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**Date:** 02/22/22                          **Course:** Calculus II (38336)          **Assignment:** HW 10

1. Give the appropriate form of the partial fraction decomposition for the following function.

$$\frac{8x^2 + 3}{\left(x^2 - 4x + 4\right)\left(x^2 + 2x + 3\right)}$$

What is the appropriate form of the partial fraction decomposition for the given function?

○ **A.** $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{Cx+D}{x^2 + 2x + 3}$

○ **B.** $\dfrac{A}{(x-2)^2} + \dfrac{Bx+C}{x^2 + 2x + 3}$

○ **C.** $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x^2 + 2x + 3}$

○ **D.** $\dfrac{A}{(x-2)^2} + \dfrac{B}{x^2 + 2x + 3}$

○ **E.** $\dfrac{A}{x-2} + \dfrac{Bx+C}{x^2 + 2x + 3}$

2. Express the given rational function in terms of partial fractions. Watch out for any preliminary divisions.

$$\frac{11x + 26}{x^2 + 6x + 8}$$

$\dfrac{11x + 26}{x^2 + 6x + 8} = $ [ ]

3. Write the partial fraction decomposition of the following rational expression.

$$\frac{3x^2 - 7x + 20}{x(x + 4)(x + 5)}$$

$\dfrac{3x^2 - 7x + 20}{x(x + 4)(x + 5)} = $ [ ]

4. Evaluate the following integral.

$$\int \frac{20}{x^2 - 25}\, dx$$

$\displaystyle\int \frac{20}{x^2 - 25}\, dx = $ [ ]

5. Evaluate the following integral.

$$\int \frac{x^2 + 24x - 9}{x^3 - 9x} \, dx$$

$$\int \frac{x^2 + 24x - 9}{x^3 - 9x} \, dx = \boxed{\phantom{xxxxxxxxx}}$$

6. Evaluate the following definite integral.

$$\int_0^2 \frac{3}{y^2 - 2y - 15} \, dy$$

$$\int_0^2 \frac{3}{y^2 - 2y - 15} \, dy = \boxed{\phantom{xxxxxxxxx}}$$

(Type an exact answer.)

7. Evaluate the following integral.

$$\int \frac{48x^2}{(x - 24)(x + 8)^2} \, dx$$

$$\int \frac{48x^2}{(x - 24)(x + 8)^2} \, dx = \boxed{\phantom{xxxxxxxxx}}$$

8. Evaluate the following integral.

$$\int_{-1}^1 \frac{5x}{(x + 5)^2} \, dx$$

$$\int_{-1}^1 \frac{5x}{(x + 5)^2} \, dx = \boxed{\phantom{xxxxxxxxx}}$$

(Type an exact answer.)

9. Evaluate the following integral.

$$\int \frac{15}{x^3 + x^2} \, dx$$

$$\int \frac{15}{x^3 + x^2} \, dx = \boxed{\phantom{xxxxxxxxx}}$$

10.  Evaluate the following integral.

$$\int \frac{2x^2 + x + 3}{(x+1)\left(x^2+1\right)}\, dx$$

$$\int \frac{2x^2 + x + 3}{(x+1)\left(x^2+1\right)}\, dx = \boxed{\phantom{xxxxxxxxxxxx}}$$

11.  Evaluate the following integral.

$$\int \frac{x^4 + 3}{x^3 + 8x}\, dx$$

$$\int \frac{x^4 + 3}{x^3 + 8x}\, dx = \boxed{\phantom{xxxxxxxx}}$$

12. Determine whether the following statements are true and give an explanation or counterexample.

Answer parts **a.**–**d.** below.

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**a.** Is the following statement true? To evaluate $\int \dfrac{6x^6}{x^4 + 7x^2}\,dx$, the first step is to find the partial fraction decomposition of the integrand. Choose the correct answer below.

- ○ **A.** Yes, because the denominator can be factored into simple linear factors, including a repeated linear factor.
- ○ **B.** Yes, because the degree of the numerator is greater than the degree of the denominator.
- ○ **C.** No, because the denominator is not yet in factored form.
- ○ **D.** No, because the degree of the numerator is greater than the degree of the denominator.

**b.** Is the following statement true? The easiest way to evaluate $\int \dfrac{12x + 3}{6x^2 + 3x}\,dx$ is with a partial fraction decomposition of the integrand. Choose the correct answer below.

- ○ **A.** Yes, because partial fraction decomposition takes fewer steps than trigonometric substitution or u-substitution.
- ○ **B.** No, because the integrand is an improper fraction and, therefore, cannot be put into a partial fraction decomposition.
- ○ **C.** No, because it is easier to evaluate with a u-substitution with $u = 6x^2 + 3x$.
- ○ **D.** Yes, because it is a proper fraction, and the denominator can be factored into simple linear factors.

**c.** Is the following statement true? The rational function $f(x) = \dfrac{1}{x^2 - 11x + 24}$ has an irreducible quadratic denominator. Choose the correct answer below.

- ○ **A.** No, because the trinomial $x^2 - 11x + 24$ can be factored.
- ○ **B.** Yes, because the trinomial $x^2 - 11x + 24$ cannot be factored.
- ○ **C.** Yes, because the trinomial $x^2 - 11x + 24$ is a quadratic.
- ○ **D.** No, because the trinomial $x^2 - 11x + 24$ is not a quadratic.

**d.** Is the following statement true? The rational function $f(x) = \dfrac{1}{x^2 - 11x + 31}$ has an irreducible quadratic denominator. Choose the correct answer below.

- ○ **A.** No, because the trinomial $x^2 - 11x + 31$ is not a quadratic.
- ○ **B.** Yes, because the trinomial $x^2 - 11x + 31$ cannot be factored.
- ○ **C.** No, because the trinomial $x^2 - 11x + 31$ can be factored.
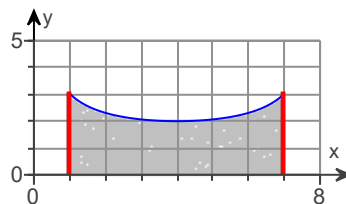- ○ **D.** Yes, because the trinomial $x^2 - 11x + 31$ is a quadratic.

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13.

Find the area of the region bounded by the curve $y = \dfrac{16}{x^2 - 6x - 55}$, the x-axis, and the lines $x = -3$ and $x = 3$.

The area of the region is $\boxed{\phantom{xxxxxxx}}$.

(Type an exact answer.)

14.

Find the volume of the solid generated by revolving the shaded region about the x-axis.

The curve is $y = \dfrac{8}{\sqrt{8x - x^2}}$; $x_1 = 1$, $x_2 = 7$.



The volume of the solid is $\boxed{\phantom{xxxxxxx}}$.

(Type an exact answer, using $\pi$ as needed.)

15.  The following integral requires a preliminary step such as long division or a change of variables before using the method of partial fractions. Evaluate the following integral.

$$\int \frac{e^x}{\left(e^x - 1\right)\left(e^x + 7\right)}\, dx$$

$\displaystyle\int \frac{e^x}{\left(e^x - 1\right)\left(e^x + 7\right)}\, dx = \boxed{\phantom{xxxxxxx}}$

(Use integers or fractions for any numbers in the expression.)