Energy aware scheduling

March 23, 2016

1 Problem formulation

We consider a problem of scheduling applications in a center with limited power and resources.

This data center runs two different applications : active applications (also called web applications) and batch applications. Those aplications must be executed on servers, each server having a limited resources availability.

1.1 Time intervals

The scheduling is peformed over 24 1-hour long intervals.

Each time interval has its limited power capacity for the center. For this reason we need to consider, on each interval, the actual energy consumed by the applications executed at this interval.

1.2 Active applications

We consider n active applications $\{A_{i \in 1..n}\}$, which run continuously over the time intervals.

Each application A_i has 3 different executions modes. Each application's mode produces a given power consumption and a given profit when applied over an interval, and the application must be assigned an execution mode for each time interval, with

$M_{i,j} \in 1..3$ mode of activity i at time j.

$Ea_{i,j} \in 0..1000$ power consumption of activity i at time j.

$Pa_{i,j} \in 0..1000$ profit for running activity i at time j.

Those informations can be stored in tables, eg :

	Mode 1	Mode 2	Mode 3
A_1	50	60	70
A_2	40	45	55

Table 1: Energy consumption for two active applications

	Mode 1	Mode 2	Mode 3
A_1	100	110	120
A_2	90	100	110

Table 2: Profit for two active applications

The power and profit of an active application A_i can be comptued with following formula :

- element ($M_{i,j}$, Row_i^{Energy}, $E_{i,j}$) % E_i=$Row_i^{Energy}[M_i]$
- element ($M_{i,j}$, Row_i^{Profit}, $P_{i,j}$) % P_i=$Row_i^{Profit}[M_i]$

1.3 Batch applications

We consider m batch jobs $\{B_{i\in 1..m}\}$, each with its own specifications :

$Duration_i$ ≤ 24, the number of intervals this job must be run before being finished,

$Deadline_i$ ≤ 24, the interval at which the job must be finished,

$Pbmax_i$ $\in 0..1000$ is the profit earnt only if the whole job is finished before or at deadline,

Eb_i is the power consumed by the job at each interval it is executed.

	Duration	Deadline	Profit	Power
B_1	3	12	100	60
B_2	4	20	150	80

Table 3: examples of Batch applications

Each batch job B_i can be decomposed in as many subjobs $b'_{i,k}$ as its duration, with

- $k \in 1..Duration_i$,
- $Duration_{b'_{i,k}} = 1$,
- $E_{b'_{i,k}} = Eb_i$
- $S_{i,k}$ the interval at which $b'_{i,k}$ is executed.

Since each subjob must be executed over a different time interval,

$$0 < S_{i,1} < S_{i,2} ... < S_{i,Duration_i}$$

We note Bd_i the boolean value indicating if B_i meets its deadline.

$$Bd_i \iff S_{i,Duration_i} \leq Deadline_i$$

The energy consumption of batch job B_i at interval j, noted $Eb_{i,j}$, can be deduced from the execution of a subjob :

$$\exists! k, S_{i,k} = j \implies Eb_{i,j} = Eb_i$$

$$\nexists k, S_{i,k} = j \implies Eb_{i,j} = 0$$

Finally, the actual profit of the job i, noted Pb_i, is deduced from the interval of its last subjob :

$$Pb_i = element(S_{i,Duration_i}, [Q_i, .., Q_i, 0, ..0])$$

1.4 Application placement and migration

In the context of a data center containing more than one server, the applications must also be placed at each interval on a server. An application can be migrated from one server to another. We need to consider the extra energy cost induced by migration in our model.

We introduce the placement variables and a migration cost for both active and batch jobs. This cost is added to an interval's energy use when this interval requires the migration of the job.

1.4.1 Active job migration

Active jobs are always running on an host. We name

$HA_{i,j}$ the host of the application A_i during interval j,

$PHA_{i,j}$ the cost of moving the application A_i on the interval j.

- If $HA_{i,j-1}$ is defined, i.e. application A_i was running during interval $j-1$, and $HA_{i,j-1} = HA_{i,j}$ then $P(HA_{i,j}) = 0$.
- If $HA_{i,j-1}$ is undefined, then $P(HA_{i,j}) = 0$.
- If $HA_{i,j-1}$ is defined and $HA_{i,j-1} \neq HA_{i,j}$ then $P(HA_{i,j})$ is a function of $HA_{i,j-1}$ and $HA_{i,j}$.

1.4.2 Batch job migration

Batch jobs can be stopped fo as long as required. However, even when stopped they are still present on the server, as their memory state is saved. We name

$HB_{i,j}$ the host of the application B_i during interval j,

$PHB_{i,j}$ the cost of moving the application B_i on the interval j.

- If $HB_{i,j-1}$ is defined i.e. application B_i was not yet completed during interval $j-1$, and $HB_{i,j-1} = HB_{i,j}$ then $P(HB_{i,j}) = 0$.
- If $HB_{i,j-1}$ is undefined, then $P(HB_{i,j}) = 0$.
- If $HB_{i,j-1}$ is defined and $HB_{i,j-1} \neq HB_{i,j}$ then $P(HB_{i,j})$ is a function of $HB_{i,j-1}$ and $HB_{i,j}$.

1.5 Energy cap

We introduce the maximum available energy in the data center at interval j as $Capacity_j$, where $j \in$ [1,24].

In cumulative constraint modeling, the limit of maximum available energy can not be changed over time. We thus introduce fake jobs in each interval j to match with the $Capacity_j$. The maximum capacity is defined as $max(Capacity_h)$, $h \in$ [1,24]. The total energy cost at interval j is

$$E_j = \sum_{i \in 1..n} Ea_{i,j} + \sum_{i \in 1..m} Eb_{i,j} + Migcost_j < Capacity_j$$

1.6 Memory use

The data center contains l servers $\{S_k\}, k \in 1..l$. Each server S_k has a memory capacity M_k which limits the number of applications this server can execute.

Executing an application A_i or a job B_i on a server S_k consumes a fixed amount of memory on the server over the execution interval. Reciprocally, each application must be run on a server at any time and any job executed during an interval must be run on a server.

We note

$HA_{i,j}$ the host of the application A_i during interval j

$HB_{i,j}$ the host of the job B_i during interval j

$Load_{k,j}$ the memory load of the server k during interval time j

$mem(C)$ the memory use of the job or application C.

then we know that

$$Load_{k,j} = \sum_{A_i} (mem(A_i) if HA_{i,j} = k) + \sum_{B_i} (mem(B_i) if HB_{i,j=k} \qquad (1)$$

2 Solution formulation

A solution to such a problem consists in :

- $\forall$ Active application A_i and time interval j, the execution mode $M_{i,j}$ at which to run the appliction during the interval
- $\forall$ Batch job B_i, a series time interval $S_{i,1}..S_{i,Duration_i}$ at which to execute the subjobs.
- $\forall$ time interval j, $\forall$ server S_k, the set of Applications and jobs hosted on the server during the time interval.

with respect to the previoulsy specified conditions.

3 Objective

Our objective function is to maximize P,
$P = \sum_{j=1}^{24} \left(\sum_{i=1}^{n} Pa_{i,j} \right) + \sum_{i=1}^{m} Pb_i$