

Ignorance Is Strength: Improving Performance of Matching Markets by Limiting Information[Alternatively: When Should Matching Platforms Limit Information?]*

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Abstract

This paper develops a model for studying the problem of information disclosure faced by a platform that matches buyers and sellers. Buyers search for sellers and are time-sensitive, while sellers have limited capacity for serving buyers and derive heterogeneous payoffs from being matched with different buyers. The platform controls the information the sellers observe about the buyers before forming a match. I show that full information disclosure is inefficient because of excessive rejections by the sellers.

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When the platform observes the sellers' payoff function, it can restore the full efficiency using a coarse disclosure policy, which recommends an action to each seller. For the case where seller preferences are unknown to the platform, I characterize the disclosure policy that maximizes the total welfare. Tighter capacity constraints or a higher buyer-to-seller ratio requires coarser disclosure. In a linear payoff environment with a uniform distribution of seller attributes, the efficient disclosure policy is *upper-coarsening*. For a general distribution of seller attributes, I develop an approach to solving the disclosure problem with heterogeneous and forward-looking sellers.

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1 Introduction

[Piquing reader’s curiosity] Provision of information about goods or trading partners is a main challenge for the efficiency of platforms that match heterogeneous buyers and sellers of products or services, such as platforms for housing rental, labor contracting, and transportation. Providing more information allows participants to identify and pursue the most valuable matches, avoids adverse selection and moral hazard, and matching platforms spend significant resources eliciting match-relevant information from users¹. However, sometimes this information is not fully revealed to the users. For example, Uber currently does not show drivers the passenger’s destination until after the driver has accepted the ride, even if the passenger has entered it into the application. Airbnb hosts who turn on the “Instant Book” feature commit to accepting all requests without knowing the details of prospective guests. Why do such platforms choose not to disclose all the relevant information fully? What is the optimal information intermediation policy for platforms that care about both sides of the market? What does it depend on?

[explain the setting I am working in, asymmetric treatment of buyers and sellers] I study the information intermediation problem in the context of the following model. Buyers seek sellers’ services to complete their jobs. Buyers contact sellers, who then review buyer attributes and choose whether to accept or reject the buyer job. Buyers are short-lived and must match quickly, but they are indifferent as to which seller completes their task. Sellers are long-lived and are indifferent to the match speed, but they have heterogeneous preferences over buyers. In particular, not all jobs are profitable. Sellers have limited capacity for serving buyers. If a seller accepts a job, he becomes unavailable for a fixed period of time and cannot accept new jobs. The platform designs an information disclosure policy that governs which buyer attributes are disclosed to sellers before they

¹**EFL16** Einav *et al.* (2016) name eliciting and aggregating the dispersed user information a key objective of peer-to-peer platforms. **Lew11** Lewis (2011) studies auto sales on eBay and finds that buyers are skeptical of the sellers who post few pictures of the car they are selling. **TZ15** Tadelis and Zettelmeyer (2015) show that in wholesale automobile auctions information disclosure helps match heterogeneous buyers to cars of varying quality.

decide whether to accept buyer requests. I consider the general platform's objective of maximizing a weighted average of buyer surplus and seller profits. Note that in this model, buyers and sellers are asymmetric, which focuses the information disclosure problem on the seller side of the market. If buyers had preferences about a match, the disclosure problem would be two-sided; see [subsec:Discussion-of-the-assumptions](#) [Section 2.4](#) for a discussion of this.

[explain why full disclosure need not be optimal] In this model, a key tradeoff arises between match quality and match rate. Full disclosure of information need not be optimal, because information disclosure can negatively affect the match rate and partially or wholly offset the positive effect on the match quality. Seller profit is a product of match quality and match rate, and buyer surplus is an increasing function of the match rate. So, seller profit function does not align with the platform's objective. Specifically, profit-maximizing sellers will *cream-skim* and reject inefficiently many jobs. Unlike sellers, the platform cares about welfare on both sides of the market, and so efficient disclosure is coarse if the negative effects of information disclosure on the match rate are strong. This paper considers how the optimal disclosure policy depends on the details of the seller side of the market.

[explain why there is a tradeoff between match quality/match rate] Efficiency requires that both match quality and match rate be high. Seller profit is larger when both are high, whereas buyer surplus is large when the match rate is high. However, match quality and match rate are in conflict. Enforcing higher match rates compromises quality, because sellers are forced to accept inferior jobs. Similarly, allowing sellers to cherry-pick the most valuable buyers leads to higher rejection rates and lower match rates. For example, if Uber drivers reject passenger requests more often, passengers will have longer wait times. On Airbnb, if hosts reject guest inquiries more frequently, guests must spend more time searching.

[Explain the three effects of info disclosure] The optimal disclosure policy must balance three effects: the positive effect on the seller quality of the match and the negative static and dynamic effects on the match rate. First, the effect of disclosure on the match

quality is straightforward. From a seller’s point of view, more information increases his set of attainable payoffs. Holding the match rate fixed, he benefits from more information about buyers. Second, information disclosure reduces the platform’s ability to induce sellers to accept jobs. A key observation is that the platform can increase a seller’s expected marginal profit by limiting the information revealed to him. To see why, note that pooling marginally profitable jobs (which are rejected because the seller would prefer to wait for a better job) with inframarginal jobs (which are accepted) alters a seller’s expected marginal profit.. Ignoring dynamic effects, higher marginal profit induces sellers to accept more jobs, which leads to higher match rates. Third, making more information available to a seller increases returns on his search. Indeed, since acceptance precludes further search, the opportunity cost of accepting a given offer is higher. As a result, sellers reject jobs more often, and the match rate goes down. This dynamic effect on the match rate exists only when sellers have limited capacity and are forward looking. The match-quality effect provides a motive for the platform to disclose information, while the match-rate effects provide a motive to limit information.

[benchmark with identical sellers] If sellers are identical, then some information coarsening is always optimal. In fact, such coarsening is necessary, regardless of whether the platform maximizes total welfare, buyer surplus, or the joint seller profits (see [Figure 2](#)). The disclosure policy that maximizes total welfare is coarse but has a simple form. It sends one of two recommendations to sellers, “accept” or “reject,” which are chosen in such a way that the sellers have incentives to follow them. The welfare-maximizing policy is the intermediate case between the buyer-optimal disclosure policy and the seller-optimal disclosure policy. To maximize the buyer surplus, coarsening is necessary, because sellers underweight the match rate relative to buyers in their payoff functions. The higher the buyer search costs (and thus the costlier the rejections), the coarser the optimal disclosure policy. Perhaps more surprisingly, when maximizing the joint seller profits, the optimal disclosure is also coarse.

[Seller coordination failure] The adverse effect of disclosure on joint seller profits is

a form of seller coordination failure. In a marketplace where sellers act independently, each seller keeps his schedule open by rejecting low-value jobs to increase his own chances of getting high-value jobs. As a result, sellers spend significant time waiting for high-value jobs. Collectively, this behavior is suboptimal. because all profitable jobs have to be completed in order to maximize the joint profits. I attribute the source of the coordination failure to what I call the *cream-skimming externality*: By rejecting a job, a seller remains available on the marketplace and attracts a fraction of subsequent buyers who otherwise would move to other sellers. As a result, the other sellers face fewer profitable jobs and realize lower profits. The cream-skimming externality arises only when sellers have limited capacity and are forward looking. When this externality is present, sellers underestimate the effect of their own acceptance rate on the platform-wide match rate and collectively resolve the tradeoff between match quality and match rate suboptimally. Limiting information decreases the opportunity cost of rejection and can increase the match rate to the seller-optimal level.

[Heterogeneous sellers] When sellers are unobservably heterogeneous, the optimal disclosure policy is finer than in the case of identical sellers. With heterogeneous sellers, the effect of information disclosure on match quality is stronger, but there are important subtleties. Coarse disclosure tailored to increase one seller’s acceptance rate can drive another seller’s average profit below zero and violate his individual rationality constraint. If any coarsening is optimal in this case, the degree of coarsening is now not obvious. The optimal policy should accommodate the possibly opposite reactions of different sellers to disclosure and will depend on the shape of the seller type distribution.

[Linear payoff environment] To understand how seller heterogeneity affects the optimal disclosure policy, I study a linear payoff environment with vertically differentiated buyers and sellers. In this case, the optimal disclosure policy depends on the shape of the seller type distribution, the intensity of buyer traffic, and the tightness of seller capacity constraints. In this case, the seller match payoff is linear in the buyer and seller characteristics, and buyer match payoff is constant. I first consider the case of uniform distribution of seller types,

where I can fully characterize the optimal disclosure policy. A key result in this case, is that the disclosure policy that maximizes the match rate is *upper-coarsening*: High buyer types are pooled, and low buyer types are revealed truthfully. This is in stark contrast to the case of unconstrained sellers, in which information disclosure does not affect the match rate (cf. ^{KLMZ-wp}[Kolotilin et al. \(2015\)](#)). If the buyer-to-seller ratio is high or the sellers are more capacity constrained, then the efficient disclosure is also upper-coarsening. And conversely, if the buyer-to-seller ratio is low and the sellers are loosely constrained, the full disclosure is efficient.

Turning to a general (non-uniform) distribution of sellers, I find that the optimal disclosures can have a variety of qualitatively different shapes, depending on the distribution. The heuristic in the case of unconstrained sellers is to pool buyer types on the increasing part of the buyer probability density function g , and reveal buyer types on the decreasing part of g . In the case of capacity-constrained sellers, this heuristic should be further qualified to reflect seller utilization rates. In this case, the design of the optimal information disclosure is a nontrivial problem. To see why, note that with forward-looking sellers, the information disclosure policy determines not only the seller's stage payoff but also the distribution of his potential future payoffs. As a result, a seller's decision to accept depends not only on the posterior mean of the state but also on the entire signaling structure. This makes the cavification approach of ^{KG11}[Kamenica and Gentzkow \(2011\)](#), as well as the linear programming approach of ^{Ko115}[Kolotilin \(2015\)](#), unsuitable for analysis of my model. I approach this problem by representing signaling structures as a particular class of convex functions and then using the calculus of variations to find the first-order necessary conditions (^{lem:main-lemma}[Lemma 3](#)).

[applications] The model is motivated mainly by the matching problems of digital marketplaces. To again use Airbnb as an example, guests (buyers) are differentiated by age, gender, race, personality, etc. Hosts (sellers) have preferences regarding the number of guests, their gender, race, lifestyle, etc. While guests prefer to minimize the time spent

searching and book a listing instantly,² hosts want to avoid offensive or inconvenient guests. Airbnb introduced the “Instant Book” feature to satisfy the guests’ demand for convenience. In my model, this corresponds to a coarse disclosure policy. In the current version, a host can specify the types of guests can book his listing instantly but the set of restrictions is limited (includes pets and smoking controls but does not include gender control).³ The problem of designing optimal guest segmentation is equivalent to the problem of the optimal information disclosure policy, and, as argued above, has important tradeoffs. Uber’s matching system is another notable example. Uber directs passenger (buyer) requests to drivers (sellers), and the requests include information about the passenger. In the current version of UberX, the passenger’s destination is not disclosed, even though it is relevant to the drivers’ payoff. One final example is TaskRabbit, an on-demand labor platform. On this platform, freelancers (sellers) commit to an hourly rate over a broad category of tasks, such as moving. The problem of the optimal category breadth is equivalent to the problem of the optimal disclosure policy of client task characteristics, and it can be analyzed within the same framework.

The rest of the paper is organized as follows. The next subsection relates this paper to the existing literature and highlights my contributions. [Section 2](#) introduces the model of the matching market, and establishes the existence and uniqueness of equilibrium. [Section 2.4](#) contains a discussion of the key assumptions of the model. [Section 3](#) sets up the platform’s information disclosure problem, and solves and explains it in different settings. [Section 3.1](#) explains the seller coordination failure, [Section 3.2](#) studies the setting in which the sellers are identical, and [Section 3.3](#) discusses the competing effects of information disclosure. [Section 3.4](#) studies the setting with heterogeneous sellers and presents the main characterization result of the paper. [Section 4](#) describes the technique used in proving the main theoretical result. In [Section 5](#) I discuss the implications for design of matching markets and conclude.

²[Fra15](#)
[Fradkin](#) (2015) reports that on Airbnb, an initial rejection decreases by 51% the probability that the guest eventually books any listing.

³For more details on the Instant Book, see <https://www.airbnb.com/host/instant>.

Related literature. The paper primarily contributes to search-and-matching in markets and information design strands of economic literature. Matching markets have been extensively studied in the literatures of economics and operations research. To the best of my knowledge, no prior work has explored in detail the design of information intermediation for matching platforms.

The role of information in matching and peer-to-peer markets is an active area of economic research. A closely related paper is [HMS09 Hoppe *et al.* \(2009\)](#) that shows that the option of disclosing information leads to wasteful signaling, that in certain cases can offset the benefits of improved matching. They consider a black-box matching function and only two information structures, full and no disclosure, while my paper allows for any possible information structure and microfound the matching function with search. [SS01 Shimer and Smith \(2001\)](#) also study search and matching in heterogeneous environment. Similarly to my model, the matching friction arises because a seller has a private benefit of waiting. However, they study taxes as the intermediary’s intervention policy. In contrast to many search-and-matching models that have the main source of inefficiency is mis-coordination due to simultaneity and unavailability ([BSW01, Kir09, AJK14 Burdett *et al.* \(2001\)](#); [Kircher \(2009\)](#); [Arnosti *et al.* \(2014\)](#)), I assume away the simultaneity and unavailability driven frictions to focus on cream-skimming. [LM10 Levin and Milgrom \(2010\)](#) show that in online advertising markets, standardization, or “conflation,” helps to address at least two problems associated with excessive targeting—adverse selection and reduced competition. [EFL16 Einav *et al.* \(2016\)](#) point out that in peer-to-peer markets there is an important tradeoff between eliciting dispersed information and minimizing various transactions costs to keep the user experience convenient. My paper studies a related question of what buyer information, even if elicited, should be reported to sellers. Taking into account that elicitation may be costly, my paper can inform the designers on how detailed data should be collected from the users.

The paper also contributes to the literature on information design ([AMS95 Aumann *et al.* \(1995\)](#); [GH80 Grossman and Hart \(1980\)](#); [Mil81 Milgrom \(1981\)](#); [KG11 Kamenica and Gentzkow \(2011\)](#); [RS10 Rayo and Se](#)

gal (2012); ^{BM16-TE}Bergemann and Morris (2016); ^{Ko115}Kolotilin (2015)), as it extends the model of the disclosure problem with a heterogeneous audience to the case with endogenously available and forward-looking receivers. Although forward-looking, the receivers receive one signal per job, which differs my work from the gradual learning settings of ^{Ely15}Ely (2015) and ^{Smo15}Smolin (2015). ^{RS10}Rayo and Segal (2012) study the setting with arbitrary complementarities between the sender’s and receiver’s payoffs and show that the sender-optimal disclosure policy pools “non-ordered prospects”. Compared to them, I have a simpler sender’s payoff function, all prospects are “ordered,” but the receivers have strategic interactions and optimize dynamically.

My work differs from the theory of centralized matching (^{Rot08}~~Roth~~ (2008)) in that I do not assume that agents know their true preferences over potential partners prior to engaging in a match. Critically, the participants of matching platforms have to inspect potential matches to identify the valuable ones. Papers that study the role of uncertainty in centralized dynamic matching include ^{AJM13,ALG16}~~Ashlagi~~ *et al.* (2013); Akbarpour *et al.* (2016). They show that waiting to thicken improves matching in kidney exchange because individual agents do not fully incorporate the benefits of waiting. I emphasize the “reversed” effect: individual sellers wait too long because they do not fully incorporate the buyer loss from rejections (or buyer costs of waiting).

2.1 Setup

There is mass 1 of sellers, who always stay on the platform; they never leave or arrive. The sellers do not actively look for jobs, but instead screen buyer requests: Each worker is presented with a sequence of job offers at a Poisson rate, and decides whether to accept or reject them in order to maximize his discounted profit flow. At each moment in time, a seller is either available and waiting for new jobs, or busy working on a job. An accepted job takes time τ to complete, during which time the seller cannot receive new jobs. This is the main source of matching friction. In what follows, I also refer to τ as the seller capacity constraint, because the higher value of τ , the fewer jobs the seller can complete during the same period of time.

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job. See the detailed discussion of the assumptions on buyer search and buyer arrival rate in [Section 2.4](#). See figure [1a](#) for the illustration of the matching process.

Buyer and seller preference heterogeneity. There are two dimensions of heterogeneity in the market. First, each seller has a heterogeneous match payoff across buyers. Second, different sellers have different payoff functions. For the platform’s information disclosure problem, I use the following notation: Let x denote a buyer type, with the interpretation that x is comprised of buyer characteristics observed by the platform.⁴ The space of buyer types, X , is a compact subset of a Euclidean space. The distribution of buyer types x is F , with full support. Let y denote a seller type, with the interpretation that y is comprised of seller characteristics unobserved by the platform.⁵ The space of seller types, Y , is a compact subset of a Euclidean space. The distribution of y is G , with full support that admits density g , where g is differentiable on Y . The seller’s profit for one match is $\pi(x, y)$. Assume π is continuous and, for any y , there is some x such that $\pi(x, y) > 0$. The buyer’s net match payoff is $u(x, y)$. Assume that all incoming buyers have a nonnegative match payoff:

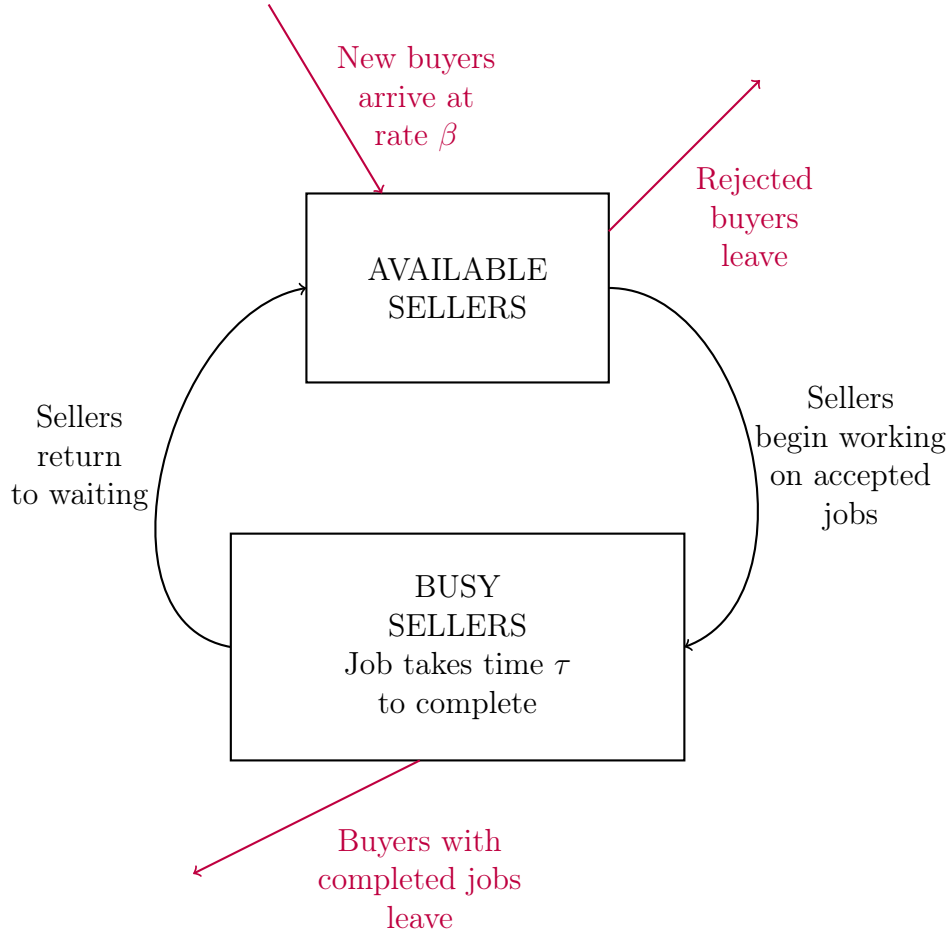
$$u(x, y) \geq 0 \quad \forall x, y. \tag{1}$$

{eq:buyer-r

Platform: Information intermediation. Before the matching process starts running, the platform designs and commits to a disclosure policy that governs which buyer characteristics are disclosed to sellers. The platform observes buyer type x and sends a signal about x to the seller. The seller does not receive any information about x other than the platform’s signal. Let $S = \Delta(X)$ be the set of all posterior distributions over X . An *information*

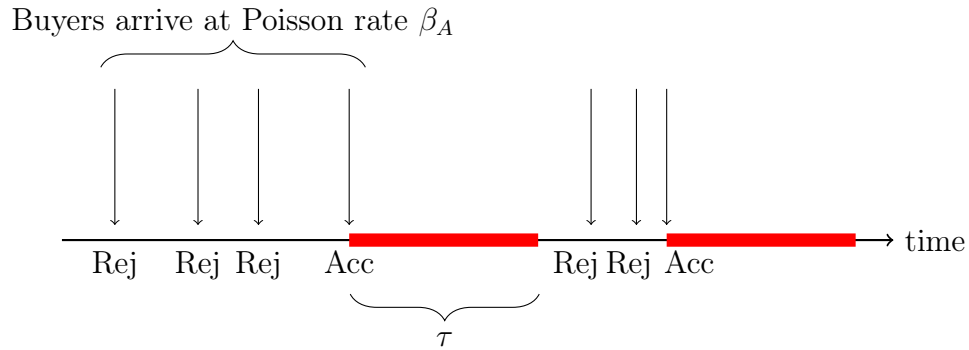
⁴Buyer type x captures the payoff-relevant information the platform elicits from the buyer about the job, whether passively from the buyer’s cookies and queries or actively by asking questions. For example, on Uber, x would include the rider’s destination; on Airbnb, x would include the guest’s race, age, and gender.

⁵Seller type y captures the payoff-relevant information that, for whatever reason—costly, unethical, etc.—the platform did not elicit from the sellers. For example, on Uber, y would include the driver’s preference for long rides and traffic; on Airbnb, y would include the host’s preference for his guest’s age, gender, socio-economic status, race, etc.



(a) Spot matching process. Buyers arrive at exogenous rate β and contact available sellers. If rejected, a buyer leaves the platform. If accepted, the buyer forms a match which lasts for time τ . After that time elapses, the buyer leaves the platform, and the seller returns to waiting.

Search-and-matching



(b) Seller dynamic optimization problem with screening and waiting. An available seller receives buyer requests at Poisson rate β_A . If a request is accepted, the seller becomes busy for time τ , during which he does not receive new requests.

ers-repeated-search

Figure 1: The model of the matching market has two main components: the spot matching process and the seller dynamic optimization problem.

disclosure policy $\lambda \in \Delta(S)$ is a probability distribution of posteriors.⁶ The interpretation is that $s \in S$ is the platform's signal to the seller, and so $\lambda(S')$ is the fraction of buyers with signals $S' \subset S$.⁷ Note that a disclosure policy can be seen as a two-stage lottery on X whose reduced lottery is the prior F . The set of possible disclosure policies is then

$$\left\{ \lambda \in \Delta(S) : \int s d\lambda(s) \sim F \right\}.$$

When a buyer of type x arrives, the platform draws a signal according to λ and shows it to the seller. The seller knows the platform's choice of λ , and so his interpretation of a signal as a posterior is correct. The full disclosure policy, denoted by λ^{FD} , perfectly reveals buyer type x to the sellers. The no disclosure policy fully conceals x . Disclosure policy λ' is *coarser* than λ'' if λ' is a Blackwell garbling of λ'' . That is, the platform can obtain λ' from λ'' by taking λ'' and pooling some x 's.

Steady-state distribution of sellers. The matching process is the dynamic system in which sellers become repeatedly busy and available. A steady state of the matching process is characterized by the fraction of available sellers of every type and their acceptance rates. Formally, let $\alpha(y) \geq 0$ be the *acceptance rate*, the fraction of buyers accepted by type- y sellers. Let $\rho(y)$ be the *utilization rate*, the fraction of type- y sellers who are busy. Denote the average utilization rate by $\bar{\rho} := \int_Y \rho(y) dG(y)$. Since the total mass of sellers is 1, $\bar{\rho}$ is also the mass of busy sellers.

In a steady state, the flow of sellers who begin working is equal to the flow of sellers who finished the job and return to waiting. The flow of beginning sellers is equal to the product of the buyer flow that type- y sellers receive and type- y sellers' acceptance rate.

⁶When X is a compact subset of a Euclidean space, $\Delta(S)$ is the set of Borel probability distributions with the weak-* topology on $\Delta(X)$.

⁷I focus on the "public" signaling, where the same λ applies to all sellers. I am not studying the mechanism design problem where the platform elicits or learns the seller's type y and tailor the disclosure policy to the seller's type. [Kolotilin et al. \(2015\)](#) find that in the one-shot persuasion problem with linear payoffs, public signaling is equivalent to private signaling.

Since buyers distribute uniformly across the available sellers, the buyer flow to type- y sellers is $\beta \frac{(1-\rho(y))g(y)}{1-\bar{\rho}}$. Thus, the beginning flow is $\beta \frac{(1-\rho(y))g(y)}{1-\bar{\rho}} \alpha(y)$. The flow of returning sellers is $g(y)\rho(y)/\tau$, because the mass of busy type- y sellers is $g(y)\rho(y)$ and jobs are completed in time τ . In a steady state, the flow of beginning sellers is equal to the flow of returning sellers:

$$\beta \frac{(1-\rho(y))g(y)}{1-\bar{\rho}} \alpha(y) = \frac{g(y)\rho(y)}{\tau}, \quad \forall y \in Y. \quad (2) \quad \boxed{\text{\{eq:SS\}}}$$

Seller dynamic optimization problem. Denote by β_A the Poisson rate at which buyers request an available seller. Since buyers contact only available sellers, β_A depends on the mass of available sellers. In a steady state, the mass of available sellers is $1 - \bar{\rho}$, and so

$$\beta_A := \frac{\beta}{1-\bar{\rho}}. \quad (3) \quad \boxed{\text{\{eq:beta_A\}}}$$

Note that β is the flow rate at which buyers arrive at the platform, while β_A is the Poisson rate at which buyers arrive to available sellers.⁸ The particularly simple form of the relationship between the two which is given in Eq. (3) follows from the uniform distribution of buyers across available sellers. Sellers take β_A as given because there is a continuum of sellers on the platform, and any individual seller's actions do not affect β_A .

A risk-neutral seller solves the dynamic optimization problem of maximizing the average profit flow. The seller faces the sequence of jobs arriving at Poisson rate β_A , and for each job observes the platform's signal s and chooses to accept or reject it. See [fig:Sellers-repeated-search](#) for an illustration. Let $\pi(s, y) = \int_X \pi(x, y) ds(x)$ be seller y 's expected profit if he accepts a job with signal s , and let $V(y)$ be the average profit flow when a seller of type y acts optimally (the value function)⁹. Let $v(s, y)$ be the value of a new job with signal s , where v includes

⁸On the one hand, an individual available seller faces a stochastic arrival process such that the probability of arrival of a new buyer over the time interval dt is $\beta_A dt + o(dt)$. On the other hand, the available sellers jointly face the deterministic arrival process of buyers, where over time interval dt the mass of buyers that arrive is $\beta_A dt$.

⁹For example, if a seller earns \$1 on each job, and the time interval between starting a pair of consequent jobs is 2, then $V(y) = 1/2$.

the option value of rejecting the job and the opportunity cost of accepting it. The value of a new job is 0 if the seller rejects it, and $\pi(s, y) - \tau V(y)$ if he accepts it, where $\tau V(y)$ is the opportunity cost of accepting, which is due to being unavailable for time τ . Therefore, $v(s, y) = \max\{0, \pi(s, y) - \tau V(y)\}$. The average profit per unit time equals the product of the expected value from one job and the expected number of new jobs: $V(y) = \beta_A \mathbb{E}[v(s, y)]$.¹⁰ Thus together, the seller optimization problem is given by

$$V(y) = \beta_A \int \max\{0, \pi(s, y) - \tau V(y)\} d\lambda(s). \quad (4)$$

The seller's strategy is a function $\sigma(\cdot, y): S \rightarrow [0, 1]$ that for every seller of type y maps a signal to the probability of accepting it. The seller's acceptance rate is the ex ante probability of accepting a job:

$$\alpha(y) = \int \sigma(s, y) d\lambda(s). \quad (5)$$

{eq:alpha}

2.2 Examples of marketplaces

In this section, I explain how the model fits the marketplaces of Uber and Airbnb and on-line temp agencies such as TaskRabbit. I will return to these applications in the discussion section after I state my main results. Recall that y captures the seller payoff heterogeneity unobserved by the platform, and x captures the buyer heterogeneity observed by the platform.

Uber. When idle, drivers receive requests from passengers. Type y includes driver's home location, preference for long rides, and tolerance for congestion, while x includes the passenger's destination and ride history. The prices per mile and per minute are fixed (conditional on aggregate multipliers, such as surge pricing). Drivers do not like very short

¹⁰I consider the time-average payoff rather than the discounted sum, because discount rate is not essential for my argument. However, the results immediately generalize to the case, where the seller has discount rate r , by replacing τ with $\tau_r = \frac{1-e^{-r\tau}}{r}$.

rides or rides to remote neighborhoods. Passengers do not like waiting.¹¹ Concealing the passenger’s destination from drivers is an example of information coarsening.

Airbnb. Hosts are capacity constrained in rooms: Once a room is booked for a given date, the host cannot accept a better guest. Type y includes the host’s preference for age, race, personality, and daily schedule. Type x includes the guest’s gender, age, socio-economic status, and lifestyle. Every host sets a price, which applies to all guests, but he may prefer to reject guests who he expects will be a bad fit. The “Instant Book” feature, if adopted by a host, acts as a coarse disclosure policy because the host pre-commits to accepting all guests filtered by only few characteristics (pets, smoking, infants).¹²

TaskRabbit. The service providers are capacity constrained in the number of tasks they can do per week. Once a service provider agrees to do one task, he is limited in picking new tasks. Type y includes the service provider’s skill and work ethic. Type x includes the job category, job difficulty, the client’s professionalism and the location. Service providers set an hourly rate that applies to all tasks in the same category. Getting service providers to commit to an hourly rate that applies to a broad category of tasks is a form of information coarsening.

2.3 Equilibrium definition and existence

A steady-state equilibrium is a market outcome in which the sellers take the buyer arrival rate β_A as given and optimize independently, and the seller’s busy-available flows balance out. Formally, a pair $(\sigma, \bar{\rho})$ constitutes a *steady-state equilibrium* if the following hold

1. [Optimality] For all y and every type- y seller, $\sigma(\cdot, y)$ is an optimal strategy given the buyer Poisson arrival rate $\beta_A = \beta/(1 - \bar{\rho})$.

¹¹~~HKM~~ Hall *et al.* (2015) study a natural experiment in Uber marketplace, that shows the passengers are very sensitive to delays.

¹²Instant Book is interesting because instead of imposing coarsening, Airbnb offers the feature as an option. This way, Instant Book also serves as a screening device for the platform. For more details about the Instant Book, visit <https://www.airbnb.com/host/instant>.

2. [Steady state] The average utilization rate $\bar{\rho}$ arises in a steady state when sellers play σ , as shown in [\(eq:SS 2\)](#) and [\(eq:alpha 5\)](#).

exists-and

Proposition 1. *A steady-state equilibrium exists, and it is unique up to the acceptance of marginal jobs in the following sense: If $(\sigma^i, \bar{\rho}^i)$, $i = 1, 2$, are two steady-state equilibria, then (1) $\bar{\rho}^1 = \bar{\rho}^2$, and (2) for any $y \in Y$, $\sigma^1(\cdot, y)$ and $\sigma^2(\cdot, y)$ coincide except on $\{s: \pi(s, y) = \tau V(y)\}$.*

To prove this result, first, I show that for an arbitrary vector of acceptance rates $\alpha(y)$, there is a unique steady-state value of $\bar{\rho}$ ([lem:rho-and-monotonicities Lemma 5](#)). Then, the average utilization rate $\bar{\rho}$ is increasing and continuous in $\alpha(y)$ for any $y \in Y$. The uniqueness of the equilibrium follows from monotonicity of the reaction curves of α in $\bar{\rho}$ and $\bar{\rho}$ in α . Namely, if the average utilization rate $\bar{\rho}$ increases, then buyer traffic to each available worker increases; as a result sellers become pickier and the acceptance rate $\alpha(y)$ decreases. As $\alpha(y)$ increases, sellers become less available, and $\bar{\rho}$ goes down. For details, see the proof in [\(proof:egm-exists Appendix B\)](#).

assumptions

2.4 Discussion of the modeling assumptions

In this subsection, I discuss in detail the critical assumptions of my model. The assumptions are motivated by the stylized facts about the online platforms.

ch-attempt

Assumption 1. *Buyers make a single search attempt.*

Rejection intolerance on the part of buyers is a simplifying assumption but captures an aspect of real-life matching markets in that rejections are costly to buyers (wasted time, wasted search effort, bidding costs). Moreover, buyers often do not continue searching after a rejection. For example, [Fra15 Fradkin \(2015\)](#) reports that on Airbnb, an initial rejection decreases by 51% the probability that the guest eventually books any listing.¹³

Assumption 2. *Buyers contact available sellers only.*

¹³The results will generalize if buyers return for consequent search attempts. If buyer makes several search attempts, then buyer arrival rate is effectively increased. Sellers still cream-skim but information coarsening is even more effective because higher buyer arrival rate means higher seller option value of rejection.

The goal of the paper is to explore the matching friction that pertains to preference heterogeneity and screening. Therefore, I assume away the coordination friction owing to simultaneity, when several buyers request the same seller at the same time. Also, I assume away the coordination friction due to unavailability, which arises when buyers request unavailable sellers who did not update their status or do not have the means to do so. The coordination frictions in matching markets have been extensively studied in the theoretical literature ([BSW01, Kir09, Halaburda2013, AJK14](#); [Burdett *et al.* \(2001\)](#); [Kircher \(2009\)](#); [Halaburda *et al.* \(2015\)](#); [Arnosti *et al.* \(2014\)](#)), and digital platforms usually have good technological means of resolving the simultaneity-driven friction¹⁴.

assignment

Assumption 3. *Buyers contact an available seller chosen uniformly at random.*

This assumption implies that buyers do not search for better sellers and holds, for instance, in the situation when buyers are indifferent between what seller completes their job. The assumption allows me to analyze the match quality - match rate tradeoff in a cleaner model, in which one side of the market (buyers) has homogenous match quality. The assumption has two implications to the seller optimization problem. First, any seller type faces the same buyer arrival rate, β_A . And second, each seller faces the same distribution of buyer types, F .

If buyers have heterogeneous match quality and search for better matches, then the platform faces the disclosure problem on both sides of the market. In this case, in addition to the disclosure policy on the seller side, the platform designs the disclosure policy on the buyer side.

Disclosure to buyers can play several roles, depending on the buyer search behavior. First, the platform can use disclosure to direct buyers to the sellers with higher acceptance rates. Imagine buyers observe seller acceptance rates imperfectly. Then they request selective sellers too often compared to what would be efficient. The platform can pool selective and non-selective sellers to shift buyer attention more to non-selective sellers. Second, the platform

¹⁴E.g. [Fra15](#) [Fradkin \(2015\)](#) finds that on Airbnb the coordination friction explains only 6% of failed matches.

can use information disclosure to direct buyers to the sellers who value them the most. Imagine, the prices do not reflect the seller match payoffs perfectly. Then the disclosure policy that pools matches that are less valuable for buyers and more valuable for sellers with those that are, conversely, more valuable for buyers and less valuable for sellers, will be efficiency-improving.¹⁵ Whether the combination of seller side disclosure with the buyer side disclosure has interesting interactions is an interesting venue for the future research.

cess-demand

Assumption 4 (No Excess Demand). *Collectively, it is physically possible for sellers to complete every buyer job: $\beta\tau < 1$*

This assumption makes the exposition cleaner and relaxing it yields nothing additional in the way of insights. Relaxing it requires more notation to deal with either automatic rejections or queues. I do this in ^{sec:Extensions}Appendix D and show that qualitatively the results do not change.

¹⁵^{RS10}Rayo and Segal (2012) study in a static match model the last situation when both buyers and sellers derive heterogeneous payoffs from matches. They find in the case of the uniform distribution of receiver reservation values, that the optimal disclosure policy pools “non-ordered prospects,” where two prospects are non-ordered in the sense described above.

3 Market Design: Information Disclosure Policy

This section studies how the platform's information disclosure policy affects the equilibrium market outcome. I consider the general platform's objective of maximizing a weighted average of the buyer and seller surpluses, which includes as special cases maximization of welfare, maximization of the joint seller profits and maximization of the match rate. I show full disclosure produces a suboptimal market outcome, because information disclosure aggravates sellers' cream-skimming behavior. I start with the benchmark case of identical sellers, and then move on to a more general case of vertically differentiated sellers. In the latter, the optimal disclosure policy depends nontrivially on the shape of the seller distribution, the buyer arrival rate, and the seller capacity constraint.

3.1 Full disclosure: Seller coordination failure

This subsection establishes that full disclosure leads to a Pareto suboptimal market outcome. The reason behind this is the sellers' coordination failure. The inefficiency arises from the sellers' decentralized decision-making and their capacity constraint.

To present the results of this and the next sections, I need to define Pareto optimality in my setting. The market outcome $O = (V(\cdot), U)$ is a combination of seller profits and buyer surplus. I say that a market outcome is *feasible* if there is a seller strategy profile that generates it and $V(y) \geq 0$ for all y . A feasible outcome O is Pareto optimal if there is no other feasible O' such that $V(y)' \geq V(y)$ for all y and $U' \geq U$, and either there is at least one seller type or buyers who are strictly better off. The Pareto frontier is the set of all Pareto-optimal outcomes. Market outcome O is *implementable* if there is a disclosure policy such that the equilibrium outcome is O . The welfare is the sum of the buyer surplus and joint seller profits. A market outcome is *efficient* if there is no other feasible outcome with higher welfare.

Write $V^\sigma(y)$, $\alpha^\sigma(y)$, U^σ for the steady-state seller profits, acceptance rates, and consumer

surplus when strategy profile σ is played. Imagine that the platform starts with full disclosure as its default disclosure policy. The next proposition shows that there is a strategy profile under which sellers are strictly better off than in the full-disclosure equilibrium, they complete more jobs, and the buyers are also better off. The result implies that in matching markets, sellers face a coordination problem.

suboptimal

Proposition 2. *Let σ^{FD} be the equilibrium strategy profile under full disclosure. Then there exists $\tilde{\sigma}$ such that for all y the following hold*

$$\begin{aligned}\tilde{V}(y) &> V^{FD}(y), \\ \tilde{U} &> U^{FD}, \\ \tilde{\alpha}(y) &> \alpha^{FD}(y).\end{aligned}$$

A full proof of this is given in the [proof:FD-suboptimal](#) [B](#). A very high-level intuition for the coordination problem is the following: A seller keeps his schedule open by rejecting low-value jobs in order to increase his own chances of getting high-value jobs. As a result, in equilibrium, sellers spend a lot of time waiting for high-value jobs. Collectively, this behavior is suboptimal, because all profitable jobs have to be completed. I call this behavior *cream-skimming*.

Here I give a proof sketch for the case of identical sellers, which gives further insight into the nature of the seller coordination problem. Under full disclosure, a seller's profit V^{FD} is strictly positive, because sellers accept only profitable jobs. The opportunity cost of accepting equals τV^{FD} , and so it, too, is strictly positive. In equilibrium, the profit on every job that is accepted is $\pi(x) \geq \tau V^{FD}$ (see Eq. [\(4\)](#)^{[eq:V-via-pi](#)}). However, the profitable jobs are those with $\pi(x) \geq 0$. Hence, some profitable jobs are rejected. Consider a strategy $\tilde{\sigma}$ that maximizes the joint seller profits. Such a strategy requires every seller to accept all jobs with $\pi(x) \geq 0$. . Naturally, $\tilde{\sigma}$ yields $\tilde{V} > V^{FD}$. The acceptance rate is also higher: $\tilde{\alpha} > \alpha^{FD}$. Consumers are better off, because the consumer surplus is increasing in the acceptance rate.

Interestingly, unlike price competition that benefits buyers and improves market effi-

ciency, seller competition for better buyers hurts buyers and decreases market efficiency. Thus, cream-skimming is a market failure. I attribute the source of the failure to what I call the *cream-skimming externality*. By rejecting a job, a seller remains available in the marketplace and attracts a fraction of subsequent buyers who otherwise would go to other sellers. As a result, the other sellers face fewer profitable jobs and realize lower profits. The necessary condition for the cream-skimming externality to arise is that sellers be forward-looking and capacity constrained.

The fundamental reason behind coordination failure is that collectively, the sellers are not capacity constrained (in time), while individually they *are* capacity constrained. When an individual seller accepts a job, he is off the market for time τ and cannot accept new jobs. Therefore, when he is available, he is afraid of missing high-value future jobs and therefore rejects low-value jobs. However, there are always some available sellers in the market, and it is feasible to accept all profitable jobs. Mathematically, the distinction between individual and collective capacity constraints is captured by having a continuum of sellers, so that while buyer traffic to each individual seller is stochastic, the aggregate buyer traffic is deterministic.

[prop:FD-suboptimal](#)
[Proposition 2](#) shows that full-disclosure equilibrium is Pareto dominated by some strategy profile $\tilde{\sigma}$. Is there an information disclosure policy that induces $\tilde{\sigma}$? In the next section, I give an affirmative answer to this question for the case of identical workers. The situation with heterogeneous sellers is trickier, and I study that in [subsec:heterogeneous-sellers](#)
[Section 3.4](#).

3.2 Benchmark: Optimal information disclosure with identical sellers

I start by considering the case of identical sellers, that is where Y is a singleton. The next proposition establishes that any Pareto-optimal outcome (in seller profits-buyer surplus space) is implementable by a disclosure policy.

Proposition 3. *Suppose the sellers are identical. Then for any Pareto-optimal outcome*

(V, U) , there is a disclosure policy that implements it. Furthermore, an optimal disclosure policy has a binary structure.

The proof relies on the Revelation Principle. Since there is one seller type and two actions, it suffices to consider only disclosure policies that send two signals, where a signal is an “action recommendation” (in this case, either “accept” or “reject”) . With such a binary signaling structure, the seller dynamic optimization problem reduces to a static optimization problem. Indeed, since there is only one signal with a recommendation to accept, there is only one type of acceptable job. All profitable jobs are the same, and there is no return to cream-skimming. Since a Pareto-optimal outcome is feasible (i.e. $V \geq 0$), sellers have an incentive to follow the platform’s recommendation to accept. For the details of the proof, see [proof:id-sellers-Pareto-frontier](#).

[prop:Pareto-frontier](#)
[Proposition 3](#) characterizes at once the range of possible objective functions. Indeed, any point on the Pareto frontier maximizes $\gamma U + (1 - \gamma)V$ for some $\gamma \in [0, 1]$. The welfare-maximization policy corresponds to $\gamma = 1/2$, and so the first-best efficient outcome is implementable by a disclosure policy. [fig:Outcomes-implementable-by](#)
[Figure 2](#) illustrates the result. Note that by [prop:FD-suboptimal](#)
[Proposition 2](#) the full-disclosure equilibrium outcome lies below the Pareto frontier.

[prop:Pareto-frontier](#)
[Proposition 3](#) is related to the result of [BBM15](#)
[Bergemann et al. \(2015\)](#), who show that segmentation of a monopolistic market can achieve every feasible combination of consumer and producer surplus. Their segmentation problem is a static signaling game with a single receiver (monopolist), who also sets the price, while my model is a signaling game with dynamically optimizing receivers whose prices are fixed.

effects-of

3.3 Three effects of information disclosure

Disclosure of buyer characteristics to sellers has three competing effects on welfare. The strongest of these effects determines the form and coarseness of the optimal disclosure.

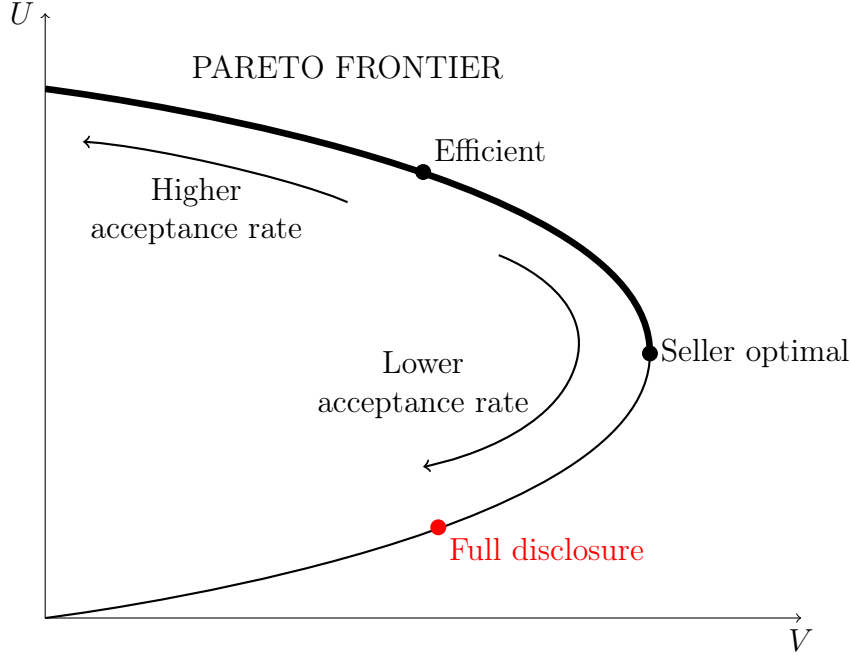


Figure 2: Graphical illustration of the set of feasible and implementable market outcomes in the case of identical workers. U stands for buyer surplus, V stands for seller surplus. A disclosure policy can implement any point on the Pareto frontier (thick solid line). The full-disclosure outcome is suboptimal.

The welfare is the sum of buyer surplus and seller profits,

$$W = U + V.$$

In the case of identical sellers, I am going to represent the welfare as a function of seller match quality and match rate, and explain how information disclosure affects both of them.

Denote the match rate by M , that is M is the number of matches that form on the platform over a unit time interval. Higher M implies that more buyers are served, and so the buyer surplus is the increasing function of the match rate, $U = U(M)$.

Denote the seller (average) match quality by v . We have:

$$V = Mv.$$

The welfare can be represented as

$$W = U(M) + Mv.$$

Obviously, W increases in both match rate M and seller match quality v .

First, information disclosure has the positive effect on the seller match quality. From a seller's point of view, more information increases his set of attainable payoffs. Therefore, holding the match rate fixed, he individually benefits from more information about buyers. Formally,

quality-effect

Claim 1 (Match quality effect). Let λ' be coarser than λ'' . Holding M fixed, the match quality under λ' is lower than under λ'' , or $v' \leq v''$.

Second, information disclosure reduces the platform's ability to induce sellers to accept more jobs. Namely, the platform can coarsen information to increase the seller expected marginal profit. Ignoring dynamic effects, higher marginal profit induces sellers to accept more jobs, which leads to higher match rates. Formally,

match-effect

Claim 2 (Static match rate effect). Suppose $\tau = 0$. Let λ'' be such that there is $s \in \text{supp } \lambda''$ with $\pi(s) < 0$. Then there is λ' coarser than λ'' such that $M' > M''$.

Third, more information available to sellers increases their return to search. Therefore, the opportunity cost of accepting is higher, because acceptance precludes further search. As a result, sellers reject more often, and the match rate goes down. In equilibrium, it hurts seller profits, because it is not collectively optimal for sellers to reject profitable jobs. Formally,

match-effect

Claim 3 (Dynamic match rate effect). Let λ' be coarser than λ'' . Holding M fixed, the option value of rejecting a marginal job under λ' is lower than under λ'' .

The match quality effect provides a motive for the platform to disclose information, while the effects on match rate provide a motive to coarsen information. Which effect is stronger depends on the primitives of the economic environment.

With identical sellers, too many efficient jobs are rejected under full disclosure. Denote the efficient disclosure policy by $\hat{\lambda}$ and the set of buyers that are efficient to accept by $\hat{X}_a := \{x \in X : \pi(x) + u(x) \geq 0\}$. Disclosure $\hat{\lambda}$ sends the recommendation “accept” for all jobs in \hat{X}_a and the recommendation “reject” for all jobs in $X \setminus \hat{X}_a$. Under full disclosure, the accepted jobs are $X_a^{FD} := \{x \in X : \pi(x) \geq \tau V^{FD}\}$. Clearly, X_a^{FD} is a proper subset of \hat{X}_a .

When sellers are identical, the match rate effects win over the match quality effect. Indeed, the jobs in $\{x : -u(x) \leq \pi(x) \leq 0\}$ are rejected, because sellers fail to internalize the effect of their acceptance decisions on buyers. This is the static match rate effect. The jobs in $\{x : 0 \leq \pi(x) \leq \tau V\}$ are rejected, because sellers fail to internalize the cream-skimming externality they impose on each other. This is the dynamic match rate effect. The match quality effect is weak, because the platform observes seller preferences and can finely control their surplus. Therefore, $\hat{\lambda}$ is coarse.

The next result gives comparative statics of the efficient disclosure policy with respect to buyer traffic β , seller capacity constraint τ , and the buyer cost of rejection.

sellers-CS

Corollary 1. *Suppose the sellers are identical. The efficient information disclosure policy $\hat{\lambda}$ is independent of the intensity of buyer traffic β and the seller capacity constraint τ . When the buyer cost of rejection is higher, $\hat{\lambda}$ prescribes pooling more of the marginally profitable jobs with the inframarginal jobs.*

Interestingly, $\hat{\lambda}$ does not depend on β or τ . This implies that the same $\hat{\lambda}$ would be optimal if sellers were able to complete jobs instantaneously ($\tau = 0$). The reason why $\hat{\lambda}$ is independent of β and τ is the binary structure of $\hat{\lambda}$. The rate of arrival to the available sellers, β_A , matters to sellers only to the extent that a higher arrival rate magnifies the opportunity cost of accepting. With a binary $\hat{\lambda}$, the opportunity cost of accepting a profitable job is 0 because all profitable jobs are the same.

The next section shows that with heterogeneous sellers, the match quality effect can be the strongest. Moreover, the optimal disclosure policy depends on β and τ .

3.4 Optimal information disclosure with heterogeneous sellers

This section studies the information disclosure problem in the linear payoff environment when sellers have payoff heterogeneity unobserved by the platform. This is a leading case of my analysis, because in most marketplaces, the intermediary observes seller preferences imperfectly. I characterize the optimal disclosure policy and show that it is qualitatively different from the statically optimal disclosure found in the prior literature. Moreover, unlike the case of identical sellers, the Pareto frontier is generally not attainable. Finally, under certain conditions, which I provide, full disclosure is the only optimal policy.

With heterogeneous sellers, the match quality effect is stronger. To see why, imagine that there are two types of sellers: professionals and amateurs. Professionals can profitably complete a larger set of jobs than amateurs. The platform does not observe the seller type. When the platform designs the disclosure policy, it should take into account that the same disclosure policy has different impacts on professionals than on amateurs. On the one hand, amateurs can profitably complete only a small subset of jobs, and so they need finer disclosure to distinguish profitable jobs from unprofitable ones. If the disclosure is too coarse, amateurs reject all jobs. On the other hand, professionals have a large set of profitable jobs, and so their average profit per job is higher. Pooling of more marginally profitable jobs with profitable jobs keeps professionals' average profits positive but induces a higher acceptance rate. Thus whether it is optimal to coarsen the information disclosure or not depends on the relative population sizes of amateurs and professionals. If there are more professionals than amateurs, then coarser disclosure is optimal, as it increases the total acceptance rate even though the amateurs stop working. If there are more amateurs, finer disclosure is optimal. In the rest of this section, I rigorously study this problem for the general continuous distribution of seller types.

Consider the setting that I call the *linear payoff* environment. The space of buyer types is $X = [0, 1]$, with the interpretation that x is the level of difficulty of the job. The space of seller types is $Y = [0, \bar{y}]$, $\bar{y} \geq 1$, with the interpretation that y is the seller's skill level.

The seller profit function is $\pi(x, y) = y - x$.¹⁶ The high- y sellers are “professionals,” and the low- y sellers are “amateurs.” The buyer match value is $u(x, y) = u$. I impose the regularity condition $f(1) > 0$, which is necessary for the results presented in what follows.

The platform’s objective is to maximize a weighted average of the buyer surplus and the joint seller profits.

$$\mathcal{J}(\gamma) = \gamma U + (1 - \gamma)V, \quad (6)$$

{eq:J-heter}

where $V = \int_Y V(y)dG(y)$ is the joint seller profits, and $U = u \cdot M$ is the buyer surplus; M is the total number of matches formed over unit of time. The general objective $\mathcal{J}(\gamma)$ includes as special cases maximization of welfare ($\gamma = 1/2$), maximization of the joint seller profits ($\gamma = 0$) and maximization of the number of matches ($\gamma = 1$).

The next proposition is the second main result of the paper. It characterizes the disclosure policy that maximizes $\mathcal{J}(\gamma)$ for the case of uniform distribution of seller types.

Definition. The disclosure policy λ is x^* -upper-coarsening for some $x^* \in [0, 1]$ if λ fully reveals $x < x^*$ and pools all $x > x^*$.¹⁷

prop:max-J

Proposition 4. Suppose $G = U[0, \bar{y}]$, $\bar{y} \geq 1$. Then for any $\gamma \in [0, 1]$, there is unique $x_\gamma^* \in [0, 1]$ such that x_γ^* -upper-coarsening maximizes $\mathcal{J}(\gamma)$. Furthermore, x_γ^* is decreasing in γ .

- There is $\gamma^* \in [0, 1]$ and there exist $\beta\tau$ and \bar{y} that are large enough so that $x_\gamma^* < 1$ for $\gamma > \gamma^*$ (some coarsening is strictly optimal).
- If $0 < \beta\tau < 1/2$, then $x_\gamma^* = 1$ for any γ (full disclosure is strictly optimal).

I reserve use of the notation x_γ^* for the cutoff in the upper-coarsening disclosure policy that maximizes $\mathcal{J}(\gamma)$. As such, x_0^* , x_1^* and $x_{1/2}^*$ are the highest truthfully revealed buyer types under the seller profits maximizing disclosure, the match rate maximizing disclosure and the efficient disclosure, respectively.

¹⁶The linear payoff environment was used in the prior literature on disclosure with heterogeneous audience, e.g. in [KLMZ-wp Kolotilin et al. \(2015\)](#).

¹⁷This terminology is borrowed from [KLMZ-wp Kolotilin et al. \(2015\)](#).

First, [Proposition 4](#) shows that the optimal disclosure involves pooling of hard tasks and truthfully revealing easy tasks. As we will see in the discussion below, the reason behind this form of disclosure is to reduce the professionals' opportunity cost of accepting.

Second, when the platform puts more weight on the buyer surplus, the optimal disclosure policy is coarser. In particular, the match-maximizing disclosure is coarser than the efficient disclosure, which is in turn coarser than the seller profits maximizing disclosure (i.e. $x_0^* \geq x_{1/2}^* \geq x_1^*$).

Third, [Proposition 4](#) shows that the optimal disclosure policy in the case of uniform distribution of seller types depends on the intensity of buyer traffic β , the tightness of the seller capacity constraints τ , and the spread of seller types \bar{y} . If $\beta\tau$ is large enough and there are sufficiently high seller types, then upper-coarsening is optimal. If, on the other hand, buyer traffic is low or capacity constraints are loose, the full disclosure is strictly optimal.

I am going to contrast the optimal disclosure policy found in [Proposition 4](#) with the solution of the static disclosure problem. The *static disclosure problem* admits only a one-shot interaction between the platform and the sellers. My model reduces to the static disclosure problem when $\tau = 0$. Indeed, when $\tau = 0$, sellers are always available, has zero opportunity cost of accepting, and so act myopically.

The upper-coarsening form of the optimal policy is qualitatively different from the solution to the static disclosure problem in the same environment.

Fact 1. Suppose $\tau = 0$ and consider the platform's objective of maximizing the number of matches ($\gamma = 1$).

- If g is decreasing, then the full disclosure policy is optimal;
- If g is increasing, then the no disclosure policy is optimal;
- If g is constant, then any disclosure policy is optimal.

The static disclosure problem has been extensively studied in the prior literature, and the above result appears, for example in [Kolotilin et al. \(2015\)](#). The implied concavification

reasoning goes back to ^{AMS95}Aumann *et al.* (1995) and ^{KG11}Kamenica and Gentzkow (2011). The proof of ^{fact:tau=00003D0}Fact 1 follows from ^{cor:tau=00003D0}Corollary 4 in the next section.

In the static disclosure problem, if the distribution of seller types is uniform, the the disclosure policy does not affect the number of matches. Any coarsening of the full disclosure policy decreases the acceptance rate of lower-type sellers and increases the acceptance rate of higher-type sellers. When the distribution of seller types is uniform, these two forces cancel out and the total number of matches is unchanged.

In dynamic setting, in contrast, ^{prop:max-J}Proposition 4 shows that information disclosure does affect the number of matches. In particular, the upper-coarsening is strictly optimal. The additional affects that arise when $\tau > 0$ are the dynamic match quality effect and the availability effect.

To get a sense of the forces that make the upper-coarsening optimal, consider two simplifications of the model, each of which alters a part of the original model. Recall that the model has two main components: the matching system (call it \mathcal{M}) and the seller optimization problem (call it \mathcal{O}). The first simplification keeps \mathcal{O} in its exact form as in ^{eq:V-via-pi}(4) but has the alternative matching system \mathcal{M}' that replenishes any available seller immediately after he becomes busy with a new seller of the same type. In $(\mathcal{O}, \mathcal{M}')$, the mass of available sellers is always 1, and the distribution is always G . The second simplification keeps \mathcal{M} but has the alternative \mathcal{O}' in which sellers act myopically. In $(\mathcal{O}', \mathcal{M})$, the sellers accept all profitable jobs, but it still takes time τ to complete them.

replenished

Corollary 2. *Suppose the distribution of available seller types is exogenous, that is, $G = U[0, 1]$ (simplification $(\mathcal{O}, \mathcal{M}')$). Then only the no disclosure policy maximizes the number of matches.*

This result demonstrates the workings of the dynamic match quality effect in the case of heterogeneous sellers. Under the premises of ^{cor:replenished}Corollary 2, the dynamic match quality effect completely dominates the match quality effect. Now compare the no disclosure to the full disclosure. If seller are myopic, then by ^{fact:tau=00003D0}Fact 1 coarsening does not affect the number

of matches. However, with forward-looking sellers there is a difference. The coarsening of information increases the acceptance rate of professionals disproportionately, because it decreases their option value of rejection. As a result, the no disclosure policy is optimal.

ic-sellers

Corollary 3. *Suppose $G = U[0, 1]$. If sellers are myopic (simplification $(\mathcal{O}', \mathcal{M})$), then full disclosure is strictly optimal in maximization of the matching rate.*

In this result, I shut down the dynamic match quality effect and instead focus on the effect of endogenous distribution of seller types. In the equilibrium, the pdf of available seller types is decreasing, because higher-seller types have higher acceptance rates and so are less available. Results from the static disclosure problem ([Fact 1](#)) suggest that in this case the full disclosure policy must be optimal. [Corollary 3](#) confirms that this is indeed the case.

In the original model, $(\mathcal{O}, \mathcal{M})$, effects of both [Corollary 2](#) and [Corollary 3](#) are present. The resulting optimal policy is a partial coarsening. In the case of uniform G , as [Proposition 4](#) shows, it is upper-coarsening.

For general G , the shape of the optimal disclosure can be quite complex, even in the static case. The heuristic is to pool x on the increasing part of the pdf g , and reveal x on the decreasing part of g . With forward-looking agents, it is even less tractable. Nevertheless, I give a first-order condition in [Lemma 3](#) that can be used to analyze the general case.

[Figure 3](#) illustrates [Proposition 4](#) and contrasts it with the case of identical sellers. By [Proposition 2](#), we know that the full disclosure outcome is suboptimal. In the case of identical sellers, it was possible to implement any point on the Pareto frontier by information disclosure ([Proposition 3](#)), and therefore coarsening was necessary for optimality. In the case of heterogeneous sellers, the Pareto frontier is not implementable in the generic case, because sellers have private information. Therefore, the “implementability frontier” in [Figure 3](#) is below the Pareto frontier. Furthermore, if $\beta\tau$ and \bar{y} are large enough, then the full disclosure outcome is below the implementability frontier, and efficiency requires some coarsening. The optimal disclosures in this case are described in [Proposition 4](#). If $\beta\tau$ is sufficiently small, the implementability frontier consists of only the full disclosure outcome. That is, the full dis-

closure policy is optimal for welfare maximization, match-rate maximization, or joint seller profits maximization.

4 Main lemma and proof outline of [Proposition 4](#)

In this section I sketch the proof of [Proposition 4](#). The main idea behind the proof is to represent information disclosure λ as some bounded convex function $\Lambda(\cdot)$, and then use the calculus of variations to find the optimal Λ . The main technical result of the paper is [Lemma 3](#).

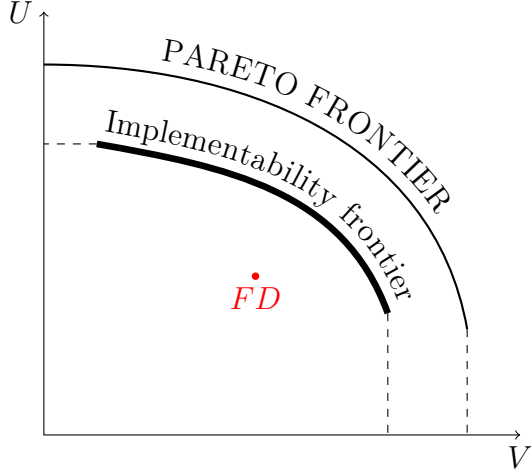
The proof of [Proposition 4](#) relies on a sequence of four lemmas. [Lemma 1](#) establishes a one-to-one correspondence between information disclosure policies and convex functions from some set. [Lemma 2](#) finds a convenient representation of the seller dynamic optimization problem. [Lemma 3](#) finds a variational derivative of the platform's objective \mathcal{J} . [Lemma 4](#) provides a necessary condition for optimality of a disclosure policy.

Denote the posterior mean of x conditional on signal s by $z(s) := \int_X x ds(x)$. I reserve use of the notation z for a typical posterior mean of x . Denote by F^λ the distribution of $z(s)$ when the platform uses disclosure policy λ . We have $F^\lambda(\zeta) = \lambda\{z(s) \leq \zeta\}$. Define the *option value function* $\Lambda: [0, \infty) \rightarrow \mathbb{R}_+$ as follows:

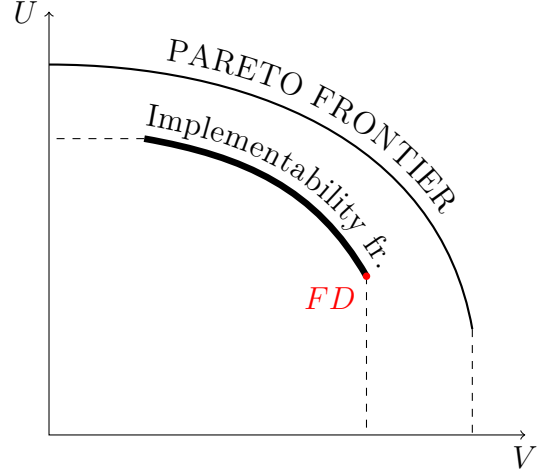
$$\Lambda(z; \lambda) := \int_0^z F^\lambda(\zeta) d\zeta. \quad (7) \quad \{\text{eq:Lambda}\}$$

As we will see from [Lemma 2](#), $\Lambda(z)$ is proportional to the option value of rejecting a job with expected difficulty z . Let $\bar{\Lambda}$ be the option value function under full disclosure, that is $\bar{\Lambda}(z) := \int_0^z F(\zeta) d\zeta$. Similarly, let $\underline{\Lambda}$ be the option value function under no disclosure, that is $\underline{\Lambda}(z) := \max\{0, z - \mathbb{E}[x]\}$. Let

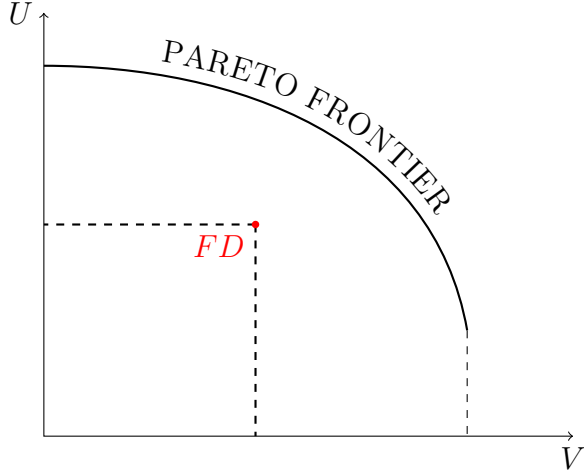
$$\mathcal{L} := \{\Lambda(z): \Lambda(z) \text{ is increasing, convex, and pointwise between } \bar{\Lambda}(z) \text{ and } \underline{\Lambda}(z)\}.$$



(a) $\beta\tau$ is large enough and \bar{y} is large enough. Full disclosure is not efficient, and it maximizes neither the joint seller profits nor the match rate.



(b) The full disclosure policy maximizes seller profits, but it is not efficient.



(c) $\beta\tau \in (0, 1/2)$. The implementability frontier is degenerate and consists of only the full-disclosure outcome.

Figure 3: Graphical illustration of the set of feasible market outcomes and the limits of implementability using information design in the case of heterogeneous workers in the linear payoff environment with the uniform seller type distribution. U stands for buyer surplus, V stands for seller surplus. FD stands for the full-disclosure outcome.

rog-sellers

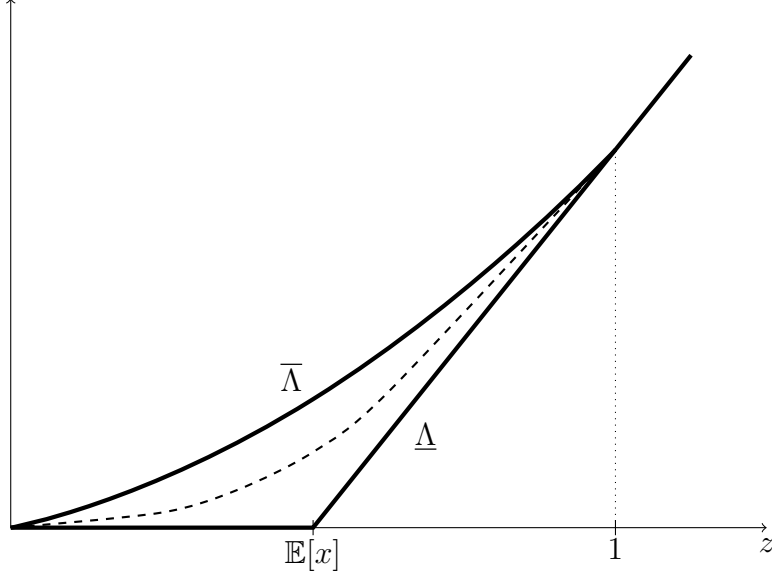


Figure 4: Any disclosure policy λ can be represented as an nondecreasing, convex function Λ point-wise between $\underline{\Lambda}$ and $\bar{\Lambda}$.

The next lemma establishes a one-to-one correspondence between the functions in \mathcal{L} and the option value functions defined in [\(7\)](#).

Lemma 1. $\ell \in \mathcal{L}$ if and only if there is $\lambda \in \Delta(S)$ such that $\Lambda(\cdot, \lambda) = \ell$.

The power of [Lemma 1](#) is that it shows that any disclosure policy can be represented as some nonnegative, nondecreasing and convex function from \mathcal{L} . See [Figure 4](#) for an illustration. I am going to use Λ to denote a typical element of \mathcal{L} . Optimization of \mathcal{J} with respect to $\lambda \in \Delta(S)$ is equivalent to optimization with respect to $\Lambda \in \mathcal{L}$. The latter allows me to use the calculus of variations to find the optimality condition below.

The next lemma characterizes the optimal seller strategy and demonstrates that the seller optimization problem depends on λ only through Λ .

Lemma 2. *For any disclosure policy λ , the seller's optimal strategy has a cutoff form with cutoff $\hat{z}(y)$ such that a type- y seller accepts all jobs with expected difficulty $z < \hat{z}(y)$ and rejects all jobs with $z > \hat{z}(y)$. Furthermore, for any β_A , the seller payoff $V(y)$ and the cutoff*

$\hat{z}(y)$ are solutions to the following system of equations:

$$V(y) = \frac{y - \hat{z}(y)}{\tau} = \beta_A \Lambda(\hat{z}(y)). \quad (8) \quad \{\text{eq:V-zhat}\}$$

This lemma implies that the platform's objective \mathcal{J} depends on λ only through Λ . This result is used in the next lemma, which is the main technical result of the paper.

Consider a functional $\mathcal{I}(\Lambda): \mathcal{L} \rightarrow \mathbb{R}$ and variation $\delta\Lambda(y)$. The first variation of \mathcal{I} is $\delta\mathcal{I} = \mathcal{I}(\Lambda + \delta\Lambda) - \mathcal{I}(\Lambda)$. The variational derivative of \mathcal{I} with respect to Λ is a function $\phi(y)$ such that $\delta\mathcal{I} = \int \phi(y)\delta\Lambda(y)dy + o(\delta\Lambda)$ as $\delta\Lambda \rightrightarrows 0$. If the variational derivative exists, it is denoted by $\delta\mathcal{I}/\delta\Lambda$.

Denote by $\nu(y)$ the fraction of type- y sellers who are available (i.e. $\nu(y) = 1 - \rho(y)$).

main-lemma

Lemma 3 (Main Lemma). *For any initial $\Lambda \in \mathcal{L}$, the variational derivative of the flow of matches M with respect to Λ exists and equals*

$$\frac{\delta M}{\delta \Lambda} = K_1 \cdot [g(y)\nu'(y) - (g(y)\nu^2(y))'], \quad (9) \quad \{\text{eq:dM/dW}\}$$

where $K_1 = \beta / \int [\nu^2(y) - \tau V(y)\nu'(y)] dG(y) > 0$. Similarly, the variational derivative of the joint seller profits V with respect to Λ exists and equals

$$\frac{\delta V}{\delta \Lambda} = \frac{\delta M}{\delta \Lambda} \cdot K_2 + \beta_A \nu(y)g(y),$$

where $K_2 = \tau \int \nu(y)V(y)dG(y)/\bar{\nu} > 0$.

To see the contribution of [Lem:main-lemma](#) [Lemma 3](#), compare it to the static disclosure problem.

tau=00003D0

Corollary 4. *Suppose $\tau = 0$. Then*

$$\frac{\delta M}{\delta \Lambda} = -\beta g'(y). \quad (10) \quad \{\text{eq:dM/dW-s}\}$$

If g is decreasing, then full disclosure is optimal. If g is increasing, then no disclosure is

optimal.

This result follows easily from [Lemma 3](#), using the fact that when $\tau = 0$, $\nu(y) = (1 + \tau\beta_A\alpha(y))^{-1} = 1$. When sellers are not capacity constrained ($\tau = 0$), the original formula [\(10\)](#) has to be adjusted, as shown in [\(9\)](#).

I now explain the additional terms in [\(9\)](#) in more detail. Consider the case of uniform distribution of seller types, $G = U[0, 1]$. By [\(9\)](#),

$$\frac{\delta M}{\delta \Lambda} \propto -(\nu^2(y) - \nu(y))'. \quad (11)$$

When $\tau = 0$, this reduces to

$$\frac{\delta M}{\delta \Lambda} = 0.$$

Therefore, the disclosure policy has no effect on the number of matches in the static case, but does have an effect in the dynamic case.

The term $-\nu(y)$ in [\(11\)](#) corresponds to the match quality effect of information disclosure. As discussed above, it arises because disclosure policy affects the capacity-constrained seller's option value of rejection. Unlike the case of unconstrained sellers, limiting of information has an additional effect on acceptance rate by decreasing the sellers' option value of rejection. By [Corollary 4](#), $-\nu(y)$ acts as a density function, and since $\nu(y)$ is decreasing, this creates a motive for the intermediary to use a coarser disclosure policy.

The term $\nu^2(y)$ in [\(11\)](#) corresponds to the availability effect. The availability effect arises because in equilibrium, higher seller types are less available than lower types. Therefore, the pdf of the available seller types is decreasing. By [Corollary 4](#), $\nu^2(y)$ acts as a density function, and since $\nu(y)$ is decreasing, this creates a motive for the intermediary to use a finer disclosure policy.

The next lemma provides a necessary condition for optimality of a disclosure policy.

Lemma 4. *If λ_0 maximizes \mathcal{J} , and $\delta\mathcal{J}/\delta\Lambda$ evaluated at λ_0 crosses 0 from above at most once, then λ_0 is upper-coarsening.*

Now I sketch the key steps of the proof of [Proposition 4](#). The complete proof relies on more technical lemmas and is deferred to [C](#).

Proof sketch of [Proposition 4](#). By [Lemma 3](#),

$$\begin{aligned}\frac{\delta \mathcal{J}}{\delta \Lambda} &= \gamma u \frac{\delta M}{\delta \Lambda} + (1 - \gamma) \left(\frac{\delta M}{\delta \Lambda} K_2 + \nu(y) g(y) \beta_A \right) \\ &= (\gamma u + K_2(1 - \gamma)) \frac{\delta M}{\delta \Lambda} + (1 - \gamma) \beta_A \nu(y) g(y),\end{aligned}\tag{12} \quad \boxed{\text{eq:12}}$$

where $K_2 > 0$. Evaluating with $G = U[0, \bar{y}]$,

$$\frac{\delta M}{\delta \Lambda} = K_1 \bar{y}^{-1} (\nu(y) - \nu^2(y))' = K_1 \bar{y}^{-1} (1 - 2\nu(y)) \nu'(y),\tag{13} \quad \boxed{\text{eq:13-1}}$$

where $K_1 > 0$. Since $\nu(0) = 1$ and $\nu(y)$ is nonnegative and decreasing, $\delta \mathcal{J} / \delta \Lambda$ either is positive for all $y \in [0, \bar{y}]$ or crosses 0 once from above. Denote by λ_γ^* the disclosure policy that maximizes $\mathcal{J}(\gamma)$. By [Lemma 4](#), λ_γ^* is upper-coarsening.

To see that the cutoff x_γ^* is decreasing in γ , note that a larger γ puts more weight on the positive term $\beta_A \nu(y)$ in [eq:12](#). Therefore, the region of Y with negative $\delta \mathcal{J} / \delta \Lambda$ is smaller.

Whether $x_\gamma^* = 1$ or strictly less than 1 depends on the existence of $y \leq \bar{y}$ with $\nu(y) < 1/2$. Indeed, from [eq:13-1](#), if $\nu(y) > 1/2$ for all $y \leq \bar{y}$, then $\delta \mathcal{J} / \delta \Lambda \geq 0$ for all y . For the details, refer to the proof in [C](#). □

5 Discussion and Conclusion

This paper studies the role of information intermediation in matching markets and shows that strategic information disclosure can be an effective tool for balancing of the match quality against the match rate. When the two sides of the market are not symmetric in their preferences for match quality and match rate, the more patient and more selective side of the market (sellers in my model) tends to cream-skim. As a platform’s policy, strategic limiting of information can be used to decrease cream-skimming and improve total welfare. Additionally, limiting of information can alleviate seller coordination failure by offsetting the cream-skimming externality.

This paper provides a case for platform’s control over matching process by means of information design. Strategic information disclosure is less intrusive than direct control and often more accessible than sophisticated pricing schemes. For example, a way to make sellers internalize the costs of rejection they impose on buyers is to charge sellers a fee for every rejection. However, such a policy creates an unsafe environment for sellers, with too large downside risks.¹⁸ Strategic information limitation is relevant in practice and can be found in the design of many platforms. Now I continue the discussion of the information design choices on the successful digital marketplaces.

Uber employs limiting of information to improve the efficiency of the marketplace. ^{prop:FD-suboptimal}[Proposition 2](#) shows that if drivers have full information about passenger rides and full discretion over accepting, they cream-skim: acceptance rates are low, and the demand-supply fit is suboptimal. The platform’s policy of hiding passenger destination from drivers is used to nudge drivers to accept more rides and avoid the cream-skimming along the destination attribute.

Hiding destination works because the increase in the driver expected marginal profit is large enough to make driver accept rides with unknown destination, but the decline in the expected average profit is small enough not to discourage drivers from participating on the platform. Buyers significantly benefit from resulting shorter wait times, and so the coarsening

¹⁸^{Rot08}[Roth \(2008\)](#) names safety as one of three key requirements for a well-functioning market.

improves quality of the service for passengers.

In the current version of UberX, the passenger destination is fully concealed and is not used in pricing. This paper shows that partial coarsening would be more efficient. For example, my results show that the drivers with higher outside option (low type sellers) require finer disclosure to stay profitable on the platform while drivers with low outside option (high type sellers) can be squeezed. In such situation, the following partial coarsening can work better. The destination of a passenger is discretized into three categories, close, medium-range, and far, where the medium-range is wide to include most of the rides.

Currently Uber uses a simple pricing rule combined with information disclosure coarsening and other requirements on drivers, such as minimal acceptance rates, for its UberX product. A natural question is why Uber does not adopt a flexible pricing scheme that would condition price on ride destination. There are several reasons for doing this. First, consumers appreciate transparent and simple pricing. Both because it is easier to assess the cost of a ride before actually looking it up in the app, and because complex pricing feels like price gauging. Second, flexible pricing that conditions on destination is a non-trivial development task that requires significant development resources.¹⁹ Third, and related, it is not feasible to make price dependent on every possible variable relevant in the transportation market. There is some limit for the designer beyond which increasing the complexity of pricing is not justified for practical purposes.²⁰ In particular, the destination variable should be discretized to be practically useful for pricing. Finally, not regarding just the destination, Uber has sufficient data on its users to introduce personalized pricing when each passenger receives his individual price. Because of a risk of price-discrimination litigation, personalized pricing is not actually implemented. Understanding the benefits of information disclosure

¹⁹UberPool already conditions prices on many variables, and as far as I am aware from private conversations, Uber is testing similarly flexible pricing scheme on its UberX product. Currently, the upfront UberX price is calculated using the expected time and distance of the trip and local traffic, as well as how many riders and nearby drivers have their Uber app open at that moment. See <https://newsroom.uber.com/upfront-fares-no-math-and-no-surprises/>.

²⁰See Segal (2007) for the study of communication requirements of social choice rules. It demonstrates that in some social choice problems, the space of prices that must be discovered is prohibitively large.

over flexible transfer schemes is an important venue for the future research.

For Airbnb, ^{prop:FD-suboptimal}[Proposition 2](#) shows that hosts cream-skim if they have detailed information about guests. This results in low acceptance rates that do not satisfy guest demand for convenient booking service, and the marketplace performance is removed from its full potential. The Instant Book feature in its current version acts as a coarse information disclosure policy, and incentivizes hosts to accept more guest requests. As a first order effect, guests benefit because booking is fast. As a second order effect, sellers also benefit, because better user experience attracts more guests to the platform. As a result, total welfare increases, as well as participation on the platform.

Optimal guest segmentation on the Instant Book should remain coarse. Too fine segmentation of guests brings back the problem of cream-skimming. The problem of optimal guest segments is equivalent to the problem of the optimal information disclosure, and can be analyzed within the framework presented in this paper. Specifically, the optimal guest segmentation balances the hosts' demand for transparency and guest demand for convenience. Furthermore, markets with higher buyer-to-seller ratio require coarser guest segmentation than the markets with low buyer-to-seller ratio.

For TaskRabbit and other temp agencies, ^{prop:FD-suboptimal}[Proposition 2](#) shows that if service providers see through the economics of tasks, they cream-skim. A service provider may prefer to reject the tasks that are too far, or come at inconvenient time, or those he does not find interesting. The cream skimming results in low utilization of labor force, which is inefficient from the platform's perspective. Buyers like when they can hire worker on demand. TaskRabbit's solution was to redesign the matching system in such a way that the service providers commit to an hourly rate over a broad category of tasks. The benefit of this coarsening is two-fold. First, as in the other examples above, it nudges the service providers to accept more tasks by increasing their expected marginal profit. Second, since only the type of task (e.g. Home Cleaning) is used for the client's request, the platform elicits limited information from clients. This further simplifies client experience on the platform, that increases their client benefit

from participation.

This paper necessarily focuses on the limited set of the platform’s design choices. Other mechanisms also play an important role in these markets. Flexible pricing and matching mechanisms, such as real-time auctions, surge pricing, recommender systems, are frequently used to ensure a good supply-demand fit. The relative effectiveness of various design choices is an exciting area for further examination.

6 Appendix

A Lemmas

monotonicities

Lemma 5. Fix the arbitrary increasing $\alpha(y) \in [0, 1]$, $y \in Y$.

- Average utilization rate $\bar{\rho} \in [0, 1]$ is a solution to

$$1 = \int \frac{dG(y)}{1 - \bar{\rho} + \beta\tau\alpha(y)}. \quad (14) \quad \{\text{eq:nubar}\}$$

- The solution $\bar{\rho}$ exists, is unique, increases in $\alpha(y)$ for any $y \in Y$, in β and in τ .
- Utilization rate of type y $\rho(y)$ increases in $\alpha(y')$ for any $y' \in Y$.

Proof. Find from [\(2\)](#) ^{eq:SS} that

$$1 - \rho(y) = \frac{1}{1 + \tau\beta\alpha(y)/(1 - \bar{\rho})}. \quad (15) \quad \{\text{eq:14}\}$$

Take the integral using cdf $G(y)$ to obtain $1 - \bar{\rho} = \int dG(y)/(1 + \tau\beta\alpha(y)/(1 - \bar{\rho}))$. Rearrange and get [\(14\)](#) ^{eq:nubar}. The right-hand side of [\(14\)](#) ^{eq:nubar} is increasing in $\bar{\rho}$. Evaluated with $\bar{\rho} = 0$, it equals $\int \frac{dG(y)}{1 + \beta\tau\alpha(y)} \leq 1$. Evaluated with $\bar{\rho} = 1$, it equals $\int \frac{dG(y)}{\beta\tau\alpha(y)} \geq 1$, using Assumption [Assumption 4](#) ^{assu:no-excess-demand}. Therefore, the solutions exists and is unique. Monotonicity of $\bar{\rho}$ is straightforward.

Finally, if $\alpha(y')$ increases (at the neighborhood of y'), $\bar{\rho}$ increases. Therefore, the right hand side of [\(15\)](#) ^{eq:14} decreases. Therefore, $\rho(y)$ increases. \square

beta-bounds

Lemma 6. In the steady state,

$$\beta \leq \beta_A \leq \frac{\beta}{1 - \tau\beta}.$$

Proof. From [\(3\)](#) ^{eq:beta_A-definition}, $\beta_A = \beta/(1 - \rho)$. Therefore, $\beta_A \geq \beta$.

Find from [\(2\)](#) ^{eq:SS} that $\tau\beta \frac{1-\rho(y)}{1-\bar{\rho}} \alpha(y) = \rho(y)$. Take expectation wrt y and obtain $\bar{\rho} = \tau\beta \int \frac{1-\rho(y)}{1-\bar{\rho}} \alpha(y) dG(y) \leq \tau\beta \int \frac{1-\rho(y)}{1-\bar{\rho}} dG(y) = \tau\beta$. Therefore, $\beta_A \leq \frac{\beta}{1-\tau\beta}$. \square

lem:alpha

Lemma 7. Assume the linear payoff environment. In the steady state equilibrium with disclosure policy λ , $\alpha(y) = \Lambda'(\hat{z}(y))$ for sellers of mass 1. Furthermore,

- $\alpha(y)$ is increasing in y .
- For any $\Lambda \in \mathcal{L}$, there is $\check{y} > 0$ such that $\alpha(y) = 1$ for $y \geq \check{y}$.
- Considering exogenous variations of β_{AT} in the seller optimization problem $(\text{eq:V-zhat-Lambda}, \alpha(y))$ is decreasing in $\tau\beta_A$ for any $y \in Y$.

Proof. By $(\text{lem:Vmu}, \text{Lemma } 2)$, the the accepted jobs are those with $z < \hat{z}(y)$. Using definition of Λ from $(\text{eq:Lambda}, \text{eq:V-zhat-Lambda})$, probability of accepting is

$$\begin{aligned}\lambda\{z(s) < \hat{z}(y)\} &\leq \alpha(y) \leq \lambda\{z(s) \leq \hat{z}(y)\} \\ F^\lambda(\hat{z}(y)-) &\leq \alpha(y) \leq F^\lambda(\hat{z}(y)+) \\ \Lambda'(\hat{z}(y)) &\leq \alpha(y) \leq \Lambda'(\hat{z}(y))\end{aligned}$$

Points of discontinuity of F^λ make a set of measure zero. Since seller types have full support on $[0, \bar{y}]$, $\alpha(y) = \Lambda'(\hat{z}(y))$ for sellers of mass 1.

From $(\text{eq:V-zhat-Lambda}, \text{eq:V-zhat-Lambda})$, $\hat{z}(y)$ is increasing in y . Since Λ is convex, $\alpha(y)$ is increasing in y .

Since $X = [0, 1]$, $\Lambda'(z) = 1$ for $z \geq 1$. And for y large enough, $\hat{z}(y) > 1$.

From $(\text{eq:V-zhat-Lambda}, \text{eq:V-zhat-Lambda})$, $\hat{z}(y)$ is decreasing in $\tau\beta_A$ for any $y \in Y$. Since Λ is convex, $\alpha(y)$ is decreasing in $\tau\beta_A$. □

B Proofs of Propositions 1-3

eqm-exists

Proof of $(\text{prop:Equilibrium-exists-and}, \text{Proposition } 1)$. Step 1. Restricting set of strategies to cutoff strategies. Fix $y \in Y$.

By $(\text{eq:V-via-pi}, \text{eq:V-via-pi})$ the optimal seller strategy is such that all $S_a(y) = \{s: \pi(s, y) > \tau V(y)\}$ are accepted, all $S_r(y) = \{s: \pi(s, y) < \tau V(y)\}$ are rejected, and the seller is indifferent between accepting and rejecting signals from $S_m(y) := \{s: \pi(s, y) = \tau V(y)\}$. Therefore any optimal strategy

has the cutoff form:

$$\sigma(s, y) = \begin{cases} 1, & s \in S_a(y) \\ \in [0, 1], & s \in S_m(y) \\ 0, & s \in S_r(y) \end{cases}$$

Denote the cutoff for type- y seller by $\hat{\pi}(y) := \tau V(y)$. Using [\(eq:V-via-pi\)](#), [\(4\)](#),

$$\begin{aligned} V(y) &= \beta_A \int (\pi(s, y) - \tau V(y)) \sigma(s, y) \lambda(ds) \\ \hat{\pi}(y) &= \tau \beta_A \int (\pi(s, y) - \hat{\pi}(y)) \sigma(s, y) \lambda(ds) \\ \hat{\pi}(y) &= \tau \beta_A \int (\pi(s, y) - \hat{\pi}(y)) I\{\pi(s, y) > \hat{\pi}(y)\} \lambda(ds) \end{aligned} \tag{16} \quad \boxed{\text{eq:17}}$$

Given λ and β_A , the cutoff of the optimal strategy is a solution to [\(eq:17\)](#). The solution is unique because the left-hand side of [\(eq:17\)](#) is strictly increasing in $\hat{\pi}(y)$ while the left hand side is decreasing in $\hat{\pi}(y)$.

Step 2. Existence. Consider a correspondence $\psi: [0, 1] \rightrightarrows [0, 1]$, which maps $\bar{\rho}$ to a set of “reaction” $\bar{\rho}$ ’s by the following procedure. To find $\psi(\bar{\rho})$, first find the unique cutoff function $\hat{\pi}$ from [\(eq:17\)](#) using $\beta_A = \beta/(1 - \bar{\rho})$. Cutoff $\hat{\pi}$ does not pin down the acceptance rates α uniquely because marginal signals from S_m can have positive probability under λ . The acceptance rates congruent with $\hat{\pi}(y)$ are integrable $\alpha(y)$ such that

$$\lambda(S_a(y)) \leq \alpha(y) \leq \lambda(S_a(y)) + \lambda(S_m(y)), \quad \forall y \in Y.$$

Take all α that are congruent with $\hat{\pi}$, call this set \mathcal{A} . For any $\alpha \in \mathcal{A}$, find $\bar{\rho}$ as shown in [lem:rho-and-monotonicities](#) [Lemma 5](#). Going over all \mathcal{A} will produce the set of $\bar{\rho}$. This will be $\psi(\bar{\rho})$.

Clearly, \mathcal{A} is convex. Thus, $\psi(\bar{\rho})$ is also convex. Since $\psi(\bar{\rho})$ is an interval subset of $[0, 1]$, it is easy to see that $\psi(\bar{\rho})$ is also closed. ψ is upper hemi-continuous because $\hat{\pi}$ is continuous in β_A according to [\(eq:17\)](#). Therefore, by Kakutani’s theorem, ψ has a fixed point.

Step 3. Uniqueness. Suppose $\bar{\rho}^1$ and $\bar{\rho}^2$ are two distinct fixed points of ψ . Suppose

$\bar{\rho}^1 > \bar{\rho}^2$. Then $\beta_A^1 > \beta_A^2$. By [\(eq:17\)](#) $\hat{\pi}^1(y) > \hat{\pi}^2(y)$ for any $y \in Y$. If so, then whenever $\pi(s, y) \geq \hat{\pi}^1(y)$ we also have $\pi(s, y) > \hat{\pi}^2(y)$. But this means that $S_a^2(y) \supseteq S_a^1(y) \cup S_m^1(y)$. Therefore $\alpha^2(y) \geq \lambda(S_a^2) \geq \lambda(S_a^1 \cup S_m^1) \geq \alpha^1(y)$ for all y . By [Lemma 5](#), this implies $\bar{\rho}^2 \geq \bar{\rho}^1$. A contradiction.

We showed that there is a unique $\bar{\rho}$ that can arise in a steady-state equilibrium. By the argument below [\(eq:17\)](#), the strategy cutoffs $\hat{\pi}(\cdot)$ are also pinned down uniquely in the steady-state equilibrium. \square

suboptimal

Proof of [Proposition 2](#). By [\(4\)](#), [prop:FD-suboptimal](#) [\(eq:V-via-pi\)](#)

$$\begin{aligned} V(y) &= \beta_A \int (\pi(s, y) - \tau V(y)) \sigma(s, y) d\lambda(s) \\ V(y) &= \beta_A \nu(y) \int \pi(s, y) \sigma(s, y) d\lambda(s), \end{aligned}$$

where $\nu(y) = (1 + \tau \beta_A \alpha(y))^{-1}$. In a steady state, by [\(eq:SS\)](#), $\beta_A \nu(y) = \rho(y) / (\tau \alpha(y))$. Therefore,

$$\begin{aligned} V(y) &= \frac{\int \pi(s, y) \sigma(s, y) d\lambda(s)}{\alpha(y)} \frac{\rho(y)}{\tau} = \\ &= \mathbb{E}[\pi(s, y) \mid \text{acc.}] \frac{\rho(y)}{\tau}. \end{aligned}$$

Now σ^{FD} prescribes that jobs $\{x: \pi(x, y) > \tau V(y)\}$ are accepted with probability 1. Consider an alternative strategy profile

$$\tilde{\sigma}(x, y) = \begin{cases} 1, & \pi(x, y) > 0 \\ 0, & \pi(x, y) \leq 0 \end{cases}$$

Since $V(y) > 0$, we have $\mathbb{E}[\pi(s, y) \mid \text{acc. with } \sigma^{FD}] < \mathbb{E}[\pi(s, y) \mid \text{acc. with } \tilde{\sigma}]$. Also, $\alpha^{FD}(y) < \tilde{\alpha}(y)$. Finally, by [Lemma 5](#), $\rho^{FD}(y) < \tilde{\rho}(y)$. Therefore, $V^{FD}(y) < \tilde{V}(y)$. \square

co-frontier

Proof of [Proposition 3](#). Take any Pareto optimal pair $O = (V, CS)$. Since O is feasible, there is seller strategy profile σ that induces O . Since there is one seller type and two actions, it

is sufficient to consider only the binary signaling structures (Revelation principle). A binary signaling structure has two signals, where a signal is “action recommendation”. Let s_a be the recommendation to accept, and s_r be the recommendation to reject. Denote this signaling structure by $\hat{\lambda}$. We need to check the incentive constraints, that is, to make sure that the sellers would follow the recommendations of $\hat{\lambda}$.

From [\(4\)](#) [leg:V-via-pi](#) we have that $v(s_a) = \pi(s_a) - \tau V$, $v(s_r) = 0$, and $V = \beta_A(\pi(s_a) - \tau V)\hat{\lambda}(s_a)$. The incentive constraints require that $\pi(s_a) \geq \tau V$ and $\pi(s_r) \leq 0$. For the former,

$$\tau V = \frac{\tau \beta_A \hat{\lambda}(s_a)}{1 + \tau \beta_A \hat{\lambda}(s_a)} \pi(s_a) < \pi(s_a).$$

For the latter, recall that O is Pareto optimal, hence σ accepts all profitable jobs. This implies that

$$\pi(s_r) \leq 0 < \tau V.$$

□

Proof of [Corollary 1](#). [cor:id-sellers-CS](#) The set of efficient matches is $\hat{X}_a := \{x \in X : \pi(x) + u(x) \geq 0\}$. The efficient disclosure policy $\hat{\lambda}$ sends signal “accept” for $x \in \hat{X}_a$ and signal “reject” for $x \notin \hat{X}_a$. Therefore, $\hat{\lambda}$ does not depend on either β or τ .

If $u(x)$ increases for some $x \in X$, then \hat{X}_a (weakly) expands. Since the set of the buyers accepted in equilibrium, $X_a^{FD} := \{x \in X : \pi(x) \geq \tau V^{FD}\}$, does not change in response to changes in u , $\hat{\lambda}$ has to pool more buyer types into the “accept” signal. □

C Proof of [Proposition 4](#) [prop:max-J](#)

The proof of [Proposition 4](#) [prop:max-J](#) relies on a sequence of lemmas.

Lemma 8. *Suppose the linear payoff environment with $G = U[0, \bar{y}]$. For any (large) $L > 0$ there is $\beta\tau < 1$ large enough and \check{y} (large enough) such that if $\bar{y} \geq \check{y}$, we have $\tau\beta_A \geq L$.*

Proof. Since $\beta_A = \beta/\bar{\nu}$, rewrite [\(14\)](#) as [eq:nubar](#)

$$\beta\tau = \int_0^{\bar{y}} \frac{dy/\bar{y}}{\frac{1}{\tau\beta_A} + \alpha(y; \tau\beta_A)}, \quad (17) \quad \{\text{eq:13}\}$$

where I made explicit that α depends on $\tau\beta_A$ in equilibrium. Treat the right hand side of [\(17\)](#) as a function of $\tau\beta_A$ and \bar{y} and denote it by $\psi(\tau\beta_A, \bar{y})$. First, ψ is strictly increasing in $\tau\beta_A$ because by [Lemma 7](#) α is decreasing in $\tau\beta_A$. Second, ψ is decreasing in \bar{y} because by [Lemma 7](#) α is increasing in y . Third, using the second part of [Lemma 7](#) and uniformity of G , $\lim_{\bar{y} \rightarrow \infty} \psi = ((\tau\beta_A)^{-1} + 1)^{-1} < 1$. Pick \check{y} such that $\psi(L, \check{y}) < 1$. Let $(\beta\tau)^* := \psi(L, \check{y})$. We have that $\beta_A\tau > L$ whenever $\beta\tau > (\beta\tau)^*$ or $y > \check{y}$. \square

=00003Deps

Lemma 9. Suppose the linear payoff environment with $G = U[0, \bar{y}]$. For any (small) $\varepsilon > 0$ there is $\beta\tau < 1$ large enough and \bar{y} large enough, such that $\nu(\bar{y}) = \varepsilon$.

Proof. Note that $\nu(y) = \frac{1}{1+\tau\beta_A\alpha(y)}$. Pick $L > \varepsilon^{-1} - 1$. By [Lemma 8](#) for any $L > 0$, we can find $\beta\tau$ and \check{y} such that if $\bar{y} \geq \check{y}$, $\tau\beta_A \geq L$. Since $\text{supp } F = [0, 1]$, we have from [\(8\)](#) that $\alpha(\bar{y}) = 1$ for \bar{y} large enough. Therefore, $\nu(\bar{y}) = \frac{1}{1+\tau\beta_A\alpha(\bar{y})} < \frac{1}{1+L} < \varepsilon$. \square

g-function

Lemma 10. $\lambda \in \Delta(S)$ is x^* -upper-coarsening if and only if the corresponding Λ has the following form:

$$\Lambda(z) = \begin{cases} \bar{\Lambda}(z), & z \in [0, x^*] \\ \bar{\Lambda}(x^*) + F(x^*)(z - x^*), & z \in (x^*, \mathbb{E}[x|x > x^*]) \\ \underline{\Lambda}(z), & z \in [\mathbb{E}[x|x > x^*], 1] \end{cases} \quad (18) \quad \{\text{eq:Lambda-}$$

Proof of [Lemma 1](#). The result is shown in e.g. in [Kolotilin et al. \(2015\)](#) and [Gentzkow and Kamenica \(2016\)](#). Here I provide a complete proof using the primitives of my model.

Sufficiency. Take any $\lambda \in \Delta(S)$. According to [\(7\)](#), the corresponding function is $\Lambda(z; \lambda) := \int_0^z F^\lambda(\zeta) d\zeta$. Since F^λ is a cdf, $\Lambda(z; \lambda)$ is increasing and convex. Since F^λ is the distribution of posterior means of x , F is a mean-preserving spread of F^λ . Therefore,

F second-order stochastically dominates F^λ . That is, for any z , $\int_0^z F(\zeta) d\zeta \geq \int_0^z F^\lambda(\zeta) d\zeta$. Similarly, F^λ is a mean-preserving spread of $\delta_{\mathbb{E}[x]}$. Therefore, $\underline{\Lambda}(z) \leq \Lambda(z; \lambda) \leq \overline{\Lambda}(z)$ for any z . Summarizing, we have that $\Lambda(z; \lambda) \in \mathcal{L}$.

Necessity. Take any $\ell \in \mathcal{L}$. Define $H(z) = \ell'(z)$, where ℓ' denotes the right-derivative. Observe that $\overline{\Lambda}'(0) = \underline{\Lambda}'(0) = 0$ and $\overline{\Lambda}'(1) = \underline{\Lambda}'(1) = 1$, hence $H(0) = 0$ and $H(1) = 1$. Also, H is increasing because ℓ is convex. Therefore, H is a cdf. Furthermore,

$$\int_0^z H(\zeta) d\zeta = \ell(z) \leq \overline{\Lambda}(z) = \int_0^z F(\zeta) d\zeta \quad \forall z \in [0, 1].$$

Thus, F is the mean-preserving spread of H . It follows that there is a garbling of λ^{FD} , call it $\lambda' \in \Delta(S)$, such that $F^{\lambda'} = H$ (Blackwell (1953)). \square

Proof of Lemma 2. The first steps of the proof of Proposition 1 show in the case of arbitrary X and Y that the seller optimal strategy has a cutoff form. They also show that the cutoff profit value is $\hat{\pi}(y) = \tau V(y)$, and moreover (see Eq. (16)),

$$\hat{\pi}(y) = \tau \beta_A \int (\pi(s, y) - \hat{\pi}(y)) I\{\pi(s, y) > \hat{\pi}(y)\} \lambda(ds) \quad (19) \quad \{\text{eq:19}\}$$

It the linear payoff environment, $\pi(s, y) = y - z(s)$ and $\hat{\pi}(y) = y - \hat{z}(y)$ for some \hat{z} . Plug it back into (19) and obtain

$$\begin{aligned} y - \hat{z}(y) &= \tau \beta_A \int (\hat{z}(y) - z(s)) I\{z(s) < \hat{z}(y)\} d\lambda(s) \\ &= \tau \beta_A \int (\hat{z}(y) - z) I\{z < \hat{z}(y)\} dF^\lambda(z) \\ &= \tau \beta_A \int_0^{\hat{z}(y)} (\hat{z}(y) - z) dF^\lambda(z) = \tau \beta_A \int_0^{\hat{z}(y)} F^\lambda(z) dz \\ &= \tau \beta_A \Lambda(\hat{z}(y)). \end{aligned}$$

\square

proof2

Proof of [Lemma 3](#). ^{[lem:main-lemma](#)} Step 1. The equilibrium values of $\alpha(y)$ and $\bar{\rho}$ are found using the system of equations ^{[eq:V-zhat](#)} (8) and ^{[eq:Lambda](#)} (14). Since $\beta_A = \beta/(1 - \bar{\rho})$, rewrite them as:

$$y - \hat{z}(y) = \tau\beta_A\Lambda(\hat{z}(y)), \quad \forall y \in Y; \quad (20) \quad \{\text{eq:20}\}$$

$$\beta_A \int \frac{dG(y)}{\tau\beta_A\alpha(y) + 1} = \beta. \quad (21) \quad \{\text{eq:21}\}$$

Denote the fraction of available sellers by

$$\nu(y) := \frac{1}{\tau\beta_A\alpha(y) + 1} = 1 - \rho(y).$$

Differentiating ^{[eq:20](#)} (20) wrt y , find that

$$\hat{z}'(y) = \nu(y).$$

Step 2. Consider a feasible variation $\delta\Lambda(y)$, that is the one such that $\Lambda + \delta\Lambda \in \mathcal{L}$. We are going to track variation in all endogenous variables to eventually solve for δM and δV .

Differentiate ^{[eq:20](#)} (20):

$$-\delta\hat{z}(y) = \tau\delta\beta_A\Lambda(\hat{z}(y)) + \tau\beta_A\delta\Lambda(\hat{z}(y)) + \tau\beta_A\alpha(y)\delta\hat{z}(y),$$

from where

$$\delta\hat{z}(y) = -\nu(y) [\tau\delta\beta_A\Lambda(\hat{z}(y)) + \tau\beta_A\delta\Lambda(\hat{z}(y))]. \quad (22) \quad \{\text{eq:delta-z}\}$$

Differentiate with respect to y :

$$\begin{aligned}
\delta \hat{z}'(y) &= -\nu'(y) [\tau \delta \beta_A \Lambda(\hat{z}(y)) + \tau \beta_A \delta \Lambda(\hat{z}(y))] \\
&\quad - \nu(y) [\tau \delta \beta_A \alpha(y) \hat{z}'(y) + \tau \beta_A \delta \Lambda'(\hat{z}(y)) \hat{z}'(y)] \\
\delta \hat{z}'(y) &= -\tau \nu'(y) [\delta \beta_A \Lambda(\hat{z}(y)) + \beta_A \delta \Lambda(\hat{z}(y))] \\
&\quad - \tau \nu^2(y) [\delta \beta_A \alpha(y) + \beta_A \delta \Lambda'(\hat{z}(y))]
\end{aligned}$$

From [\(eq:20-1\)](#),
[\(25\)](#),

$$\begin{aligned}
\delta \beta_A \bar{\nu} &= -\beta_A \delta \bar{\nu} = -\beta_A \int \delta \nu(y) dG(y) = -\beta_A \int \delta \hat{z}'(y) dG(y) \\
&= \tau \beta_A \int \delta \beta_A [\Lambda(\hat{z}(y)) \nu'(y) + \alpha(y) \nu^2(y)] dG(y) \\
&\quad + \tau \beta_A \int [\nu'(y) \beta_A \delta \Lambda(\hat{z}(y)) + \nu^2(y) \beta_A \delta \Lambda'(\hat{z}(y))] dG(y). \tag{23}
\end{aligned}$$

{eq:21-1}

Manipulate separately the integral

$$\begin{aligned}
\int \nu^2(y) \delta \Lambda'(\hat{z}(y)) dG(y) &= \\
&= \int \nu^2(y) g(y) d(\delta \Lambda(\hat{z}(y))) = \\
&= \nu^2(y) g(y) \delta \Lambda(\hat{z}(y))|_0^\infty - \int \delta \Lambda(\hat{z}(y)) (\nu^2(y) g(y))' dy,
\end{aligned}$$

where we used the integration by parts in the third line. Since feasible variations have $\delta \Lambda|_0^\infty = 0$, the first term in the expression above is zero, and so,

$$\int \nu^2(y) \delta \Lambda'(\hat{z}(y)) dG(y) = - \int \delta \Lambda(\hat{z}(y)) (\nu^2(y) g(y))' dy.$$

Put it back into (23), collect terms with $\delta\beta_A$ in the left hand side, and obtain the equality

$$\delta\beta_A \int [\nu(y) - \tau\beta_A\Lambda(\hat{z}(y))\nu'(y) + \tau\beta_A\alpha(y)\nu^2(y)] dG(y) = \tau\beta_A^2 \int \delta\Lambda(\hat{z}(y)) [\nu'(y)g(y) - (\nu^2(y)g(y))'] dy.$$

Use the fact that $\tau\beta_A\alpha(y) = 1/\nu(y) - 1$ to simplify the expression.

$$\delta\beta_A \int [\nu^2(y) - \tau\beta_A\Lambda(\hat{z}(y))\nu'(y)] dG(y) = \tau\beta_A^2 \int \delta\Lambda(\hat{z}(y)) [\nu'(y)g(y) - (\nu^2(y)g(y))'] dy.$$

By the definition of variational derivative,

$$\frac{\delta\beta_A}{\delta\Lambda} = \frac{\tau\beta_A^2}{\int [\nu^2(y) - \tau V(y)\nu'(y)]} [\nu'(y)g(y) - (\nu^2(y)g(y))']. \quad (24) \quad \boxed{\text{eq:21-2}}$$

Step 3. The flow of type- y sellers who become available has rate $\rho(y)g(y)/\tau$. In a steady state, each returning seller is equivalent to one completed match. Hence the number of matches per unit of time is

$$M = \int \frac{\rho(y)g(y)}{\tau} dy = \int \frac{1 - \nu(y)}{\tau} dG(y) = \frac{1 - \bar{\nu}}{\tau}.$$

Since $\beta_A = \beta/(1 - \bar{\rho}) = \beta/\bar{\nu}$,

$$\beta_A\bar{\nu} = \beta.$$

Take the differential,

$$\delta\beta_A\bar{\nu} + \beta_A\delta\bar{\nu} = 0. \quad (25) \quad \boxed{\text{eq:20-1}}$$

Combine [\(eq:21-2\)](#) and [\(eq:20-1\)](#):

$$\begin{aligned}\frac{\delta M}{\delta \Lambda} &= \frac{\bar{\nu}}{\tau \beta_A} \frac{\delta \beta_A}{\delta \Lambda} = \\ &= \frac{\bar{\nu} \beta_A}{\int [\nu^2(y) - \tau V(y) \nu'(y)]} \left[\nu'(y) g(y) - (\nu^2(y) g(y))' \right] = \\ &= \frac{\beta}{\int [\nu^2(y) - \tau V(y) \nu'(y)]} \left[\nu'(y) g(y) - (\nu^2(y) g(y))' \right].\end{aligned}$$

Step 4. By [Lem:Vmu](#) [Lemma 2](#),

$$V(y) = \frac{y - \hat{z}(y)}{\tau},$$

the differential of joint profits is

$$\delta V = \frac{1}{\tau} \int (-\delta \hat{z}(y)) dG(y).$$

Use [\(eq:delta-zhaty\)](#) to find:

$$\begin{aligned}\delta V &= \frac{1}{\tau} \int \nu(y) [\tau \delta \beta_A \Lambda(\hat{z}(y)) + \tau \beta_A \delta \Lambda(\hat{z}(y))] dG(y) = \\ &= \int \nu(y) \left[\frac{\tau \beta_A}{\bar{\nu}} \delta M \Lambda(\hat{z}(y)) + \beta_A \delta \Lambda(\hat{z}(y)) \right] dG(y) = \\ &= \delta M \frac{\tau \beta_A}{\bar{\nu}} \int \nu(y) \Lambda(\hat{z}(y)) dG(y) + \int \nu(y) \beta_A \delta \Lambda(\hat{z}(y)) dG(y) = \\ &= \delta M \frac{\tau}{\bar{\nu}} \int \nu(y) V(y) dG(y) + \int \nu(y) \beta_A \delta \Lambda(\hat{z}(y)) dG(y)\end{aligned}$$

By the definition of variational derivative,

$$\frac{\delta V}{\delta \Lambda} = \frac{\delta M}{\delta \Lambda} \cdot \frac{\tau}{\bar{\nu}} \int \nu(y) V(y) dG(y) + \nu(y) \beta_A g(y).$$

□

Proof of [Lem:optimality-of-up-cen](#) [Lemma 4](#). Suppose the contrary, λ_0 is not upper-coarsening. I will show that there is a deviation from λ_0 that increases \mathcal{J} .

Let $\Lambda_0 \in \mathcal{L}$ which corresponds to λ_0 . Let y^* be the zero of $\delta\mathcal{J}/\delta\Lambda$, and the corresponding cutoff in the seller optimization problem be $z^* = \hat{z}(y^*; \Lambda_0)$. I am now going to construct a feasible variation Λ^ε from Λ_0 that increases \mathcal{J} .

Let $\tilde{\Lambda}$ be the upper-coarsening (of the form [\(eq:Lambda-upper-coarsening\)](#)) that passes through point $(z^*, \Lambda_0(z^*))$. There is only one such function because there is only one line passing through $(z^*, \Lambda_0(z^*))$ and is tangential to $\bar{\Lambda}$. We have that $\Lambda_0(z) \geq \tilde{\Lambda}(z)$ on $z > z^*$, and $\Lambda_0(z) \leq \tilde{\Lambda}(z)$ on $z < z^*$. Moreover, since Λ_0 is not upper-coarsening, for some $z \in [0, 1]$ one of these inequalities is strict. Consider variation

$$\Lambda^\varepsilon(z) = (\tilde{\Lambda}(z) - \Lambda_0(z))\varepsilon.$$

Since $\delta\mathcal{J}/\delta\Lambda < 0$ on $z > z^*$ and $\delta\mathcal{J}/\delta\Lambda > 0$ on $z < z^*$, Λ^ε increases \mathcal{J} . □

proof:prop-5

Proof of [Proposition 4](#). Here I provide the complete proof. Step 1. By [Lemma 3](#), [lem:main-lemma](#)

$$\begin{aligned} \frac{\delta\mathcal{J}}{\delta\Lambda} &= \gamma u \frac{\delta M}{\delta\Lambda} + (1 - \gamma) \frac{\delta V}{\delta\Lambda} \\ &= \gamma u \frac{\delta M}{\delta\Lambda} + (1 - \gamma) \left(\frac{\delta M}{\delta\Lambda} K_2 + \nu(y)g(y)\beta_A \right) \\ &= (\gamma u + K_2(1 - \gamma)) \frac{\delta M}{\delta\Lambda} + (1 - \gamma)\beta_A \nu(y)g(y), \end{aligned} \tag{26}$$

{eq:26}

where $K_2 > 0$. Evaluating with $G = U[0, \bar{y}]$,

$$\frac{\delta M}{\delta\Lambda} = K_1 \bar{y}^{-1} (\nu(y) - \nu^2(y))' = K_1 \bar{y}^{-1} (1 - 2\nu(y))\nu'(y), \tag{27}$$

{eq:27}

where $K_1 > 0$. Since $\nu(0) = 1$ and $\nu(y)$ is non-negative and decreasing, $\delta M/\delta\Lambda$ is either positive for all $y \in [0, \bar{y}]$ or crosses zero from above once. Since $\delta\mathcal{J}/\delta\Lambda$ is a convex combination of $\delta M/\delta\Lambda$ and positive terms, then, similarly, $\delta\mathcal{J}/\delta\Lambda$ is either positive for all $y \in [0, \bar{y}]$ or crosses zero from above once. Let λ_γ^* denote the disclosure policy that maximizes $\mathcal{J}(\gamma)$. By [Lemma 4](#), λ_γ^* is upper-coarsening.

Step 2. We have that $\frac{\delta V}{\delta\Lambda} = \frac{\delta M}{\delta\Lambda} K_2 + \nu(y)\beta_A/\bar{y}$. Therefore, whenever $\frac{\delta M}{\delta\Lambda} \geq 0$, it is

true that $\frac{\delta V}{\delta \Lambda} > 0$. Therefore, the disclosure policy that maximizes the seller profits is finer than the disclosure policy that maximizes the match rate. By step 1, the optimal policy is upper-coarsening, and so $x_1^* \leq x_0^*$. For arbitrary γ , $\frac{\delta \mathcal{J}}{\delta \Lambda}$ is a linear combination of $\frac{\delta M}{\delta \Lambda}$ and $\frac{\delta V}{\delta \Lambda}$. Therefore, for any $\gamma < 1$, whenever $\frac{\delta M}{\delta \Lambda} \geq 0$, it is true that $\frac{\delta \mathcal{J}}{\delta \Lambda} > 0$. Therefore, x_γ^* is decreasing in γ .

Step 3. Suppose $\beta\tau < 1/2$. Using [Lemma 6](#), $\beta_A\tau \leq \beta\tau/(1 - \beta\tau) < 1$. Next, $\nu(y) = (1 + \beta_A\tau\alpha(y))^{-1} > (1 + 1 \cdot 1)^{-1} = 1/2$ for any y . Using [eq:27](#), $\frac{\delta M}{\delta \Lambda} \geq 0$ for any y , and so it $\delta \mathcal{J}/\delta \Lambda$. Therefore, full disclosure is optimal for any γ .

Step 4. For any γ , expand Eq. [\(26\)](#):

$$\frac{\delta \mathcal{J}}{\delta \Lambda} = (\gamma u + K_2(1 - \gamma))K_1\bar{y}^{-1}(1 - 2\nu(y))\nu'(y) + (1 - \gamma)\beta_A\nu(y)/\bar{y}.$$

We need to show that

$$\frac{\delta \mathcal{J}}{\delta \Lambda} < 0$$

for some y . Use the equality

$$\nu'(y) = -W''(\hat{z}(y))\nu^3(y)\tau\beta_A$$

to simplify the target inequality into

$$(1 - 2\nu(y))\nu^2(y)\tau W''(\hat{z}(y))K_1(\gamma u + (1 - \gamma)K_2) > 1 - \gamma.$$

If $\gamma = 1$, all we need to show is

$$(1 - 2\nu(y))\nu^2(y)\tau W''(\hat{z}(y))K_1 u > 0. \tag{28}$$

[{eq:28-1}](#)

By [Lemma 9](#), for any small $\varepsilon > 0$ I can find $\beta\tau$ and \bar{y} large enough so that $\nu(\bar{y}) = \varepsilon$. If so, then the inequality in [\(26\)](#) holds. By continuity, there is a range $[\gamma^*, 1]$ with some $\gamma^* \in [0, 1)$

such that if $\gamma > \gamma^*$, then $\frac{\delta \mathcal{J}}{\delta \Lambda} < 0$ at $y = \bar{y}$. By Step 1, the optimal disclosure policy is the upper-coarsening, and some coarsening is necessary because $\frac{\delta \mathcal{J}}{\delta \Lambda} < 0$ at $y = \bar{y}$. \square

Extensions

D Technical Extensions

Excess-Demand

D.1 No Excess Demand Assumption relaxed

Allow for the case when $\beta\tau \geq 1$. If $\beta\tau > 1$, then sellers get overwhelmed by the buyer requests and can't respond to all of them to the extent that they can't even reject them. To cover this situation we assume that if there are no available sellers to reject a pending buyer request, the platform rejects it automatically.

Since some requests can be rejected by the platform, the acceptance rate as perceived by buyers does not coincide with the acceptance rate α generated by sellers. Denote by α^e the effective acceptance rate that buyers face. Let at some moment of time there is $x \in [0, 1]$ mass of available sellers, and let buyers arrive to the platform at rate β . Then within the next time interval dt , there are βdt new request, and $x + \left(\frac{1-x}{\tau} dt\right)$ available sellers. What is α^e when sellers use acceptance rate α ? Consider three cases.

1. $x > 0$. There is plenty of available sellers, $x + \left(\frac{1-x}{\tau} dt\right) > \beta dt$. Fraction α of buyers are accepted, therefore $\alpha^e = \alpha$.
2. $x = 0$ and $\alpha\beta < \frac{1}{\tau}$. There are few sellers that just became available but in sufficient number to process all buyers. In the same fashion as in case 1, $\alpha^e = \alpha$.
3. $x = 0$ but $\alpha\beta \geq \frac{1}{\tau}$. Not sufficient sellers to process all buyers, some buyers are rejected by the platform. The number of accepted jobs is $\frac{1}{\tau} dt$. The acceptance rate is therefore $\alpha^e = \frac{1/\tau}{\beta}$.

Combining all there cases, we have that

$$\alpha^e = \min\left\{\alpha, \frac{1}{\tau\beta}\right\}.$$

The adjusted definition of equilibrium is then the following.

1.

$$\alpha \in [F(c^*(\beta_A)-), F(c^*(\beta_A)+)].$$

2.

$$\beta_A = \frac{\beta(\alpha^e)}{1 - \beta(\alpha^e)\alpha^e\tau} = \begin{cases} \frac{\beta(\alpha)}{1 - \beta(\alpha)\alpha\tau}, & \alpha\beta(\alpha)\tau < 1 \\ +\infty, & \alpha\beta(\alpha)\tau \geq 1 \end{cases}$$

$\beta_A = +\infty$ reflects the fact that when the demand is overwhelming, buyers line up for sellers so sellers start a new job immediately after they finish the previous one. The No Excess Demand assumption makes sure that this never happens, $\alpha\beta(\alpha)\tau < 1$ for all $\alpha \in [0, 1]$.

The next result shows that in the equilibrium there are no lines. By this reason for the clarity of exposition we decided to restrict the analysis to the case of no lines to begin with.

Claim. In equilibrium, $\beta_A < \infty$.

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