

Mathematics Course Descriptions

Algebra

- A **Linear Algebra I (Honors)**, *Fall 2019*.
Vector spaces, linear transformations, change of basis, eigenvalues, diagonalizability, inner product spaces, adjoint, spectral theorem, unitary operators.
Hoffman and Kunze: *Linear Algebra* (ch. 2-6, 8)
- A **Linear Algebra II**, *Winter 2020*.
Infinite-dimensional vector spaces, dual spaces, quotient spaces, Cayley–Hamilton theorem, singular value decomposition, generalized eigenspaces, Jordan canonical form.
Freidberg, Insel, and Spence: *Linear Algebra* (ch. 5-7)
Axler: *Linear Algebra Done Right* (ch. 3, 7-8) (supplemental self-study)
- A **Group Theory (Honors)**, *Fall 2020*.
Integers, groups, cosets, Lagrange's theorem, isomorphism theorems, group actions, Sylow theorems, Noetherian groups, finitely generated groups, universal properties.
Elman: *Lectures on Abstract Algebra*
Aluffi: *Algebra: Chapter 0* (supplemental self-study)
- A **Ring and Module Theory (Honors)**, *Winter 2021*.
Rings, ideals, PIDs, Zorn's lemma, integral domains, UFDs, polynomial rings, Nullstellensatz, modules, Noetherian modules, Smith normal form, rational and Jordan canonical forms.
Elman: *Lectures on Abstract Algebra*
Aluffi: *Algebra: Chapter 0* (supplemental self-study)
- A- **Field and Galois Theory**, *Spring 2021*.
Field extensions, algebraic closure, separability, squaring the circle and related problems, fundamental theorem of Galois theory, Abel–Ruffini theorem.
Elman: *Lectures on Abstract Algebra*

Analysis

- B+ **Multivariable Calculus and Analysis I (Honors)**, *Fall 2019*.
Continuity, compactness, linear algebra, big-O and little-o notation, total derivative, Hessian matrix, extreme values, inverse and implicit function theorems, Lagrange multipliers.
Shurman: *Calculus and Analysis in Euclidean Space*
- A **Multivariable Calculus and Analysis II (Honors)**, *Winter 2020*.
Riemann integrability, volume zero, gamma/beta integral, Feynman's trick, Fubini's theorem, connectedness, change of variable, differential forms, generalized Stokes' theorem.
Shurman: *Calculus and Analysis in Euclidean Space*
- A **Nonlinear Differential Equations**, *Winter 2020*.
1- and 2-dimensional differential equations, fixed points and stability, bifurcations, phase plane, existence and uniqueness theorems, conservative and reversible systems, limit cycles.
Strogatz: *Nonlinear Dynamics and Chaos* (ch. 1-7)

- A- **Complex Analysis (Honors)**, *Winter 2020*.
 Goursat's theorem, Cauchy's integral theorem, Cauchy's integral formula, residue formula, argument principle, logarithm, Möbius transformations, Riemann mapping theorem.
 Stein and Shakarchi: *Complex Analysis* (ch. 1-3, 8)
- A **Real Analysis I (Honors)**, *Winter 2020*.
 Peano axioms, equivalence relations, construction of reals, sequences and series, Riemann rearrangement theorem, cardinality, metric spaces, open and closed sets, completeness.
 Rudin: *Principles of Mathematical Analysis*
- A **Real Analysis II (Honors)**, *Spring 2020*.
 Continuity, compactness, connectedness, Arzelà–Ascoli theorem, differentiation, integration, uniform convergence, trigonometric functions, Fourier series, contraction mapping.
 Rudin: *Principles of Mathematical Analysis*
- B- **Measure Theory and L^p Spaces (Graduate)**, *Fall 2020*.
 Measures, measurable functions, Lebesgue integration, Fubini's theorem, absolute continuity, L^p spaces, convex functions, Riesz representation theorem.
 Evans and Gariepy: *Measure Theory and Fine Properties of Functions* (ch. 1)
 Lieb and Loss: *Analysis* (ch. 2)

- A **Real Analysis III**, *Spring 2021*.
 Banach spaces, Stone–Weierstrass theorem, ODEs, Picard–Lindelöf theorem, review of Riemann integral, classical Stokes' theorem, measure and Lebesgue integration.
 No textbook.

Combinatorics

- A **Graph Theory**, *Spring 2020*.
 Graph score, Eulerian graphs, 2-connectivity, triangle-free graphs, trees, planar graphs, Euler's formula, 5 color theorem, matrix-tree theorem, Ramsey theory.
 Matoušek and Nešetřil: *An Invitation to Discrete Mathematics* (ch. 4-6, 8)
- A **Probabilistic Method (Graduate)**, *Fall 2021*.
 Linearity of expectation, method of alterations, second moment method, Lovasz local lemma, Szemerédi regularity lemma, various problems from recent research.
 Alon and Spencer: *The Probabilistic Method*
- A+ **Geometric Combinatorics (Graduate)**, *Fall 2021*.
 Polytopes, root systems, triangulations, zonotopes, hyperplane arrangements, matroids, Tutte polynomial, oriented matroids.
 Ziegler: *Lectures on Polytopes*
 Stanley: *An Introduction to Hyperplane Arrangements*
- in progress **Enumerative Combinatorics (Graduate)**, *Winter 2022*.
 Partition identities, random partitions, partition bijections, \mathbb{N} -rational functions, algebraic functions, Lagrange inversion, symmetric functions.
 Stanley: *Enumerative Combinatorics*

in progress **Incidence Combinatorics (Graduate)**, *Winter 2022*.
Finite field Kakeya problem, Bezout's theorem, Szemerédi–Trotter theorem, Zarankiewicz problem, Erdős distinct distance problem, polynomial partitioning, multilinear Kakeya.
No textbook.

Computer Science

- A **Parameterized Algorithms (Graduate)**, *Spring 2020*.
FPT, kernelization, crown decomposition, bounded search trees, iterative compression, randomized methods, treewidth.
Cygan, et al.: *Parameterized Algorithms*
- A **Computability and Complexity (Graduate)**, *Fall 2020*.
Turing machines, NP, uncomputability, oracle computation, space bounds, polynomial hierarchy, randomized complexity, circuit complexity, interactive proofs, PCP Theorem.
Arora and Barak: *Computational Complexity* (ch. 1-8, 11)
- A **Communication Complexity (Graduate)**, *Winter 2021*.
Determinism, nondeterminism, randomness, lower bound methods, polynomial hierarchy, Yao's min-max principle, multiparty communication, quantum communication.
Kushilevitz and Nisan: *Communication Complexity* (ch. 1-3, 6)
- A **Greatest Theory Hits of the 21st Century (Graduate)**, *Winter 2021*.
SL = L, graph sparsification, interlacing polynomials, extender formulations, sensitivity conjecture, sunflower conjecture, log-concave polynomials, mixing-in matroids.
No textbook.
- P **Algorithmic Machine Learning (Graduate)**, *Spring 2021*.
Analysis of gradient descent and variants, SVD, online learning, graphical models, GLMs, GANs, streaming algorithms, frequency estimation.
No textbook.
- B **Cryptography (Graduate)**, *Fall 2021*.
One-way functions, hard-core bits, pseudorandom generators, semantic security, encryption, digital signatures, interactive/zero-knowledge proofs, hash functions, commitment protocols.
No textbook.

Geometry

- A **Topology**, *Winter 2021*.
Metric spaces, point-set topology, separation axioms, continuity, compactness, connectedness, product spaces, Tychonoff's theorem, homotopic paths, fundamental group, covering spaces.
Gamelin and Greene: *Introduction to Topology* (ch. 1-3)
- A- **Differential Geometry**, *Spring 2021*.
Curves and surfaces in 3-space, Frenet formulas, Gaussian curvature, congruence of curves and surfaces, intrinsic geometry of surfaces, isometries, geodesics, Gauss–Bonnet theorem.
Shifrin: *Differential Geometry* (ch. 1-3)

Other

P **Advanced Problem Solving**, *Fall 2019 and Fall 2020*.

Various problems from across undergraduate mathematics for Putnam preparation.

Andreescu and Gelca: *Putnam and Beyond*

A **Optimization**, *Winter 2020*.

Newton's method, gradient methods and convergence, conjugate direction methods, linear programming, simplex algorithm, duality, Lagrange condition, KKT condition.

Chong and Zak: *An Introduction to Optimization* (ch. 6-11, 15-17, 20-21)

A- **Mathematical Logic**, *Spring 2020*.

Propositional logic, first order logic, Tarski conditions, expressibility, soundness, completeness, compactness, Tarski's theorem, Gödel's first incompleteness theorem.

No textbook.

in progress **Introduction to Set Theory**, *Winter 2021*.

ZFC axioms, construction of the reals, cardinality, ordinals, transfinite induction, axiom of choice, independence proofs.

No textbook.