MIDLANDS STATE UNIVERSITY

FACULTY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS AND STATISTICS

HMT109: MATHEMATICAL DISCOURSE AND STRUCTURES/

HCSE214: DISCRETE MATHEMATICS AND APPLICATIONS

SESSIONAL EXAMINATIONS
NOVEMBER 2018

DURATION: 3 HOURS

INSTRUCTIONS

1. *Answer **ALL** Questions from Section A and any **THREE** Questions from Section B.*
2. *All questions carry marks as indicated.*

ADDITIONAL MATERIAL(S)

1. *Scientific Calculators*

SECTION A: 40 MARKS.

A1 Define the following terms as used in Mathematical Discourse and Structures:

a) Theorem.

b) Ring.

c) Equivalence class.

d) Set difference.

e) Symmetric relation.

[2,2,2,2,2]

A2 a) A function $f(x)$ is defined so that,

$$f(x) = \frac{x+3}{2-x};$$

i) State the domain and range of $f(x)$.

ii) Find $f(3)$.

iii) Find $f^{-1}(x)$.

b) Let R be a relation xRy iff $xy > 0$, $x, y \in \mathbf{R}$. Prove that R is an equivalence relation.

[(2,2,2),(6)]

A3 a) Let $*$ be a binary operation in $\mathbf{R}$ defined by $a * b = (a+b)^2$. Verify whether:

i) $\forall a, b, c \in \mathbf{R}, (a * b) * c = a * (b * c)$.

ii) $\forall a, b \in \mathbf{R}, a * b = b * a$.

b) Define a commutative group by listing the group axioms.

[4,3,5]

A4 Let p and q be statements and $p \rightarrow q$ be an implication. Give the following statements in terms of p and q with respect to the implication:

a) the inverse,

b) the converse,

c) the contrapositive.

[2,2,2]

SECTION B: 60 MARKS.

B5 a) Let A,B,C and D be sets. Prove the following theorems.

i) $A_1 \subseteq A$ and $B_1 \subseteq B$ implies $(A_1 \times B_1) \subseteq (A \times B)$.

ii) $A \subset B$ implies $A \cap B = A$.

iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

b) Let $\{A_i\}_{i \in I}$ be an indexed family of sets and B be any set. Prove that:
$B - (\bigcup_{i \in I} A_i) = \bigcap_{i \in I}(B - A_i)$.

[(5,5,5),5]

B6 a) With respect to functions, define the terms:

i) f is surjective.

ii) f is one-to-one.

b) For the function, $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^2 + 4$, find the range with justification and verify whether the function is surjective, injective or bijective.

c) Find whether the following relations on the set of real numbers are reflexive, symmetric or transitive.

i) xRy such that $x - y$ is an integer.

ii) xRy such that $x + y$ is an integer.

[(2,2),6,(5,5)]

B7 a) Let A be a family of sets. Prove that the relation in A defined by "X is a subset of Y" is a partial order of A.

b) Let - (subtraction) be a binary operation on the set of integers. Show that it is neither commutative nor associative.

c) In a Boolean Algebra, answer the following:

i) State the dual of the statement, $(a+b)' = a' * b'$.

ii) Construct a circuit for the Boolean Polynomial, $[A' \wedge (C \wedge B) \vee C] \wedge [D \vee (A' \wedge B)]$.

[5,5,(3,7)]

B8 a) Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction.

b) Prove the following, stating the method of proof used:

i) $\sqrt{2}$ is irrational.

ii) For all integers $n \geq 1, 1(2) + 2(3) + 3(4) + ... + n(n+1) = \frac{n(n+1)(n+2}{3}$

iii) Using the axioms of multiplication for the set of real numbers, show that $-(-x) = x$.

[5,(5,5,5)]

END OF QUESTION PAPER