Opinion Dynamics for Vaccination Models

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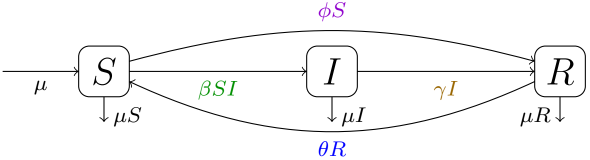
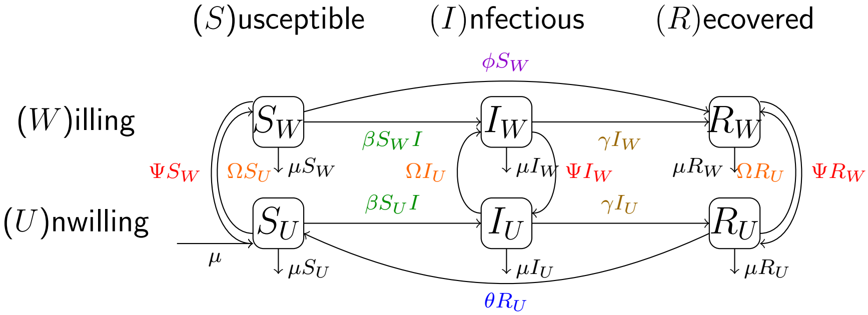
Standard SIR with Vaccination and Loss of Immunity



- ϕ and θ can be modified to reflect vaccine hesitancy,
  - but even if ϕ is small, everyone gets vaccinated eventually.
    - This is an unrealistic modeling assumption!
- Our experience with COVID-19 shows:
  - Many people never get vaccinated.
  - Most people stop getting boosters.
  - Individual opinion on these choices changes over time.
- Models need to account for willing and unwilling subgroups.
  - Preferably with flexible composition.

SIR-WU Model



- Newborns are initially unable to be vaccinated.
- Willing Susceptibles are vaccinated at rate ϕS_W.
- Unwilling Recovereds lose immunity at rate θR_U.
- Unwilling become Willing with rate coefficient $\Omega(I)$.
- Willing become Unwilling with rate coefficient $\Psi(I)$.

Opinion Dynamics

The Unwilling to Willing and Willing to Unwilling rate coefficients depend on the distribution of opinion x toward vaccination.

- How do we model the distribution of opinion?

1. Individual opinion $x(t)$ is determined from an ODE $\dot{x} = g(x, t)$ that combines attractors from various influences.
2. Community opinion is indicated by an opinion density function $u(x, t) \geq 0$ s.t. $\int_{-\infty}^{\infty} u(x, t)\, dx = 1$:

$$P\{x_0 \leq x \leq x_1, t\} = \int_{x_0}^{x_1} u(x, t)\, dx.$$

3. $u(x, t)$ is determined from $g(x, t)$ using the PDE for convective-diffusive transport.

Individual Opinion Change

We assume

$$\dot{x} = g(x,t) = -ax - \sum_{j=1}^{J} b_j \cdot k_j(x - x_j) \cdot (x - x_j),$$

where

- ax (with small a) is a weak global attractor that pulls individuals toward the neutral opinion $x = 0$;
- $b_j \cdot k_j(x - x_j) \cdot (x - x_j)$ is a strong local attractor that pulls individuals toward opinion x_j with maximum strength b_j;
- $k_j(h)$ indicates the relative strength of influence on individuals whose opinion differs from that of the "influencer" by h.
  - The influence is local because $\lim_{|h|\to\infty} k_j = 0$.

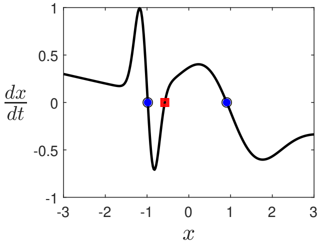
A Relative Strength Function $k(h)$

- We assume k has these properties:
  1. It is even about $h = 0$ so that influence strength does not depend on direction.
  2. $k \geq 0$ so that influencers always attract.
  3. $k' \leq 0$ for $h > 0$ so that the relative strength does not increase with distance.
  4. $\int_{-\infty}^{\infty} |hk(h)|\, dh = 1$ to separate influence "shape" k from influence strength b.
     - Influence is **local**; ie, it falls to 0 as the subject-influencer opinion difference increases.
- Example: Gaussian form ("width" σ)

$$k(h) = \frac{1}{\sigma^2} e^{-(h/\sigma)^2}.$$

Example

- Experts: aggregate influence is strong and broad
  - Local attractor of opinion 1, strength 1, width 1
- Deniers: aggregate influence is moderate and narrow
  - Local attractor of opinion -1, strength 0.5, width 0.25
- Slight tendency toward moderation:
  - Global attractor of strength 0.1



- Two stable and one unstable equilibria.

Aggregate Opinion Change

Let $u(x,t)$ be the population density function for opinion.

- Aggregate opinion changes by convection (gradient of $-gu$) and diffusion (gradient of u_x), where $g(x,t) = \dot{x}(t)$:

$$u_t = Du_{xx} - (gu)_x = F[u]_x, \qquad F[u] \equiv Du_x - gu,$$

- We assume boundary conditions

$$F(\pm X) = 0 \quad \text{for some large } X$$

  - The single global attractor $g_0 = -ax$ means that u and u_x vanish as $|x| \to \infty$.
  - $\int_{-X}^{X} u(x,t)\,dx = 1 \quad \to \quad F(X) = F(-X).$

Equilibrium Opinion Density

At equilibrium, $\bar{F}[u^*] = 0$ where

$$\bar{F}[u] = Du_x - \bar{g}u, \qquad \bar{g}(x) = \lim_{t\to\infty} g(x,t)$$

Also,

$$\int_{-X}^{X} u^*(x)\,dx = 1.$$

Let

$$G(x) = \int_0^x \bar{g}(y)\,dy, \quad \eta(x) = e^{D^{-1}G(x)}, \quad I = \int_{-X}^{X} \eta(x)\,dx.$$

The equilibrium opinion density is then

$$u^*(x) = I^{-1}\eta(x),$$

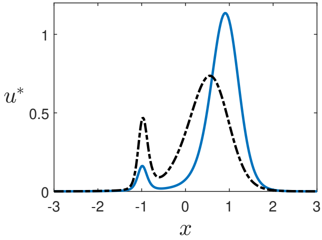
$$\bar{F}\left[I^{-1}\eta(x)\right] = D(I^{-1}\eta)' - \bar{g}\cdot I^{-1}\eta = DI^{-1}\cdot D^{-1}\bar{g}\eta - I^{-1}\bar{g}\eta = 0$$

Example Scenarios

- The Secretary of the US Department of Health and Human Services has said that people should *do their own research* before deciding about vaccination.
- From the NY Times (paraphrased for brevity)
  - The FDA announced **on 2025/05/20** that they would *require additional studies* before approving more COVID boosters for healthy people under age 65.
- Scenario 1 is as before:
  - Strength 1, width 1, opinion 1
  - Strength 0.5, width 0.25, opinion -1
- In scenario 2, expert opinion has split into
  - a pro-vaccination medical establishment (as before, but with strength 0.6), and
  - a neutral government position of strength 0.4 and opinion 0.

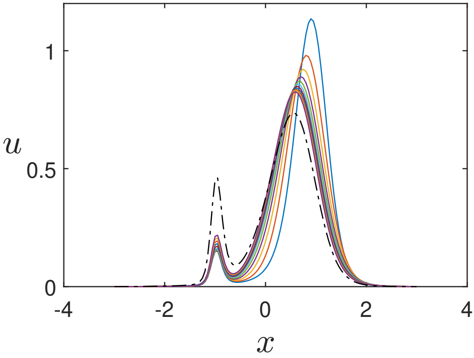
Example Scenarios

- Scenario 1 (solid) is as before:
  - Strength 1, width 1, opinion 1
  - Strength 0.5, width 0.25, opinion -1
- In scenario 2 (dash-dot), expert opinion has split into
  - a pro-vaccination medical establishment (as before, but with strength 0.6), and
  - a neutral government position of strength 0.4 and opinion 0.



PDE Example

- Initial condition is scenario 1.
- g is set to match scenario 2.



- We see a smooth transition, with a quick drop of the peak near $x = 1$ and slow growth of the peak at $x = -0.9$.

Equilibrium Distribution Stability Problem

Assume $u(x,t) = u^*(x) + w(x,t), \quad g(x,t) = \bar{g}(x)$.

- PDE: $w_t = [Dw_x - g(x)w]_x$
- BC: $[Dw_x - g(x)w](\pm X) = 0$
- IC: $w(x,0) = w_0(x), \qquad \int_{-X}^{X} w_0(x)\,dx = 0$

Assume solutions $w_n(x,t) = \psi_n(t)\phi_n(x)$.

- $\psi_n(t): \;\; \psi_n' = -\lambda_n \psi_n$
- $\phi_n(x): \;\; (D\phi_n' - g\phi_n)' + \lambda_n \phi_n = 0, \quad [D\phi_n' - g\phi_n](\pm X) = 0$

The ϕ problem is of regular Sturm-Liouville type.

- $w(x,t) = \sum_{n=0}^{\infty} c_n e^{-\lambda_n t}\phi_n(x)$
- $\lambda_0 < \lambda_1 < \cdots$ and ϕ_n has n zeros.

Equilibrium Distribution Stability

1. $(D\phi_n' - g\phi_n)' + \lambda_n \phi_n = 0, \quad [D\phi_n' - g\phi_n](\pm X) = 0$
    - u^* satisfies the ϕ problem with $\lambda = 0$:
        - u^* is an eigenfunction with eigenvalue 0.

2. ϕ_n has n zeros.
    - u^* has no zeros:
        - $\phi_0 = u^*$ and $\lambda_0 = 0$.
        - All other eigenvalues are positive.

3. $w(x,t) = \sum_{n=0}^{\infty} c_n e^{-\lambda_n t} \phi_n(x)$
    - $\lim_{t\to\infty} w(x,t) = c_0 u^*(x)$.

4. $\int_{-X}^{X} w(x,t)\, dx = 0 \quad \forall t$
    - $0 = \int_{-X}^{X} \left[\lim_{t\to\infty} w(x,t)\right] dx = c_0$.
        - $\lim_{t\to\infty} w(x,t) = 0$.

Additional Modeling Needed

- What is missing from the modeling described here is a model component that links opinion dynamics with vaccination.
- Transition rates between Willing and Unwilling should depend on the opinion distribution **and** the current epidemiological status.
- For a model in which epidemiological status alone determines the transition rates, see Yi Jiang's talk **at 2:20pm EDT**.

Acknowledgements

- This work was begun as part of a REUF workshop.

- The REUF workshop and our subsequent group meetings were funded by AIM and ICERM.

References

- For incorporation of vaccination into short-term and long-term epidemiology models, see
  - Ledder (2022), Incorporating mass vaccination into compartment models for infectious diseases, *Mathematical Biosciences and Engineering*
    - https://doi.org/10.3934/mbe.2022440
- For an SIR-WU model without opinion dynamics, see
  - Jiang, Kurianski, Lee, Ma, Cicala, Ledder (2025), Incorporating changeable attitudes toward vaccination into compartment models for infectious diseases, *Mathematical Biosciences and Engineering*
    - https://doi.org/10.3934/mbe.2025011