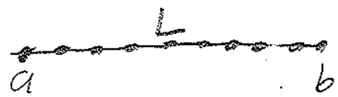
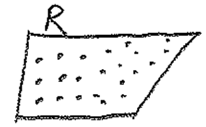
The Definite Integral Concept





	1D	2D	3D
Regions have infinitely-many infinitesimal points, each with ___	thickness dx	area dA	volume dV
The amount of "stuff" per unit ___ is given by ___	thickness, $f(x)$	area, $f(x,y)$	volume $f(x,y,z)$
The amount of stuff at an arbitrary point is ___	$f(x)\,dx$	$f(x,y)\,dA$	$f(x,y,z)\,dV$
The total amount of stuff in the region is ___	$\int_L f(x)\,dx$	$\iint_R f(x,y)\,dA$	$\iiint_W f(x,y,z)\,dV$
We calculate the integral as ___	$\int_a^b f(x)\,dx$	See 16.2 + 16.4	See 16.3, 16.5a, 16.5b

Example 1: Area

$$A = \int_L \frac{\text{area}}{\text{Thickness}}\, dx = \int_L \underbrace{\text{length}\, dx}_{\text{length}^2}$$

$$A = \iint_R \frac{\text{area}}{\text{area}}\, dA = \iint_R \underbrace{1\ dA}_{\text{length}^2}$$

N/A

Example 2: Volume

$$V = \int_L \frac{\text{volume}}{\text{Thickness}}\, dx = \int_L \underbrace{\text{area}\ dx}_{\text{length}^3}$$

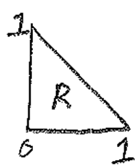
$$V = \iint_R \frac{\text{volume}}{\text{area}}\, dA = \iint_R \underbrace{\text{length}\ dA}_{\text{length}^3}$$

$$V = \iiint_W \frac{\text{volume}}{\text{volume}}\, dV = \iiint_W \underbrace{1\ dV}_{\text{length}^3}$$

Key Fact: For any integral (1D, 2D, 3D, curve in 2D or 3D, surface in 3D),

$$\int_\square 1\ d\square = \text{length}, \quad \iint_\square 1\ d\square = \text{area}, \quad \iiint_\square 1\ d\square = \text{volume}$$

The integral of 1 is the size of the region.

example: 

$\Rightarrow \iint_R 1\ dA = A(R) = \frac{1}{2}$

Example 3: Mass

$$m = \int_L \underbrace{\frac{\text{mass}}{\text{Thickness}}}_{\underline{\text{not}}\ \text{density}} dx \qquad m = \iint_R \underbrace{\frac{\text{mass}}{\text{area}}}_{\underline{\text{not}}\ \text{density}} dA \qquad m = \iiint_W \frac{\text{mass}}{\text{volume}} dV = \iiint_W \text{density}\ dV$$

Note: In most physics, chemistry, engineering books, density is ρ.
Calculus books use ρ for the distance from $(0,0,0)$ to (x,y,z).
Most calculus books use δ for material density.

Example 4: Average Density

$$\text{Average density} = \frac{\text{Total Mass}}{\text{Total Volume}} = \frac{\iiint_W \text{density}\ dV}{V(W)} = \frac{\iiint_W \text{density}\ dV}{\iiint_W 1\ dV}$$

In general, the average of a function f in any dimension is the integral of f divided by the integral of 1:

$$\bar{f} = \frac{\int_\square f\ d\square}{\int_\square 1\ d\square}$$