Muon Lifetime

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Abstract

This experiment measures the lifetime of Muons created in the upper-atmosphere via the interaction between high-energy cosmic ray and atmospheric molecules [1]. Due to their high kinetic energy when produces, Muons travel at relativistic speeds which allow them to reach the surface of Earth before they decay. These energetic Muons are attenuated in a scintillator tube where they decay. When a Muon enters the scintillator tube, and promptly decays, a brief flash of light is produced, which is then amplified by a photomultiplier tube. The time between flashes is recorded into a histogram. Fitting a curve to this histogram will yield an estimate for the value of a Muon lifetime. In the case of this experiment, the Muon lifetime was estimated to be (2.0955 ± 0.0135) µs, which is within 5% of the actual value of (2.1969811 ± 0.0000022) µs [4].

1 Introduction

Muons are fundamental particles of nature, and their existence plays a significant role in the formation of modern physics, and the standard model. The Muon was first discovered in 1937, and their lifetime was first determined in 1941 by F. Rasetti [1]. Their behavior is similar to that of an electron, however they are several times more massive, as well as unstable. Muons are so unstable in fact that they have an average lifetime of only about 2.197 µs [4]. Although they exist for a very short amount of time, these particles can still be detected on the surface of Earth, as they can ionize materials that they pass through [1]. One of the main sources of Muons that reach Earth are from showers of particles formed by the collision of high-energy cosmic rays colliding with the atmosphere. These collisions are so energetic that the nuclei of air molecules are broken apart, releasing an array of particles, one of which being the Muon [1]. The resulting particles from this process posses large amounts of kinetic energy, which causes them to exhibit relativistic behavior. In the case of the Muon, these relativistic speeds cause lengths to be contracted from the perspective of the Muon, which allow them to reach the surface of the Earth before they decay. This is the driving mechanism that allows these Muons to be detected on the surface of Earth, and what will be taken advantage of during this experiment to measure their lifetime.

2 Theory

2.1 Muons

Muons are fundamental particles that are classified as Leptons. Leptons are characterized by their point-like representation, meaning that they have no constituent particles like quarks responsible for their properties. Furthermore, Leptons can exist in two configurations; as a particle or anti-particle [2]. A "regular" Muon (μ^-) has the same charge as an electron, $-e$, however it has a mass $\sim$200 times greater than that of the electron [4]. An anti-Muon (μ^+) has the same mass, but opposite charge, $+e$.

As mentioned in section 1, the Muons detected in this experiment originate from high-energy collisions in the upper atmosphere. A vast amount of particles are created from these collisions, and in particular Pions ($\pi^{\pm}$ which are classified as Mesons) are produced in large quantities [2]. Pions have a mean lifetime of $\sim$26.03 ns [4]. Once a Pion decays, it often decays as follows [2]

$$\begin{aligned} \pi^{+} &\rightarrow \mu^{+} + \nu_{\mu} \\ \pi^{-} &\rightarrow \mu^{-} + \overline{\nu}_{\mu} \end{aligned} \tag{1}$$

Where ν_{μ}, $\overline{\nu}_{\mu}$ are the Muon Neutrino, and anti-Muon Neutrino respectively. After a Muon has existed for a sufficient time, it will then decay as follows [2]

$$\begin{aligned} \mu^{-} &\rightarrow W^{-} + \nu_{\mu} \rightarrow e^{-} + \overline{\nu}_{e} + \nu_{\mu} \\ \mu^{+} &\rightarrow W^{+} + \overline{\nu}_{\mu} \rightarrow e^{+} + \nu_{e} + \overline{\nu}_{\mu} \end{aligned} \tag{2}$$

Here, $W^{\pm}$ is a W boson, and $\nu_{e}, \overline{\nu}_{e}$ are the Electron Muon and anti-Electron Muon respectively. The W boson that is created quickly decays into an Electron and Electron Neutrino as shown in equation 2 [2]. Since Neutrinos interact extremely weakly with matter, the Electrons produced from this decay chain are the only particle capable of being measured in this experiment, as they will ionize matter that they interact with. Similarly, Muons that interact with matter will also cause ionization that can be measured [2].

2.2 Time Distribution

Imagine a population of Muons that varies with time, $N(t)$, and that any given Muon has a probability of decay given by λdt, where dt is a small interval of time and λ is the rate at which a Muon decays. The change in population of Muons can be characterized by [1]

$$dN = -N(t)\lambda dt \tag{3}$$

Note that the right hand side of this relation is negative due to the population decreasing in size. This differential equation can be solved simply by separating the terms and integrating [1]

$$\frac{dN}{N(t)} = -\lambda dt, \quad \rightarrow \quad \int_{N_0}^{N} \frac{dN}{N(t)} = \int_{0}^{t} -\lambda dt, \quad \rightarrow \quad N(t) = N_0 e^{-\lambda t} \tag{4}$$

Since λ is the decay rate, the value $\tau = \frac{1}{\lambda}$ can be defined to be the average lifetime of a Muon [1]. Furthermore, this experimental setup, which will be explained in section 3, examines the time between a Muon decaying in a detector and it entering the detector, which can be written as $t_i - t_f = -\Delta t$. Finally, a background term B is also considered, giving the final equation which will characterize the distribution of decay times of a Muon

$$N(\Delta t) = N_0 e^{\frac{-\Delta t}{\tau}} + B \tag{5}$$

2.3 Relativistic Effects

Due to the high-energy of these atmospheric collisions, the Pions that are produced have large amounts of kinetic energy when they are created, which then is transferred to the Muon and Neutrino that they decay into. The resulting kinetic energy of the Muon is so high in fact that it is traveling at close to the speed of light, which causes it to exhibit relativistic effects. Most importantly, the close-to-c velocity of the Muon causes distances to contract relative to the frame of the Muon as follows

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}, \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \quad L = \frac{L_0}{\gamma} \tag{6}$$

Here, c is the speed of light, and γ is known as the *Lorentz Factor* for a relativistic object, and L_0 is a distance in the Muon's rest frame. Assuming that a Muon is traveling at 0.99 c, it can be calculated that distances from the frame of the Muon will be scaled by a factor of $\sim$0.14. This means that for a Muon created in the upper atmosphere, it will only have to travel $\sim$2.1 km, as apposed to the $\sim$15 km it would need travel classically [2].

3 Experimental Setup

In order to detect the decays of atmospheric Muons, a Scintillator and Photo-multiplier Tube (PMT) are connected to a computer (see figure 1). When a charged particle passes through the Scintillator, a flash of light is produced which is them amplified by the PMT. When a Muon passes through the Scintillator, a flash of light is produced, and another is produced when the Muon slows in the detector and decays into a charged Electron or Positron, as seen in equation 2 [2]. An electronics box then convert the analog signal from the PMT into a digital signal which the computer can then process. An oscilloscope is used to help debug the system, as well as a multimeter to find an adequate threshold voltage for the electronics box. The computer then measures the time between flashes, and records this information into a histogram [3].

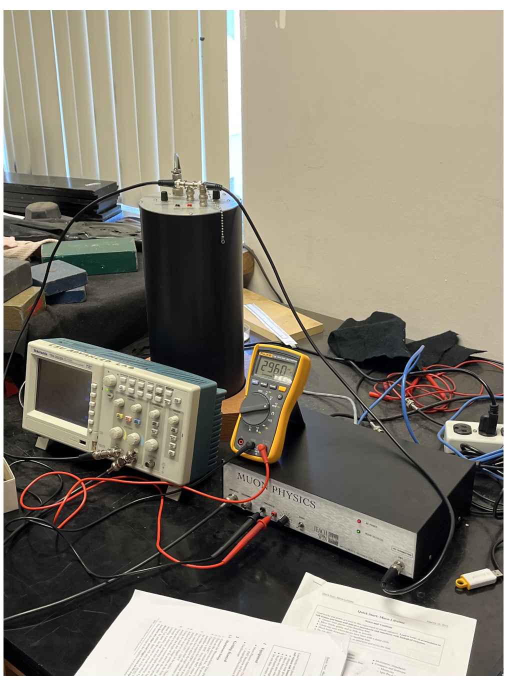

Figure 1: Image of experimental apparatus, showing the Scintillator and PMT (black cylinder) as well as an oscilloscope, and an electronics box.

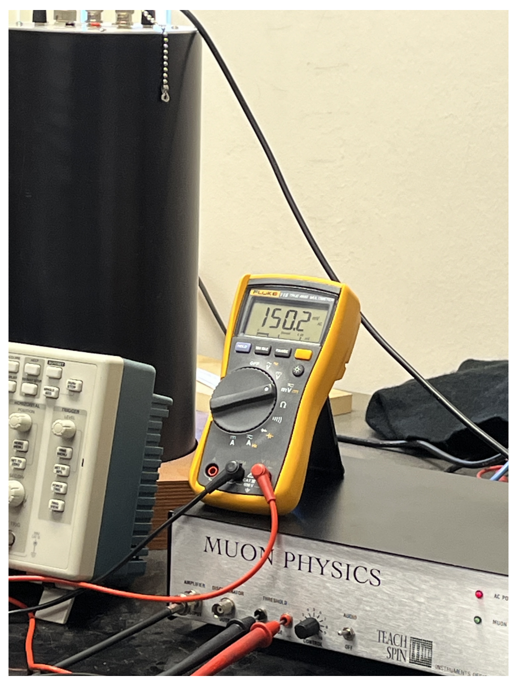

Figure 2: Multimeter showing a threshold voltage value of $\sim$150 mV.

Often during this experiment, Muons will enter the Scintillator, but not slow down enough to decay inside of it. This means that there will not be a second flash corresponding to a decay inside of the detector. To mitigate this problem, the computer has a maximum Δt lockout time of 40 μs. These values are simply discarded in the data analysis, as they do not correspond a decay event. During testing, a similar amount of 20 μs event were also recorded, and promptly discarded, as these values correspond to the same scenario as the 40 μs events.

Finally, some runs included an array of led bricks above the detector apparatus. These led bricks slow down the Muons before they reach the detector. This means that Muons that were decaying inside of the detector before now get stopped in the led, and some of the Muons that were traveling fast enough to pass through the detector are now slow enough to stop inside of the detector and decay. This effectively shifts the range of kinetic energies of Muons being studied, which allows for a more broad dataset. Each of the runs in this experiment spanned over the course of at least 1 week, a the rate at which Muon decay events were recorded was relatively slow. This also allowed for more data to be collected overall.

4 Results and Discussion

Data for this experiment was taken over the course of 4 weeks, with a PMT high voltage of ∼1200 V, as well as a electronics box threshold voltage of 150 mV. 3 runs were conducted regularly, and 1 run was conducted with a led brick covering as discussed in section 3. Each of the histograms produced by the computer had an exponential fit applied to them according to equation 5 in order to extra a value of τ, which is ultimately the average Muon decay time recorded during this experiment. Additionally, values of N_0 and B were extracted as well. An example of an exponential fit is shown in figure 3.

Run #	τ [μs]	B	N_0
1 (no led)	2.133 ± 0.030	12.60 ± 1.04	5441.72 ± 63.66
2 (no led)	2.092 ± 0.022	13.68 ± 0.81	5693.78 ± 49.60
3 (no led)	2.058 ± 0.029	12.97 ± 1.08	5783.85 ± 67.72
4 (with led)	2.088 ± 0.027	10.54 ± 0.95	6035.93 ± 65.50
Combined	2.090 ± 0.013	53.67 ± 1.90	22960.30 ± 117.12

Table 1: Runs with estimated values of τ, B, and N_0. Also includes all datasets combined.

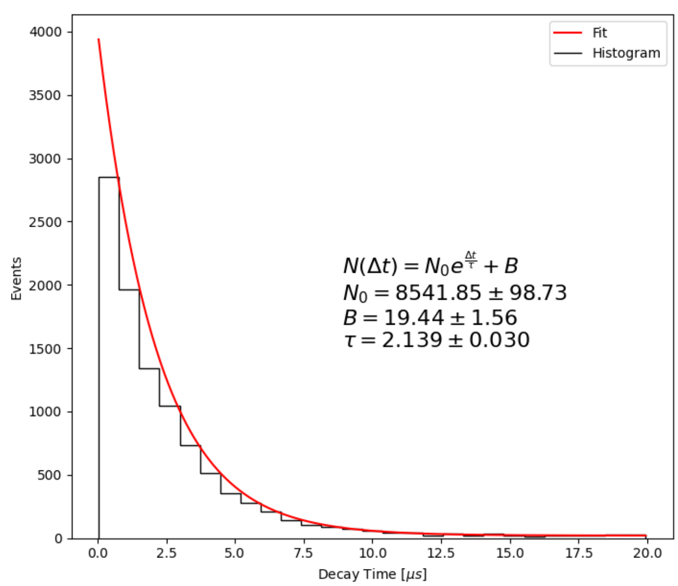

Figure 3: Exponential fit applied to the Δt histogram provided by the computer.

Each run from table 1 was averaged to arrive at a final value for τ of (2.0955 $\pm$ 0.0135) μs, giving an error of 0.64%. This value is within 5% of the actual value of (2.1969811 $\pm$ 0.0000022) μs [4].

4.1 Analysis

Throughout all of the runs, the estimated value of τ was consistently low compared to the accepted value of τ. More precisely, the estimated value is over 7 standard deviations away from the accepted value. This suggests that there is some source of systematic error within either the experimental setup or the curve fitting procedure.

The first possible source of systematic error checked was the fitting parameters as a function of bin number used in the histogram. Although it's somewhat obvious that a small bin number would affect the fitting parameters, it was unclear if an arbitrary choice of a larger bin number would bring the final value of τ lower or higher. Along with this, the statistical error on each of the fitting parameters was analysed in the same way. Consider figures 4 and 5.

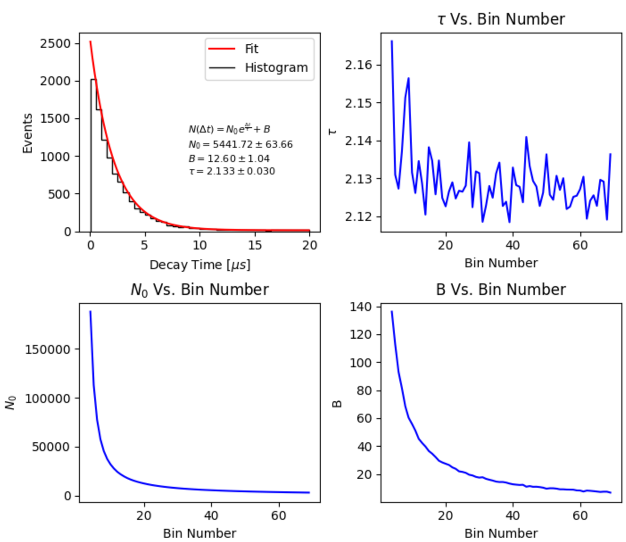

Figure 4: Fitting parameters versus number of bins

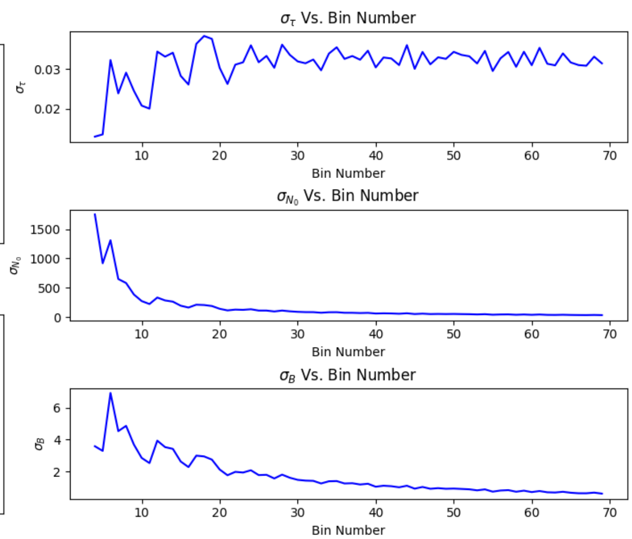

Figure 5: Errors on fitting parameters versus number of bins

As seen above, the number of bins being used for the fitting procedure has at least some affect on the final values of the parameters, however the affect is relatively small. The value of τ only varies by $\sim \pm 0.02$ μs for bin values above 20. The error on the fitting parameters follows similarly. This means that the systematic error present in the experiment is coming from another source, as the value of τ is *still* low if its value were to be maximized by selectively choosing a number of bins.

If the systematic error present is not coming from the data analysis, then it must be coming from somewhere in the experimental setup itself. Although it is unclear where this could occur, there are several possible candidates, such as the room the experiment is housed in affecting the Muons in some way, or the voltages of the electronics box threshold or the high voltage of the PMT. Ultimately, it was determined that this discrepancy is due to the fact that the Muons interact with carbon and hydrogen nuclei within the Scintillator, which causes the measured lifetime to be slightly lower than in vacuum [1].

5 Conclusions

Ultimately, this experiment was successful in estimating a value for the average decay time of Muons, as well as indirectly verifying aspects of special relativity. Although the final value for τ was within 5% of the accepted value, it is clear that there are sources systematic errors affecting this experiment. Even after being narrowed down to an error within the experimental setup itself, it is still relatively unclear where the actual source of error is. If this experiment were to be done again, the runs would be shorter, allowing for more parameters to be changed between runs in order to further understand the carbon capture phenomena that is causing τ to be consistently low. Additionally, more experimentation would be done with the fitting procedure, such as measuring how excluding bins from the front or back of the histogram changes the values of the fitting parameters.

References

[1] T.e. Coan and J. Ye. Muon physics. *TeachSpin*, 2018.

[2] Colorado State University. Lifetime of the muon, 2020.

[3] Colorado State University. Quick start: Muon lifetime, 2020.

[4] R. L. Workman and Others. Review of Particle Physics. *PTEP*, 2022:083C01, 2022.