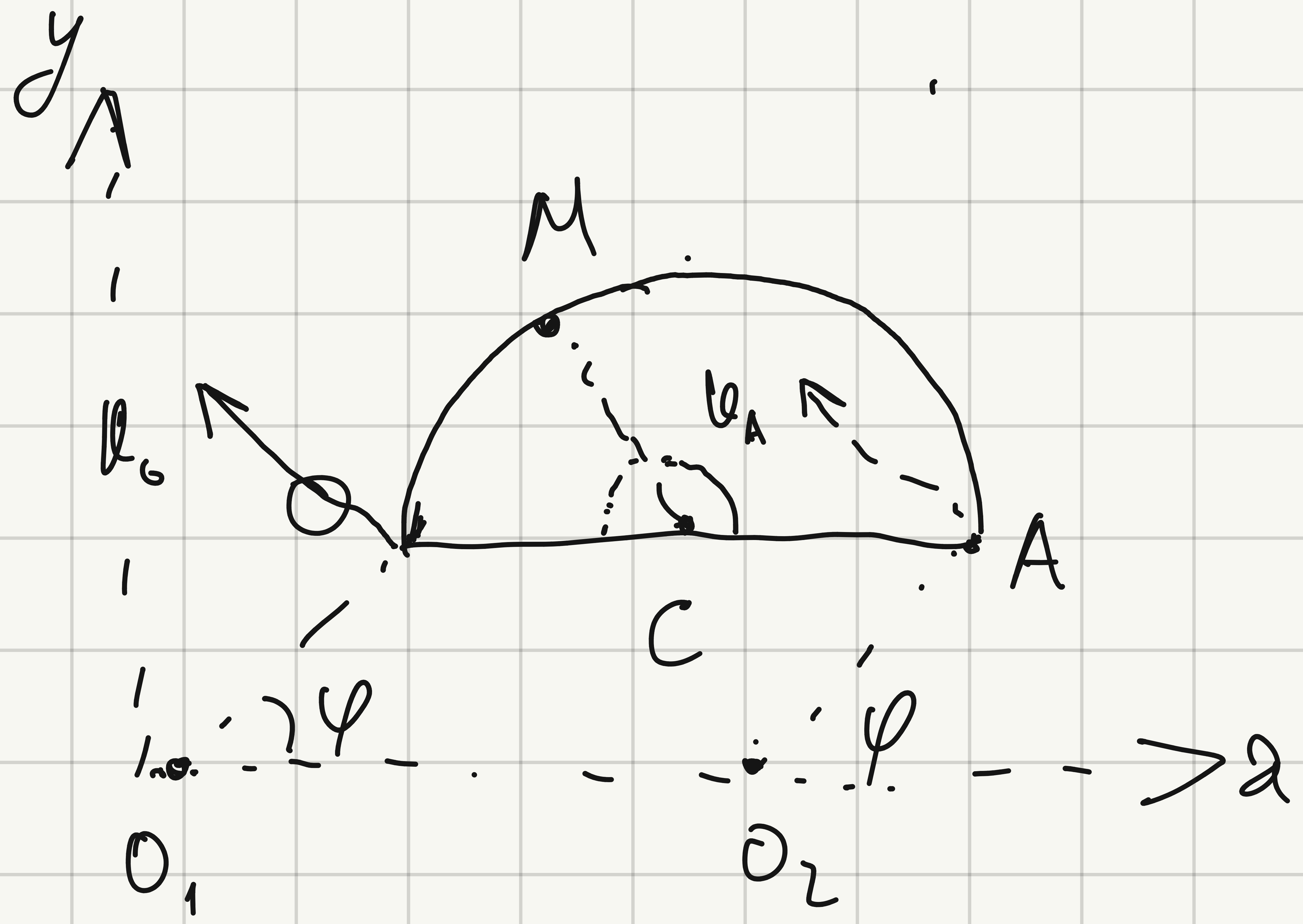
Task 1



Since $OO_1 = O_2A$: $v_O = v_A$, so $\omega_{OA} = 0$. Thus, semicircle OMA will have traslatory motion, and coriolis acceleration will be equal to 0.

Let's use point O_1 as a starting point of coordinate system. So:

$$O_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad O_2 = \begin{bmatrix} 2 \cdot R \\ 0 \end{bmatrix}$$

Point O is moving along $\omega(O_1, O_1O)$, so:

$$O = \begin{bmatrix} O_1O \cdot \cos\varphi \\ O_1O \cdot \sin\varphi \end{bmatrix}$$

Point A is moving along $\omega(O_2, O_2A)$, so

$$A = \begin{bmatrix} O_2A \cdot \cos\varphi \\ O_2A \cdot \sin\varphi \end{bmatrix} + O_2 = \begin{bmatrix} 2 \cdot R + O_2A \cos\varphi \\ O_2A \cdot \sin\varphi \end{bmatrix}$$

Point C is a center of semicircle OMA, and this is a center of segment OA. Therefore:

$$C = \frac{O + A}{2} = \begin{bmatrix} O_1O \cos\varphi + R \\ O_1O \sin\varphi \end{bmatrix}$$

$\omega(O, r)$ – circle with center at point O and radius r

M is moving along $\omega(C, R)$. So, its trajectory may be represented as:

$$M = C + \begin{bmatrix} R \cdot \cos(\angle MCA) \\ R \cdot \sin(\angle MCA) \end{bmatrix} = \begin{bmatrix} OO_1 \cos\varphi + R\cos(\angle MCA) + R \\ OO_1 \sin\varphi + R\sin(\angle MCA) \end{bmatrix}$$

$\frac{OM}{R}$ – is an $\angle OCM$ (in radians),

So, $\angle MCA = \pi - \angle OCM$. Therefore, $\cos(\angle MCA) = -\cos(\angle OCM)$, and $\sin(\angle MCA) = \sin(\angle OCM)$

$$M = \begin{bmatrix} OO_1 \cos\varphi + R - R\cos\left(\frac{OM}{R}\right) \\ OO_1 \sin\varphi + R\sin\left(\frac{OM}{R}\right) \end{bmatrix}$$

We obtain positions for all points, so we can simulate this mechanism

Velocities and accelerations

Relative motion:

$$v_r = \dot{OM} = \frac{OM(t_i) - OM(t_{i-1})}{t_i - t_{i-1}} \text{ (for small } (t_i - t_{i-1}))$$

$$a_r = \ddot{OM} = \frac{v_r(t_i) - v_r(t_{i-1})}{t_i - t_{i-1}} \text{ (for small } (t_i - t_{i-1}))$$

$$|a_r^n| = \frac{v_r^2}{R}$$

Since point M moving along $\omega(C,R)$, a_r^n will be always directed from point M to point C.

unit vector for a_r^n: $\bar{u}_r^n = \dfrac{C-M}{|C-M|}$

In code $\bar{u}_r^n(t) = \dfrac{C(t)-M(t)}{|C(t)-M(t)|}$

$\bar{a}_r^\tau \perp \bar{a}_r^n$, so we can find unit vector for $\bar{a}_r^\tau$:

$$\bar{u}_r^\tau = \begin{bmatrix} -(u_r^n)_y \\ (u_r^n)_x \end{bmatrix} \cdot \operatorname{sign}(\bar{a}_r^\tau(t))$$

acceleration may be negative

such vector since motion is clockwise

unit vector for v_r:

$$\bar{u}_r^v = \begin{bmatrix} -(u_r^n)_y \\ (u_r^n)_x \end{bmatrix} \cdot \operatorname{sign}(v_r(t))$$

Velocity may become negative at some point

initial motion is clockwise

Transport motion:

$$\omega(t) = \dot{\varphi}(t) = \frac{\varphi(t_i) - \varphi(t_{i-1})}{t_i - t_{i-1}} \quad (\text{for small } (t_i - t_{i-1}))$$

$$\varepsilon(t) = \dot{\omega}(t) = \frac{\omega(t_i) - \omega(t_{i-1})}{t_i - t_{i-1}} \quad (\text{for small } (t_i - t_{i-1}))$$

$$v_A(t) = \omega(t) \cdot O_1O = \frac{\varphi(t_i) - \varphi(t_{i-1})}{t_i - t_{i-1}}$$

Since we have translatory motion, $\bar{v}_{Mt} = \bar{v}_A$; $\bar{a}_{Mt} = \bar{a}_{At}$

$$v_M(t) = \frac{\varphi(t_i) - \varphi(t_{i-1})}{t_i - t_{i-1}}$$

$$a_t^{\tau} = a_A^{\tau} = \varepsilon \cdot O_1O = \frac{\omega(t_i) - \omega(t_{i-1})}{t_i - t_{i-1}} \quad \left(\text{for small } (t_i - t_{i-1})\right)$$

$$a_t^{n} = a_A^{n} = \omega^2(t) \cdot O_1O = \frac{\left(\varphi(t_i) - \varphi(t_{i-1})\right)^2}{(t_i - t_{i-1})^2} \cdot O_1O \quad \left(\text{for small } (t_i - t_{i-1})\right)$$

Direction of unit vector of a_A^n:

$$\bar{u}_t^n = \bar{u}_A^n(t) = \frac{O_1 - O(t)}{|O_1 - O(t)|} = \bar{u}_t^n(t)$$

$\bar{u}_A^{\tau} \perp \bar{u}_A^n$, so:

$$\bar{u}_t^{\tau} = \bar{u}_A^{\tau} = \begin{bmatrix} (u_A^n)_y \\ -(u_A^n)_x \end{bmatrix} \cdot \operatorname{sign}(a_A^{\tau})$$

since motion is counterclockwise

$u_A^{\tau} \parallel v_A$, so:

$$\bar{u}_A^{v} = \bar{u}_t^{v} = \begin{bmatrix} (u_A^n)_y \\ -(u_A^n)_x \end{bmatrix} \cdot \operatorname{sign}(v_A)$$

All vectors of velocities and accelerations are represented as:

$$\underbrace{V_{ec}}_{\text{Vector we need to find}} = \underbrace{\bar{u}}_{\text{unit vector}} \cdot \underbrace{m}_{\text{magnitude}}$$

Absolute Velocity and acceleration:

$$\bar{v} = |v_r| \cdot \bar{u}_r^{v} + |v_t| \cdot \bar{u}_t^{v}$$

$$\bar{a} = |a_r^{\tau}| \cdot \bar{u}_r^{\tau} + |a_r^{n}| \cdot \bar{u}_r^{n} + |a_t^{\tau}| \cdot \bar{u}_t^{\tau} + |a_t^{n}| \cdot \bar{u}_t^{n}$$

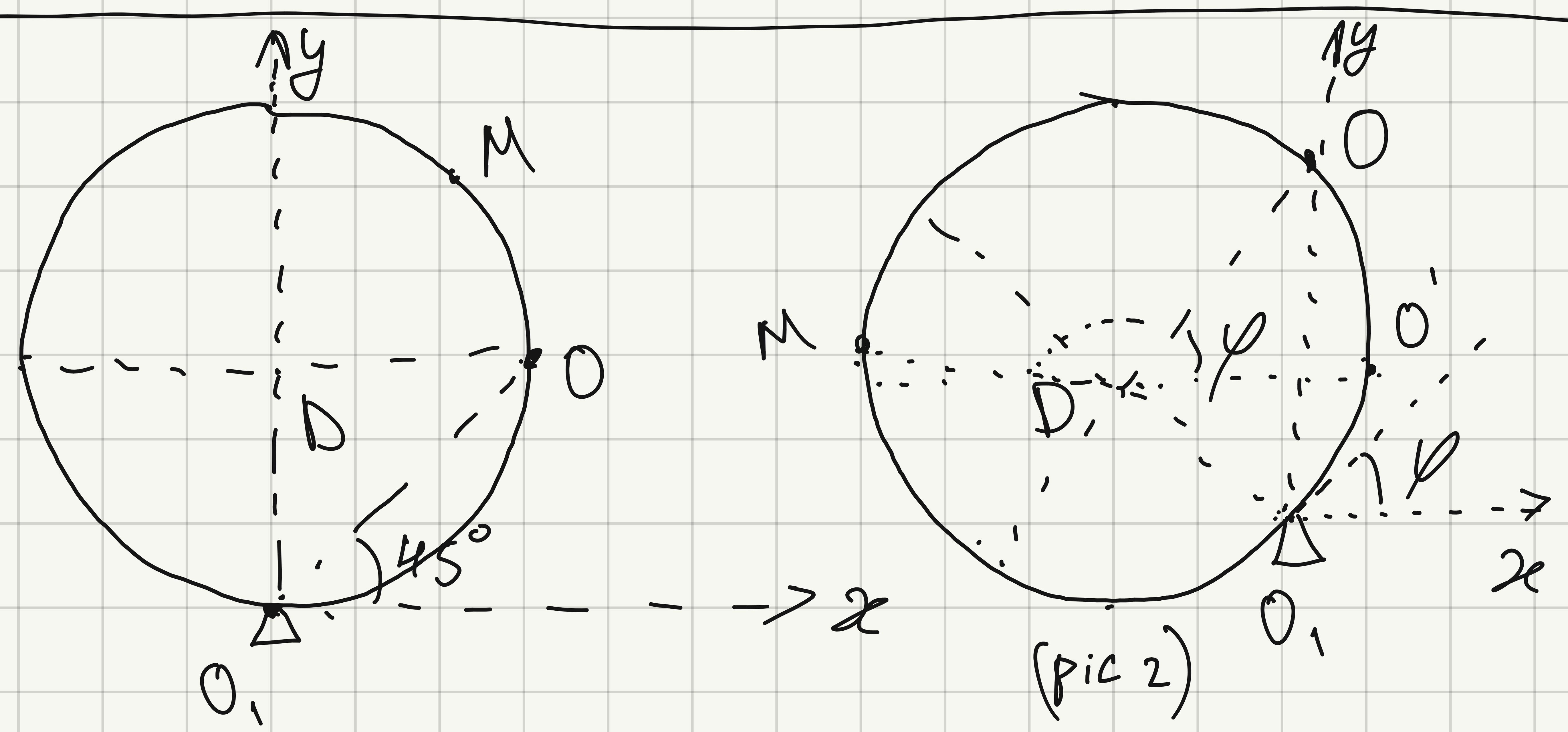
All values for velocities and accelerations were found numerically using python!!!

When M reaches A:

If M reaches A, $OM = \pi \cdot R$, so:

$$6\pi t^2 = 18 \Rightarrow t = \pm\sqrt{\frac{3}{\pi}}$$

Task 2



(pic 2)

Let's use O_1 as a starting position of coordinates, so:

$$O_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Point D is moving along $\omega(O_1, R)$, so:

$$D = \begin{bmatrix} R \cdot \cos(\varphi + pi/2) \\ R \cdot \sin(\varphi + pi/2) \end{bmatrix}$$

since initial angle for D is $pi/2$

Point O is moving along $\omega(O_1, O_1O)$, so:

$O_1O = R \cdot \sqrt{2}$

initial angle for O is $\frac{pi}{4}$

$$O = \begin{bmatrix} R\sqrt{2} \cdot \cos(\varphi + pi/4) \\ R\sqrt{2} \cdot \sin(\varphi + pi/4) \end{bmatrix}$$

M moving along $\omega(D, R)$. But this circle is rotating.

Let's draw a line through D parallel to axis x (line O'D)

We know angle of rotation of circle, it is φ.

Since OD parallel to tangent line to the circle in point O_1, $\angle ODO' = \varphi$ (since φ - is angle between initial tangent line, which was x axis, and current tangent line)

So, $\angle MDO' = \angle MDO + \angle ODO' = \frac{OM}{R} + \varphi$

Thus, M can be represented as:

$$M = D + \begin{bmatrix} R \cdot \cos\left(\frac{OM}{R} + \varphi\right) \\ R \cdot \sin\left(\frac{OM}{R} + \varphi\right) \end{bmatrix} = \begin{bmatrix} R\cos\left(\varphi + \frac{pi}{2}\right) + R\cos\left(\varphi + \frac{OM}{R}\right) \\ R\sin\left(\varphi + \frac{pi}{2}\right) + R\sin\left(\varphi + \frac{OM}{R}\right) \end{bmatrix}$$

We found positions for all interesting points, now we can simulate

Velocities and accelerations

Relative motion:

$$v_r = \dot{OM} = \frac{OM(t_i) - OM(t_{i-1})}{t_i - t_{i-1}} \quad (\text{for small } (t_i - t_{i-1}))$$

$$a_r = \ddot{OM} = \frac{v_r(t_i) - v_r(t_{i-1})}{t_i - t_{i-1}} \quad (\text{for small } (t_i - t_{i-1}))$$

$$|a_r^n| = \frac{v_r^2}{R}$$

Since point M moving along $\omega(D,R)$, a_r^n will be always directed from point M to point D.

unit vector for a_r^n: $\bar{u}_r^n = \dfrac{D - M}{|D-M|}$

In code $\bar{u}_r^n(t) = \dfrac{D(t)-M(t)}{|D(t)-M(t)|}$

$\bar{a}_r^\tau \perp \bar{a}_r^n$, so we can find unit vector for $\bar{a}_r^\tau$:

$$\bar{u}_r^\tau = \underbrace{\begin{bmatrix} (u_r^n)_y \\ -(u_r^n)_x \end{bmatrix}}_{\text{such vector since motion is counterclockwise}} \cdot \underbrace{\operatorname{sign}(\bar{a}_r^\tau(t))}_{\text{acceleration may be negative}}$$

unit vector for v_r:

$$\bar{u}_r^v = \underbrace{\begin{bmatrix} (u_r^n)_y \\ -(u_r^n)_x \end{bmatrix}}_{\text{initial motion is clockwise}} \cdot \underbrace{\operatorname{sign}(v_r(t))}_{\text{Velocity may become negative at some point}}$$

Transport motion:

$$\omega(t) = \dot{\varphi}(t) = \frac{\varphi(t_i) - \varphi(t_{i-1})}{t_i - t_{i-1}} \; (\text{for small } |t_i - t_{i-1}|)$$

$$\varepsilon(t) = \dot{\omega}(t) = \frac{\omega(t_i) - \omega(t_{i-1})}{t_i - t_{i-1}} \; (\text{for small } (t_i - t_{i-1}))$$

Since rotation is around point O_1:

$$v_t(t_i) = O_1M(t_i) \cdot \omega(t_i) = \|M(t_i) - O_1\| \cdot \frac{\varphi(t_i) - \varphi(t_{i-1})}{t_i - t_{i-1}}$$

$$a_t^\tau(t_i) = O_1M(t_i) \cdot \varepsilon(t_i) = \|M(t_i) - O_1\| \cdot \frac{\omega(t_i) - \omega(t_{i-1})}{t_i - t_{i-1}}$$

$$a_t^n(t_i) = \omega^2(t_i) \cdot O_1M(t_i) = O_1M(t_i) \cdot \left(\frac{\varphi(t_i) - \varphi(t_{i-1})}{t_i - t_{i-1}}\right)^2$$

Direction of unit vector of a_t^n:

$$\bar{U}_t^n(t) = \frac{O_1 - M(t)}{|O_1 - M(t)|} = \bar{u}_t^n(t)$$

$\bar{u}_t^\tau \perp \bar{u}_n^n$, so:

$$\bar{U}_t^\tau = \begin{bmatrix} (u_t^n)_y \\ -(u_t^n)_x \end{bmatrix} \cdot \operatorname{sign}(a_t^\tau)$$

since motion is counterclockwise

$u_t^\tau \parallel \bar{v}_t$, so:

$$\bar{U}_t^v = \begin{bmatrix} (u_t^n)_y \\ -(u_t^n)_x \end{bmatrix} \cdot \operatorname{sign}(v_t)$$

Coriolis acceleration:

$$\bar{a}_c = 2\bar{\omega}(t) \times \bar{v}_r(t) = 2 \cdot \omega(t) \cdot v_r(t) \cdot \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \bar{u}_r^v(t)\right) = 2 \cdot \omega(t) \cdot v_r(t) \cdot (-\bar{u}_r^n(t))$$

All vectors of velocities and accelerations are represented as:

$$v_{ec} = \bar{u} \cdot m$$

Vector we need to find

unit vector

magnitude

Absolute velocity and acceleration:

$$\bar{v} = |v_r| \cdot \bar{U}_r^v + |v_t| \cdot \bar{u}_t^v$$

$$\bar{a} = |a_r^\tau| \cdot \bar{u}_r^\tau + |a_r^n| \cdot \bar{u}_r^n + |a_t^\tau| \cdot \bar{u}_t^\tau + |a_t^n| \cdot \bar{u}_t^n + \bar{a}_c$$

All values for velocities and accelerations were found numerically using python!!!

When M reaches O point second time:

if M reaches O second time, $OM = 2\pi R \Rightarrow$

$$75\cancel{\pi}(0.1t + 0.3t^2) = 60\cancel{\pi}$$

$$0.1t + 0.3t^2 = 0.8$$

$$3t^2 + t - 8 = 0$$

$$t = \frac{-1 \pm \sqrt{97}}{6}$$