

Fish density

X

E

ω

Visual model

$$X_i = \lambda_i \cdot E_i^{-1}$$

$$N_i \sim \text{Poisson}(\lambda_i)$$

(1)

Water model

$$X_i = W_i \cdot \omega$$

(2)

W_i

qPCR water model

$$Z_{it} \sim \text{Bernoulli}(\psi_i)$$

$$\psi_i = 1 - \exp(-W_i \cdot \phi)$$

$$Y_{itp} \sim \text{Normal}(\mu_{ip}, \sigma_i) \quad \text{if } Z_{it} = 1$$

$$\mu_{ip} = \beta_0 + \beta_{1p} \cdot \ln(W_i)$$

$$\sigma_i = \exp(\gamma_0 + \gamma_1 \cdot \ln(W_i))$$

(4)

Air model

$$\ln(W_i) = \eta_j + \ln(A_{ijb}) + \varepsilon_{ij} + \delta_{ijb}$$

$$\varepsilon_{ij} \sim \mathcal{N}(0, \tau_j)$$

$$\delta_{ijb} \sim \mathcal{N}(0, \rho_j)$$

(3)

A_{ijb}

qPCR air model

$$Z_{ijbt} \sim \text{Bernoulli}(\psi_{ijb})$$

$$\psi_{ijb} = 1 - \exp(-A_{ijb} \cdot \phi)$$

$$Y_{ijbtp} \sim \text{Normal}(\mu_{ijb}, \sigma_{ijb}) \quad \text{if } Z_{ijbt} = 1$$

$$\mu_{ijb} = \beta_0 + \beta_{1p} \cdot \ln(A_{ijb})$$

$$\sigma_{ijb} = \exp(\gamma_0 + \gamma_1 \cdot \ln(A_{ijb}))$$

(5)

ϕ_0, ϕ_1
 β_0, β_{1p}
 γ_0, γ_1

$$Z_{kr} \sim \text{Bernoulli}(\psi_k)$$

$$(\psi_k) = 1 - \exp(-K_k \cdot \phi)$$

$$Y_{kr} \sim \text{Normal}(\mu_k, \sigma_k) \quad \text{if } Z_{kr} = 1$$

$$\mu_k = \beta_0 + \beta_{1p} \cdot \ln(K_k)$$

$$\sigma_k = \exp(\gamma_0 + \gamma_1 \cdot \ln(K_k))$$

qPCR Standard model (6)

Data

N	number of counted fish
E	days between counting (effort)
Z	qPCR amplification (yes=1; no=0)
Y	qPCR Ct values
K	Known concentration in copies/ μL

Subscripts

i	time
j	filter type
b	biological replicate
t	technical replicate
p	qPCR plate
k	qPCR standard sample

Parameters

λ	expected fish accumulated over E days
ψ	probability of positive qPCR amplification
ϕ	detection probability intercept function and DNA concentration (K and W or A)
μ	mean Ct values of qPCR
β_0, β_{1p}	intercept and slope between μ and DNA concentration (K and W or A)
σ	standard deviation of qPCR Ct values
γ_0, γ_1	intercept and slope between σ and DNA concentration (K and W or A)
ω	conversion parameter between fish density and DNA concentration
η	dilution factor of DNA concentration from water to air
ε	error term (residual)
δ	biological replicate error (bio-rep residual)

State variables

W	unknown eDNA concentration in water samples (copies/ μL)
A	unknown eDNA concentration in air samples (copies/ μL)
X	unknown fish density (fish/day)