1) $\begin{cases} x = 3t \\ y = 4t^2 + 1 \end{cases} \Rightarrow \begin{cases} t = \frac{x}{3}, \\ y(x) = 4\left(\frac{x}{3}\right)^2 + 1, \end{cases}$

$$y(x) = \frac{4}{9}x^2 + 1$$

Calculation of velocity and acceleration can be done through differentiation:

$$\bar{V}(t) = \dot{\bar{r}} = \begin{bmatrix} (3t)' \\ (4t^2+1)' \end{bmatrix} = \begin{bmatrix} 3 \\ 8t \end{bmatrix}$$

$$64 - \frac{64^2 t^2}{9+64t^2}$$

$$\bar{a}(t) = \dot{\bar{V}} = \ddot{\bar{r}} = \begin{bmatrix} 3' \\ (8t)' \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$

$$\frac{8^2 \cdot \cancel{9} + 8^2 \cdot \cancel{64t^2}}{\cancel{9+64t^2}}$$

To find tangential acceleration:

$$|\bar{a}_t(t)| = \frac{\bar{a} \cdot \bar{V}}{|\bar{V}|} = \begin{bmatrix} 0 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 8t \end{bmatrix} \cdot \frac{1}{\sqrt{9+64t^2}} = \frac{|64t|}{\sqrt{9+64t^2}}$$

Since tangential acceleration has the same direction as velocity:

$$\bar{a}_t(t) = a_t \cdot \frac{\bar{V}}{|\bar{V}|} = \frac{64t}{\sqrt{9+64t^2}} \cdot \frac{1}{\sqrt{9+64t^2}} \cdot \begin{bmatrix} 3 \\ 8t \end{bmatrix} =$$

Normal acceleration as a vector may be found by:

$$\bar{a}_n = \bar{a} - \bar{a}_t$$

Magnitude can be found by formula:

$$|\bar{a}_n| = \frac{|a \times V|}{|V|} = \left| \begin{bmatrix} 0 \\ 3 \\ 8t \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix} \right| \cdot \frac{1}{\sqrt{9+64t^2}} = \frac{24}{\sqrt{9+64t^2}}$$

Curvature can be found by such formula:

$$k(t) = \frac{1}{\rho} = \frac{|a_n|}{|V|^2} = \frac{24}{\sqrt{9+64t^2}} \cdot \frac{1}{9+64t^2} = \frac{24}{(9+64t^2)^{\frac{3}{2}}}$$

Answer:

$$y(x) = \frac{4}{9}x^2 + 1$$

$$V(t) = \begin{bmatrix} 3 \\ 8t \end{bmatrix}$$

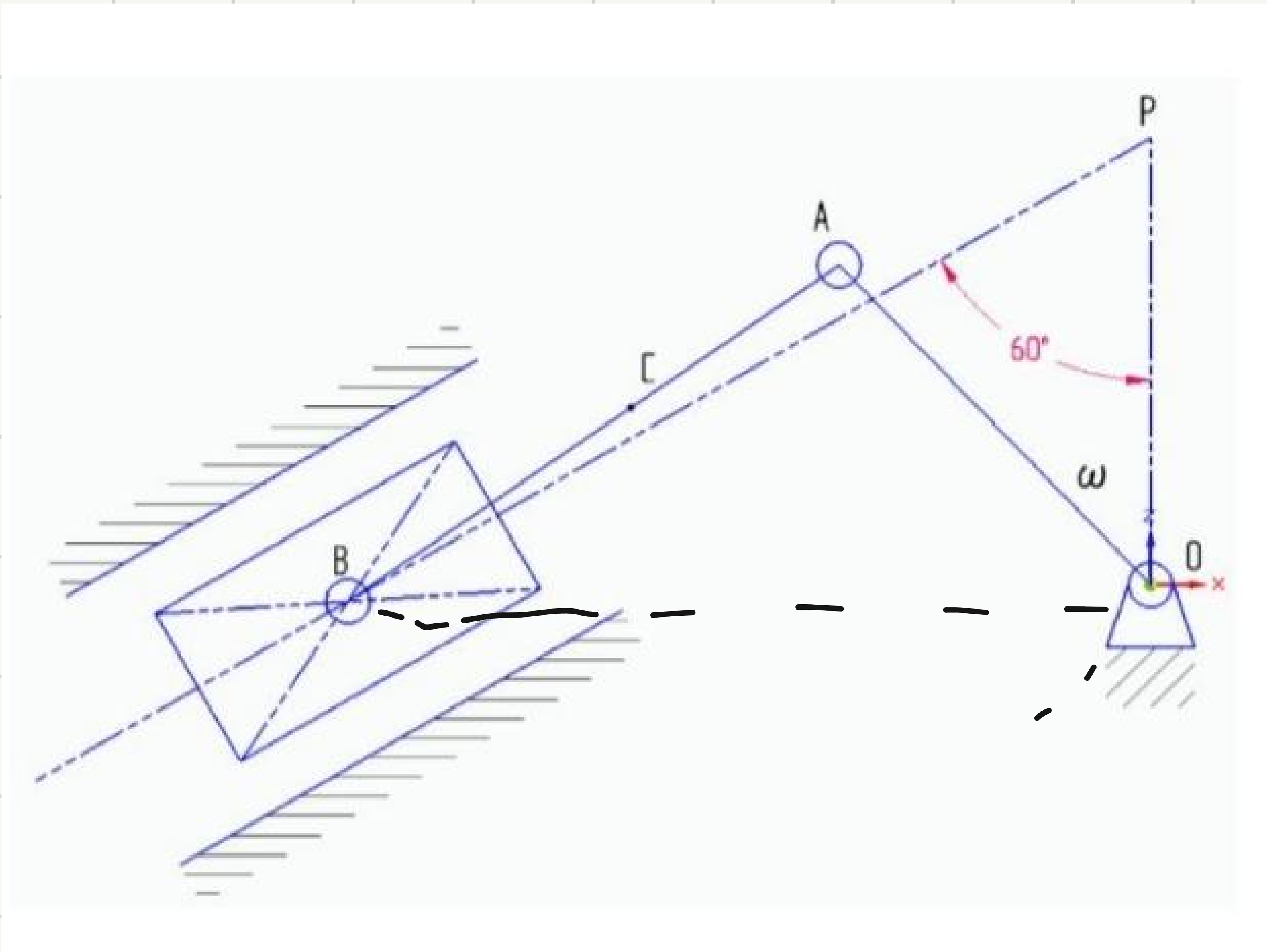
$$a(t) = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$

$$a_t(t) = \frac{64t}{\sqrt{9+64t^2}}$$

$$a_n(t) = \frac{24}{\sqrt{9+64t^2}}$$

$$k(t) = \frac{24}{(9+64t^2)^{\frac{3}{2}}}$$

②



Let the initial position of point A is P: then $\omega t = 0$ at this point.

locus of point A is a circle with center at O and radius OA.

So, $A(t) = OA \cdot \begin{bmatrix} -\sin \omega t \\ \cos \omega t \end{bmatrix}$

Initial coordinates of point B are:

$$\begin{bmatrix} -AB \cdot \sin 60^\circ \\ OP - AB \cos 60^\circ \end{bmatrix}$$

Since B is moving along line with an angle of 30° relative to the axis x:

$$\bar{B}(t) = \begin{bmatrix} BP \cdot \sin 60^\circ \\ OP - BP \cos 60^\circ \end{bmatrix}$$

We know that the distance between A and B is constant. So, we can describe it as an intersection of a circle around point A with radius AB and line BP. Since we know trajectory of A, we can obtain trajectory of B

After we find trajectory of point B and A, we can simply find trajecory of point C:

$$\bar{C} = \bar{A} + (\bar{B} - \bar{A}) \cdot \frac{AB}{AC}$$

Velocities for B and C can simply be found by differentiation of trajectories of B and C:

$$\bar{v}_B(t) = \dot{\bar{B}}(t); \qquad \bar{v}_C(t) = \dot{\bar{C}}(t)$$

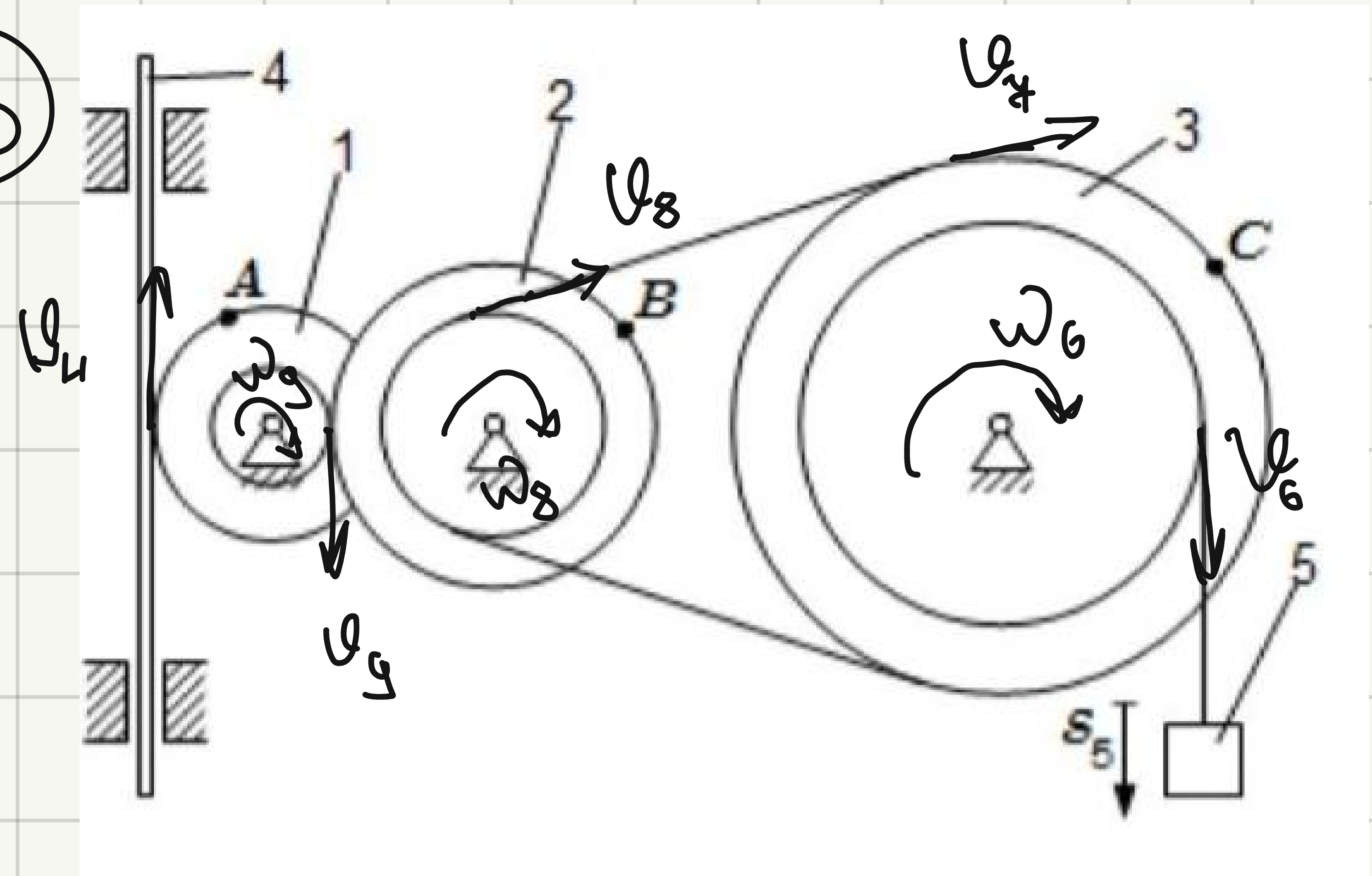
Similarly accelerations may be found by differentiating velocities of v_B and v_C:

$$\bar{a}_B(t) = \dot{\bar{v}}_B(t) = \ddot{\bar{B}}(t); \qquad a_C(t) = \dot{\bar{v}}_C(t) = \ddot{\bar{C}}(t)$$

Values for $B(t)$, $C(t)$, $v_B(t)$, $v_C(t)$, $a_B(t)$, $a_C(t)$ are obtained numerically using python

$$A(t) = OA \begin{bmatrix} -\sin\omega t \\ \cos\omega t \end{bmatrix}$$

③



Given: $r_1 = 2$, $R_1 = 4$

$r_2 = 6$, $R_2 = 8$

$r_3 = 12$, R_3

$s_5 = t^3 - 6t$

$v_A - ?$, $v_c - ?$ $\varepsilon_3 - ?$, $a_B - ?$ $a_4 - ?$ (for $t = 2$)

Since s_5 is connected to circle r_3:

$$v_6 = v_5 = \dot{s} = (t^3 - 6t)' = 3t^2 - 6$$

$$v_6 = \omega_6 \cdot r_3 \Rightarrow \omega_6 = \frac{v_6}{r_3} = \frac{3t^2 - 6}{r_3}$$

Since wheel r_3 is connected to R_3:

$$\omega_6 = \omega_3 = \frac{3t^2 - 6}{r_3}$$

$$\varepsilon_3 = \dot{\omega}_3 = \frac{6t}{r_3} = \frac{6 \cdot 2}{12} = 1$$

v_c can be found from ω_3:

$$v_c = \omega_3 \cdot R_3 = \frac{(3t^2 - 6)}{r_3} \cdot R_3 = \frac{(3 \cdot 2^2 - 6)}{12} \cdot 16 = 8$$

$v_7 = v_c$ (since these points are on the same wheel)

$v_7 = v_8$ (points are connected) $\Rightarrow v_8 = v_c = \frac{(3t^2 - 6) \cdot R_3}{r_3}$

$$v_8 = \omega_8 \cdot R_2 \Rightarrow \omega_8 = \frac{v_8}{r_2} = \frac{(3t^2 - 6)}{r_3} \cdot \frac{R_3}{r_2}$$

$$v_9 = v_B = \omega_8 \cdot R_2 = \frac{(3t^2 - 6)}{r_3 \cdot r_2} \cdot R_3 \cdot R_2$$

a_B^τ can be found by differentiation of ϑ_B:

$$a_B^\tau = \dot{\vartheta}_B = (3t^2-6)\frac{R_2R_3}{r_2r_3} = \frac{6t\,R_2R_3}{r_2r_3} \quad \text{(tangential)}$$

a_B^n can be found by such formula:

$$a_B^n = \frac{\vartheta_B^2}{R_2} = \frac{(3t^2-6)^2\cdot R_2^{\cancel{2}}\cdot R_3^2}{r_2^2\cdot r_3^2\cdot \cancel{R_2}} = \frac{(3t^2-6)^2\cdot R_2\cdot R_3^2}{r_2^2\cdot r_3^2}$$

$$a_B = \sqrt{{a_B^\tau}^2 + {a_B^n}^2} = \sqrt{36t^2\cdot\frac{R_2^2\cdot R_3^2}{r_2^2\cdot r_3^2} + \frac{(3t^2-6)^4 R_2^2\cdot R_3^4}{r_2^4\cdot r_3^4}} =$$

$$= \frac{R_2R_3}{r_2r_3}\sqrt{36t^2 + \frac{(3t^2-6)^4 R_3^2}{r_2^2\cdot r_3^2}}$$

Since $\vartheta_9 = \omega_8\cdot R_2$, but on the other hand $\vartheta_9 = \omega_9\cdot r_1$:

$$\omega_9 = \frac{\vartheta_9}{r_1} = \frac{\omega_8 R_2}{r_1} = \frac{(3t^2-6)R_2R_3}{r_1r_2r_3}$$

$$\vartheta_A = \omega_9\cdot R_1 = \frac{(3t^2-6)R_1R_2R_3}{r_1r_2r_3}$$

$\vartheta_A = \vartheta_4$ (since points are on the same wheel)

a_4 can be found by differentiation of ϑ_4:

$$a_4 = \dot{\vartheta}_4 = \dot{\vartheta}_A = \left(\frac{(3t^2-6)R_1R_2R_3}{r_1r_2r_3}\right)' = \frac{6t\,R_1R_2R_3}{r_1r_2r_3}$$

Answer: $v_A = \frac{(3t^2-6)\cdot R_1R_2R_3}{r_1r_2r_3} = \frac{(3\cdot 4-6)\cdot 4\cdot 8\cdot 16}{2\cdot \cancel{6}\cdot 12} \approx 21.33$

$v_C = \frac{(3t^2-6)\cdot R_3}{r_3} = \frac{(3\cdot 2^2-6)\cdot 16}{12} = \frac{6\cdot 16}{12} = 8$

$\varepsilon_3 = \frac{6t}{r_3} = \frac{6\cdot 2}{12} = 1$

$a_B = \frac{R_2R_3}{r_2r_3}\sqrt{36t^2 + \frac{(3t^2-6)^4 R_3^2}{r_1^2\cdot r_2^2}} = \frac{8\cdot 16}{6\cdot 12}\cdot\sqrt{36\cdot 2^2 + \frac{6^4\cdot 16}{6^2\cdot 12^2}} =$

$\frac{4\cdot 4}{3\cdot 3}\cdot\sqrt{208} \approx 25.64$

$a_{L_1} = \frac{6t\,R_1R_2R_3}{r_1r_2r_3} = \frac{\cancel{6}\cdot\cancel{2}\cdot 4\cdot 8\cdot 16}{\cancel{2}\cdot\cancel{6}\cdot 12} = \frac{8\cdot 16}{3} = \frac{128}{3} = 42.67$