1.1-1.2 **R.O.:** A bullet (material point.)

**Methodology:** Differential representation of $2^{nd}$ Newton's law.

**Force analysis:**


$$\boxed{F_y = m\ddot{y}}$$

**Conditions:**

$x_0 = 0$ $\quad$ $x_f = 1500$

$y_0 = 0$ $\quad$ $y_f = 0$

$\dot{x}_0 = V_0 \cdot \cos\alpha$ $\quad$ $\dot{x}_f = ?$

$\dot{y}_0 = V_0 \sin\alpha$ $\quad$ $\dot{y}_f = ?$

$t_0 = 0$ $\quad$ $t_f = ?$

**Solution:**

From $2^{nd}$ Newton's law we get:

$$m\ddot{x} = \sum F_x = 0 \Rightarrow \dot{x} = 0$$

$$m\ddot{y} = \sum \vec{F}_y = -mg \Rightarrow \ddot{y} = -g$$

By integration of these equations we get:

$$\int_{t_0}^{t} \ddot{x}\, d\tau = \int_{t_0}^{t} 0\, d\tau$$

$$\dot{x}(\tau) = \dot{x}(t_0) = V_0 \cos\alpha$$

$$\int_{t_0}^{t_f} \dot{x}\, d\tau = \int_{t_0}^{t_f} V_0 \cos\alpha \, d\tau$$

$$x(\tau) - \cancel{x(t_0)}^{0} = V_0 \cos\alpha \cdot \tau - \cancel{V_0 \cos\alpha \cdot t_0}^{0}$$

So, $x(t_f) = V_0 \cos\alpha \cdot t_f \Rightarrow V_0 \cos\alpha \cdot t_f = 1500$

$$\int_{t_0}^{t} \ddot{y}\, d\tau = \int_{t_0}^{t} -g\, d\tau$$

$$\dot{y}(t) - \dot{y}(t_0) = -gt + \cancel{gt_0}^{0} \Rightarrow \dot{y}(t) = -gt + V_0 \sin\alpha$$

$$\int_{t_0}^{t} \dot{y}\, d\tau = \int_{t_0}^{t} (-g\tau + V_0 \sin\alpha)\, d\tau$$

$$y(t) - \cancel{y(t_0)}^{0} = -\frac{gt^2}{2} + V_0 \sin\alpha\, t + \cancel{\frac{gt_0^2}{2}}^{0} - \cancel{V_0 \sin\alpha \cdot t_0}^{0}$$

So, $y(t) = -\frac{gt^2}{2} + V_0 \sin\alpha \cdot t$

$$y(t_f) = -\frac{gt_f^2}{2} + V_0 \sin\alpha \cdot t_f = 0$$

We obtain such system of equations:

$$\begin{cases} V_0 \cos\alpha \cdot t_f = 1500, \\ -\frac{gt_f^2}{2} + V_0 \sin\alpha \cdot t_f = 0. \end{cases}$$

By solving it using python we get:

$$\left[\begin{array}{l}\begin{cases}\alpha_1 = 0.009721183181162229 \\ tf_1 = 1.7242194009686 5\end{cases} \\ \begin{cases}\alpha_2 = 1.5610751436137 3 \\ tf_2 = 177.36164978871 9\end{cases}\end{array}\right.$$

Now let's find the max height of a cargo. It will be obtained, when $\dot{y}=0$.

So: $-gt_{max} + V_0 \sin\alpha = 0 \Rightarrow t_{max} = \frac{V_0 \sin\alpha}{g}$

By substituting values obtained values:

for $\alpha_1$:

$y_{max_1} = 3.645558530457287$

for $\alpha_2$:

$y_{max_2} = 38574.33609284569$

Answer: 1.1) $\alpha_1 = 0.009721183181162229$,

$\alpha_2 = 1.56107514361373$

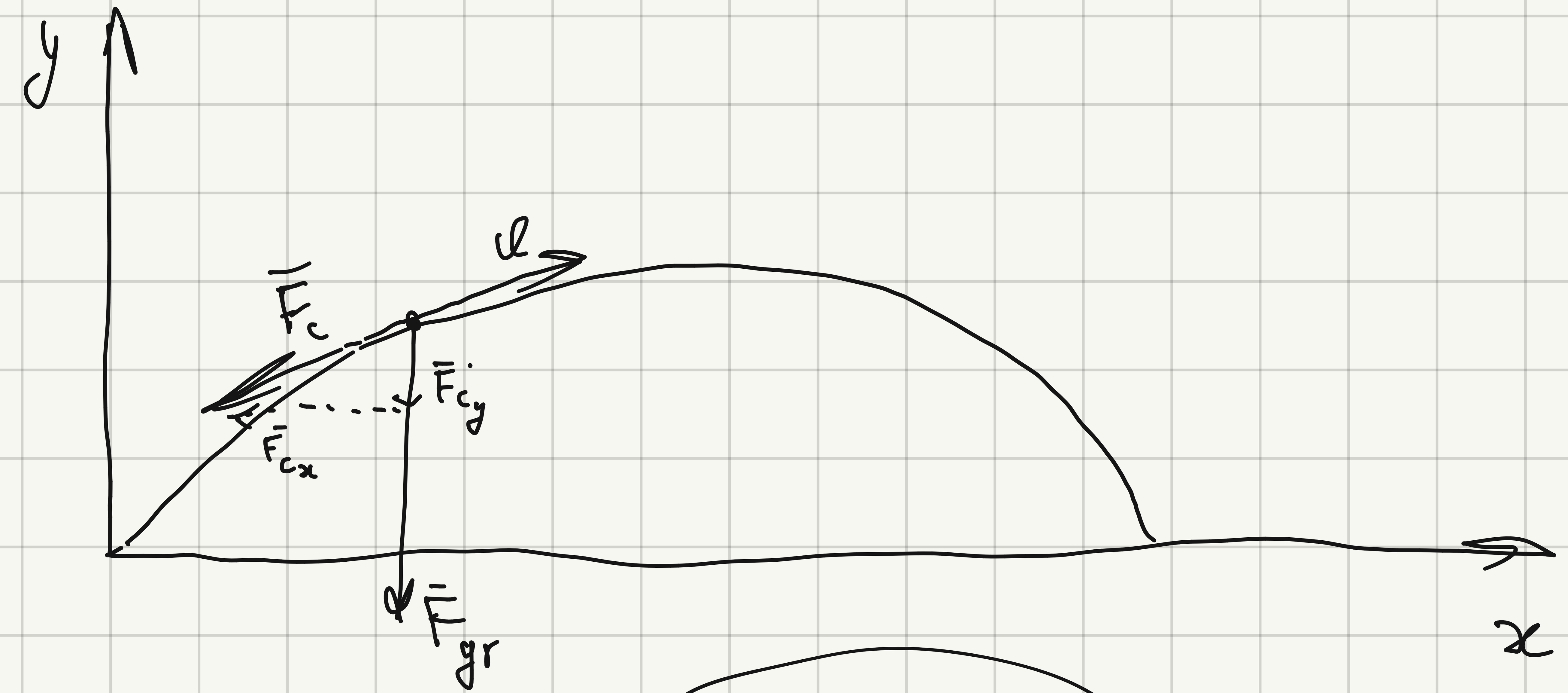
1.2) maximal height of cargo:

$y_{max} = 38574.33609284569$, for $\alpha_2$

1.3 **R.O.**: A bullet (material point)

**Methodology:** Differential representation of $2^{nd}$ Newton's law:

**Force analysis:**




$\bar{F}_c = -k v \bar{v}$ $\qquad$ $\bar{F}_{gr} = m\bar{g}$

**Conditions:**

| | |
|---|---|
| $x_0 = 0$ | $x_f = 1500$ |
| $y_0 = 0$ | $y_f = 0$ |
| $\dot{x}_0 = V_0 \cdot \cos\alpha$ | $\dot{x}_f = ?$ |
| $\dot{y}_0 = V_0 \sin\alpha$ | $\dot{y}_f = ?$ |
| $t_0 = 0$ | $t_f = ?$ |

**Solution:**

Applying second Newton's law:

$$OX: \; m\ddot{x} = \sum \bar{F}_x = \bar{F}_{c_x} = -k\sqrt{\dot{x}^2+\dot{y}^2}\cdot\dot{x}$$

$$OY: \; m\ddot{y} = \sum \bar{F}_y = \bar{F}_{c_y} + \bar{F}_{gr} = -k\sqrt{\dot{x}^2+\dot{y}^2}\cdot\dot{y} - m\cdot g$$

Now going through different angles $\alpha$ from $0$ to $\frac{\pi}{2}$. let's solve such initial value problem:

$$\begin{cases} \ddot{x} = -\frac{k}{m}\sqrt{\dot{x}^2+\dot{y}^2}\,\dot{x}. \\ \ddot{y} = -\frac{k}{m}\sqrt{\dot{x}^2+\dot{y}^2}\,\dot{y} - g. \\ x(0)=0. \\ y(0)=0, \\ \dot{x}(0) = v_0\cos\alpha. \\ \dot{y}(0) = v_0\sin\alpha. \end{cases}$$

And find angle $\alpha$, for which trajectory will go through point $y=0$; $x=1500$.

Using python we get:

$\alpha = 0.9360689412100808$

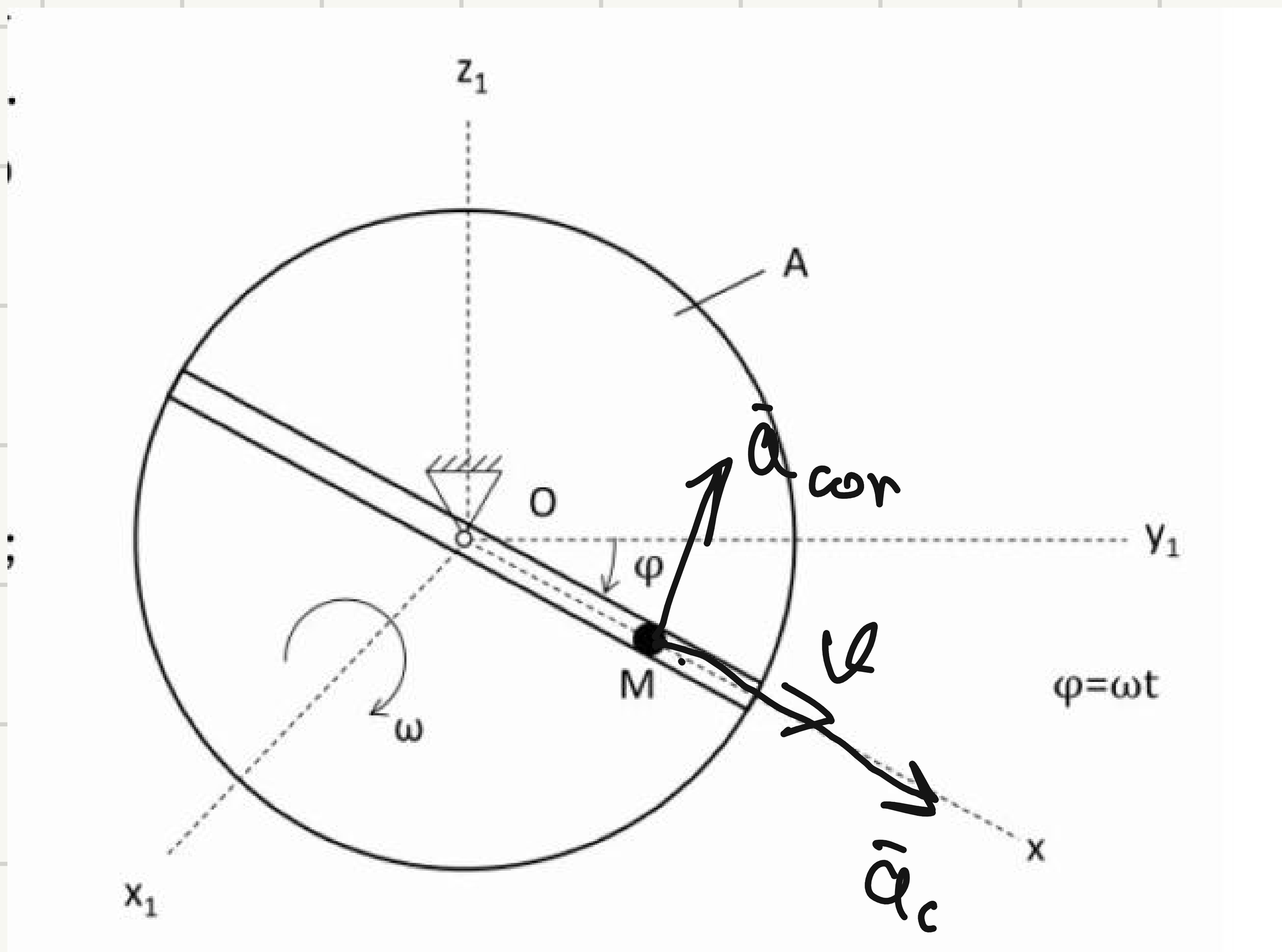
Answer: $\alpha = 0.9360689412100808$

② R.O.: particle M (material point) inside of the moving body A

M – traslatory motion; A – rotational motion.

**Methodology:** Differential representation of 2nd Newton's Law

**Kinematic analysis:**

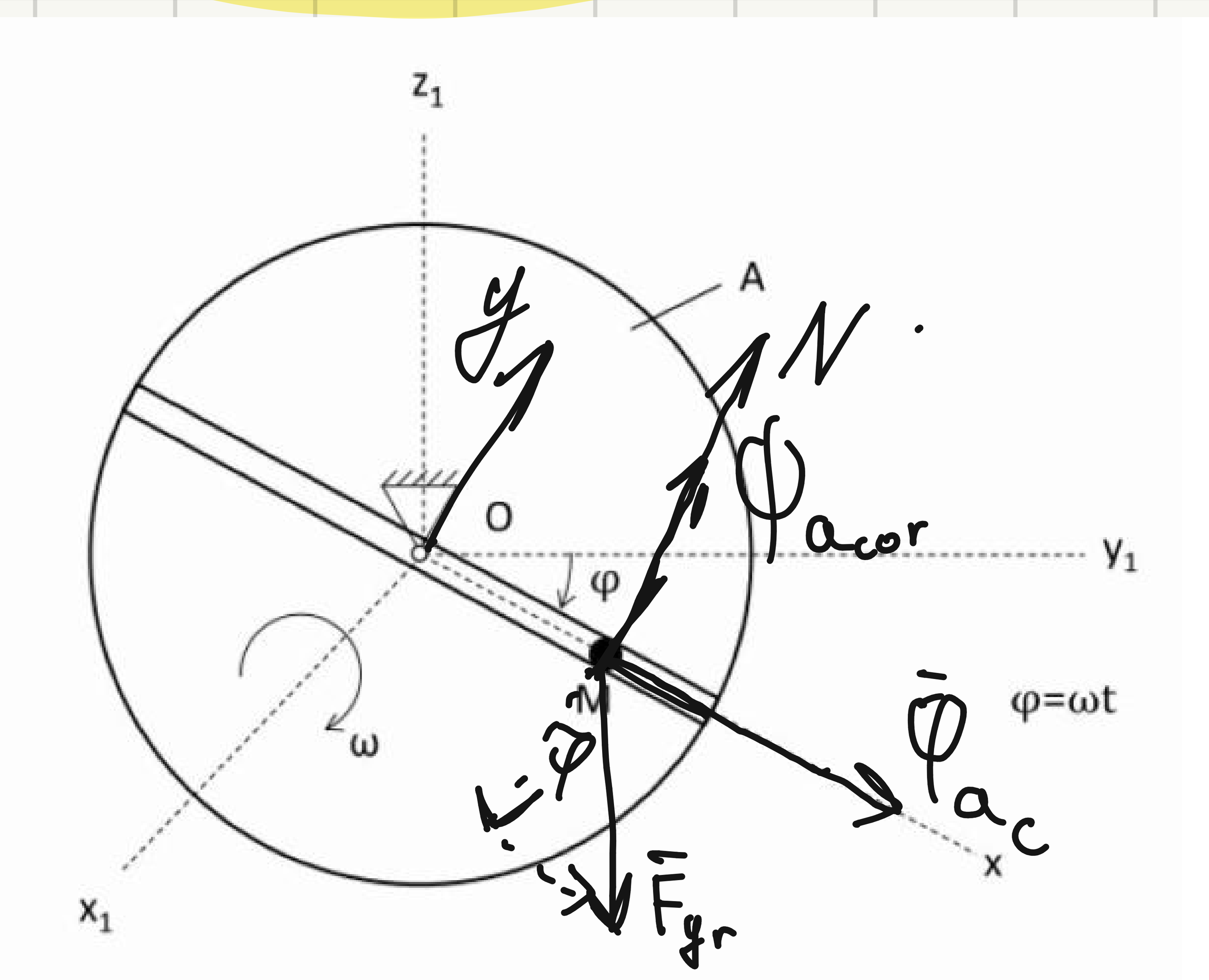



$\varphi_A = \omega t$ ; $\omega_A$ ; $\varepsilon_A = 0$ ;

$a_c = \omega^2 \cdot x$ ; $a_{cor} = \dot{x} \times \omega$

$\omega = -\pi$

**Force analysis:**




$\bar{F}_{gr} = -m \cdot \bar{g}$

$\bar{\Phi}_{a_{cor}}$ – ? inertial coriolis force

$\bar{\Phi}_{a_c}$ – ? inertical centrifugal force

$N$ – ? normal force

**Solutions:**

By applying 2nd Newton's law:

OX: $m\ddot{x} = \sum \bar{F}_x = \Phi_{a_c} + \bar{F}_{gr} \cdot \sin\varphi = m\omega^2 \cdot x + mg \cdot \sin\omega t$

OY: $m\ddot{y} = \sum \bar{F}_y = N - \Phi_{a_{cor}} - F_{gr} \cdot \cos\varphi = N - \dot{x} \cdot \omega - mg\cos\omega t = 0$

$N = \dot{x}\omega + mg\cos\omega t$

After solving this system of differential equations with such initial conditions:

$x(0) = 0$ ; $\dot{x}(0) = 0.4$

Using wolfromalpha, we get:

$$x = -0.312152 \cdot e^{-\pi t} + 0.312152 \cdot e^{\pi t} - 0.49698 \cdot \sin(\pi t)$$

$$\dot{x} = 0.980654 \cdot e^{-\pi t} + 0.980654 \cdot e^{\pi t} - 1.56131 \cos(\pi t)$$

Now we can find time, when point will leave the channel:

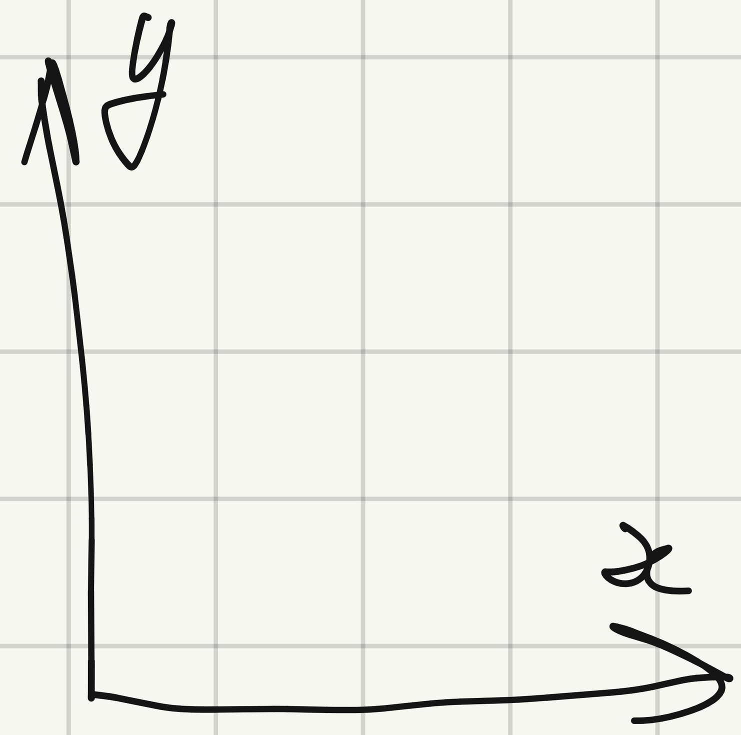
$x(t) = 0.5$

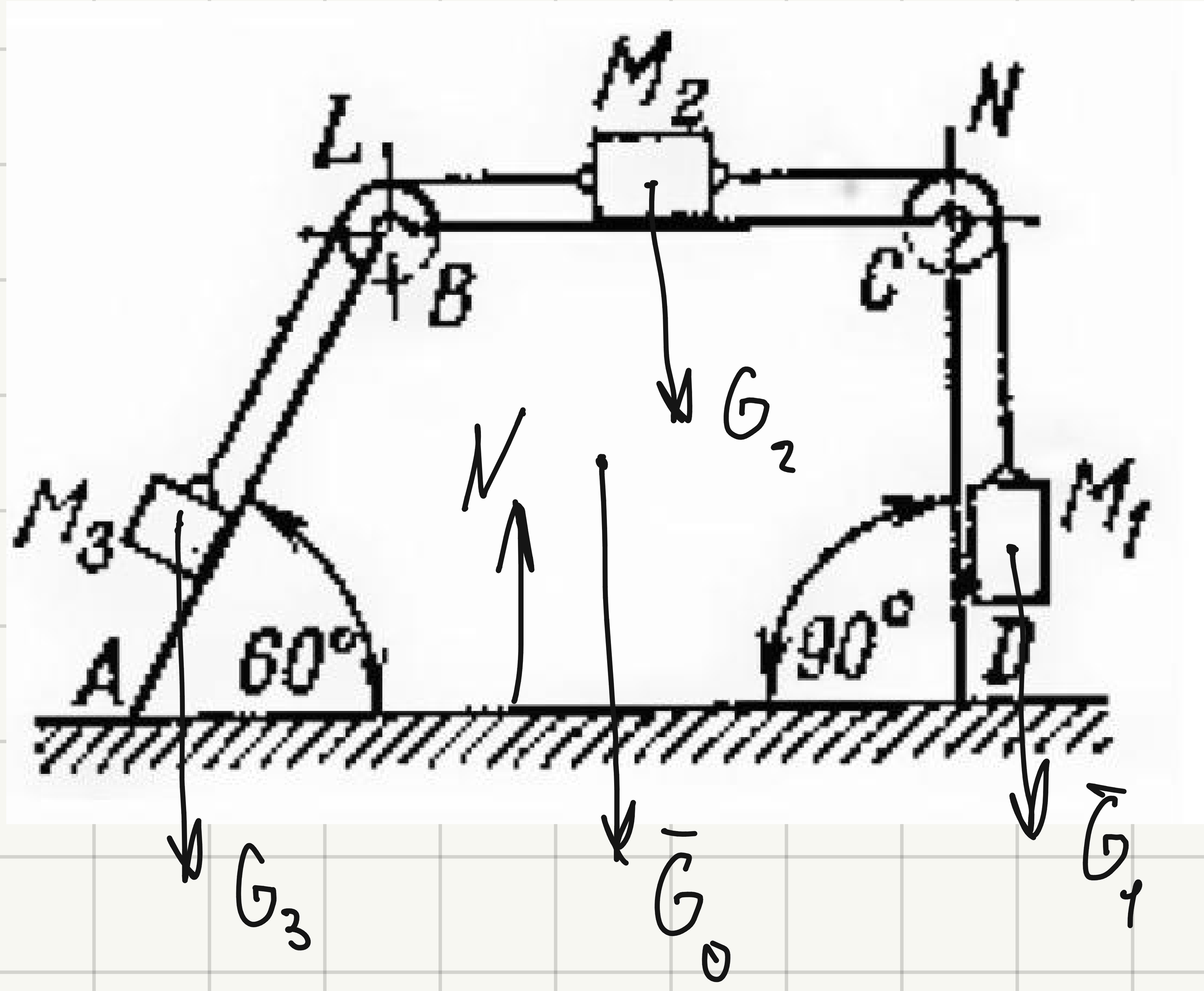
Wolfram shows: $t = 0.3889$

Now we have everything to simulate.

③ **R.O.:** System of bodies: weight 1 ($M_1 = 20kg$), weight 2 ($M_2 = 15kg$), weight 3 ($M_3 = 10kg$), body ABCD ($M_0$)

Pulleys: L, N




**Methodology:**

$2^{nd}$ Newton's law for systems of bodies

**Force analysis:**

$N - ?$ $G_3 = M_3 g$, $G_2 = M_2 g$; $G_1 = M_1 g$; $G_0 = M_0 g$

**Conditions:**

| initial | | final | |
|---|---|---|---|
| $x_0 = b_0$ | $y_0 = c_0$ | $x'_0 = b_0 + L$ | $y'_0 - ?$ |
| $x_1 = b_1$ | $y_1 = c_1$ | $x'_1 = b_1 + 0 + L$ | $y'_1 = c_1 + 1$ |
| $x_2 = b_2$ | $y_2 = c_2$ | $x'_2 = b_2 + 1 + L$ | $y'_2 = c_2$ |
| $x_3 = b_3$ | $y_3 = c_3$ | $x'_3 = b_3 + 1 \cdot \sin 60° + L$ | $y'_3 = c_3 + 1 \cdot \cos 60°$ |

**Solution:**

Applying $2^{nd}$ Newton's law:

$$OX: \; Ma_{c_x} = \sum F_x = 0$$

Since system has no initial velocity:

$$Mx_{c_1} = Mx_{c_2} \Rightarrow x_{c_1} = x_{c_2}$$

$$\frac{M_0 b_0 + M_1 b_1 + M_2 b_2 + M_3 b_3}{\cancel{M_0 + M_1 + M_2 + M_3}} = \frac{M_0(b_0 + L) + M_1(b_1 + L) + M_2(b_2 + 1 + L) + M_3(b_3 + \sin 60° + L)}{\cancel{M_0 + M_1 + M_2 + M_3}}$$

$$\cancel{M_0 b_0} + M_0 L + \cancel{M_1 b_1} + M_1 L + \cancel{M_2 b_2} + M_2 + M_2 L + \cancel{M_3 b_3} + M_3 \frac{1}{2} + M_3 L =$$

$$= \cancel{M_0 b_0} + \cancel{M_1 b_1} + \cancel{M_2 b_2} + \cancel{M_3 b_3}$$

$$M_0 L + M_1 L + M_2 L + M_3 L + M_2 + M_3 \frac{1}{2} = 0$$

$$L = \frac{-M_2 - M_3 \frac{1}{2}}{M_0 + M_1 + M_2 + M_3} = \frac{-15 - 5}{M_0 + 20 + 15 + 10} \approx \frac{-20}{M_0 + 45}$$

If we take $M_0 = 100 kg$ (taken from Meshchesky 35.20)

$$\frac{-20}{145} \approx -0.14 m = -14 cm$$

So, body ABCD will move to the left on 14cm

Answer: body ABCD will move to the left on 14cm