In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
from time import time
from sklearn.linear_model import LogisticRegression
from tqdm import tqdm
```

Problem 1

Dataset Generation

Write a function to **generate a training set** of size m

- randomly generate a weight vector $w \in \mathbb{R}^{10}$, normalize length
- generate a training set $\{(x_i, y_i)\}$ of size m
    - x_i: random vector in $\mathbb{R}^{10}$ from $\mathbf{N}(0, I)$
    - y_i: $\{0, +1\}$ with $P[y = +1] = \sigma(w \cdot x_i)$ and $P[y = 0] = 1 - \sigma(w \cdot x_i)$

In [21]:

```
def generate_data(m):
    # returns the true w as well as X, Y data
    w = np.random.normal(loc=0, scale=1, size=10)
    w = w / np.linalg.norm(w)

    X, Y = [], []
    for _ in range(m):
        x = np.random.normal(loc=0, scale=1, size=10)
        p = 1 / (1 + np.exp(-np.sum(w * x)))
        y = np.random.choice([0, 1], p=[1-p, p])
        X.append(x)
        Y.append(y)
    return w, (X, Y)
```

Algorithm 1: logistic regression

The goal is to learn w. Algorithm 1 is logistic regression (you may use the built-in method LogisticRegression for this. Use max_iter=1000).

In [3]:

```
ms = list(range(50, 300, 50))
```

In [4]:

```
tic = time()
errors_log_reg = []
for m in ms:
    errors = []
    for _ in range(10):
        lg = LogisticRegression(max_iter=1000, random_state=101)
        w, (X,Y) = generate_data(m)
        lg.fit(X, Y)
        w_pred = lg.coef_.squeeze()
        error = np.linalg.norm(w - w_pred)
        errors.append(error)
    errors_log_reg.append(np.mean(errors))

toc = time()
time_log_reg = toc - tic
```

Algorithm 2: gradient descent with square loss

Define square loss as

$$L_i(w^{(t)}) = \frac{1}{2}\left(\sigma(w^{(t)} \cdot x) - y_i\right)^2$$

Algorithm 2 is gradient descent with respect to square loss (code this up yourself -- run for 1000 iterations, use step size eta = 0.01).

In [5]:

```
def sigmoid(w, x):
    return 1 / (1 + np.exp(-np.sum(x*w)))

def grad_step(w, x, y):
    s = sigmoid(w, x)
    return (s - y) * s * (1 - s) * x
```

In [6]:

```
n_steps = 1000
eta = 0.01
```

In [10]:

```
tic = time()
errors_gd = []
for m in tqdm(ms):
    errors = []
    for _ in range(10):
        w, (X,Y) = generate_data(m)
        w_pred = np.zeros_like(w)
        for step in range(n_steps):
            grad_cum = np.mean(np.array([grad_step(w_pred, x, y) for x,y in zip
(X, Y)]), axis=0)
            w_pred = w_pred - eta * grad_cum
        error = np.linalg.norm(w - w_pred)
        errors.append(error)
    errors_gd.append(np.mean(errors))

toc = time()
time_taken_gd = toc - tic
```

```
100%|██████████| 5/5 [01:19<00:00, 15.88s/it]
```

Algorithm 3: stochastic gradient descent with square loss

Similar to gradient descent, except we use the gradient at a single random training point every iteration.

In [12]:

```
tic = time()
errors_sgd = []
for m in ms:
    errors = []
    for _ in range(10):
        w, (X,Y) = generate_data(m)
        w_pred = np.zeros_like(w)
        for step in range(n_steps):
            ind = np.random.randint(m)
            x, y = X[ind], Y[ind]
            w_pred = w_pred - eta * grad_step(w_pred, x, y)
        error = np.linalg.norm(w - w_pred)
        errors.append(error)
    errors_sgd.append(np.mean(errors))

toc = time()
time_taken_sgd = toc - tic
```

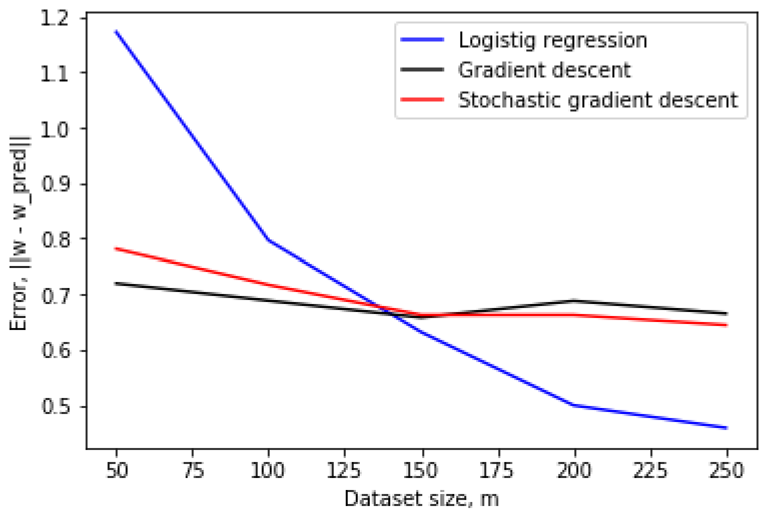
Evaluation

Measure error $\|w - \hat{w}\|_2$ for each method at different sample size. For any fixed value of m, choose many different w's and average the values $\|w - \hat{w}\|_2$ for Algorithms 1, 2 and 3. Plot the results for for each algorithm as you make m large (use $m = 50, 100, 150, 200, 250$). Also record, for each algorithm, the time taken to run the overall experiment.

In [13]:

```
print(f"Logistic regression, overall time taken: {time_log_reg:.2f} sec")
print(f"Gradient descent, overall time taken: {time_taken_gd:.2f} sec")
print(f"Stochastic gradient descent, overall time taken: {time_taken_sgd:.2f} se
c")
plt.plot(ms, errors_log_reg, c='b', label="Logistig regression")
plt.plot(ms, errors_gd, c='k', label="Gradient descent")
plt.plot(ms, errors_sgd, c='r', label="Stochastic gradient descent")
plt.legend()
plt.xlabel("Dataset size, m")
plt.ylabel("Error, ||w - w_pred||")
plt.show();
```

```
Logistic regression, overall time taken: 0.42 sec
Gradient descent, overall time taken: 79.39 sec
Stochastic gradient descent, overall time taken: 1.43 sec
```



NB: it seems that bigger step size is needed. For example, choosing step_size = 0.1 leads to GD performs as good as logistic regression.

Problem 2

In [14]:

```
from sklearn import datasets
from sklearn.ensemble import AdaBoostClassifier
from sklearn.model_selection import KFold
from sklearn.tree import DecisionTreeClassifier
from sklearn.model_selection import cross_val_score
```

In [15]:

```
cancer = datasets.load_breast_cancer()
```

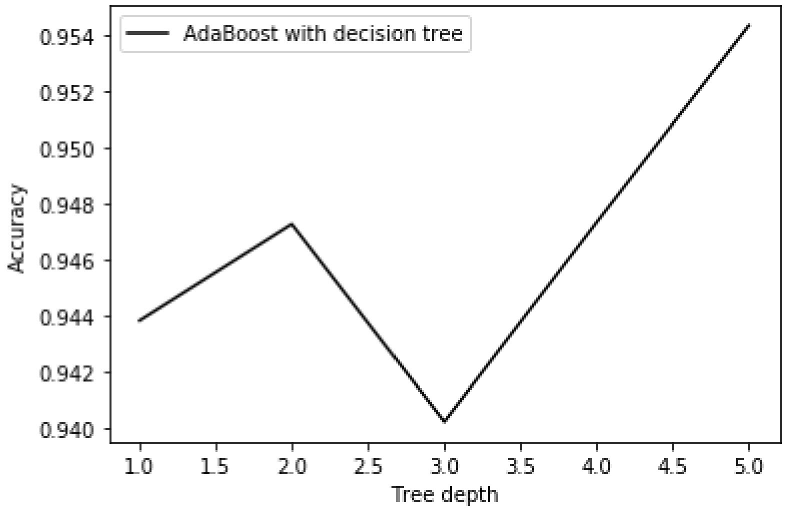
For each depth in $1, \ldots, 5$, instantiate an AdaBoost classifier with the base learner set to be a decision tree of that depth (set `n_estimators=10` and `learning_rate=1`), and then record the 10-fold cross-validated error on the entire breast cancer data set. Plot the resulting curve of accuracy against base classifier depth. Use 101 as your random state for both the base learner as well as the AdaBoost classifier every time.

In [16]:

```
depths = list(range(1, 6))
lr = 1
n_estimators = 10
random_state = 101
scores = []
cv = KFold(n_splits=10, shuffle=True, random_state=random_state)
for depth in depths:
    decision_tree = DecisionTreeClassifier(max_depth=depth, random_state=random_
state)
    ada_boost = AdaBoostClassifier(base_estimator=decision_tree,
                                   random_state=random_state,
                                   n_estimators=n_estimators,
                                   learning_rate=lr)
    scores.append(cross_val_score(ada_boost, cancer.data, cancer.target, cv=cv).
mean())
```

In [17]:

```
plt.plot(depths, scores, c='k', label="AdaBoost with decision tree")
plt.xlabel("Tree depth")
plt.ylabel("Accuracy")
plt.legend()
plt.show();
```


AdaBoost with decision tree
Accuracy
Tree depth