Art Forgery

Until the mid-1900s, the only way to judge the authenticity of an alleged old masters painting was to have the work judged by art experts. The experts were usually right, but not always. A celebrated case occurred in the aftermath of World War II. A little-known Dutch painter named H.A. Van Meegeren was arrested under a charge of collaborating with the Nazis by selling them paintings made by the 17th Century Dutch master Jan Vermeer. Van Meegeren's defense was that he did no such thing; rather, he swindled the Nazis by selling them forgeries he had painted himself. The case was passed on to a team of art historians, who concluded that all of the paintings were forgeries with the possible exception of one called "Disciples at Emmaus," which some of the experts thought to be authentic. The truth was determined in 1968 by Bernard Keisch, a chemist at Carnegie Mellon University.

Naturally occurring lead contains small amounts of radium as an impurity. The amounts can vary considerably, depending on the source of the lead. In the natural state, the radioactive isotope lead-210 maintains an equilibrium concentration because the rate at which it decays to produce polonium-210 is just matched by the rate at which it is replenished by decay of radium-226. The radium decays so slowly that its concentration is virtually unaffected. When the lead ore is refined into white lead, the radium is largely removed, so the concentration of lead-210 is no longer at equilibrium. After about 200 to 300 years, equilibrium is restored at a smaller concentration of lead.

Write down a mathematical model for the radium and lead concentrations, using the assumptions that radium $r(t)$ decays at relative rate a (producing lead at the same rate) and lead $x(t)$ decays at relative rate b. The rate constants can be determined from the half-lives of 1600 years and 22 years, respectively. This can be done later. Your model should be an initial value problem, so you will have to include parameters r_0 and x_0 for the initial amounts of radium and lead. We do not know these values, and that will make it difficult to use the model to determine the age of the paint.

Solve the model problem to obtain formulas for $r(t)$ and $x(t)$. The challenging question is this: How do we estimate the age of the paint in "Disciples at Emmaus" if we know the current concentrations r and x but not the initial concentrations? Hint: We know the ratio of radium to lead in the unrefined ore and we can estimate the fraction of radium that was removed in the refinement of the ore (roughly 97.5 to 99 percent).