

Fish density

X

E

ω

Visual model

$$\lambda_i = X_i \cdot E_i$$

$$N_i \sim \text{Negative Binomial}(\lambda_i, 20)$$

(1)

Water model

$$W_i = X_i \cdot \omega$$

(2)

W_i

qPCR water model

$$\begin{aligned} U_i &= W_i / V_{itp} \\ Z_{it} &\sim \text{Bernoulli}(\psi_i) \\ \psi_i &= 1 - \exp(-U_i \cdot \phi) \\ Y_{itp} &\sim \text{Normal}(\mu_{ip}, \sigma_i) \quad \text{if } Z_{it} = 1 \\ \mu_{ip} &= \beta_0 + \beta_{1p} \cdot \ln(U_i / G) \\ \sigma_i &= \exp(\gamma_0 + \gamma_1 \cdot \ln(U_i)) \end{aligned}$$

(4)

Air model

$$\ln(A_{ijb}) = \eta_j + \ln(W_i) + \varepsilon_{ij} + \delta_{ijb}$$

$$\varepsilon_{ij} \sim \mathcal{N}(0, \tau_j)$$

$$\delta_{ijb} \sim \mathcal{N}(0, \rho_j)$$

(3)

A_{ijb}

qPCR air model

$$\begin{aligned} Q_{ijb} &= A_{ijb} \cdot S_{ij} / V_{ijbtp} \\ Z_{ijbt} &\sim \text{Bernoulli}(\psi_{ijb}) \\ \psi_{ijb} &= 1 - \exp(-Q_{ijb} \cdot \phi) \\ Y_{ijbtp} &\sim \text{Normal}(\mu_{ijb}, \sigma_{ijb}) \quad \text{if } Z_{ijbt} = 1 \\ \mu_{ijb} &= \beta_0 + \beta_{1p} \cdot \ln(Q_{ijb}) \\ \sigma_{ijb} &= \exp(\gamma_0 + \gamma_1 \cdot \ln(Q_{ijb})) \end{aligned}$$

(5)

ϕ_0, ϕ_1
 β_0, β_{1p}
 γ_0, γ_1

$$\begin{aligned} Z_{kr} &\sim \text{Bernoulli}(\psi_k) \\ (\psi_k) &= 1 - \exp(-K_k \cdot \phi) \\ Y_{kr} &\sim \text{Normal}(\mu_k, \sigma_k) \quad \text{if } Z_{kr} = 1 \\ \mu_k &= \beta_0 + \beta_{1p} \cdot \ln(K_k) \\ \sigma_k &= \exp(\gamma_0 + \gamma_1 \cdot \ln(K_k)) \end{aligned}$$

qPCR Standard model (6)

Data

N	number of counted fish
E	days between counting (effort)
Z	qPCR amplification (yes=1; no=0)
Y	qPCR Ct values
K	Known concentration in copies/ μL
V	Reaction volume in μL
S	Surface area of air collection method cm^2

Subscripts

i	time
j	filter type
b	biological replicate
t	technical replicate
p	qPCR plate
k	qPCR standard sample

Parameters

λ	expected fish accumulated over E days
ψ	probability of positive qPCR amplification
ϕ	detection probability intercept function and DNA concentration (K and U or Q)
μ	mean Ct values of qPCR
β_0, β_1	intercept and slope between μ and DNA concentration (K and U or Q)
σ	standard deviation of qPCR Ct values
γ_0, γ_1	intercept and slope between σ and DNA concentration (K and U or Q)
ω	conversion parameter between fish density and DNA concentration
η	dilution factor of DNA concentration from water to air (in \log_e)
ε	error term (residual)
τ	standard deviation of ε
δ	biological replicate error (bio-rep residual)
ρ	standard deviation of δ

State variables

W	unknown eDNA concentration in water samples (copies/L)
U	unknown eDNA concentration in water samples (copies/ μL)
A	unknown eDNA concentration in air samples (copies/day/ cm^2)
Q	unknown eDNA concentration in air samples (copies/ μL)
X	unknown fish density (fish/day)