Information Design

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Mechanism Design and Information Design

- **Basic Mechanism Design**:
  - Fix an economic environment and information structure
  - Design the rules of the game to get a desirable outcome

- **Information Design**
  - Fix an economic environment and rules of the game
  - Design an information structure to get a desirable outcome

Mechanism Design and Information Design

- **Basic Mechanism Design**:
  - Can compare particular mechanisms..
    - e.g., first price auctions versus second price auctions
  - Can work with space of all mechanisms...
    - without loss of generality, let each agent's action space be his set of types...revelation principle
    - e.g., Myerson's optimal mechanism

- **Information Design**
  - Can compare particular information structures
    - Linkage Principle: Milgrom-Weber 82
    - Information Sharing in Oligopoly: Novshek-Sonnenschein 82
  - Can work with space of all information structures
    - without loss of generality, let each agent's type space be his set of actions......revelation principle

Information Design: Some Leading Cases

1. Uninformed information designer (or "mediator"):
   - Myerson: "Bayesian games with communication"
   - Incomplete Information Correlated Equilibrium literature of the 1980s and 1990s (Forges 93)

2. One player (a "receiver") and an informed information designer (or "sender")
   - "Bayesian Persuasion": Kamenica-Gentzkow 11 and large and important literature inspired by it

3. One player and uniformed designer: *very boring*

4. Many players *and* an informed information designer
   - Some of my recent theoretical and applied work with various co-authors....
   - ...and this lecture

This Talk

- a general framework in two slides
- leading examples at length
- applications in brief
- various elaborations if time

Setup

- Maintained Environment: Fix players $1,...,I$; payoff states Θ; prior on states $\psi \in \Delta(\Theta)$
- Basic Game $G : (A_i, u_i)_{i=1..I}$ where $u_i : A \times \Theta \to \mathbb{R}$
- Information Structure S: $(T_i)_{i=1..I}$ and $\pi : \Theta \to \Delta(T)$

Information Designer's Problem

- *Decision rule* $\sigma : T \times \Theta \rightarrow \Delta(A)$ is *obedient* for (G, S) if, for all i, t_i, a_i and a'_i,

$$\sum_{a_{-i}, t_{-i}, \theta} u_i\left((a_i, a_{-i}), \theta\right) \sigma\left(a|t, \theta\right) \pi\left(t|\theta\right) \psi\left(\theta\right)$$

$$\geq \sum_{a_{-i}, t_{-i}, \theta} u_i\left((a'_i, a_{-i}), \theta\right) \sigma\left(a|t, \theta\right) \pi\left(t|\theta\right) \psi\left(\theta\right);$$

  Obedient decision rule σ is a *Bayes correlated equilibrium*. Characterizes implementability.

- Information designer with payoff $v : A \times \Theta \rightarrow \mathbb{R}$ picks a Bayes correlated equilibrium $\sigma \in BCE(G, S)$ to maximize

$$V_S(\sigma) \equiv \sum_{a, t, \theta} \psi(\theta) \pi(t|\theta) \sigma(a|t, \theta) v(a, \theta).$$

Information Design: Three Interpretations

1. Literal: actual information designer with ex ante commitment
2. Metaphorical: e.g., adversarial / worst case
3. Informational robustness: family of objectives characterize set of attainable outcomes

One Informed Player: Benchmark Investment Example

- a firm is deciding whether to invest or not:
- binary state: $\theta \in \{B, G\}$, bad or good
- binary action: $a \in \{\text{Invest, Not Invest}\}$
- payoffs

	bad state B	good state G
Invest	-1	x
Not Invest	0	0

  with $0 < x < 1$
- prior probability of each state is $\frac{1}{2}$
- firm is uninformed (so one uninformed player)
- information designer (government) seeks to maximize probability of investment (independent of state)
- leading example of Kamenica-Gentzkow 11

Decision Rule

- p_θ is probability of investment, conditional on being in state θ

	bad state B	good state G
Invest	p_B	p_G
Not Invest	$1 - p_B$	$1 - p_G$

- interpretation: firm observes signal = "action recommendation," drawn according to (p_B, p_G)
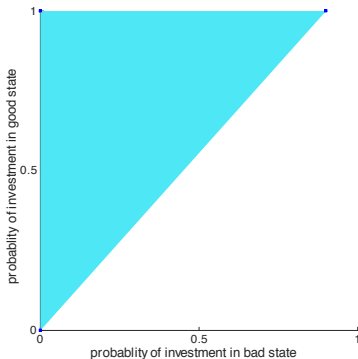
Obedience Constraints

- if "advised" to invest, invest has to be a best response:

$$-\frac{1}{2}p_B + \frac{1}{2}p_G x \geq 0 \Leftrightarrow$$
$$p_G \geq \frac{p_B}{x}$$

- because $x < 1$, investment constraint is binding one
- always invest ($p_B = 1$ and $p_G = 1$) is not a BCE
- the "full information equilibrium" has invest only in good state ($p_B = 0$ and $p_G = 1$)

Bayes Correlated Equilibria

equilibrium outcomes (p_B, p_G) for $x = 0.9$

probability of investment in good state (y-axis)

probablity of investment in bad state (x-axis)

- ► always invest ($p_B = 1$ and $p_G = 1$) is not a BCE
- ► the full information equilibrium has invest only in good state ($p_B = 0$ and $p_G = 1$)

Information Design

- recommendation maximizing the probability of investment:

$$p_B = x, \ p_G = 1$$

- best BCE

	B	G
Invest	x	1
Not Invest	$1 - x$	0

- Optimal for government to obfuscate, partially pooling good state and bad state

One Informed Player

- Firm receives a signal which is "correct" with probability $q > 1/2$.
- Formally, the firm observes a signal g or b, with signals g and b being observed with conditionally independent probability q when the true state is G or B respectively:

	bad state B	good state G
bad signal b	q	$1 - q$
good signal g	$1 - q$	q

- As before, except distriubtion over states $\theta \in \{B, G\}$ changes in response to firm's signal $t \in \{b, g\}$;
- A decision rule is then a quadruple $\left(p_B^b, p_G^b, p_B^g, p_G^g\right)$.
- For example, if firm's own information is sufficiently noisy, or $q \leq \frac{1}{1+x}$, the binding constraint is still the investment constraint for each signal

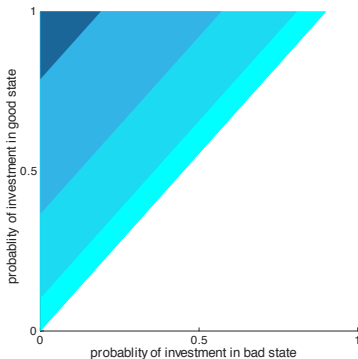$$p_G^g \geq \frac{1-q}{q} \frac{p_B^g}{x}.$$

(otherwise the not invest constraint binds)
- the interesting question: what if we project $\left(p_B^b, p_G^b, p_B^g, p_G^g\right)$ back into ex ante behavior $\left(p_B^b, p_G^b, p_B^g, p_G^g\right)$, e.g.

$$p_G = q p_G^g + (1-q) p_G^b$$

One Informed Player: Bayes Correlated Equilibrium

equilibrium set (for $x = 0.9$ and $q = 0.5, 0.575, 0.7$ and 0.875)

Two Firms

- payoffs almost as before....

$\theta = B$	I	N	$\theta = G$	I	N
I	$-1 + \varepsilon$	-1	I	$x + \varepsilon$	x
N	0	0	N	0	0

- ...up to ε term
- assume that information designer (government) wants to maximize the *sum* of probabilities that firms invest....
- if $\varepsilon = 0$, problem is exactly as before firm by firm; doesn't matter if and how signals are correlated
- we will consider what happens when $|\varepsilon| \approx 0$ (so the analysis cannot change very much)
- will now have profile of action recommendations depending on the state

Two Firms: Strategic Complementarities

- If $\varepsilon > 0$, optimal rule is

$\theta = B$	I	N
I	$\frac{x+\varepsilon}{1-\varepsilon}$	0
N	0	$\frac{1-x-2\varepsilon}{1-\varepsilon}$

$\theta = G$	I	N
I	1	0
N	0	0

- the probability of any one firm investing is still about x..
- because binding constraints are still investment constraints, slackened by having simultaneous investment...

$$\frac{x+\varepsilon}{1-\varepsilon}\left(-1+\varepsilon\right) + x + \varepsilon \geq 0$$

- ....so signals are public

Two Firms: Strategic Substitutes

- If $\varepsilon < 0$, optimal rule is

$\theta = B$	I	N	$\theta = G$	I	N
I	0	$x + \varepsilon$	I	1	0
N	$x + \varepsilon$	$1 - 2x - 2\varepsilon$	N	0	0

- the probability of any one firm investing if the state is bad is still about x....
- because binding constraints are still investment constraints, slackened by having minimally correlated investment...

$$(x + \varepsilon)(-1) + x + \varepsilon \geq 0$$

- ....and signals are private

Application 1: Information Sharing (and Strategic Substitutes)

- ▶ Classic Question: are firms better off if they share their information?
- ▶ Consider quantity competition when firms uncertain about level of demand (intercept of linear demand curve) with symmetry, normality and linear best response; two effects in conflict:
    1. Individual Choice Effect: Firms would like to be as informed as possible about the state of demand

Application 1: Information Sharing (and Strategic Substitutes)

- ▶ Classic Question: are firms better off if they share their information?
- ▶ Consider quantity competition when firms uncertain about level of demand: individual and strategic effects in conflict

- ▶ Resolution:
  - ▶ For large enough price sensitivity (and thus strategic substitutability), strategic effect wins and no information is optimal
  - ▶ For small enough price sensitivity (and thus strategic substitutability), individual choice effect wins and full information is optimal
  - ▶ For intermediate price sensitivity, there is a non-trivial trade-off and it is optimal to have firms observe noisy signals of demand, but with uncorrelated noise and thus conditionally independent signals, and thus signals which are as uncorrelated as possible conditional on their accuracy

Application 2: Aggregate Volatility (and Wacky Designer Objective)

- Classic Question: can informational frictions explain aggregate volatility?
- Consider a setting where each agent sets his output equal to his productivity which has a common component and an idiosyncratic component
- again with symmetry and normality.... common compenent y with variance σ^2; idiosyncratic component x_i with variance τ^2;
- Which information structure maximizes variance of average action?

Application 2: Aggregate Volatility

What information structure maximizes variance of average action?

- critical information structure has a confounding (c.f., Lucas 72) signal $s_i = \lambda x_i + (1 - \lambda) y$ *without noise...*

- variance of average action is maximized when

$$\lambda = \frac{\sigma}{2\sigma + \sqrt{\sigma^2 + \tau^2}}$$

  and maximum variance of average action is

$$\left(\frac{\sigma + \sqrt{\sigma^2 + \tau^2}}{2} \right)^2$$

- as $\sigma \to 0$:
  - weight on idiosyncratic component goes to 0
  - agents put a lot of weight on their signal in order to put a non-trivial weight on their idiosyncratic component
  - common component is like payoff irrelevant but common "sentiments" shock:

- can be embedded in a richer setting (Angeletos La'O 13)

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Application 3: First Price Auctions (and Information Shrinking BCE, Adversarial Information Designer, Robust Predictions and Partial Identication)

- ▶ Example: Two bidders and valuations independently and uniformly distributed on the interval $[0, 1]$
- ▶ Plot: (expected bidders' surplus, expected revenue) pairs
- ▶ green = feasible pairs, blue = unknown value pairs, red = known value pairs
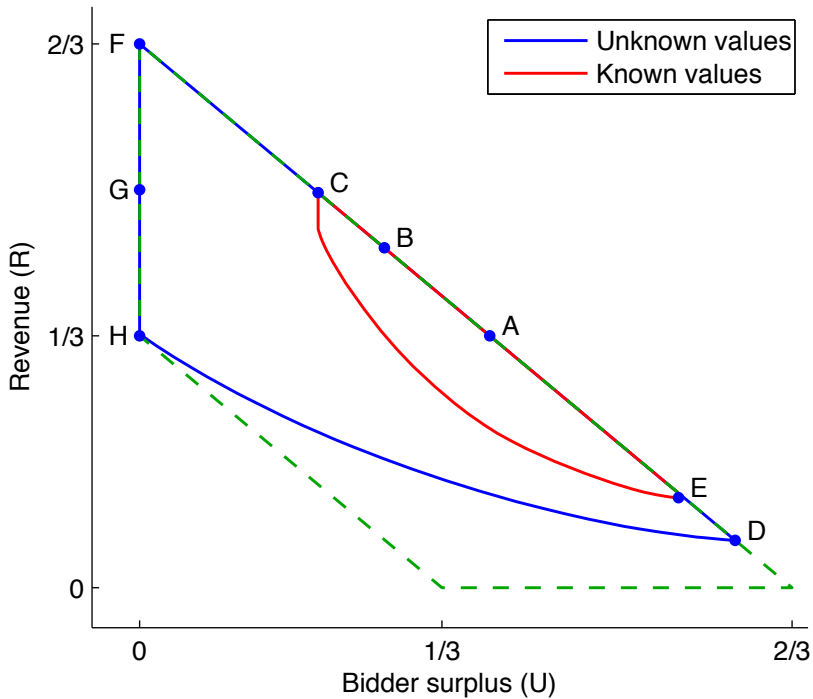
Application 3: First Price Auctions

1. Known value case (red region) is subset of unknown value case (blue region)
2. Robust Prediction: revenue has lower bound $\approx 1/10$
3. Partial Identification: Winning bid distribution $\implies$ Lower bound on Value Distribution (w/o identifying private vs. common values)

# Designer Access to Players' Information

- We want to assume that information designer knows the state θ...
- ...but what should we assume about what information designer knows about players' information? Consider three scenarios:
  1. Omniscient Designer: the designer knows all players' information too...[**maintained assumption so far**]
  2. Communicating Designer: the designer can condition his announcements about the state only on players' reports of their types
  3. Non-Communicating Designer: the designer can tell players about the state but without conditioning on players' information

Back to One Informed Player: Communicating Designer

- as before, firm observes a signal $t \in T$ and government makes a recommendation to invest p_θ^t as a function of reported signal t and state θ
- incentive constraint: add truth-telling to obedience
- to insure truth-telling, differences in recommendations must be bounded across states

Communicating Designer

- adding truth-telling constraints...($x = 0.9$, $q = 0.7$)
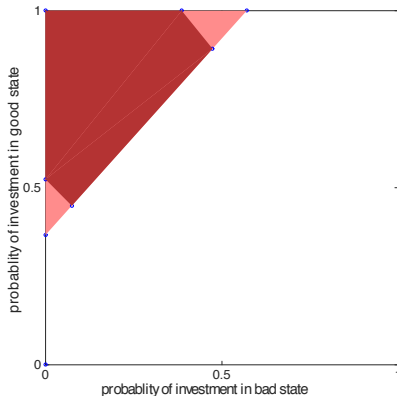
- communicating (red), omniscient (pink)
- truthtelling imposes constraints at low and high ends

Non-communicating designer

- firm observes his signal
- government offers a recommendation, independent of the signal, depending on the true state
- In our example, communicating and non-communicating designer can attain the same set of outcomes; Kotolin et al show this in a more general - but still restrictive - class on environments

Taxonomy

	Single Agent	Many Agent Uninformed Designer	Many Agent Informed Designer
Omniscient	.	Bayesian Solution	Bayes Correlated Equilibrium
Communicating	Kolotilin et al	Communication Equilibrium	.
Non Communicating	KG informed receiver	Strategic Form Correlated Equilibrium	.

Elaborations

1. Other Objectives
   - Ely 15, Arieli 15

2. Comparing Information
   - Mathevet, Perego and Taneva 16

3. Concavification and its many player generalizations
   - Kamenica-Gentzkow 11 get a lot of action out of "concavification" (Aumann-Maschler 95); many player generalization harder - Mathevet, Perego and Taneva 16 harder

4. Adversarial Information Design
   - Carroll 15, Taneva et al 16, Kajii-Morris 97

5. Incomplete information correlated equilibrium literature
   - Forges 93

6. Relation to Mechanism Design
   - Myerson 82, 87, 91

Literature

- Our methodology papers:
  - "Robust Predictions in Incomplete Information Games," Ecta 13 (includes Cournot results)
  - "Bayes Correlated Equilibrium and The Comparison of Information Structures," TE 16
  - "Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium," AER P&P 2016
  - Full paper on material in lecture in preparation

- Examples
  - Kamenica-Gentzkow 11 and Bergemann Morris 16 (see also Taneva 16)
  - "Information and Volatility" (with Tibor Heumann) JET ; see also "Information and Market Power" (working paper)
  - "First Price Auctions with General Information Structures: Implications for Bidding and Revenue" (with Ben Brooks) Ecta forthcoming; see also "The Limits of Price Discrimination" AER

1. Other Objectives

- Suppose the government was interested in maximizing the probability of at least one firm investing
- This can always be achieved with probability 1....

$\theta = B$	I	N	$\theta = G$	I	N
I	0	$\frac{1}{2}$	I	1	0
N	$\frac{1}{2}$	0	N	0	0

This is true for $\varepsilon = 0$ and by continuity for $|\varepsilon|$ independent of the sign...

- Compare Ely 15, Arieli 15

Other Objectives and a Benevolent Information Designer

- In one firm case, if government had the same objective as the firm, he would always give them full information...
- But in the two firm case, a benevolent government maximizing the (joint) profits of the two firms might still manipulate information in order to correct for externalities and coordinate behavior
- In game

$\theta = B$	I	N
I	$-1 + \varepsilon + z$	-1
N	z	0

$\theta = G$	I	N
I	$x + \varepsilon + z$	x
N	z	0

benevolent government will behave as an investment maximizing government if z is large enough

2. Ordering Information

- Intuition: more information for the player imposes more constraints on the information designer and reduces the set of outcomes she can induce
- Recall Auction Example
- Say that information structure S "is more incentive constrained than" ($=$ more informed than) S' if it gives rise to a smaller set of BCE outcomes than S' in all games
    - in one player case, this ordering corresponds to Blackwell's sufficiency ordering
    - in many player case, corresponds to "individual sufficiency" ordering
- Bergemann-Morris 16, see also Lehrer et al 10 and 11

Nice Properties of Individual Sufficiency Ordering

- Reduces to Blackwell in one player case
- Transitive
- Two information structures are each individually sufficient for each other if and only if they share the same higher order beliefs about Θ
- S is individually sufficient for S' if and only if giving extra signals to S' equals S plus an appropriate correlation device

3. Concavification

- We described two step procedure for solving information design problem (with one or many players):
  1. Characterize all implementable decision rules
  2. Pick the designer's favorite

- Concavification procedure (with one player) [Aumann-Maschler 95 and Kamenica-Gentzkow 11]
  - Identify information designer's utility for every belief of the single player
  - Identify utility from optimal design by concavification, identifying information design only implicitly

- Many player generalization: Mathevet al 16

- Always nice interpretation, sometimes (but not always) useful in solving information design problem

4. Adversarial Equilibrium Selection

- Suppose that an information designer gets to make a communication $\Phi : T \times \Theta \rightarrow \Delta(M)$; new game of incomplete information (G, S, Φ)
- Write $E(G, S, \Phi)$ for the set of Bayes Nash equilibria of (G, S, Φ) and Write $V_S^*(C, \beta)$ for the information designer's utility
- We have been studying the maxmax problem

$$\max_C \max_\beta V_S^*(C, \beta)$$

using a revelation principle argument to show that this equals

$$\max_{\sigma \in BCE(G,S)} V_S(\sigma)$$

- The maxmin problem

$$\max_C \min_\beta V^*(S, C, \beta)$$

does not have a revelation principle characterization

- maxmin problem relates to Carroll 15, Taneva et al 16, Kaji Morris 97

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5. Incomplete Information Correlated Equilibrium

▶ Decision rule $\sigma : T \times \Theta \rightarrow A$ is *incentive compatible* for (G, S) if, for each i, t_i and a_i, we have

$$\sum_{a_{-i}, t_{-i}, \theta} u_i \left((a_i, a_{-i}), \theta \right) \sigma (a|t, \theta) \pi (t|\theta) \psi (\theta) \qquad (1)$$

$$\geq \sum_{a_{-i}, t_{-i}, \theta} u_i \left((\delta (a_i), a_{-i}), \theta \right) \sigma \left(a| (t_i', t_{-i}), \theta \right) \pi (t|\theta) \psi (\theta);$$

for all t_i' and $\delta_i : A_i \rightarrow A_i$.

▶ Decision rule $\sigma : T \times \Theta \rightarrow A$ is *join feasible* for (G, S) if $\sigma (a|t, \theta)$ is independent of θ, i.e., $\sigma (a|t, \theta) = \sigma (a|t, \theta')$ for each $t \in T$, $a \in A$, and $\theta, \theta' \in \Theta$.

▶ Solution Concepts:
  ▶ Bayes correlated equilibrium = obedience
  ▶ Communication equilibrium = incentive compatibility (and thus obedience) and join feasibility
  ▶ etc...

6. Mechanism Design and Information Design

- ▶ Myerson Mechanism Design:
    - ▶ Dichotomy in Myerson (1991) textbook
        - ▶ Bayesian games with communication (game is fixed)
        - ▶ Bayesian collective choice problems (mechanism is chosen by designer)
    - ▶ both combined in Myerson (1982, 1987)
- ▶ Truth-telling (honesty) and obedience constraints always maintained
- ▶ "information design" = "Bayesian games with communication" − truth-telling + informed information designer/mediator
- ▶ compare also informed principal literature