Networked Life HW 6

Glen Choo 1000472

Loo Juin 1000546

Tan Hao Qin 1000521

Question 1

Node	1	2	3	4	5	6
$t=0$	0	∞	∞	∞	∞	∞
$t=1$	0	4	1	5	∞	8
$t=2$	0	4	1	5	6	7

Question 2

1

Without statistical multiplexing, one assumes that all users are busy all the time. N_d is thus the maximum value of N that satisfies $N <= C$, i.e. $N_d = C$.

2

Since $N_d = C$,

$$P(X > C) < \gamma$$

Using

$$\begin{aligned} P(X > C) &= 1 - P(X \leq C) \\ &= 1 - (\sum_{i=0}^{C} p^i (1-p)^{N_s - i} \binom{N_s}{i}) \end{aligned}$$

We select the highest value of N_s that satisfies this relationship.

Given $p = 0.1, \gamma = 0.01$,
for $C = 10, 20, 30, N_s = 50, 122, 301$. Thus N_s grows more quickly than C and a link of $2C$ accommodates more than $2N_s$ users.

$\sum_{i=N_d+1}^{N_s} P(X = i)$ is the probability that the link is congested.

3

For a link of capacity $2C, N_{s2} > 2N_s$. This is greater than two links of capacity, C which can accommodate $N_s + N_s = 2N_s$.

```
def nCr(n, r):
    f = math.factorial
    return f(n) / f(r) / f(n-r)

def pXC(Ns, k, p):
    return (p ** k) * ((1- p) ** (Ns - k)) * nCr(Ns, k)

def pXgeC(Ns, C, p):
    p_masses = tuple(pXC(Ns, i, p) for i in range(0, C+1))
    return 1 - sum(p_masses)

def computeNs(C, p, gamma):
    ns = C
    while True:
        if pXgeC(ns, C, p) > gamma:
            break
        ns += 1
    return ns - 1

for i in [10,20,30]:
```

```
print computeNs(i, 0.1, 0.01)
# 50, 122, 200
```

Question 3

$$E[n,p] = \frac{\frac{p^n}{n!}}{}$$

Sub $tp = p$ into Equation 1

$$e^{-t\rho}\sum_{k=0}^{n}(t\rho)^k/k! = \int_{tp}^{\infty}\frac{x^n e^{-x}}{n!}dx$$

Sub $t\rho = \rho, tn = n$ into $1/E[n,\rho]$

$$\begin{aligned}\frac{1}{E[tn,t\rho]} &= \frac{\sum_{i=0}^{tn}\frac{t\rho^i}{i!}}{\frac{(t\rho)^n}{(tn)!}} \\ &= \frac{(tn)!}{(t\rho)^n}e^{t\rho}\int_{tp}^{\infty}\frac{x^{tn}e^{-x}}{(tn)!}dx \\ &= \frac{(tn)!}{(t\rho)^n}\frac{1}{(tn)!}e^{t\rho}\int_{tp}^{\infty}x^{tn}e^{-x}dx \\ &= \frac{e^{t\rho}}{(t\rho)^n}\int_{tp}^{\infty}x^{tn}e^{-x}dx\end{aligned}$$

Reindex $t\rho$ to 0

$$\frac{1}{E[tn,t\rho]} = \frac{e^{t\rho}}{(t\rho)^n}\int_{0}^{\infty}x^{tn}e^{-x}dx$$

Integrating $x^{tn}e^{-x}$, we get

$$\frac{1}{E[tn,t\rho]} = \frac{e^{t\rho}}{(t\rho)^n}\Gamma(tn+1)$$

Since an exponential function increases at a far greater rate than a polynomial function, $\frac{e^{t\rho}}{(t\rho)^n}$ is an increasing function.

Since the gamma function is an increasing function as well, this implies that $\frac{1}{E[tn,t\rho]}$ is an increasing function as well.

As such, $E[tn,t\rho]$ is a strictly decreasing function in t, which implies that $E[2n, 2\rho] < E[n, \rho]$.

> Written with StackEdit.