workers wait too long because they do not fully incorporate the buyer loss form rejections (or buyer waiting costs).

The paper contributes to the broader economic question of the span of control of planner/intermediary. Thus, information coarsening implies more control. Hagiu and Wright (2015) explains the role of the intermediary using the contract theory, and the answer depends on who, the platform or sellers, engages in complimentary activities, such as marketing and advertising.

There is extensive operations research literature on staffing and queuing problems for platforms. Gurvich *et al.* (2015); Banerjee *et al.* (2015) study price incentives schemes for staffing problem under uncertain demand but in both papers the supply side does not cream-skim. Cachon *et al.* (2015) shows that surge-price practices of ride-sharing platforms is nearly revenue-maximizing but also generates higher welfare than in the fixed-price or fixed-wage contract. Taylor (2016) and Arnosti *et al.* (2014) study how congestion affect the platform performance. Taylor (2016) show that congestion may have a counter-intuitive effect on optimal pricing in two-sided markets. Namely, in the presence of the uncertainty congestion may increase the optimal customer price and decrease the optimal wage. Arnosti *et al.* (2014) study the matching market when applicants can send multiple applications and employer's screening is costly. They show that if the screening cost is large enough, restricting the number of applications is strict Pareto improvement for both applicants and employers.

2 The Model of Decentralized Matching Market

In this section I lay out a model of a decentralized matching market, in which buyers arrive gradually over time and pursue sellers by proposing a job; sellers have heterogenous preferences over buyers, seller capacities are limited, and sellers independently choose what matches to accept.

2.1 Setup

Spot matching process. There are three parties involved in the search and matching process: sellers, buyers and the platform itself.

There is mass 1 of sellers, who always stay on the platform, never leave or arrive. The sellers do not actively look for jobs, but instead screen the buyer requests: each worker is presented with a sequence of job offers at Poisson rate, and he decides whether to accept or reject them to maximize discounted profit flow. An accepted job takes time τ to complete, during which time the seller cannot receive new jobs. At each moment a seller is either available and waits for new jobs, or busy with working on a job.

There is a continuum of potential buyers who gradually arrive over time at flow rate β. That is, within a time interval dt, mass βdt of buyers arrive to the platform. Each buyer has a single job he wants to be completed. Upon arrival, the buyer proposes the job to one of the available sellers. I assume that the target seller is drawn uniformly at random from the pool of available workers. If the buyer's job is accepted, the buyer stays until the job is completed, otherwise he leaves the platform.

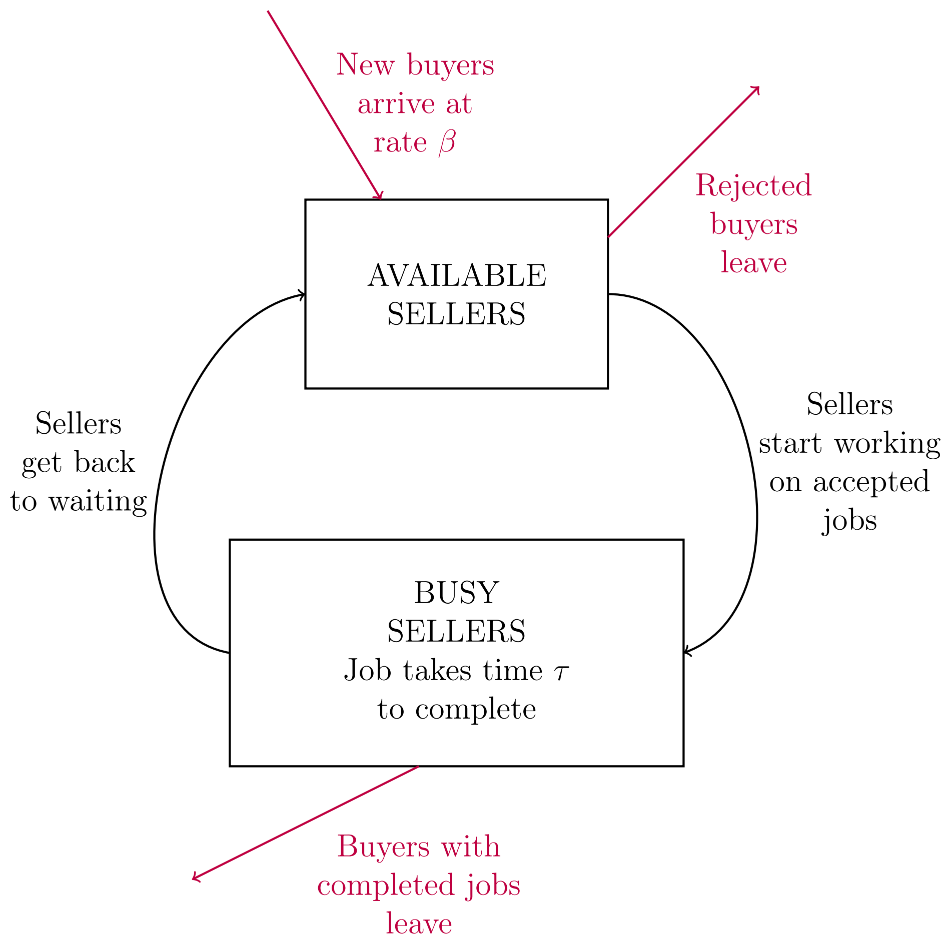
Assume $\beta\tau < 1$ which implies that collectively, it is physically possible for sellers to complete every buyer job. The assumption makes the exposition easier and relaxing it requires more notation to deal with either automatic rejections or queues. I do this in Appendix C and show that qualitatively results do not change.

The platform opts out of the centralized matching process, instead it makes disclosure of buyer characteristics part of its design, as described below. See Figure 1a for the illustration of the matching process.

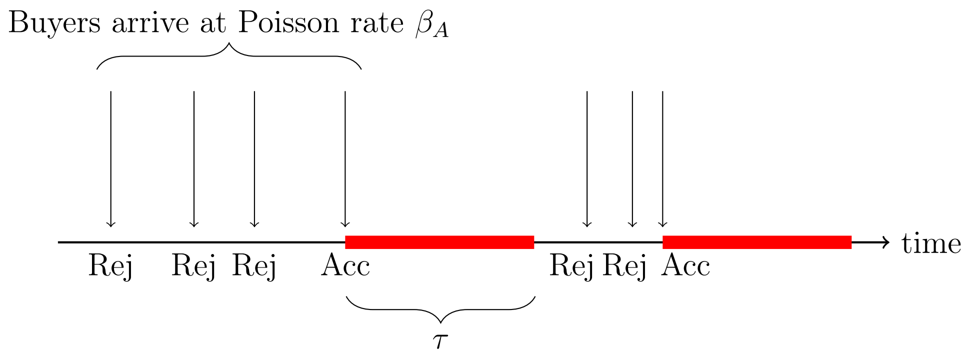
Buyer and seller preference heterogeneity. There are two dimensions of heterogeneity in the matching market. First, each seller has heterogeneous match payoff across buyers. Second, different seller have different payoff functions over jobs. Concerning the platform's information disclosure problem, I need the following pieces of notation. Let x be buyer type, with the interpretation that x is the buyer characteristics observed by the platform.[2] The space of buyer types X is a compact subset of Euclidean space. Distribution of x is F with full support. Let y be seller type, with the interpretation that y is the seller characteristics unobserved by the platform.[3] The space of seller types Y is a compact subset of Euclidean

[2]Buyer type x captures the payoff-relevant information the platform elicits from the buyer about the job, passively from the buyer's cookies and queries or actively by asking questions. For example, on Uber, x would include rider's destination; on Airbnb, x would include guest's race, age and gender.

[3]Seller type y captures the payoff-relevant information the platform did not elicit from sellers by whatever reason – costly, unethical, etc. For example, on Uber, y would include the driver's preference for long rides and traffic; on Airbnb, y would include the host's preference for his guest's age, gender, socio-economic



(a) Spot matching process. Buyers arrive at exogenous rate β, and contact available sellers. If rejected, a buyer leaves the platform. If accepted, the buyer forms a match which lasts for time τ. After the time elapses, the buyer leaves the platform, and the seller returns to waiting.



(b) Seller dynamic optimization problem with repeated search and waiting. An available seller receives requests at Poisson rate β_A. If a request is accepted, the seller becomes busy for time τ when he does not receive new requests.

Figure 1: Search and matching process.

space. Distribution of y is G with full support that admits density g, g is differentiable on Y. Seller profit for one match is $\pi(x, y)$. Assume π is continuous and for any y there is x such that $\pi(x, y) > 0$. Buyer net match payoff is $u(x, y)$. Assume that all incoming buyers have non-negative match payoff:

$$u(x, y) \geqslant 0. \tag{1}$$

Platform: Information intermediation. The platform plays a signaling game with sellers. The platform observes buyer type x and chooses how to reveal it to sellers. Sellers do not receive any additional information about x outside of what the platform tells them. Let $S = \Delta(X)$ be the set of all posterior distributions over X. *Information disclosure policy* $\lambda \in \Delta(S)$ is a probability distribution of posteriors.[4] The interpretation is $s \in S$ is the platform's signal to the seller, and so $\lambda(S')$ is the fraction of buyers with signals $S' \subset S$.[5] Note that a disclosure policy can be seen as a two-stage lottery on X whose reduced lottery is prior F.[6] The set of possible disclosure policies is then:

$$\left\{ \lambda \in \Delta(S) \colon \int s\lambda(ds) \sim F \right\}.$$

Full disclosure policy, denoted by λ^{FD}, perfectly reveals buyer type x to the sellers. No disclosure policy fully conceals x. Disclosure policy λ' is *coarser* than λ'' if λ' is a Blackwell garbling of λ''. That is, the platform can obtain λ' from λ'' by taking λ'' and partially concealing some x's. When a buyer of type x arrives, the platform draws a signal according to λ and shows it to the seller. The seller knows the platform's choice of λ, and so his interpretation of a signal as a posterior is correct.

Steady state distribution of sellers. A steady state of the matching process is characterized by the fraction of available sellers of every type and their acceptance rates. Formally, let $\alpha(y) \geqslant 0$ be the acceptance rate of type-y available sellers, and let $\rho(y)$ be the type-y *utilization rate* – the fraction of type-y sellers who are busy. Denote the total fraction of busy sellers by $\bar{\rho} := \int_Y \rho(y)dG(y)$.

In a steady state, the flow of sellers who become busy is equal to the flow of workers who become available. The available-to-busy flow is equal to the buyer flow type-y sellers receive times their acceptance rate. Since buyer get distributed uniformly across the available sellers, the buyer flow to type-y sellers is $\beta\frac{(1-\rho(y))g(y)}{1-\bar{\rho}}$. The acceptance rate is $\alpha(y)$. Thus, the available-to-busy flow is $\beta\frac{(1-\rho(y))g(y)}{1-\bar{\rho}}\alpha(y)$. The busy-to-available flow is $g(y)\rho(y)/\tau$ because the mass of busy y-sellers is $g(y)\rho(y)$, and jobs are completed in time τ. Hence, in a steady

status, race, etc.

[4] When X is a compact subset of a Euclidean space, $\Delta(S)$ is the set of Borel-measurable probability distributions with the weak-* topology on $\Delta(X)$.

[5] I focus on the "public" signaling when the same λ applies to all seller. I am not studying the mechanism design problem where the platform tries to elicit or learn the seller's type y and tailor the disclosure policy to seller type. Kolotilin *et al.* (2015) find in the one-shot signaling game with linear payoffs that public signaling is equivalent to private signaling.

[6] The two-stage lottery framework for disclosure is adopted from Bergemann *et al.* (2015).

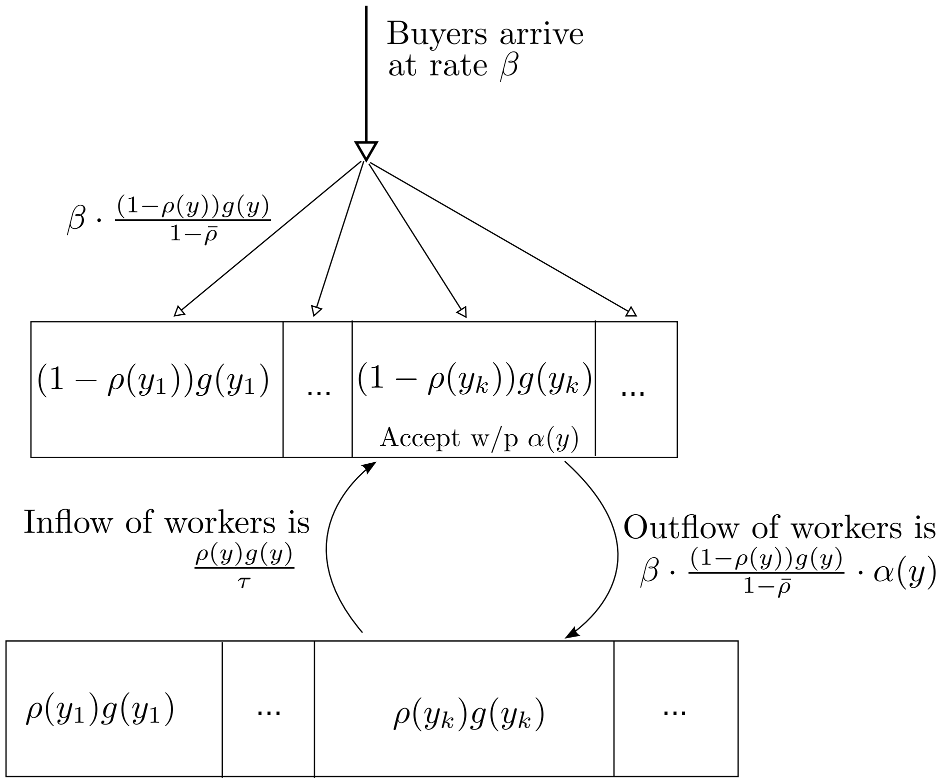


Figure 2: Matching process in a steady state. $g(y)$ is the mass of y-sellers, $\rho(y)$ is utilization rate, $\bar{\rho}$ is the average utilization rate.

state, we have:

$$\beta\frac{(1-\rho(y))g(y)}{1-\bar{\rho}}\alpha(y) = \frac{g(y)\rho(y)}{\tau}, \quad \forall y \in Y. \tag{2}$$

See Figure 2 for the illustration.

Seller dynamic screening problem. Buyers arrive at flow rate β and get distributed uniformly at random across all available sellers. Therefore, each individual available seller experiences a Poisson arrival process of buyers, where the rate of the arrival is equal to the buyer-to-seller ratio:

$$\beta_A := \frac{\beta}{1-\bar{\rho}} \tag{3}$$

(Myerson (2000)). There is thus a contrast between individual arrival process and aggregate arrival process in that the individual arrival process is stochastic while the aggregate arrival process is deterministic (a consequence of law of large numbers). More concretely, an individual seller faces a stochastic arrival process, such that over an infinitesimal time interval dt the probability that one buyer arrives is $\beta_A dt + o(dt)$. Jointly, available sellers face the deterministic buyer arrival process, such that over time interval dt the continuum mass $\beta_A dt$ of buyers arrive. As we show in the rest of this section, stochasticity of individual arrival process combined with independent decision-making creates seller coordination problem which results in matching inefficiency.

Each seller takes β_A as given[7] and solves the dynamic optimization problem where for each incoming job he observes the platform's signal s, and based on s, chooses to accept or reject the job in order to maximize his average profit flow. See Figure 1b for the illustration of the seller dynamic optimization problem. The expected profit for seller y of a job with signal s is $\pi(s,y)=\int_X \pi(x,y)s(dx)$. Denote by $V(y)$ the average profit flow when seller of type y acts optimally (value function)[8]. Denote by $v(s,y)$ the value of one new job with signal s, where v includes the option value of rejecting the job and the opportunity cost of being unavailable. Value of a new job is zero if the seller rejects it, and $\pi(s,y)-\tau V(y)$ if he accepts it, where $\tau V(y)$ is the opportunity cost of accepting due to being unavailable for time τ. There is no discounting. Seller optimization problem is then the system of the following two equations:[9]

$$V(y) = \beta_A \int v(s,y)\lambda(ds) \tag{4}$$

$$v(s,y) = \max\{0, \pi(s,y)-\tau V(y)\} \tag{5}$$

Seller strategy is function $\sigma(\cdot,y)\colon S\to[0,1]$ that for every seller type maps a signal to the probability of accepting it. The seller acceptance rate is the ex ante probability of accepting a job:

$$\alpha(y)=\int \sigma(s,y)\lambda(ds). \tag{6}$$

2.2 Equilibrium definition and existence

I am interested in a steady state equilibrium of the market in which the sellers take buyer arrival rate β_A as given and optimize independently, and the seller busy-available flows balance out. Recall that the exogenous objects in my model are: buyer flow rate β, profit function $\pi(x,y)$, distribution of types $G(y)$ and $F(x)$, and completion time τ. Based on the exogenous objects, a tuple $(\sigma,\bar\rho)$ constitute a *steady-state equilibrium* if

1. [Optimality] For every type-y seller for all y, $\sigma(\cdot,y)$ is an optimal strategy given buyer Poisson arrival rate $\beta_A=\beta/(1-\bar\rho)$.

2. [Steady state] Average utilization rate $\bar\rho$ arises in a steady state when sellers play σ, as shown in {(2), (6)}.

Proposition 1. *Steady-state equilibrium exists. It is unique up to the acceptance of marginal jobs in the follwing sense. If $(\sigma^i,\bar\rho^i)$, $i=1,2$ are two steady-state equilibria, then (1) $\bar\rho^1=\bar\rho^2$, and (2) for any $y\in Y$, $\sigma^1(\cdot,y)$ and $\sigma^2(\cdot,y)$ coincide except on $\{s\colon \pi(s,y)=\tau V(y)\}$.*

[7] This follows from the fact that there is a continuum of sellers on the platform, and so any individual seller's actions does not affect β_A.

[8] For example, if a seller earns \$1 on each job, and time interval between starting consequent jobs is 2, then $V(y)=1/2$.

[9] I consider time average payoff rather than discounted sum because discount rate is not essential for my argument. However, the results immediately generalize to the case when the seller has discount rate r by replacing τ with $\tau_r=\frac{1-e^{-r\tau}}{r}$.

In the proof in the Appendix, first, I show that for an arbitrary vector of acceptance rates $\alpha(y)$, there is a unique steady state value of $\bar{\rho}$. Then, average utilization $\bar{\rho}$ is increasing and continuous in $\alpha(y)$ for any $y \in Y$. The uniqueness of equilibrium obtains by monotonicity of reaction curves of α in $\bar{\rho}$ and $\bar{\rho}$ in α. Namely, in (6), as buyer traffic intensity β_A increases, sellers become more picky and acceptance rate $\alpha(y)$ decreases. As $\alpha(y)$ increases, buyers get directed to a smaller set of sellers, hence β_A increases. For details see the proof in the Appendix on page 28.

2.3 Examples of Marketplaces

In this section I discuss how the model maps to the reality in the case of Uber, Airbnb and labor platform such as TaskRabbit. Recall that y captures the seller's unobserved heterogeneity; and x captures buyer characteristics observed by the platform but not directly observed by the sellers.

Uber. Drivers receive requests from passenger when they are idle. Type y includes driver home location, preference for long rides, tolerance to congestion; x includes passenger's destination and his ride history. Price per mile and minute is fixed (conditional on aggregate multiplies, such as surge pricing). Drivers do not like very short rides or the rides to the neighborhoods far removed from busy areas. Passengers do not like waiting. Proposition 2 shows that if drivers have full discretion over accepting, they screen excessively: acceptance rates are low, and the demand-supply fit is suboptimal.

Airbnb. Hosts are capacity constrained in rooms. Once a room is booked for a specific date, the host cannot accept a better guest. Type y includes host's preference for age, race, personality, daily schdule. Type x includes guests' gender, age, socio-economic status, life style. Every host sets his own price that applies to all guests but he may prefer to reject certain guests who he expects can be a bad fit. Proposition 2 shows that if hosts have full discretion over accepting or rejecting guest requests, hosts screen excessively: acceptance rates are too low to satisfy guest demand for hassle-free service, and the marketplace is removed from its full potential.

TaskRabbit. The service providers are capacity constrained because they can do only that many tasks per week. Once a service provider agreed to do some job, he is constrained in picking new tasks. Type y includes seller skill and work ethics. Type x includes job category, job difficulty, client's professionality. Service providers set hourly rate that apply to all tasks in the same category. The service provider may prefer to reject the tasks which he does not find to be a good fit to his skill, or those that are too far, or those with bad timing. Proposition 2 shows that if service providers have full discretion over accepting or rejecting client leads, they screen excessively: acceptance rates a low, the utilization of labor force on the platform is inefficiently low, and the matching efficiency is suboptimal.

2.4 Discussion of the Modeling Assumptions

In this subsection I discuss in detail the important assumptions of my model.

Assumption 1. *Buyers make a single search attempt.*

Rejection-intolerant buyers is a simplifying assumption that help to avoid endogenous distribution of buyer types. It captures the real aspect of matching markets that rejections are costly to buyers, e.g. wasted time, wasted search effort, bidding costs, etc. Moreover, buyers often do take rejections badly. For example, Fradkin (2015) reports that on Airbnb, an initial rejection decreases the probability that the guest eventually books any listing by 50%.

Assumption 2. *Buyers contact available sellers only.*

The goal of the paper is to explore the matching frictions that pertain to preference heterogeneity and screening. Therefore, I assume away the coordination frictions owing to several buyers requesting the same seller and the coordination frictions owing to buyer requesting unavailable sellers. All three types of matching frictions are present in the matching marketplaces[10]. The coordination frictions in matching markets have been extensively studied in the theoretical literature (Burdett *et al.* (2001); Kircher (2009); Halaburda *et al.* (2015); Arnosti *et al.* (2014)), and I assume away this channel of matching inefficiency to focus on screening.

Assumption 3. *Buyers contact an available seller chosen uniformly at random*

This assumption has two implications. First, seller of any type faces the same intensity of buyer traffic. Second, each seller faces the same distribution of buyer jobs. This is a simplifying assumption that helps me to focus sellers excessive screening problem. In the extension **??** I show how to deal with a more general demand system where demand for each seller type can be different.

Assumption 4 (No Excess Demand)**.** *Collectively, it is physically possible for sellers to complete every buyer job:* $\beta\tau < 1$

The assumption makes the exposition easier and relaxing it requires more notation to deal with either automatic rejections or queues. I do this in Appendix C and show that qualitatively results do not change.

[10] Fradkin (2015)

3 Market Design: Information Disclosure Policy

This section studies the general platform's objective of maximizing the weighted average of buyer and seller surplus, which incorporates welfare maximization, profit maximization and maximizing the number of matches.

3.1 Full disclosure: Seller coordination problem

This subsection establishes that the full disclosure does not yield a Pareto optimal market outcome. In particular, it is not efficient. The reason behind it is the sellers' coordination problem. The inefficiency arises due to decentralization and dynamic nature of the matching.

To state the results of this and next sections, I need to define Pareto-optimality in my setting. Market outcome $O = (\{V(y)\}, CS)$ is a combination of seller profits and consumer surplus that arises in a steady-state equilibrium. I say that a market outcome is *feasible* if there is a seller strategy profile that generates it, and $V(y) \geqslant 0$ for all y. A feasible outcome O is Pareto-optimal if there is no other feasible O' such that $V(y)' > V(y)$ for all y, and $CS' > CS$. The Pareto frontier is the set of all Pareto-optimal outcomes. We say that market outcome O is *implementable* if there is a disclosure policy such that the equilibrium outcome is O. The welfare is the sum of consumer surplus and seller profits. A market outcome is *efficient* if there is no other feasible outcome with higher welfare.

Write $V^{\sigma}(y)$, $\rho^{\sigma}(y)$, CS^{σ} for steady-state profits, utilization rates and consumer surplus when strategy profile σ is played. Imagine the platform starts with the full disclosure as its default disclosure policy. The next proposition shows that there is a feasible outcome at which sellers are strictly better off than under the full disclosure, they complete more jobs, and additionally, the buyers are also better off. The result implies that there is a coordination problem among sellers.

Proposition 2. *Let σ^{FD} be the equilibrium strategy profile under full disclosure. Then there exists $\tilde{\sigma}$ such that for all y:*

$$\begin{aligned} \tilde{V}(y) &> V^{FD}(y), \\ \tilde{\rho}(y) &> \rho^{FD}(y), \\ \widetilde{CS} &> CS^{FD}. \end{aligned}$$

The full proof is in the Appendix. Here I give the sketch of the proof for the case of identical sellers, which gives insights into the nature of the seller coordination problem. Under the full disclosure, seller profit V^{FD} is strictly positive because sellers accept only profitable jobs. Therefore, the opportunity cost of accepting τV^{FD} is strictly positive, too (see Eq. (5)). As a result, in equilibrium, the accepted jobs are those with profits $\pi \geqslant \tau V^{FD}$. However, the profitable jobs are those with $\pi \geqslant 0$. Hence, some profitable jobs are rejected. Consider strategy $\tilde{\sigma}$ that maximizes the joint seller profits. It prescribes every seller to accept jobs with $\pi \geqslant 0$ (and this is feasible by the No Excess Demand assumption). Naturally, $\tilde{\sigma}$ yields $\tilde{V} > V^{FD}$. It also increases acceptance rate, and thus the utilization rates, $\tilde{\rho} > \rho^{FD}$. Consumers are better off because consumer surplus is increasing in the acceptance rate.

A very high-level intuition for the coordination problem is the following. A seller keeps his schedule open by rejecting low-value jobs to increase his individual chances of getting high-value jobs. As a result, in equilibrium, sellers spend a lot of time waiting for the high-value jobs. Collectively, this behavior is suboptimal because all profitable jobs have to be completed.

Interestingly, that unlike in the price competition, that benefits buyers and improves market efficiency, seller coordination problem in this setting results in inefficient outcome with both lower buyer and seller surplus. This is a market failure. I attribute the source of the failure to what I call the *cream-skimming externality*. By rejecting a job, a seller remains available on the marketplace and attracts a fraction of subsequent buyers, who otherwise would go the other sellers. As a result, the other sellers face fewer profitable jobs and obtain lower profits. The key necessary conditions for the cream-skimming externality to arise is the dynamic nature of matching and dispersed seller capacities.

The fundamental reason behind the coordination failure is that collectively, the sellers are not capacity constrained (in time) while individually, the sellers *are* capacity constrained. When an individual seller accepts a job, he is off the market for time τ and cannot accept new jobs. He is afraid to miss valuable future jobs and therefore rejects low-value jobs. However, there are always some available workers in the market, and it is feasible to accept all profitable jobs. Mathematically, the distinction between individual and collective capacity constraints is captured by having a continuum of sellers, so that while buyer traffic to each individual seller is stochastic, the aggregate buyer traffic is deterministic.

Now we know that the full disclosure outcome is Pareto dominated by some strategy profile $\tilde{\sigma}$. A natural question is whether there is an information disclosure policy that induces $\tilde{\sigma}$? In the next section we give the affirmative answer to this question in the case of identical workers.

3.2 Benchmark: Optimal information disclosure with identical sellers

I start by considering the case of identical sellers, i.e. singleton Y. The next proposition establishes that any Pareto optimal outcome (in profit-surplus space) is implementable by a disclosure policy.

Proposition 3. *Suppose the sellers are identical. Then for any Pareto optimal outcome* (V, CS) *there is a disclosure policy that implements it. Furthermore, an optimal disclosure policy has binary structure.*

The proof relies on the Revelation principle. Since there is one seller type and two actions, it is sufficient to consider only disclosure policies that send two signals, where a signal is "action recommendation". With such a binary signaling structure, the seller dynamic optimization problem reduces to the static optimization problem. Indeed, with only one type of acceptable jobs, there is no option value of rejecting a profitable jobs. All profitable jobs are the same! Since a Pareto optimal outcome is feasible with $V \geqslant 0$, sellers have incentives to follow the platform's recommendations. For the details of the proof, see the Appendix.

Proposition 3 characterizes at once the range of possible objective functions. Indeed, any point on the Pareto frontier maximizes $\gamma V + (1-\gamma)CS$ for some $\gamma \in [0, 1]$. Figure 3 illustrates the result. From Proposition 2 we know that the full disclosure outcome is suboptimal. The welfare maximization policy corresponds to $\gamma = 1/2$.

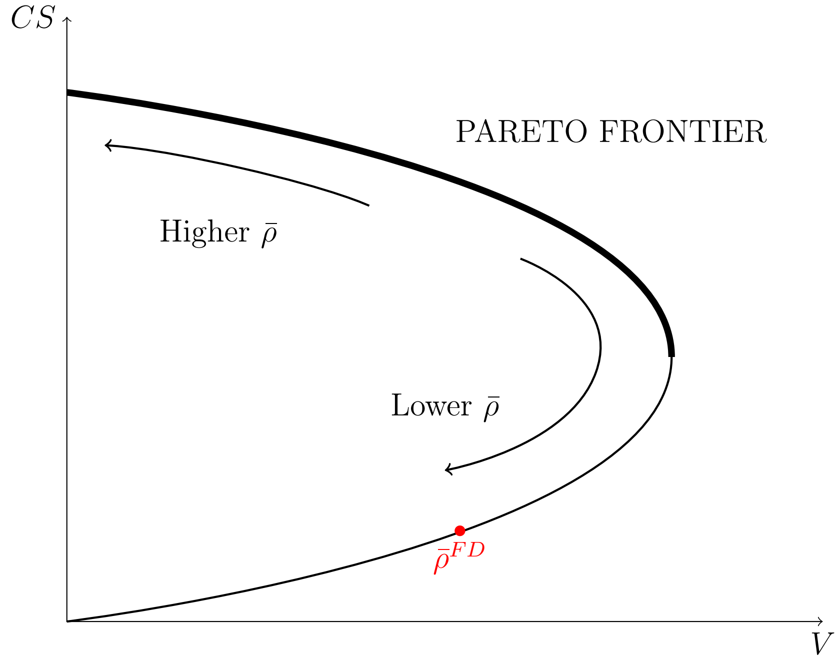


Figure 3: Graphical illustration of the set of feasible and implementable market outcomes in the case of identical workers. CS stands for buyer surplus, V stands for seller surplus. Disclosure policy can implement any point on the Pareto frontier (thick solid line). The full disclosure outcome, marked with $\bar{\rho}^{FD}$, is suboptimal.

Proposition 3 is related to the result of Bergemann *et al.* (2015) who show that segmentation of a monopolistic market can achieve every feasible combination of consumer and producer surplus. Their segmentation problem is a static signaling game with single receiver (monopolist) while my model is a signaling game with dynamically optimizing receivers. This dynamism is the novel element not present in the prior literature on information disclosure.

3.3 Effects of information disclosure

Disclosure of buyer characteristics to sellers has three competing effects on welfare. The first is the standard *Individual Choice effect* on the seller side. From an individual seller's point of view, more information increases his set of attainable payoffs (Blackwell (1953)). Therefore, holding fixed the other sellers' behavior, he individually benefits from more information about buyers. Formally,

Claim 1 (Seller Individual Choice effect). Fix β_A in the seller optimization problem (4). Let λ' be coarser than λ''. Then the profits $V' \leqslant V''$.

The second effect of information disclosure is the *Buyer-side effect*. More information available to sellers reduces the platform's ability to induce sellers to accept buyer-valuable jobs. It results in higher rejection rates that "slows down" the buyer side of the market. Formally,

Claim 2 (Buyer-side effect). Fix $\tau = 0$. Let λ'' be such that there is $s \in \operatorname{supp} \lambda''$ with $\pi(s) < 0$. Then there is coarsening λ' such that $CS' > CS''$.

The third effect is the *Seller Option Value effect*. More information available to sellers increases their ability to "cherry-pick" the best jobs, and so creates the positive option value of rejecting. In equilibrium, it hurts seller surplus because it is not efficient to reject profitable jobs. Formally,

Claim 3 (Seller Option Value effect). Fix β_A in the seller optimization problem (4). Let λ' be coarser than λ''. Then the option values of rejection $\tau V' \leqslant \tau V''$.

Both the Buyer-side and the Option Value effects lead to sellers' *excessive screening* — rejection rates are inefficiently high. The Individual choice effect gives a motive to the platform to release information while the Buyer-side and the Option Value effects give a motive to conceal information. The Buyer-side and the Option Value effects are the consequences of the general principle that more information imposes more constraints on the designer and reduces the set of outcomes he can induce (Bergemann and Morris (2016)).

When workers are identical, achieving efficiency requires coarse information disclosure policy because the Buyer-side effect and the Option Value effects win over the Individual Choice effect. Denote the efficient disclosure policy by $\hat{\lambda}$ and the set of efficient matches by $\hat{X}_a := \{x \in X \colon \pi(x) + u(x) \geqslant 0\}$. Disclosure $\hat{\lambda}$ sends the recommendation "accept" for any jobs in $\hat{X}_a$ and recommendation "reject" for jobs in $X \backslash \hat{X}_a$. Under full disclosure, the accepted jobs are $X_a^{FD} := \{x \in X \colon \pi(x) \geqslant \tau V^{FD}\}$. Note that X_A^{FD} is a proper subset of $\hat{X}_a$. There are two sources of inefficiency under full disclosure. Jobs $\{x \colon -u(x) \leqslant \pi(x) \leqslant 0\}$ are rejected because sellers fail to internalize the effect of their acceptance decisions on buyers. This is the Buyer-side effect. Jobs $\{x \colon 0 \leqslant \pi(x) \leqslant \tau V\}$ are rejected because sellers fail to internalize the cream-skimming externality they impose on each other. This is the Option Value effect.

How does the efficient disclosure policy $\hat{\lambda}$ depends on the economic primitives of the market? What is interesting is that $\hat{\lambda}$ does not depend on the intensity of buyer traffic β (optimal disclosure for other γ have this property, too). This implies that $\hat{\lambda}$ is the same disclosure policy as would be optimal in a static case, when $\tau = 0$.

Corollary 1. *Suppose the sellers are identical. Efficient information disclosure policy $\hat{\lambda}$ is independent of the intensity of buyer traffic β and seller capacity constraints τ. When buyer search costs are higher, $\hat{\lambda}$ prescribes pooling more of marginal unprofitable jobs with inframarginal profitable jobs.*

Independence of $\hat{\lambda}$ from β and τ happens because the arrival rate to available sellers β_A matters only to the extent that it creates option value of rejection. When the sellers are identical, the efficient $\hat{\lambda}$ induces the posterior mean distribution $F^{\hat{\lambda}}$ of payoffs that has only one acceptable value. Therefore, the option value is zero, and so β does not matter.

3.4 Optimal information disclosure with heterogeneous sellers

Typically, the platform observes seller preferences imperfectly, and there is seller preference component that is known only to the seller (skill level, willingness to do long rides, etc). If there is privately known match value held by sellers, the intermediary's ability to influence the market outcome using information design is limited. Depending on the distribution of seller types, there may or may not exist a coarse disclosure policy that improves acceptance rates. In this section I provide sufficient conditions for this.

To illustrate the new tradeoff the platform faces when sellers have private information, imagine two types of sellers: professionals and amateurs. Professionals can profitably complete a larger set of jobs than amateurs. When the platform designs the disclosure policy, it should take into account that the same disclosure policy have different impacts on professionals an on amateurs. Amateurs can profitably complete only a small subset of jobs, and so they need more information to tell them apart from the unprofitable jobs. Otherwide, if insufficient information is provided, amateurs cannot do that and reject all jobs. Professionals have a large set of profitable jobs, and so their average profit per job is high. Pooling more unprofitable marginal jobs with profitable inframarginal jobs keeps professionals' average profits positive but induces higher acceptance rate. Whether it is optimal to coarsen the information disclosure or not thus depends on the relative sizes of amateur and professional populations. If there are more professionals than amateurs, then coarser disclosure increases the total acceptance rates even though amateurs stop working. If there are more amateurs, coarse disclosure policy decreases the total acceptance rate. In the rest of this section I rigorously study this problem for the general continuous distribution of seller types.

Let the space of buyer types be $X = [0, 1]$, with the interpretation x is the difficulty of the job. Let the space of seller types be $Y = [0, \overline{y}]$, $\overline{y} \geqslant 1$, with the interpretation that y is the seller skill level. The seller profit function is $\pi(x, y) = y - x$. "Professionals" are the sellers with high y, and "amateurs" are the sellers with low y. Buyer match value is $u(x, y) = u$. Assume regularity condition necessary for the results below: $f(0) > 0$. I call this *linear payoff* environment.

Consider the general platform's objective of maximizing the weighted average of buyer surplus and joint seller profits.

$$\mathcal{J}(\gamma) = \gamma CS + (1 - \gamma)V,$$

where $V = \int_Y V(y)dG(y)$ is the joint seller profits, and $CS = u \cdot M$ is buyer surplus, where M is the total number of matches formed over unit of time. The welfare maximizing platform has $\gamma = 1/2$. Seller maximizing has $\gamma = 0$. Maximizing buyer surplus corresponds to $\gamma = 1$ and is equivalent to maximizing the number of matches on the platform because all buyers have the same matching value.

I start by stating my second main result that characterizes the optimal disclosure policy for maximizing the weighted average of buyer and seller surplus $\mathcal{J}$. Then I will consider the special case of maximizing the number of matches ($\gamma = 1$) and compare it to the result in the prior literature.

Definition. Disclosure λ is x^**-upper-coarsening* for some $x^* \in [0, 1]$ if λ fully reveals $x < x^*$ and pools all $x > x^*$.[11]

Proposition 4. *Suppose $G = U[0, \overline{y}]$, $\overline{y} \geqslant 1$. Then for any $\gamma \in [0, 1]$, there is unique $x^*_\gamma \in [0, 1]$ such that x^*_γ-upper-coarsening maximizes $\mathcal{J}(\gamma)$. Furthermore, x^*_γ is decreasing in γ.*

[11] The terminology is borrowed from Kolotilin *et al.* (2015).

- *For any $\gamma \in [0,1]$, there exist $\beta\tau$ and $\overline{y}$ large enough such that $x^*_\gamma < 1$ (some coarsening is strictly optimal).*

- *If $0 < \beta\tau < 1/2$, then $x^*_\gamma = 1$ for any γ (full disclosure is strictly optimal).*

I reserve notation x^*_γ to denote the cutoff in the upper-coarsening disclosure policy that maximizes $\mathcal{J}(\gamma)$. This way, x^*_0, x^*_1 and $x^*_{1/2}$ are the highest truthfully revealed buyer types under profits-maximizing disclosure, match-maximizing disclosure and the efficient disclosure, respectively. Note that $x^*_0 \geqslant x^*_{1/2} \geqslant x^*_1$. That implies that when the platform puts more weight on buyer surplus, the optimal disclosure policy is coarser.

Proposition 4 shows that the optimal disclosure policy in the case of the uniform distribution of seller types depends on the intensity of buyer traffic β, tightness of seller capacity constraints τ and the spread of seller types $\overline{y}$. For any platform's objective of maximizing the weighted average of buyer and seller surplus, if $\beta\tau$ is large enough and there are sufficiently high seller types, then information coarsening by pooling large x is the optimal. If buyer traffic is low or capacity constraints are weak, full disclosure is strictly optimal.

The goal of this subsection is to demonstrate that the dynamism of seller optimization problem creates qualitatively new effects compared to the static matching. This goal motivates the choice of the linear payoff environment. From the prior literature it is known that when the distribution of seller types is uniform, the disclosure policy has no effect on the total number of matches. For the non-uniform distributions, the optimal disclosure policy depends on whether the probability density function is decreasing or increasing.

Fact 1. *Suppose $\tau = 0$ and consider the platform's objective of maximizing the number of matches ($\gamma = 1$).*

- *If g is decreasing, then full disclosure is optimal;*

- *If g is increasing, then no disclosure is optimal;*

- *If g is constant, then any disclosure is optimal.*

The result appears e.g. in Kolotilin *et al.* (2015), and the implied concavification reasoning goes back to Aumann *et al.* (1995) and Kamenica and Gentzkow (2011). In this paper, the result follows from the main Lemma 3 in the next section.

Fact 1 says that in case of uniform G and static matching, the information disclosure has no effect on number of matches. In contrast, in dynamic setting, Proposition 4 shows that information disclosure does affect the number of matches. If buyer traffic β and seller capacity constraints τ are small, then the full disclosure is optimal. If $\tau\beta$ is large and there are sufficiently high seller types, coarsening high x is strictly optimal.

To better understand the mechanics behind the result and the role of the dynamic screening, consider two simplifications of the model. In the first simplification, sellers optimize dynamically but are perfectly replenished after leaving, so that the mass of available sellers is always 1, and the distribution is always G. In the second simplification, sellers are myopic, i.e. they accept all profitable jobs.

Corollary 2. *Suppose the distribution of available seller types is exogenous $G = U[0,1]$. Then only the policy of no disclosure maximizes the number of matches.*

The results explains the interplay between the Seller Individual Choice effect, Buyer-side effect and the Seller Option Value effect. In one-shot matching setting, coarsening information decreases the acceptance rat of low-type sellers and increases the acceptance rate of high-type sellers. With uniform distribution of seller types, these two effects cancel out and the total number of matches is unchanged (Fact 1). However, when sellers optimize dynamically, less information also decreases their option value of rejection and increases the acceptance rate. Therefore, no disclosure becomes optimal with dynamic matching.

Corollary 3. *Suppose $G = U[0, 1]$. If sellers are myopic, then full disclosure is strictly optimal to maximize the matching rate.*

In this result, we shut down the Option Value effect of information disclosure and focus on the effect of endogenous distribution of seller types. In the equilibrium, the pdf of available seller types is decreasing because high seller types have higher acceptance rates. By the result from the static matching (Fact 1) one may suggest that that the full disclosure maximizes matching rate. 3 confirms that this is indeed the case.

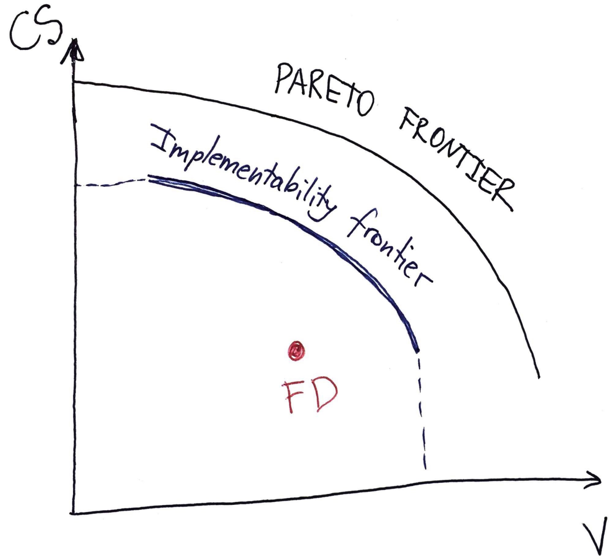
When we take $\gamma = 0$ in Proposition 4, we find that profit maximization may also involve coarsening of the disclosure policy. The coarsening is necessary to alleviate the seller coordination problem when $\beta\tau$ is large. Large $\beta\tau$ implies large option value for sellers, and the option value is the largest for high type sellers. Large x are the marginal buyers for the high type sellers. Therefore coarsening at the right end of X decreases the sellers' option and increases the fraction of accepted profitable jobs.

Figure 4 illustrates Proposition 4 and contrasts it with the case of identical sellers. From Proposition 2 we know that the full disclosure outcome is suboptimal. In the case of identical sellers, it was possible to implement any point on the Pareto frontier by information disclosure (Proposition 3), and therefore coarsening was necessary for optimality. In the case of heterogeneous sellers, the Pareto frontier is not implementable because sellers have private information. Therefore, the "implementability frontier" in Figure 4 is below the Pareto frontier. Now, if $\beta\tau$ and $\overline{y}$ are large enough then the full disclosure outcome is below the implementability frontier, and efficiency requires some coarsening. The optimal disclosures in this case are described in Proposition 4. If $\beta\tau$ is small, the implementability frontier consists of only the full disclosure outcome. That is, the full disclosure is optimal for welfare maximization, match-rate maximization or joint profits maximization.

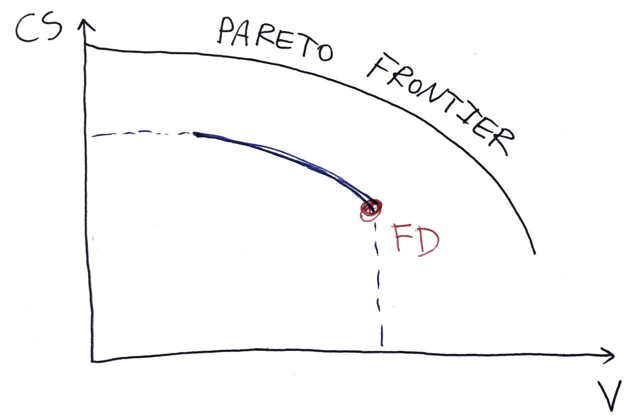
3.5 Main Lemma and the Proof of Proposition 4

In this subsection I sketch the proof of Proposition 4. The main idea behind the proof is to represent information disclosure λ as a bounded convex function $\Lambda(\cdot)$, and then use the calculus of variations to find the optimal Λ. The main technical result of the paper is Lemma 3.

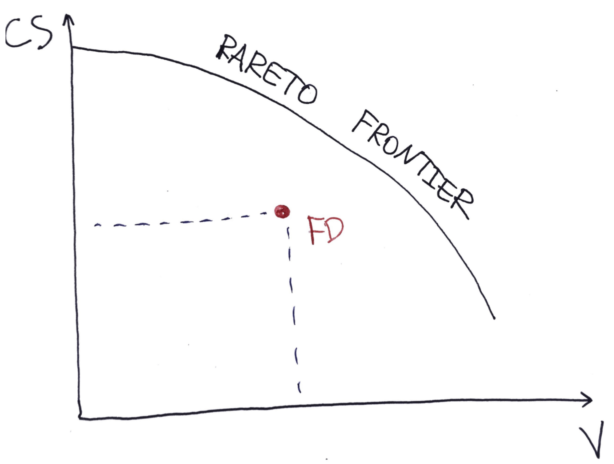
The proof of Proposition 4 relies on four lemmas. Lemma 1 establishes the one-to-one correspondence between information disclosure policies and convex functions from some set. Lemma 2 finds the convenient representation of the seller dynamic optimization problem. Lemma 3 finds a variational derivative of $\mathcal{J}$. Lemma 4 provides a necessary condition for optimality of a disclosure policy.



(a) $\beta\tau$ is large enough and $\overline{y}$ is large enough. The full disclosure is neither efficient, nor profits-maximizing, nor #matches-maximizing.



(b) The full disclosure is profits-maximizing but not efficient.



(c) $\beta\tau \in (0, 1/2)$. The implementability frontier is degenerate and consists of only the full-disclosure outcome.

Figure 4: Graphical illustration of the set of feasible market outcomes and the limits of implementability using information design in the case of heterogeneous workers in the linear payoff environment with the uniform seller type distribution. CS stands for buyer surplus, V stands for seller surplus. FD stands for the full-disclosure outcome.

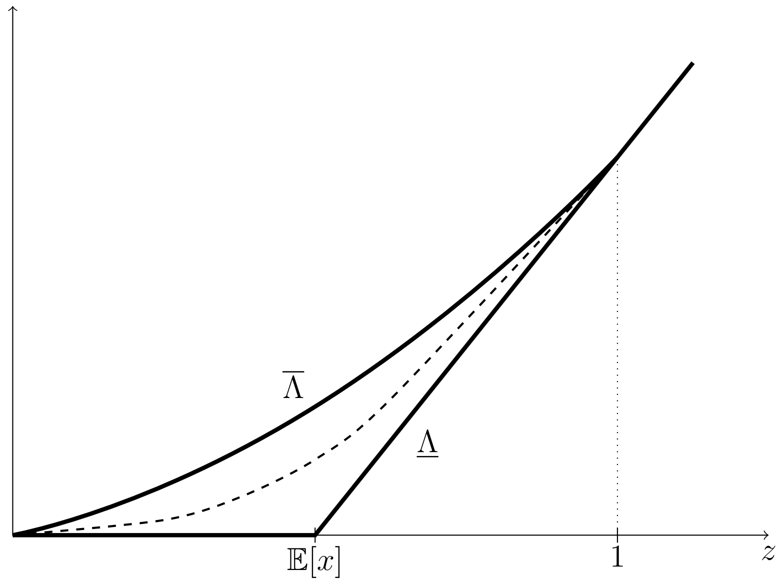


Figure 5: Any disclosure policy λ can be represented as an increasing, convex function Λ point-wise between $\underline{\Lambda}$ and $\overline{\Lambda}$.

Denote the posterior mean of x conditional on signal s by $z(s) := \int_X x\, s(dx)$. I reserve notation z for a typical posterior mean of x. Denote by F^λ the distribution of $z(s)$ when the platform uses disclosure policy λ. We have $F^\lambda(\zeta) = \lambda\{z(s) \leqslant \zeta\}$. Define the *option value function* $\Lambda\colon [0,\infty) \to \mathbb{R}_+$:

$$\Lambda(z;\lambda) := \int_0^z F^\lambda(\zeta)\, d\zeta, \tag{7}$$

As we will see from the Lemma 2, $\Lambda(z)$ is proportional to the option value of rejecting a job with expected difficulty z. Let $\overline{\Lambda}$ be the option value function under full disclosure, $\overline{\Lambda}(z) := \int_0^z F(\zeta)\, d\zeta$. Similarly, let $\underline{\Lambda}$ be the option value function under no disclosure, $\underline{\Lambda}(z) := \max\{0, z - \mathbb{E}[x]\}$. Let

$$\mathcal{L} := \left\{\Lambda(z)\colon \Lambda(z) \text{ is increasing, convex and pointwise between } \overline{\Lambda}(z) \text{ and } \underline{\Lambda}(z)\right\}.$$

The next lemma establishes a one-to-one correspondence between functions from $\mathcal{L}$ and option value functions defined in (7).

Lemma 1. *$\ell \in \mathcal{L}$ if and only if there is $\lambda \in \Delta(S)$ such that $\Lambda(\cdot, \lambda) = \ell$.*

The result is shown in e.g. Gentzkow and Kamenica (2016). The proof uses the fact that distribution of x is the mean preserving spread of the distribution of posterior means of x. The power of Lemma 1 is that it shows that any disclosure policy can be represented as some non-negative, non-decreasing and convex function from $\mathcal{L}$. See Figure 5 for illustration. I am going to use notation Λ for a typical element of $\mathcal{L}$. Therefore, the optimization of $\mathcal{J}$ with respect to $\lambda \in \Delta(S)$ is equivalent to the optimization with respect to $\Lambda \in \mathcal{L}$. The latter allows me to use the calculus of variations to find the optimality condition below.

The next lemma characterizes the optimal seller strategy and demonstrates that the seller optimization problem depends on λ only through Λ.

Lemma 2. *For any disclosure policy λ, seller's optimal strategy has a cutoff form with cutoff $\hat{z}(y)$ such that y-seller accepts all jobs with expected difficulty $z < \hat{z}(y)$ and rejects all jobs with $z > \hat{z}(y)$. Furthermore, for any β_A, seller payoff $V(y)$ and cutoff $\hat{z}(y)$ are solutions to the following system of equations:*

$$V(y) = \frac{y - \hat{z}(y)}{\tau} = \beta_A \Lambda(\hat{z}(y)). \tag{8}$$

The lemma implies that the platform's objective $\mathcal{J}$ depends on λ only through Λ. It is used in the next lemma, which is the main technical result of the paper.

Consider functional $\mathcal{I}(\Lambda) \colon \mathcal{L} \to \mathbb{R}$ and consider variation $\delta\Lambda(y)$. The first variation of $\mathcal{I}$ is $\delta\mathcal{I} = \mathcal{I}(\Lambda + \delta\Lambda) - \mathcal{I}(\Lambda)$. The variational derivative of $\mathcal{I}$ with respect to Λ is function $\phi(y)$ such that $\delta\mathcal{I} = \int \phi(y)\delta\Lambda(y)dy$. If the variational derivative exists, it is denoted by $\delta\mathcal{I}/\delta\Lambda$.

Denote by $\nu(y) := 1 - \rho(y)$ the fraction of type-y sellers who are available.

Lemma 3 (Main Lemma)**.** *For any initial $\Lambda \in \mathcal{L}$, the variational derivative of M with respect to Λ exists and equals:*

$$\frac{\delta M}{\delta \Lambda} = K_1 \cdot \left[g(y)\nu'(y) - (g(y)\nu^2(y))'\right], \tag{9}$$

where $K_1 = \beta / \int \left[\nu^2(y) - \tau V(y)\nu'(y)\right] dG(y) > 0$. Similarly,

$$\frac{\delta V}{\delta \Lambda} = \frac{\delta M}{\delta \Lambda} \cdot K_2 + \nu(y)\beta_A,$$

where $K_2 = \tau \int \nu(y)V(y)dG(y)/\bar{\nu} > 0$.

To see the contribution of Lemma 3, compare it to the static case that has been studied in the prior literature. The seller problem becomes static when $\tau = 0$.

Corollary 4. *Suppose $\tau = 0$. Then*

$$\frac{\delta M}{\delta \Lambda} = -\beta g'(y). \tag{10}$$

If G is concave, then full disclosure is optimal. If G is convex, no disclosure is optimal.

The result easily follows from Lemma Lemma 3 using the fact that when $\tau = 0$, $\nu(y) = (1 + \tau\beta_A\alpha(y))^{-1} = 1$. When matching is dynamic, the original formula (10) has to be adjusted for the dynamic effects, as shown in (9). I now explain the additional dynamic effects in more detail.

To make the contrast between the static and dynamic cases even starker, consider uniform distribution of seller types, $G = U[0, 1]$. By (9),

$$\frac{\delta M}{\delta \Lambda} \propto -(\nu^2(y) - \nu(y))' \tag{11}$$

in dynamic setting and

$$\frac{\delta M}{\delta \Lambda} = 0$$

in the static setting. Therefore, the disclosure has no effect on the number of matches in the static case but does have effect in the dynamic case.

There are two additional forces that appear in the dynamic case: *availability effect* and *patience effect*. The availability effect arises because in equilibrium, high types are less available than low types. Therefore, the pdf of available seller types is decreasing. By 4, this creates a motive for the intermediary to use finer disclosure policy. The availability effect is reflected by term $\nu^2(y)$ in (11).

The patience effect arises because sellers do not act myopically and reject low-value jobs due to dynamic optimization. High types have greater option value of rejecting a job (Claim 3), and so coarsening information has additional effect on acceptance by decreasing the sellers' option value of rejection. Note that this effect is dynamic and is distinct from the pure static effect of high types having higher acceptance rate. The patience effect is reflected by term $-\nu(y)$ in (11).

The next lemma provides a necessary condition for optimality of a disclosure policy.

Lemma 4. *If λ_0 maximizes $\mathcal{J}$, and $\delta\mathcal{J}/\delta\Lambda$ evaluated at λ_0 crosses zero from above at most once, then λ_0 is upper-coarsening.*

Now I sketch the key steps of the proof of the main result for the case of heterogeneous seller Proposition 4. The complete proof is deferred to the Appendix on page 30.

Proof sketch of Proposition 4. By Lemma 3,

$$\begin{aligned}\frac{\delta\mathcal{J}}{\delta\Lambda} &= \gamma u\frac{\delta M}{\delta\Lambda} + (1-\gamma)\left(\frac{\delta M}{\delta\Lambda}K_2 + \nu(y)\beta_A\right) = \\ &= (\gamma u + K_2(1-\gamma))\frac{\delta M}{\delta\Lambda} + (1-\gamma)\beta_A\nu(y), \end{aligned} \tag{12}$$

where $K_2 > 0$. Evaluating with $G = U[0,\overline{y}]$,

$$\frac{\delta M}{\delta\Lambda} = K_1\overline{y}^{-1}(\nu(y)-\nu^2(y))' = K_1\overline{y}^{-1}(1-2\nu(y))\nu'(y), \tag{13}$$

where $K_1 > 0$. Since $\nu(0) = 1$ and $\nu(y)$ is non-negative and decreasing, $\delta\mathcal{J}/\delta\Lambda$ is either positive for all $y \in [0,\overline{y}]$ or crosses zero once from above. Denote by λ^*_γ the disclosure policy that maximizes $\mathcal{J}(\gamma)$. By Lemma 4, λ^*_γ is upper-coarsening.

To see that the cutoff x^*_γ is decreasing in γ, note that larger γ puts more weight on the positive term $\beta_A\nu(y)$ in (12). Therefore the region of Y with negative $\delta\mathcal{J}/\delta\Lambda$ is smaller.

Whether $x^*_\gamma = 1$ or strictly less than 1 depends on the existence of $y \leqslant \overline{y}$ with $\nu(y) < 1/2$. Indeed, from (13), if $\nu(y) > 1/2$ for all $y \leqslant \overline{y}$, then $\delta\mathcal{J}/\delta\Lambda \geqslant 0$ for all y. For the details, refer to the proof in the Appendix on page 30. □

4 Conclusion

5 Appendix

A Lemmas

Lemma 5. *Fix the arbitrary increasing $\alpha(y) \in [0,1]$, $y \in Y$. Average utilization rate $\bar{\rho} \in [0,1]$ is a solution to*

$$1 = \int \frac{dG(y)}{1 - \bar{\rho} + \beta\tau\alpha(y)}. \tag{14}$$

The solution $\bar{\rho}$ exists, is unique, *increases in $\alpha(y)$ for any $y \in Y$, in β and in τ.*

Proof. Find from (2) that $1-\rho(y) = 1/(1+\tau\beta\alpha(y)/(1-\bar{\rho}))$. Take the integral using cdf $G(y)$ to obtain $1 - \bar{\rho} = \int dG(y)/(1 + \tau\beta\alpha(y)/(1 - \bar{\rho}))$. Rearrange and get (14). The right-hand side of (14) is increasing in $\bar{\rho}$. Evaluated with $\bar{\rho} = 0$, it equals $\int \frac{dG(y)}{1+\beta\tau\alpha(y)} \leqslant 1$. Evaluated with $\bar{\rho} = 1$, it equals $\int \frac{dG(y)}{\beta\tau\alpha(y)} \geqslant 1$, using Assumption Assumption 4. Therefore, the solutions exists and is unique. Monotonicity is straightforward. □

Lemma 6. *In the steady state,*

$$\beta \leqslant \beta_A \leqslant \frac{\beta}{1 - \tau\beta}.$$

Proof. From (3), $\beta_A = \beta/(1 - \bar{\rho})$. Therefore, $\beta_A \geqslant \beta$.

Find from (2) that $\tau\beta\frac{1-\rho(y)}{1-\bar{\rho}}\alpha(y) = \rho(y)$. Take expectaton wrt y and obtain $\bar{\rho} = \tau\beta\int \frac{1-\rho(y)}{1-\bar{\rho}}\alpha(y)dG(y) \leqslant \tau\beta\int \frac{1-\rho(y)}{1-\bar{\rho}}dG(y) = \tau\beta$. Therefore, $\beta_A \leqslant \frac{\beta}{1-\tau\beta}$. □

Lemma 7. *Assume the linear payoff environment. In the steady state equilibrium with disclosure policy λ, $\alpha(y) = \Lambda'(\hat{z}(y))$ for sellers of mass 1. Furthermore,*

- *$\alpha(y)$ is increasing in y.*
- *For any $\Lambda \in \mathcal{L}$, there is $\check{y} > 0$ such that $\alpha(y) = 1$ for $y \geqslant \check{y}$.*
- *Considering exogenous variations of $\beta_A\tau$ in the seller optimization problem (8), $\alpha(y)$ is decreasing in $\tau\beta_A$ for any $y \in Y$.*

Proof. By Lemma 2, the the accepted jobs are those with $z < \hat{z}(y)$. Using definition of Λ from (7), probability of accepting is

$$\begin{gathered}
\lambda\{z(s) < \hat{z}(y)\} \leqslant \alpha(y) \leqslant \lambda\{z(s) \leqslant \hat{z}(y)\} \\
F^{\lambda}(\hat{z}(y)-) \leqslant \alpha(y) \leqslant F^{\lambda}(\hat{z}(y)+) \\
\Lambda'(\hat{z}(y)) \leqslant \alpha(y) \leqslant \Lambda'(\hat{z}(y))
\end{gathered}$$

Points of discontinuity of F^{λ} make a set of measure zero. Since seller types have full support on $[0, \overline{y}]$, $\alpha(y) = \Lambda'(\hat{z}(y))$ for sellers of mass 1.

From (8), $\hat{z}(y)$ is increasing in y. Since Λ is convex, $\alpha(y)$ is increasing in y.

Since $X = [0, 1]$, $\Lambda'(z) = 1$ for $z \geqslant 1$. And for y large enough, $\hat{z}(y) > 1$.

From (8), $\hat{z}(y)$ is decreasing in $\tau\beta_A$ for any $y \in Y$. Since Λ is convex, $\alpha(y)$ is decreasing in $\tau\beta_A$. □

Lemma 8. *Suppose the linear payoff environment with $G = U[0, \overline{y}]$. For any (large) $L > 0$ there is $\beta\tau < 1$ large enough and $\breve{y}$ (large enough) such that if $\overline{y} \geqslant \breve{y}$, we have $\tau\beta_A \geqslant L$.*

Proof. Since $\beta_A = \beta/\bar{\nu}$, rewrite (14) as

$$\beta\tau = \int_0^{\overline{y}} \frac{dy/\overline{y}}{\frac{1}{\tau\beta_A} + \alpha(y; \tau\beta_A)}, \tag{15}$$

where I made explicit that α depends on $\tau\beta_A$ in equilibrium. Treat the right hand side of (15) as a function of $\tau\beta_A$ and $\overline{y}$ and denote it by $\psi(\tau\beta_A, \overline{y})$. First, ψ is strictly increasing in $\tau\beta_A$ because by Lemma 7 α is decreasing in $\tau\beta_A$. Second, ψ is decreasing in $\overline{y}$ because by Lemma 7 α is increasing in y. Third, using the second part of Lemma 7 and uniformity of G, $\lim_{\overline{y}\to\infty} \psi = ((\tau\beta_A)^{-1} + 1)^{-1} < 1$. Pick $\breve{y}$ such that $\psi(L, \breve{y}) < 1$. Let $(\beta\tau)^* := \psi(L, \breve{y})$. We have that $\beta_A\tau > L$ whenever $\beta\tau > (\beta\tau)^*$ or $y > \breve{y}$. □

Lemma 9. *Suppose the linear payoff environment with $G = U[0, \overline{y}]$. For any (small) $\varepsilon > 0$ there is $\beta\tau < 1$ large enough and $\overline{y}$ large enough, such that $\nu(\overline{y}) = \varepsilon$.*

Proof. Note that $\nu(y) = \frac{1}{1+\tau\beta_A\alpha(y)}$. Pick $L > \varepsilon^{-1} - 1$. By Lemma 8 for any $L > 0$, we can find $\beta\tau$ and $\breve{y}$ such that if $\overline{y} \geqslant \breve{y}$, $\tau\beta_A \geqslant L$. Since $\operatorname{supp} F = [0, 1]$, we have from (8) that $\alpha(\overline{y}) = 1$ for $\overline{y}$ large enough. Therefore, $\nu(\overline{y}) = \frac{1}{1+\tau\beta_A\alpha(\overline{y})} < \frac{1}{1+L} < \varepsilon$. □

Lemma 10. *$\lambda \in \Delta(S)$ is x^*-upper-coarsening if and only if the corresponding Λ has the following form:*

$$\Lambda(z) = \begin{cases} \overline{\Lambda}(z), & z \in [0, x^*] \\ \overline{\Lambda}(x^*) + F(x^*)(z - x^*), & z \in (x^*, \mathbb{E}[x|x > x^*]) \\ \underline{\Lambda}(z), & z \in [\mathbb{E}[x|x > x^*], 1] \end{cases} \tag{16}$$

Proof of Lemma 4. Suppose the contrary, λ_0 is not upper-coarsening. I will show that there is a deviation from λ_0 that increases $\mathcal{J}$.

Let $\Lambda_0 \in \mathcal{L}$ which corresponds to λ_0. Let y^* be the zero of $\delta\mathcal{J}/\delta\Lambda$, and the corresponding cutoff in the seller optimization problem be $z^* = \hat{z}(y^*; \Lambda_0)$. I am now going to construct a feasible variation Λ^ε from Λ_0 that increases $\mathcal{J}$.

Let $\tilde{\Lambda}$ be the upper-coarsening (of the form (16)) that passes through point $(z^*, \Lambda_0(z^*))$. There is only one such function because there is only one line passing through $(z^*, \Lambda_0(z^*))$ and is tangential to $\overline{\Lambda}$. We have that $\Lambda_0(z) \geqslant \tilde{\Lambda}(z)$ on $z > z^*$, and $\Lambda_0(z) \leqslant \tilde{\Lambda}(z)$ on $z < z^*$. Moreover, since Λ_0 is not upper-coarsening, for some $z \in [0, 1]$ one of these inequalities is strict. Consider variation

$$\Lambda^\varepsilon(z) = (\tilde{\Lambda}(z) - \Lambda_0(z))\varepsilon.$$

Since $\delta\mathcal{J}/\delta\Lambda < 0$ on $z > z^*$ and $\delta\mathcal{J}/\delta\Lambda > 0$ on $z < z^*$, Λ^ε increases $\mathcal{J}$. □

B Proofs omitted from section 3

Proof of Proposition 1. Step 1. Restricting set of strategies to cutoff strategies. Fix $y \in Y$. By (5) the optimal seller strategy is such that all $S_a(y) = \{s : \pi(s, y) > \tau V(y)\}$ are accepted, all $S_r(y) = \{s\colon \pi(s,y) < \tau V(y)\}$ are rejected, and the seller is indifferent between accepting and rejecting signals from $S_m(y) := \{s\colon \pi(s,y) = \tau V(y)\}$. Therefore any optimal strategy has the cutoff form:

$$\sigma(s,y) = \begin{cases} 1, & s \in S_a(y) \\ \in [0,1], & S_m(y) \\ 0, & s \in S_r(y) \end{cases}$$

Denote the cutoff for type-y seller by $\hat{\pi}(y) := \tau V(y)$. Using (4),

$$\begin{aligned} V(y) &= \beta_A \int (\pi(s,y) - \tau V(y))\sigma(s,y)\lambda(ds) \\ \hat{\pi}(y) &= \tau\beta_A \int (\pi(s,y) - \hat{\pi}(y))\sigma(s,y)\lambda(ds) \\ \hat{\pi}(y) &= \tau\beta_A \int (\pi(s,y) - \hat{\pi}(y)) I\{\pi(s,y) > \hat{\pi}(y))\}\lambda(ds) \end{aligned} \tag{17}$$

Given λ and β_A, the cutoff of the optimal strategy is a solution to (17). The solution is unique because the left-hand side of (17) is strictly increasing in $\hat{\pi}(y)$ while the left hand side is decreasing in $\hat{\pi}(y)$.

Step 2. Existence. Consider a correspondence $\psi\colon [0,1] \rightrightarrows [0,1]$, which maps $\bar{\rho}$ to a set of "reaction" $\bar{\rho}$'s by the following procedure. To find $\psi(\bar{\rho})$, first find the unique cutoff function $\hat{\pi}$ from (17) using $\beta_A = \beta/(1-\bar{\rho})$. Cutoff $\hat{\pi}$ does not pin down the acceptance rates α uniquely because marginal signals from S_m can have positive probability under λ. The acceptance rates congruent with $\hat{\pi}(y)$ are integrable $\alpha(y)$ such that

$$\lambda(S_a(y)) \leqslant \alpha(y) \leqslant \lambda(S_a(y)) + \lambda(S_m(y)), \quad \forall y \in Y.$$

Take all α that are congruent with $\hat{\pi}$, call this set $\mathcal{A}$. For any $\alpha \in \mathcal{A}$, find $\bar{\rho}$ as shown in Lemma 5. Going over all $\mathcal{A}$ will produce the set of $\bar{\rho}$. This will be $\psi(\bar{\rho})$.

Clearly, $\mathcal{A}$ is convex. Thus, $\psi(\bar{\rho})$ is also convex. Since $\psi(\bar{\rho})$ is an interval subset of $[0,1]$, it is easy to see that $\psi(\bar{\rho})$ is also closed. ψ is upper hemi-continuous because $\hat{\pi}$ is continuous in β_A accroding to (17). Therefore, by Kakutani's theorem, ψ has a fixed point.

Step 3. Uniqueness. Suppose $\bar{\rho}^1$ and $\bar{\rho}^2$ are two distinct fixed points of ψ. Suppose $\bar{\rho}^1 > \bar{\rho}^2$. Then $\beta_A^1 > \beta_A^2$. By (17) $\hat{\pi}^1(y) > \hat{\pi}^2(y)$ for any $y \in Y$. If so, then whenever $\pi(s,y) \geqslant \hat{\pi}^1(y)$ we also have $\pi(s,y) > \hat{\pi}^2(y)$. But this means that $S_a^2(y) \supseteq S_a^1(y) \cup S_m^1(y)$. Therefore $\alpha^2(y) \geqslant \lambda(S_a^2) \geqslant \lambda(S_a^1 \cup S_m^1) \geqslant \alpha^1(y)$ for all y. By Lemma 5, this implies $\bar{\rho}^2 \geqslant \bar{\rho}^1$. A contradiction.

We showed that there is a unique $\bar{\rho}$ that can arise in a steady-state equilibrium. By the argument below (17), the strategy cutoffs $\hat{\pi}(\cdot)$ are also pinned down uniquely in the steady-state equilibrium. □

Proof of Proposition 2. Consider $\tilde{\sigma}$ □

Proof of Proposition 3. Take any Pareto optimal pair $O = (V, CS)$. Since O is feasible, there is seller strategy profile σ that induces O. Since there is one seller type and two actions, it is sufficient to consider only the binary signaling structures (Revelation principle). A binary signaling structure has two signals, where a signal is "action recommendation". Let s_a be the recommendation to accept, and s_r be the recommendation to reject. Denote this signaling structure by $\hat{\lambda}$. We need to check the incentive constraints, that is, to make sure that the sellers would follow the recommendations of $\hat{\lambda}$.

From (5) we have that $v(s_a) = \pi(s_a) - \tau V$, $v(s_r) = 0$, and $V = \beta_A(\pi(s_a) - \tau V)\hat{\lambda}(s_a)$. The incentive constraints require that $\pi(s_a) \geqslant \tau V$ and $\pi(s_r) \leqslant 0$. For the former,

$$\tau V = \frac{\tau\beta_A\hat{\lambda}(s_a)}{1 + \tau\beta_A\hat{\lambda}(s_a)}\pi(s_a) < \pi(s_a).$$

For the latter, recall that O is Pareto optimal, hence σ accepts all profitable jobs. This implies that

$$\pi(s_r) \leqslant 0 < \tau V.$$

□

Proof of Lemma 3. Step 1. M and β_A are positively related. Indeed, when the mass of available sellers is X, the flow of matches is

$$M = \frac{1 - X}{\tau}.$$

The buyer arrival rate on available sellers is $\beta_A = \beta/X$. Therefore,

$$M = \frac{1 - \beta/\beta_A}{\tau}.$$

We are interested in the sign of $\delta M/\delta\Lambda$, therefore we will find $\delta\beta_A/\delta\Lambda$.

Step 2. The equilibrium values of $\alpha(y)$ and β_A are found from the system of equations (8) and (**??**), which we reproduce here:

$$y - \hat{z}(y) = \tau\beta_A\Lambda(\hat{m}(y)), \quad \forall y \in Y; \tag{18}$$

$$\int \frac{dG(y)}{\tau\alpha(y) + 1/\beta_A} = \beta. \tag{19}$$

□

Proof of Proposition 4. Here I provide the entire complete proof. Step 1. By Lemma 3,

$$\begin{aligned}\frac{\delta\mathcal{J}}{\delta\Lambda} &= \gamma u\frac{\delta M}{\delta\Lambda} + (1-\gamma)\left(\frac{\delta M}{\delta\Lambda}K_2 + \nu(y)\beta_A\right) = \\ &= (\gamma u + K_2(1-\gamma))\frac{\delta M}{\delta\Lambda} + (1-\gamma)\beta_A\nu(y),\end{aligned} \tag{20}$$

where $K_2 > 0$. Evaluating with $G = U[0, \overline{y}]$,

$$\frac{\delta M}{\delta\Lambda} = K_1\overline{y}^{-1}(\nu(y) - \nu^2(y))' = K_1\overline{y}^{-1}(1 - 2\nu(y))\nu'(y), \tag{21}$$

where $K_1 > 0$. Since $\nu(0) = 1$ and $\nu(y)$ is non-negative and decreasing, $\delta\mathcal{J}/\delta\Lambda$ is either positive for all $y \in [0, \overline{y}]$ or crosses zero from above once. Let λ^*_γ denote the disclosure policy that maximizes $\mathcal{J}(\gamma)$. By Lemma 4, λ^*_γ is upper-coarsening.

Step 2. To see that the cutoff x^*_γ is decreasing in γ, note that larger γ puts more weight on the positive term in (20). Therefore the region of Y with negative $\delta\mathcal{J}/\delta\Lambda$ is smaller.

Step 3. Suppose $\beta\tau < 1/2$. Using Lemma 6, $\beta_A\tau \leqslant \beta\tau/(1-\beta\tau) < 1$. Next, $\nu(y) = (1+\beta_A\tau\alpha(y))^{-1} > (1+1\cdot 1)^{-1} = 1/2$ for any y. Using (21), $\frac{\delta M}{\delta\Lambda} \geqslant 0$ for any y, and so it $\delta\mathcal{J}/\delta\Lambda$. Therefore, full disclosure is optimal for any γ. □

C Technical Extensions

C.1 No Excess Demand Assumption relaxed

Allow for the case when $\beta(1)\tau \geqslant 1$. If $\beta(1)\tau > 1$, then sellers get overwhelmed by the buyer requests and can't respond to all of them to the extent that they can't even reject them. To cover this situation we assume that if there are no available sellers to reject a pending buyer request, the platform rejects it automatically.

Since some requests can be rejected by the platform, the acceptance rate as perceived by buyers does not coincide with the acceptance rate α generated by sellers. Denote by α^e the effective acceptance rate that buyers face. Let at some moment of time there is $x \in [0,1]$ mass of available sellers, and let buyers arrive to the platform at rate β. Then within the next time interval dt, there are βdt new request, and $x + \left(\frac{1-x}{\tau}dt\right)$ available sellers. What is α^e when sellers use acceptance rate α? Consider three cases.

1. $x > 0$. There is plenty of available sellers, $x + \left(\frac{1-x}{\tau}dt\right) > \beta dt$. Fraction α of buyers are accepted, therefore $\alpha^e = \alpha$.

2. $x = 0$ and $\alpha\beta < \frac{1}{\tau}$. There are few sellers that just became available but in sufficient number to process all buyers. In the same fashion as in case 1, $\alpha^e = \alpha$.

3. $x = 0$ but $\alpha\beta \geqslant \frac{1}{\tau}$. Not sufficient sellers to process all buyers, some buyers are rejected by the platform. The number of accepted jobs is $\frac{1}{\tau}dt$. The acceptance rate is therefore $\alpha^e = \frac{1/\tau}{\beta}$.

Combining all there cases, we have that

$$\alpha^e = \min\{\alpha, \frac{1}{\tau\beta}\}.$$

The adjusted definition of equilibrium is then the following.

1. $$\alpha \in [F(c^*(\beta_A)-), F(c^*(\beta_A)+)].$$

2. $$\beta_A = \frac{\beta(\alpha^e)}{1-\beta(\alpha^e)\alpha^e\tau} = \begin{cases} \frac{\beta(\alpha)}{1-\beta(\alpha)\alpha\tau}, & \alpha\beta(\alpha)\tau < 1 \\ +\infty, & \alpha\beta(\alpha)\tau \geqslant 1 \end{cases}$$

$\beta_A = +\infty$ reflects the fact that when the demand is overwhelming, buyers line up for sellers so sellers start a new job immediately after they finish the previous one. The No Excess Demand assumption makes sure that this never happens, $\alpha\beta(\alpha)\tau < 1$ for all $\alpha \in [0, 1]$.

The next result shows that in the equilibrium there are no lines. By this reason for the clarity of exposition we decided to restrict the analysis to the case of no lines to begin with.

Claim. In equilibrium, $\beta_A < \infty$.

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