Learning Ising parameters for nonbinary classification

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Ising Likelihood maximisation

Let $(X, y) \in \mathcal{X}^N \times \mathcal{Y}^N$, where $\mathcal{X}$ is some measure space and $\mathcal{Y} \subset_{\text{finite}} \mathbb{R}$. Let $\sigma : \mathcal{X} \to \mathcal{Y}$.

For fixed $J : \mathcal{X}^2 \to \mathbb{R}$ let

$$H_J(X, y) = \sum_{i,j} J(X_i, X_j) y_i y_j.$$

Given such an H there is an associated probability distribution

$$\widetilde{\mathbb{P}}_J(X, y) = e^{H_J(X,y)} \text{ and } \mathbb{P}_J(X, y) = \frac{\widetilde{\mathbb{P}}(X, y)}{\sum_{\mathcal{Y}} \int_{\mathcal{X}} \widetilde{\mathbb{P}}(X, y)},$$

though the latter expression only makes sense where $\mathcal{X}$ is finite or with some infinitesimal interpretation.

This may be related to the usual Ising potential on a lattice Λ by letting $\mathcal{X} = \Lambda$, letting $J(X, X') = 1_{\{X \text{ is a neighbour of } X'\}}$ and $\mathcal{Y} = \{\pm\}$. Choosing some enumeration of Λ, a configuration

$$\sigma : \Lambda \cong \{1, \cdots, |\Lambda|\} \to \{\pm\}$$

has potential

$$H(\sigma) = H_J\left((1, \cdots, |\Lambda|), \sigma\right).$$

References