

## Fish density

$X_t$

$E$

$\omega$

## Visual model

$$\lambda_t = X_t \cdot E_t$$

$$N_t \sim \text{Negative Binomial}(\lambda_t, \phi)$$

$$\phi = 20$$

(1)

## Water model

$$W_t = X_t \cdot \omega$$

(2)

$W_t$

## qPCR water model

$$U_t = (W_t \cdot F) / V$$

$$Z_{tr} \sim \text{Bernoulli}(\psi_t)$$

$$\psi_t = 1 - \exp(-U_t \cdot \theta)$$

$$Y_{trp} \sim \text{Normal}(\mu_{tp}, \sigma_t) \quad \text{if } Z_{tr} = 1$$

$$\mu_{tp} = \beta_0 + \beta_{1p} \cdot \ln(U_t)$$

$$\sigma_t = \exp(\gamma_0 + \gamma_1 \cdot \ln(U_t))$$

(4)

## Air model

$$\ln(A_{tjb}) = \eta_j + \ln(W_t) + \varepsilon_{tj} + \delta_{tjb}$$

$$\varepsilon_{tj} \sim \mathcal{N}(0, \tau_j)$$

$$\delta_{tjb} \sim \mathcal{N}(0, \rho_j)$$

(3)

$A_{tjb}$

## qPCR air model

$$Q_{tjb} = (A_{tjb} \cdot S_{tj}) / (V \cdot P)$$

$$Z_{tjbr} \sim \text{Bernoulli}(\psi_{tjb})$$

$$\psi_{tjb} = 1 - \exp(-Q_{tjb} \cdot \theta)$$

$$Y_{tjbrp} \sim \text{Normal}(\mu_{tjb}, \sigma_{tjb}) \quad \text{if } Z_{tjbr} = 1$$

$$\mu_{tjb} = \beta_0 + \beta_{1p} \cdot \ln(Q_{tjb})$$

$$\sigma_{tjb} = \exp(\gamma_0 + \gamma_1 \cdot \ln(Q_{tjb}))$$

(5)

$\phi_0, \phi_1$   
 $\beta_0, \beta_{1p}$   
 $\gamma_0, \gamma_1$

$$Z_{kr} \sim \text{Bernoulli}(\psi_k)$$

$$(\psi_k) = 1 - \exp(-K_k \cdot \theta)$$

$$Y_{kr} \sim \text{Normal}(\mu_k, \sigma_k) \quad \text{if } Z_{kr} = 1$$

$$\mu_k = \beta_0 + \beta_{1p} \cdot \ln(K_k)$$

$$\sigma_k = \exp(\gamma_0 + \gamma_1 \cdot \ln(K_k))$$

## qPCR Standard model (6)

## Data

$N$  number of counted fish  
 $E$  days between counting (effort)  
 $Z$  qPCR amplification (yes=1; no=0)  
 $Y$  qPCR Ct values  
 $K$  Known concentration in copies/ $\mu$ L  
 $V$  Reaction volume in  $\mu$ L  
 $S$  Surface area of air collection method  $\text{cm}^2$   
 $F$  Water volume filtered in the field in L  
 $P$  Passive air filter time deployment in days

## Subscripts

$t$  time  
 $j$  filter type  
 $b$  biological replicate  
 $r$  technical replicate  
 $p$  qPCR plate  
 $k$  qPCR standard sample

## Parameters

$\lambda$  expected fish accumulated over  $E$  days  
 $\psi$  probability of positive qPCR amplification  
 $\theta$  detection probability intercept function and DNA concentration ( $K$  and  $U$  or  $Q$ )  
 $\mu$  mean Ct values of qPCR  
 $\beta_0, \beta_1$  intercept and slope between  $\mu$  and DNA concentration ( $K$  and  $U$  or  $Q$ )  
 $\sigma$  standard deviation of qPCR Ct values  
 $\gamma_0, \gamma_1$  intercept and slope between  $\sigma$  and DNA concentration ( $K$  and  $U$  or  $Q$ )  
 $\omega$  conversion parameter between fish density and DNA concentration  
 $\eta$  dilution factor of DNA concentration from water to air (in  $\log_e$ )  
 $\varepsilon$  error term (residual)  
 $\tau$  standard deviation of  $\varepsilon$   
 $\delta$  biological replicate error (bio-rep residual)  
 $\rho$  standard deviation of  $\delta$

## State variables

$W$  unknown eDNA concentration in water samples (copies/L)  
 $U$  unknown eDNA concentration in water samples (copies/ $\mu$ L)  
 $A$  unknown eDNA concentration in air samples (copies/ $\text{cm}^2/\text{day}$ )  
 $Q$  unknown eDNA concentration in air samples (copies/ $\mu$ L)  
 $X$  unknown fish density (fish/day)