Mathematics Course Descriptions

Algebra

A **Linear Algebra I (Honors)**, *Fall 2019*.
Vector spaces, linear transformations, change of basis, eigenvalues, diagonalizability, inner product spaces, adjoint, spectral theorem, unitary operators.

Hoffman and Kunze: *Linear Algebra* (ch. 2-6, 8)

A **Linear Algebra II**, *Winter 2020*.
Infinite-dimensional vector spaces, dual spaces, quotient spaces, Caley–Hamilton theorem, singular value decomposition, generalized eigenspaces, Jordan canonical form.

Freidberg, Insel, and Spence: *Linear Algebra* (ch. 5-7)
Axler: *Linear Algebra Done Right* (ch. 3, 7-8) (supplemental self-study)

A **Group Theory (Honors)**, *Fall 2020*.
Integers, groups, cosets, Lagrange's theorem, isomorphism theorems, group actions, Sylow theorems, Noetherian groups, finitely generated groups, universal properties.

Elman: *Lectures on Abstract Algebra* (ch. 1-2)
Aluffi: *Algebra: Chapter 0* (ch. 1-2, 4) (supplemental self-study)

A **Ring and Module Theory (Honors)**, *Winter 2021*.
Rings, ideals, PIDs, Zorn's lemma, integral domains, UFDs, polynomial rings, Nullstellensatz, modules, Noetherian modules, Smith normal form, rational and Jordan canonical forms.

Elman: *Lectures on Abstract Algebra* (ch. 3-4)
Aluffi: *Algebra: Chapter 0* (ch. 3, 5-6) (supplemental self-study)

A- **Field and Galois Theory**, *Spring 2021*.
Field extensions, algebraic closure, separability, squaring the circle and related problems, fundamental theorem of Galois theory, Hilbert's theorem 90, Abel–Ruffini theorem.

Elman: *Lectures on Abstract Algebra* (ch. 5-6)

Analysis

B+ **Multivariable Calculus and Analysis I (Honors)**, *Fall 2019*.
Continuity, compactness, linear algebra, big-O and little-o notation, total derivative, Hessian matrix, extreme values, inverse and implicit function theorems, Lagrange multipliers.

Shurman: *Calculus and Analysis in Euclidean Space* (ch. 2-5)

A **Multivariable Calculus and Analysis II (Honors)**, *Winter 2020*.
Riemann integrability, volume zero, gamma/beta integral, Feynman's trick, Fubini's theorem, connectedness, change of variable, differential forms, generalized Stokes' theorem.

Shurman: *Calculus and Analysis in Euclidean Space* (ch. 6-9)

A- **Complex Analysis (Honors)**, *Winter 2020*.
Goursat's theorem, Cauchy's integral theorem, Cauchy's integral formula, residue formula, argument principle, logarithm, Möbius transformations, Riemann mapping theorem.

Stein and Shakarchi: *Complex Analysis* (ch. 1-3, 8)

A **Real Analysis I (Honors)**, *Winter 2020*.
Peano axioms, equivalence relations, construction of reals, sequences and series, Riemann rearrangement theorem, cardinality, metric spaces, open and closed sets, completeness.

Rudin: *Principles of Mathematical Analysis* (ch. 1-3)

A **Real Analysis II (Honors)**, *Spring 2020*.
Continuity, compactness, connectedness, Arzelà–Ascoli theorem, differentiation, integration, uniform convergence, trigonometric functions, Fourier series, contraction mapping.

Rudin: *Principles of Mathematical Analysis* (ch. 4-8)

B- **Measure Theory and L^p Spaces (Graduate)**, *Fall 2020*.
Measures, measurable functions, Lebesgue integration, Fubini's theorem, absolute continuity, L^p spaces, convex functions, Riesz representation theorem.

Evans and Gariepy: *Measure Theory and Fine Properties of Functions* (ch. 1)
Lieb and Loss: *Analysis* (ch. 2)

A **Real Analysis III**, *Spring 2021*.
Banach spaces, Stone–Weierstrass theorem, ODEs, Picard–Lindelöf theorem, review of Riemann integral, classical Stokes' theorem, measure and Lebesgue integration.

No textbook.

Applied Mathematics

A **Nonlinear Differential Equations**, *Winter 2020*.
1- and 2-dimensional differential equations, fixed points and stability, bifurcations, phase plane, existence and uniqueness theorems, conservative and reversible systems, limit cycles.

Strogatz: *Nonlinear Dynamics and Chaos* (ch. 1-7)

A **Optimization**, *Winter 2020*.
Newton's method, gradient methods and convergence, conjugate direction methods, linear programming, simplex algorithm, duality, Lagrange condition, KKT condition.

Chong and Zak: *An Introduction to Optimization* (ch. 6-11, 15-17, 20-21)

Discrete Mathematics

A **Graph Theory**, *Spring 2020*.
Graph score, Eulerian graphs, 2-connectivity, triangle-free graphs, trees, planar graphs, Euler's formula, 5 color theorem, matrix-tree theorem, Ramsey theory.

Matoušek and Nešetřil: An Invitation to Discrete Mathematics (ch. 4-6, 8)

A- **Mathematical Logic**, *Spring 2020*.
Propositional logic, first order logic, Tarski conditions, expressibility, soundness, completeness, compactness, Tarski's theorem, Gödel's first incompleteness theorem.

No textbook.

planned **Probabilistic Methods (Graduate)**, *Fall 2021*.
Second movement method, local lemma, correlation inequalities, martingales, large deviation inequalities, Janson and Talagrand inequalities, pseudo-randomness.

No textbook.

planned **Combinatorial Theory I (Graduate)**, *Fall 2021*.
Generating functions, enumerative graph theory, partitions, matchings, duality theorems, packings, coverings, configurations, polyhedra, Ramsey theory, finite and transfinite.

No textbook.

Geometry

A **Topology**, *Winter 2021*.
Metric spaces, point-set topology, separation axioms, continuity, compactness, connectedness, product spaces, Tychonoff's theorem, homotopic paths, fundamental group, covering spaces.

Gamelin and Greene: *Introduction to Topology* (ch. 1-3)

A- **Differential Geometry**, *Spring 2021*.
Curves and surfaces in 3-space, Frenet formulas, Gaussian curvature, congruence of curves and surfaces, intrinsic geometry of surfaces, isometries, geodesics, Gauss–Bonnet theorem.

Shifrin: *Differential Geometry* (ch. 1-3)

Theoretical Computer Science

A **Parameterized Algorithms (Graduate)**, *Spring 2020*.
FPT, kernelization, crown decomposition, bounded search trees, iterative compression, randomized methods, treewidth.

Cygan, et al.: *Parameterized Algorithms* (ch. 1-5, 7)

A **Computability and Complexity (Graduate)**, *Fall 2020*.
Turing machines, NP, uncomputability, oracle computation, space bounds, polynomial hierarchy, randomized complexity, circuit complexity, interactive proofs, PCP Theorem.

Arora and Barak: *Computational Complexity* (ch. 1-8, 11)

A **Communication Complexity (Graduate)**, *Winter 2021*.
Determinism, nondeterminism, randomness, lower bound methods, polynomial hierarchy, Yao's min-max principle, multiparty communication, quantum communication.

Kushilevitz and Nisan: *Communication Complexity* (ch. 1-3, 6)

A **Greatest Theory Hits of the 21st Century (Graduate)**, *Winter 2021*.
SL = L, graph sparsification, interlacing polynomials, extender formulations, sensitivity conjecture, sunflower conjecture, log-concave polynomials, mixing-in matroids.

No textbook.

P **Algorithmic Machine Learning (Graduate)**, *Spring 2021*.
Analysis of gradient descent and variants, SVD, online learning, graphical models, GLMs, GANs, streaming algorithms, frequency estimation.

No textbook.

planned **Cryptography (Graduate)**, *Fall 2021*.
One-way functions, hard-core bits, pseudorandom generators, semantic security, encryption, digital signatures, interactive/zero-knowledge proofs, hash functions, commitment protocols.

No textbook.

Other

P **Advanced Problem Solving**, *Fall 2019 and Fall 2020.*
Various problems from across undergraduate mathematics for Putnam preparation.

Andreescu and Gelca: *Putnam and Beyond* (ch. 2-6)