t	x	forward	backward	center
0	0	0.35		
2	0.7	0.55	0.35	0.45
4	1.8	0.8	0.55	0.675
6	3.4	0.85	0.8	0.825
8	5.1	0.6	0.85	0.725
10	6.3	0.5	0.6	0.55
12	7.3	0.35	0.5	0.425
14	8	0.2	0.35	0.275
16	8.4		0.2	

ENGR 216 HW 2 Finite Differences Spring 2022

M/s

1. Use the following data to find the velocity and acceleration at t = 4, 8, and 12 seconds using a) forward finite difference, b) backward finite differences, and c) centered finite differences

t	Time t, seconds	0	2	4	6	8	10	12	14	16
f(T)	Position x, meters	0	0.7	1.8	3.4	5.1	6.3	7.3	8.0	8.4

the Rest on excel

	Forward	Backward	Center
4s	0.8	0.55	0.675
8s	0.6	0.85	0.725
12s	0.35	0.5	0.425

T=4 Forward $\frac{3.4-1.8}{2}$ Backwards $\frac{1.8-0.7}{2}$ Center $\frac{3.4-0.7}{4}$

2. The velocity v (m/s) of air flowing past a flat surface is measured at several distances y (m) away from the surface. Use Newton's viscosity law

$$\tau = \mu \frac{du}{dy}$$

to determine the shear stress τ (N/m^2) at the surface (y=0). Assume a value for the dynamic viscosity $\mu = 1.8 \times 10^{-5} Ns/m^2$. How does your answer change if you use more accurate finite differences?

$\frac{N}{m^2} = \frac{Ns}{m^2}\left(\frac{m}{s}\,\frac{1}{m}\right)$

y	y (m)	0	0.002	0.006	0.012	0.018	0.024
u(y)	u (m/s)	0	0.287	0.899	1.915	3.048	4.299

The smaller the Δy, the more accurate. it's also more accurate if you use the centered one

it gets closer to the real value if you use more accurate finite differences

$\tau = 1.8E{-5}\left(\frac{.002}{.287}\right) = 1.25E{-7}$ y=0

3. Consider the polynomial

$$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$$

slope

The true value of its derivative at x=0.5 is $f'(0.5) = -0.9125$. Use backward, forward, and centered first finite differences to estimate the derivative numerically if the step size $\Delta x = 0.25$, and determine the percent error between the true value and each of the estimated values (percent error is given by ε

$f(.75) \approx 0.6363$ $f(.25) \approx 1.1035$

$$\varepsilon = \frac{true\ value - estimated\ value}{true\ value}$$

$f(.5) = 0.925$

converted to a percentage.) What value of Δx would you have to use for the backward and forward finite differences to get the same percent error as the centered finite difference (hint: it should be less than 0.25.)

Forward $\Delta x = 0.025$
Backward $\Delta x = -0.025$

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$\frac{f(0.5+.025)-f(.5)}{.025}$

$$f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

$\Delta x = -.025$ $\frac{f(.5)-f(.5--.025)}{-.025}$

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

$\frac{f(.5--.025)-f(.5)}{.025}$

Forward $f'(0.5) = \frac{f(0.5+0.25)-f(0.5)}{0.25} \approx -1.15469$

Backward $f'(0.5) = \frac{f(0.5)-f(0.5-0.25)}{0.25} \approx -0.71406$

Centered $f'(0.5) = \frac{f(0.5+0.25)-f(0.5-0.25)}{0.5} \approx -0.934375$

0.5

Center % error $= \frac{-0.9125 - -0.934375}{-0.9125} = -0.02397 \approx -2.397\%$

$f'(x) = \frac{f(x+a)-f(x)}{a}$ $f'(x) = \frac{f(x)-f(x-b)}{b}$

$-0.02397 = \frac{-0.9125-(x)}{-0.9125}$

$f'(x) = \frac{f(x+c)-f(x-c)}{2c}$

$x = 0.934375$

$-0.934375 = \frac{f'(x)\Delta m + f(x) - f(x-\Delta x)}{2\Delta x}$

$-0.934375 = \frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{f(x)\Delta x + f(x) - f(x)}{\Delta x}$

$$-0.934375\Delta x = f(x+\Delta x) - f(x)$$

$$-0.934375\Delta x = -0.1(x+\Delta x)^4 - 0.15(x+\Delta x)^3 - 0.5(x+\Delta x)^2 - 0.25(x+\Delta x) + 1.2 - f(x)$$

$$\underbrace{(x+\Delta x)(x+\Delta x)}(x+\Delta x)(x+\Delta x)$$

$$(x^2 + 2x\Delta x + \Delta x^2)(x+\Delta x)$$

$$(x^3 + 2x^2\Delta x + x\Delta x^2 + x^2\Delta x + 2x\Delta x^2 + \Delta x^3)$$

$$(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)(x+\Delta x)$$

$$x^4 + 3x^3\Delta x + 3x^2\Delta x^2 + x\Delta x^3 + x^3\Delta x + 3x^2\Delta x^2 + 3x\Delta x^3 + \Delta x^4$$

$$x^4 + 4x^3\Delta x + 6x^2\Delta x^2 + 4x\Delta x^3 + \Delta x^4$$

$$-0.934375\Delta x = -0.1(x^4 + 4x^3\Delta x + 6x^2\Delta x^2 + 4x\Delta x^3 + \Delta x^4) - 0.15(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3) - 0.5(x^2 + 2x\Delta x + \Delta x^2) - 0.25(x+\Delta x) + 1.2 - f(x)$$

$x = .5$ $\quad 4x^3 =$

$x^2 = .25$

$x^3 = .125$

$x^4 = 0.0625$

$$f(.5) - 0.934375\Delta x = \left\{ \begin{array}{l} -0.1(0.0625 + 0.5\Delta x + 1.5\Delta x^2 + 2\Delta x^3 + \Delta x^4) \\ -0.15(.125 + .75\Delta x + 1.5\Delta x^2 + \Delta x^3) \\ -0.5(.25 + \Delta x + \Delta x^2) \\ -0.25(.5 + \Delta x) \\ +1.2 \end{array} \right.$$

$$f(.5) - 0.934375\Delta x = 0.1\Delta x^4 - .14\Delta x^3 - .875\Delta x^2 - .9125\Delta x + 0.925$$

$$0 = 0.1\Delta x^4 - .14\Delta x^3 - .875\Delta x^2 - .9125\Delta x + 0.925 - (\underbrace{f(.5)}_{0.925} - 0.934375\Delta x)$$

(graph)

$$\boxed{\Delta x = 0.025}$$

Below this line are failed

attempts

$X =$

$$-0.934375\Delta x = f(x) - f(x - \Delta x)$$

$$-0.934375\Delta x = f(x) - \left(-0.1(x-\Delta x)^4 - 0.15(x-\Delta x)^3 - 0.5(x-\Delta x)^2 - 0.25(x-\Delta x) + 1.2\right)$$

$$(x-\Delta x)(x-\Delta x)$$

$$x^2 - 2x\Delta x + \Delta x^2$$

$$\underbrace{(x-\Delta x)(x-\Delta x)}(x-\Delta x)(x-\Delta x)$$

$$(x^2 - 2x\Delta x + \Delta x^2)(x-\Delta x)$$

$$(x^3 - 2x^2\Delta x + x\Delta x^2 - x^2\Delta x + 2x\Delta x^2 - \Delta x^3)$$

$$(x^3 - 3x^2\Delta x + 3x\Delta x^2 - \Delta x^3)(x-\Delta x)$$

$$(x^4 - 3x^3\Delta x + 3x^2\Delta x^2 - x\Delta x^3 - x^3\Delta x + 3x^2\Delta x^2 - 3x\Delta x^3 + \Delta x^4)$$

$$-0.934375\Delta x = f(x) + \underbrace{0.1(x^4 - 3x^3\Delta x + 3x^2\Delta x^2 - x\Delta x^3 - x^3\Delta x + 3x^2\Delta x^2 - 3x\Delta x^3 + \Delta x^4) + 0.15(x^3 - 3x^2\Delta x + 3x\Delta x^2 - \Delta x^3) + 0.5(x^2 - 2x\Delta x + \Delta x^2) + 0.25(x-\Delta x) - 1.2}$$

$x = .5$
$x^2 = .25$
$x^3 = .125$
$x^4 = 0.0625$

$$0.1(x^4 - 3x^3\Delta x + 3x^2\Delta x^2 - x\Delta x^3 - x^3\Delta x + 3x^2\Delta x^2 - 3x\Delta x^3 + \Delta x^4)$$

$$0.15(x^3 - 3x^2\Delta x + 3x\Delta x^2 - \Delta x^3)$$

$$0.5(x^2 - 2x\Delta x + \Delta x^2)$$

$$0.25(x - \Delta x)$$

$$-1.2$$

$$0.1\Delta x^4 \quad - 0.2\Delta x^3 + 1.3\Delta x^2 - 0.9\Delta x - 0.925$$

$$0.1(-3x\Delta x^3 \quad + 3x^2\Delta x^2 \quad - x^3\Delta x)$$

$$0.1\Delta x^4 - 0.35\Delta x^3 + 1.375\Delta x^2 - .9125\Delta x - 0.925$$

0.925

$$-0.934375\Delta x = f(x) + 0.1\Delta x^4 - 0.35\Delta x^3 + 1.375\Delta x^2 - .9125\Delta x - 0.925$$

$$0 = 0.1\Delta x^4 - 0.35\Delta x^3 + 1.375\Delta x^2 - .9125\Delta x + 0.934375\Delta x$$

at $x = \Delta x$

$$-0.934375\Delta x = f(x) - \left(-0.1(x-\Delta x)^4 - 0.15(x-\Delta x)^3 - 0.5(x-\Delta x)^2 - 0.25(x-\Delta x) + 1.2\right)$$

$$-0.934375\Delta x = f(\Delta x) - 1.2$$

$$1.2 - 0.934375\Delta x = f(\Delta x)$$

$$1.2 - 0.934375\Delta x = -0.1\Delta x^4 - 0.15\Delta x^3 - 0.5\Delta x^2 - 0.25\Delta x + 1.2$$

$$-0.934375 = -0.1\Delta x^3 - 0.15\Delta x^2 - 0.5\Delta x - 0.25$$

$$-0.1\Delta x^3 - 0.15\Delta x^2 - 0.5\Delta x + 0.684375 = 0$$

0.5

$$-0.934375/2 = f'(x)\Delta m + f(x) - f(x-\Delta x)$$

$$f(x-\Delta x) = f'(x)\Delta m + f(x) + 0.934375/2$$

$$f(x-\Delta x) = f'(x)\Delta m + f(x) + 0.934375/2$$

$$\frac{f(x) - f(x-\Delta x)}{\Delta x} = f'(x)\Delta m + f(x) + 0.934375/2$$

$$\frac{f(x) - f(x-\Delta x)}{\Delta x} = f'(x)\Delta m + f(x) + f'(x)/2$$

$u = -0.934375$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = u$$

$$\frac{(f'(x)\Delta x + f(x)) - f(x)}{u} = \Delta x$$

$$\frac{f'(x)\Delta x}{u} = \Delta x$$

$$\frac{f'(x)\Delta x}{u} - \Delta x = 0$$

$$\Delta x\left(\frac{f'(x)}{u} - 1\right) = 0$$

$$\frac{f'(x)}{u} - 1 = 0$$

$$\frac{f'(x)}{u} = 1$$

$$f'(x) = u$$

$f'(x)$

$$dy = f(x+\Delta x) - f(x)$$

$$dy = \frac{f'(x)}{\Delta x} + f(x) - f(x)$$

$$dy = \frac{f'(x)}{\Delta x}$$

$$\Delta x = \frac{f'(x)}{dy} = \frac{\frac{dy}{dx}}{dy} = \frac{dy^2}{dx} = dx$$

$$dy^2 = dx^2$$

$$\Delta x = \frac{f(x+\Delta x) - f(x)}{0.934375}$$

$$-0.934375 = \frac{f'(x)\Delta x + f(x) - f(x)}{\Delta x}$$

$$-0.934375\Delta x = f'(x)\Delta x$$

$$\Delta x - \frac{f(x+\Delta x)}{.934375} = \frac{-f(x)}{.934375}$$

$$\Delta x = \frac{\frac{dy}{dx}}{dy}$$

$$\Delta x = \frac{dy\ dx^{-1}}{dy}$$

$$f'(x) = -0.934375$$

assume $a = b = c$

$$f(x+a) = f'(x)a + f(x) \qquad f(x+b) = f'(x)b + f(x)$$

$$f(x-b) = -f'(x)b + f(x)$$

$$f(x+c) = 2f'(x)c + f(x-c)$$

$$f(x-c) = f(x+c) - 2f'(x)c$$

$$f(x-b) = -f'(x)b + f(x)$$

$$f(x+b) = 2f'(x)b + f(x-b)$$

$$f(x-b) = f(x+b) - 2f'(x)b$$

$$\Delta x = \frac{1}{dx}$$

$$\Delta x^2 = 1$$

$$\Delta x = \pm 1$$

$$f(x-b) = f(x) - f'(x)b$$

$$-\quad f(x-b) = f(x+b) - 2f'(x)b$$

$$0 = f(x) - f(x+b) + f'(x)b$$

$$f(x+b) = f(x) + f'(x)b$$