

## Mathematics Course Descriptions

### Algebra

A **Linear Algebra I (Honors)**, *Fall 2019*.

Vector spaces, linear transformations, change of basis, eigenvalues, diagonalizability, inner product spaces, adjoint, spectral theorem, unitary operators.

Hoffman and Kunze: *Linear Algebra* (ch. 2-6, 8)

A **Linear Algebra II**, *Winter 2020*.

Infinite-dimensional vector spaces, dual spaces, quotient spaces, Caley–Hamilton theorem, singular value decomposition, generalized eigenspaces, Jordan canonical form.

Freidberg, Insel, and Spence: *Linear Algebra* (ch. 5-7)

Axler: *Linear Algebra Done Right* (ch. 3, 7-8) (supplemental self-study)

A **Group Theory (Honors)**, *Fall 2020*.

Integers, groups, cosets, Lagrange's theorem, isomorphism theorems, group actions, Sylow theorems, Noetherian groups, finitely generated groups, universal properties.

Elman: *Lectures on Abstract Algebra*

Aluffi: *Algebra: Chapter 0* (supplemental self-study)

A **Ring and Module Theory (Honors)**, *Winter 2021*.

Rings, ideals, PIDs, Zorn's lemma, integral domains, UFDs, polynomial rings, Nullstellensatz, modules, Noetherian modules, Smith normal form, rational and Jordan canonical forms.

Elman: *Lectures on Abstract Algebra*

Aluffi: *Algebra: Chapter 0* (supplemental self-study)

A- **Field and Galois Theory**, *Spring 2021*.

Field extensions, algebraic closure, separability, squaring the circle and related problems, fundamental theorem of Galois theory, Abel–Ruffini theorem.

Elman: *Lectures on Abstract Algebra*

directed **Intro to Algebraic and Analytic Number Theory**, *Spring 2022*.

reading Quadratic reciprocity,  $p$ -adics, Hilbert symbol, Hasse–Minkowski theorem, Lagrange four square theorem, Dirichlet theorem

Serre: *A Course in Arithmetic* (ch. 1-4, 6-7)

### Analysis

B+ **Multivariable Calculus and Analysis I (Honors)**, *Fall 2019*.

Continuity, compactness, linear algebra, big-O and little-o notation, total derivative, Hessian matrix, extreme values, inverse and implicit function theorems, Lagrange multipliers.

Shurman: *Calculus and Analysis in Euclidean Space*

A **Multivariable Calculus and Analysis II (Honors)**, *Winter 2020*.

Riemann integrability, volume zero, gamma/beta integral, Feynman's trick, Fubini's theorem, connectedness, change of variable, differential forms, generalized Stokes' theorem.

Shurman: *Calculus and Analysis in Euclidean Space*

- A **Nonlinear Differential Equations**, *Winter 2020*.  
1- and 2-dimensional differential equations, fixed points and stability, bifurcations, phase plane, existence and uniqueness theorems, conservative and reversible systems, limit cycles.  
Strogatz: *Nonlinear Dynamics and Chaos* (ch. 1-7)
- A- **Complex Analysis (Honors)**, *Winter 2020*.  
Goursat's theorem, Cauchy's integral theorem, Cauchy's integral formula, residue formula, argument principle, logarithm, Möbius transformations, Riemann mapping theorem.  
Stein and Shakarchi: *Complex Analysis* (ch. 1-3, 8)
- A **Real Analysis I (Honors)**, *Winter 2020*.  
Peano axioms, equivalence relations, construction of reals, sequences and series, Riemann rearrangement theorem, cardinality, metric spaces, open and closed sets, completeness.  
Rudin: *Principles of Mathematical Analysis*
- A **Real Analysis II (Honors)**, *Spring 2020*.  
Continuity, compactness, connectedness, Arzelà–Ascoli theorem, differentiation, integration, uniform convergence, trigonometric functions, Fourier series, contraction mapping.  
Rudin: *Principles of Mathematical Analysis*
- B- **Measure Theory and  $L^p$  Spaces (Graduate)**, *Fall 2020*.  
Measures, measurable functions, Lebesgue integration, Fubini's theorem, absolute continuity,  $L^p$  spaces, convex functions, Riesz representation theorem.  
Evans and Gariepy: *Measure Theory and Fine Properties of Functions* (ch. 1)  
Lieb and Loss: *Analysis* (ch. 2)
- A **Real Analysis III**, *Spring 2021*.  
Banach spaces, Stone–Weierstrass theorem, ODEs, Picard–Lindelöf theorem, review of Riemann integral, classical Stokes' theorem, measure and Lebesgue integration.  
No textbook.
- B **Partial Differential Equations**, *Spring 2022*.  
The wave equation, the diffusion equation, reflections and sources, boundary problems, Fourier series  
Strauss: *Partial Differential Equations* (ch. 1-5)

## Combinatorics

- A **Graph Theory**, *Spring 2020*.  
Graph score, Eulerian graphs, 2-connectivity, triangle-free graphs, trees, planar graphs, Euler's formula, 5 color theorem, matrix-tree theorem, Ramsey theory.  
Matoušek and Nešetřil: *An Invitation to Discrete Mathematics* (ch. 4-6, 8)
- A **Probabilistic Method (Graduate)**, *Fall 2021*.  
Linearity of expectation, method of alterations, second moment method, Lovasz local lemma, Szemerédi regularity lemma, various problems from recent research.  
Alon and Spencer: *The Probabilistic Method*

- A+ **Geometric Combinatorics (Graduate)**, *Fall 2021*.  
Polytopes, root systems, triangulations, zonotopes, hyperplane arrangements, matroids, Tutte polynomial, oriented matroids.

Ziegler: *Lectures on Polytopes*

Stanley: *An Introduction to Hyperplane Arrangements*

- A **Enumerative Combinatorics (Graduate)**, *Winter 2022*.  
Partition identities, random partitions, partition bijections, ( $\mathbb{N}$ -)rational functions, algebraic functions, Lagrange inversion, symmetric functions.

Stanley: *Enumerative Combinatorics*

- A **Incidence Combinatorics (Graduate)**, *Winter 2022*.  
Finite field Kakeya problem, Bezout's theorem, Szemerédi–Trotter theorem, Zarankiewicz problem, Erdős distinct distance problem, polynomial partitioning, multilinear Kakeya.

No textbook.

- in progress **Tiling Problems (Graduate)**, *Fall 2022*.  
Domino tilings, Conway tiling group, coloring methods, partitions and rim hooks, matrix methods, undecidable, NP-complete, and #P-complete tilings.

No textbook.

## Computer Science

- A **Optimization**, *Winter 2020*.  
Newton's method, gradient methods and convergence, conjugate direction methods, linear programming, simplex algorithm, duality, Lagrange condition, KKT condition.

Chong and Zak: *An Introduction to Optimization* (ch. 6-11, 15-17, 20-21)

- A **Parameterized Algorithms (Graduate)**, *Spring 2020*.  
FPT, kernelization, crown decomposition, bounded search trees, iterative compression, randomized methods, treewidth.

Cygan, et al.: *Parameterized Algorithms*

- A **Computability and Complexity (Graduate)**, *Fall 2020*.  
Turing machines, NP, uncomputability, oracle computation, space bounds, polynomial hierarchy, randomized complexity, circuit complexity, interactive proofs, PCP Theorem.

Arora and Barak: *Computational Complexity* (ch. 1-8, 11)

- A **Communication Complexity (Graduate)**, *Winter 2021*.  
Determinism, nondeterminism, randomness, lower bound methods, polynomial hierarchy, Yao's min-max principle, multiparty communication, quantum communication.

Kushilevitz and Nisan: *Communication Complexity* (ch. 1-3, 6)

- A **Greatest Theory Hits of the 21st Century (Graduate)**, *Winter 2021*.  
SL = L, graph sparsification, interlacing polynomials, extender formulations, sensitivity conjecture, sunflower conjecture, log-concave polynomials, mixing-in matroids.

No textbook.

P **Algorithmic Machine Learning (Graduate)**, *Spring 2021*.

Analysis of gradient descent and variants, SVD, online learning, graphical models, GLMs, GANs, streaming algorithms, frequency estimation.

No textbook.

B **Cryptography (Graduate)**, *Fall 2021*.

One-way functions, hard-core bits, pseudorandom generators, semantic security, encryption, digital signatures, interactive/zero-knowledge proofs, hash functions, commitment protocols.

No textbook.

in progress **Automated Reasoning (Graduate)**, *Fall 2022*.

SAT solving, DPLL algorithm, tractable circuits, BDDs/SDDs, applications to machine learning and explainable AI

No textbook.

## Geometry

A **Topology**, *Winter 2021*.

Metric spaces, point-set topology, separation axioms, continuity, compactness, connectedness, product spaces, Tychonoff's theorem, homotopic paths, fundamental group, covering spaces.

Gamelin and Greene: *Introduction to Topology* (ch. 1-3)

A- **Differential Geometry**, *Spring 2021*.

Curves and surfaces in 3-space, Frenet formulas, Gaussian curvature, congruence of curves and surfaces, intrinsic geometry of surfaces, isometries, geodesics, Gauss–Bonnet theorem.

Shifrin: *Differential Geometry* (ch. 1-3)

directed **Computational Algebraic Geometry**, *Winter 2022*.

reading Groebner bases, elimination of variables, ideal-variety correspondence, Nullstellensatz, automated geometric theorem proving.

Cox, Little, O'Shea: *Ideals, Varieties, and Algorithms* (ch. 1-4, 6)

## Logic

A- **Mathematical Logic**, *Spring 2020*.

Propositional logic, first order logic, Tarski conditions, expressibility, soundness, completeness, compactness, Tarski's theorem, Gödel's first incompleteness theorem.

No textbook.

A+ **Introduction to Set Theory**, *Winter 2021*.

ZFC axioms, construction of the reals, cardinality, ordinals, transfinite induction, axiom of choice, independence proofs.

No textbook.

in progress **First Order Logic and Model Theory (Graduate)**, *Fall 2022*.

First order logic, Gödel's completeness theorem, compactness, diagram method, quantifier elimination.

Hils and Loeser: *A First Journey Through logic*

## Other

P **Advanced Problem Solving**, *Fall 2019 and Fall 2020*.

Various problems from across undergraduate mathematics for Putnam preparation.

Andreescu and Gelca: *Putnam and Beyond*