

## Fish density

 $X_t$ 
 $E$ 
 $\omega$ 

## Visual model

$$\lambda_t = X_t \cdot E_t$$

$$N_t \sim \text{Negative Binomial}(\lambda_t, \phi)$$

$$\phi = 20$$

(1)

## Water model

$$W_t = X_t \cdot \omega$$

(2)

 $W_t$ 

## qPCR water model

$$U_t = W_t / V_{trp}$$

$$Z_{tr} \sim \text{Bernoulli}(\psi_t)$$

$$\psi_t = 1 - \exp(-U_t \cdot \theta)$$

$$Y_{trp} \sim \text{Normal}(\mu_{tp}, \sigma_t) \quad \text{if } Z_{tr} = 1$$

$$\mu_{tp} = \beta_0 + \beta_1 \cdot \ln(U_t)$$

$$\sigma_t = \exp(\gamma_0 + \gamma_1 \cdot \ln(U_t))$$

(4)

## Air model

$$\ln(A_{tjb}) = \eta_j + \ln(W_t) + \varepsilon_{tj} + \delta_{tjb}$$

$$\varepsilon_{tj} \sim \mathcal{N}(0, \tau_j)$$

$$\delta_{tjb} \sim \mathcal{N}(0, \rho_j)$$

(3)

 $A_{tjb}$ 

## qPCR air model

$$Q_{tjb} = A_{tjb} \cdot S_{tj} / V_{tjbrp}$$

$$Z_{tjbr} \sim \text{Bernoulli}(\psi_{tjb})$$

$$\psi_{tjb} = 1 - \exp(-Q_{tjb} \cdot \theta)$$

$$Y_{tjbrp} \sim \text{Normal}(\mu_{tjb}, \sigma_{tjb}) \quad \text{if } Z_{tjbr} = 1$$

$$\mu_{tjb} = \beta_0 + \beta_1 \cdot \ln(Q_{tjb})$$

$$\sigma_{tjb} = \exp(\gamma_0 + \gamma_1 \cdot \ln(Q_{tjb}))$$

(5)

 $\phi_0, \phi_1$   
 $\beta_0, \beta_1$   
 $\gamma_0, \gamma_1$ 

$$Z_{kr} \sim \text{Bernoulli}(\psi_k)$$

$$(\psi_k) = 1 - \exp(-K_k \cdot \theta)$$

$$Y_{kr} \sim \text{Normal}(\mu_k, \sigma_k) \quad \text{if } Z_{kr} = 1$$

$$\mu_k = \beta_0 + \beta_1 \cdot \ln(K_k)$$

$$\sigma_k = \exp(\gamma_0 + \gamma_1 \cdot \ln(K_k))$$

## qPCR Standard model (6)

### Data

$N$	number of counted fish
$E$	days between counting (effort)
$Z$	qPCR amplification (yes=1; no=0)
$Y$	qPCR Ct values
$K$	Known concentration in copies/ $\mu\text{L}$
$V$	Reaction volume in $\mu\text{L}$
$S$	Surface area of air collection method $\text{cm}^2$

### Subscripts

$t$	time
$j$	filter type
$b$	biological replicate
$r$	technical replicate
$p$	qPCR plate
$k$	qPCR standard sample

### Parameters

$\lambda$	expected fish accumulated over $E$ days
$\psi$	probability of positive qPCR amplification
$\theta$	detection probability intercept function and DNA concentration (K and U or Q)
$\mu$	mean Ct values of qPCR
$\beta_0, \beta_1$	intercept and slope between $\mu$ and DNA concentration (K and U or Q)
$\sigma$	standard deviation of qPCR Ct values
$\gamma_0, \gamma_1$	intercept and slope between $\sigma$ and DNA concentration (K and U or Q)
$\omega$	conversion parameter between fish density and DNA concentration
$\eta$	dilution factor of DNA concentration from water to air (in $\log_e$ )
$\varepsilon$	error term (residual)
$\tau$	standard deviation of $\varepsilon$
$\delta$	biological replicate error (bio-rep residual)
$\rho$	standard deviation of $\delta$

### State variables

$W$	unknown eDNA concentration in water samples (copies/L)
$U$	unknown eDNA concentration in water samples (copies/ $\mu\text{L}$ )
$A$	unknown eDNA concentration in air samples (copies/day/ $\text{cm}^2$ )
$Q$	unknown eDNA concentration in air samples (copies/ $\mu\text{L}$ )
$X$	unknown fish density (fish/day)