Vector Addition

What is a Vector

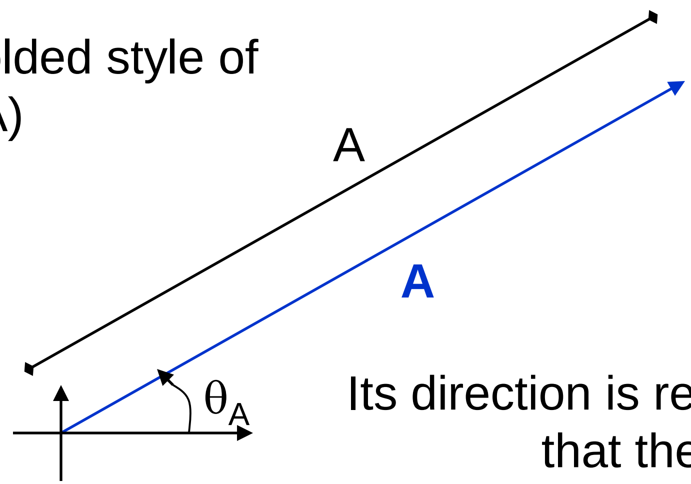
- A vector is a value that has a **magnitude** and **direction**
  - Examples
    - Force
    - Velocity
    - Displacement
- A scalar is a value that has a **magnitude**.
  - Examples
    - Temperature
    - Speed
    - Distance

Vector Representation

A vector (**A**) can be represented as an arrow.

Its magnitude is represented by the length of the arrow,

Designated by the un-bolded style of the symbol (A)



Its direction is represented by the angle that the arrow points,
Designated normally by a Greek symbol (sometimes with a subscript) (θ_A)
It is normally measured from the **positive X-axis** in a **Counter Clockwise** Manner

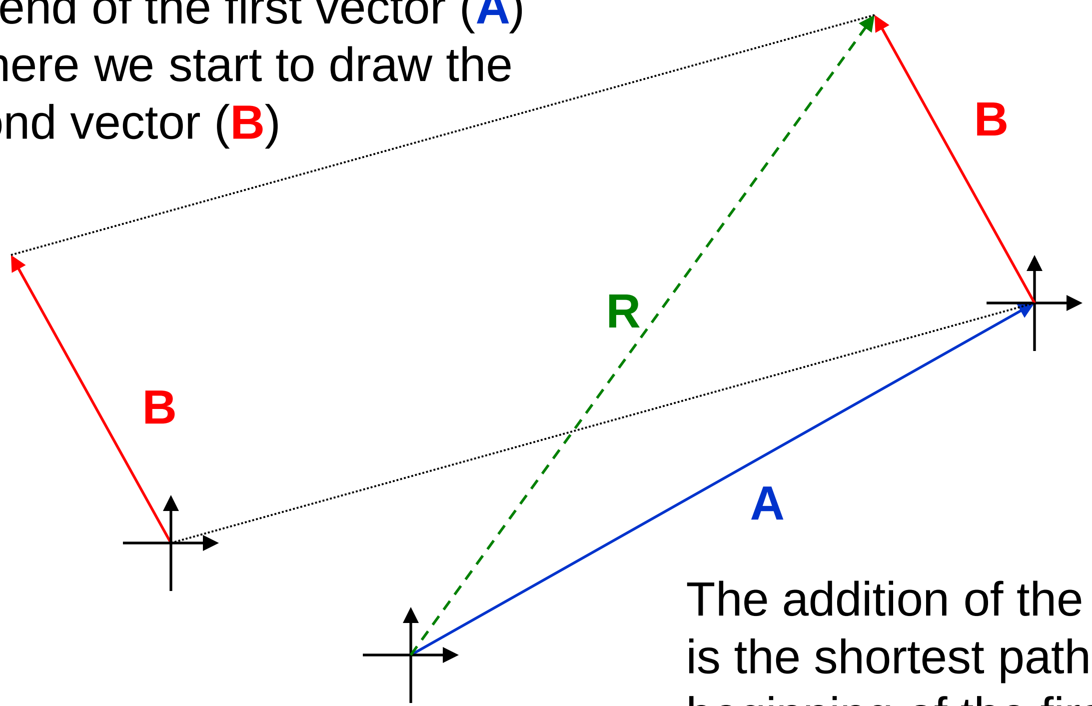
The vector can be written as an ordered pair in the following form:
(A, θ_A)

Graphical Vector Addition

To add two vectors (**A** and **B**) together to get a resultant vector (**R**) all we have to do is draw the first vector (**A**)

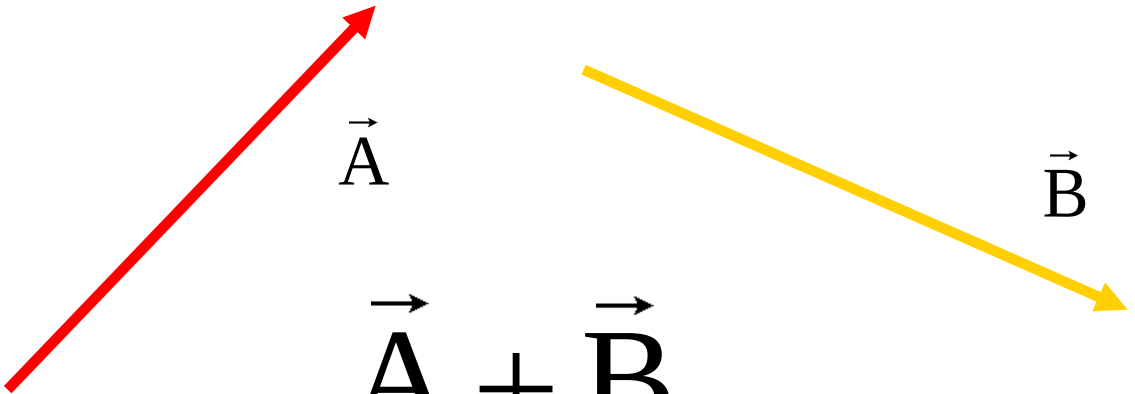
The end of the first vector (**A**) is where we start to draw the second vector (**B**)

We can see that the new vector (**B**) is parallel to the original vector as well as the same length

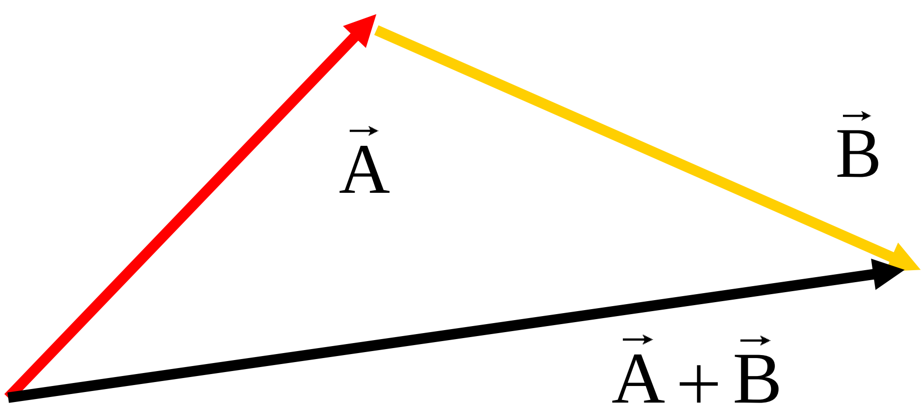


The addition of the two vectors (**A** + **B** = **R**) is the shortest path that connects the beginning of the first vector (**A**) to the end of the last vector (**B**).

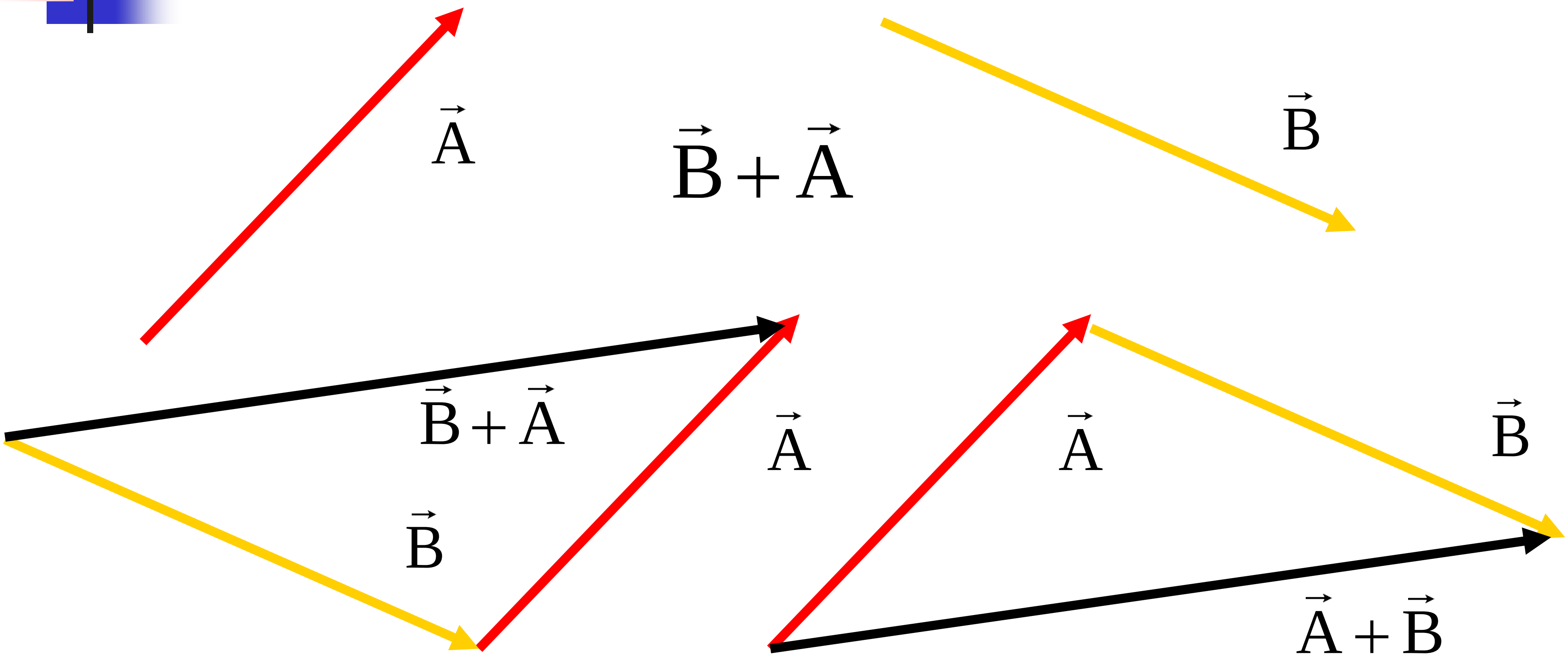
Vector addition – Tip to tail method



$$\vec{A} + \vec{B}$$



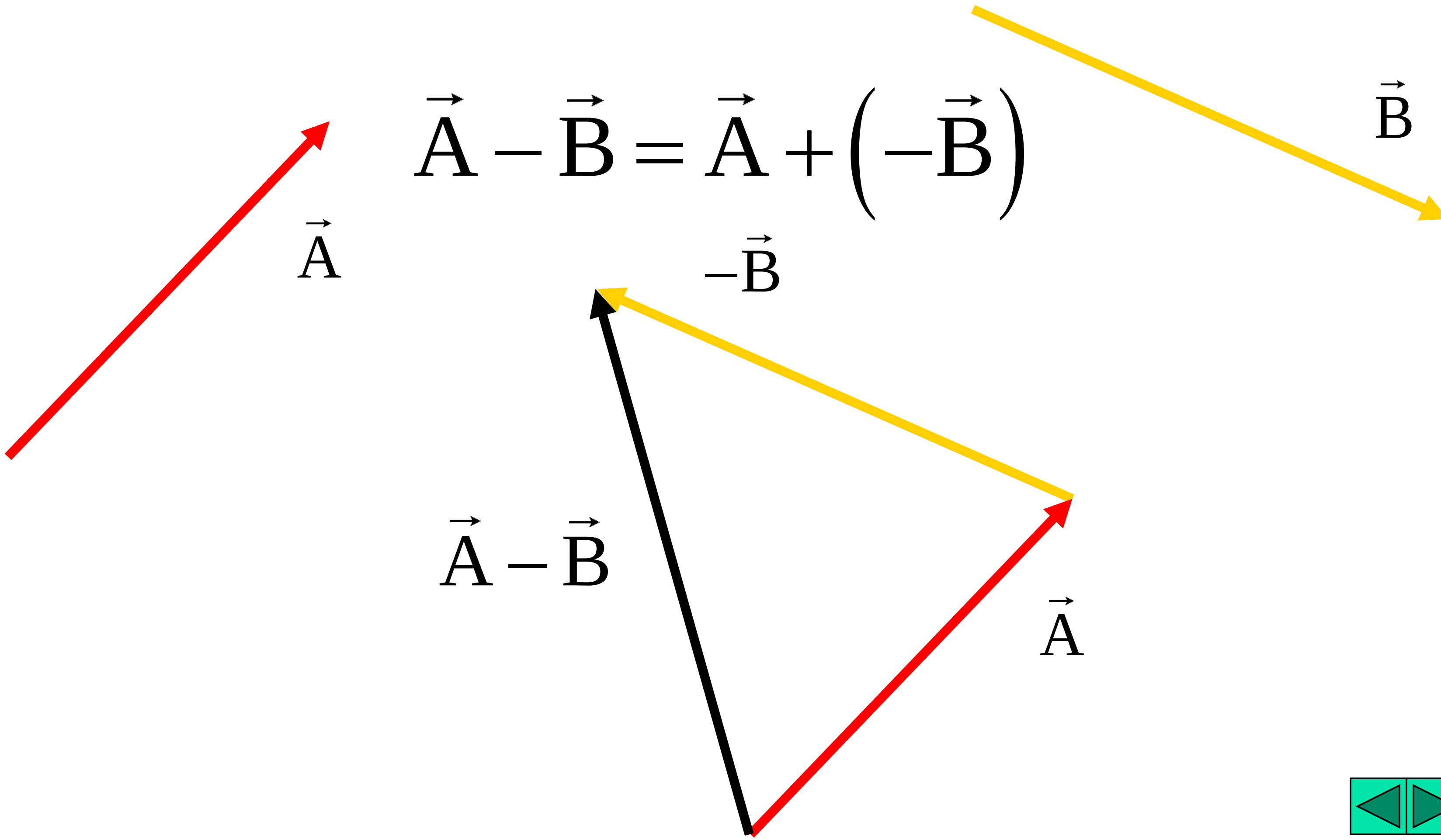
Vector addition – Commutative Law



$$\vec{B}+\vec{A}=\vec{A}+\vec{B}$$

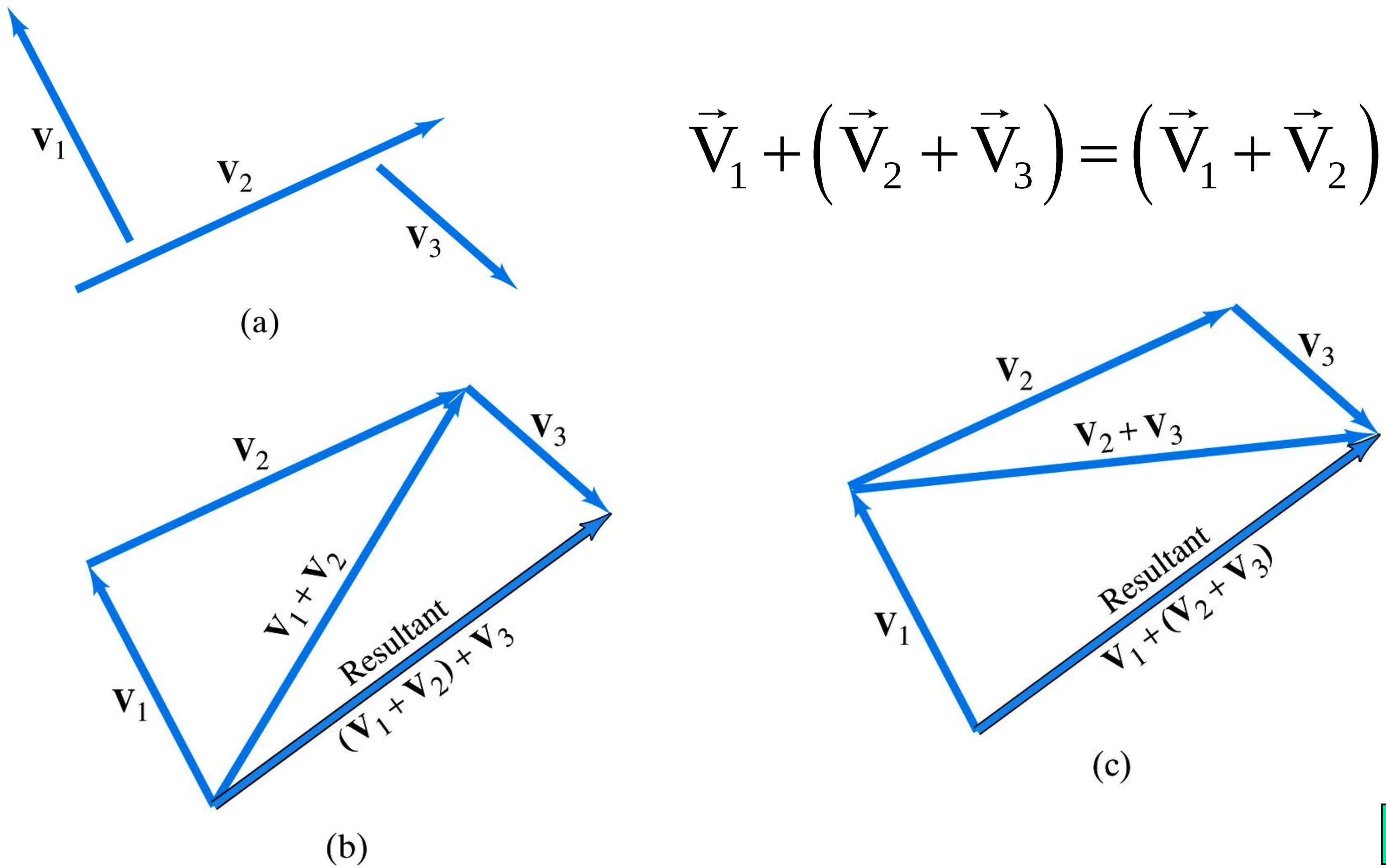
Vector subtraction
Add the negative

$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

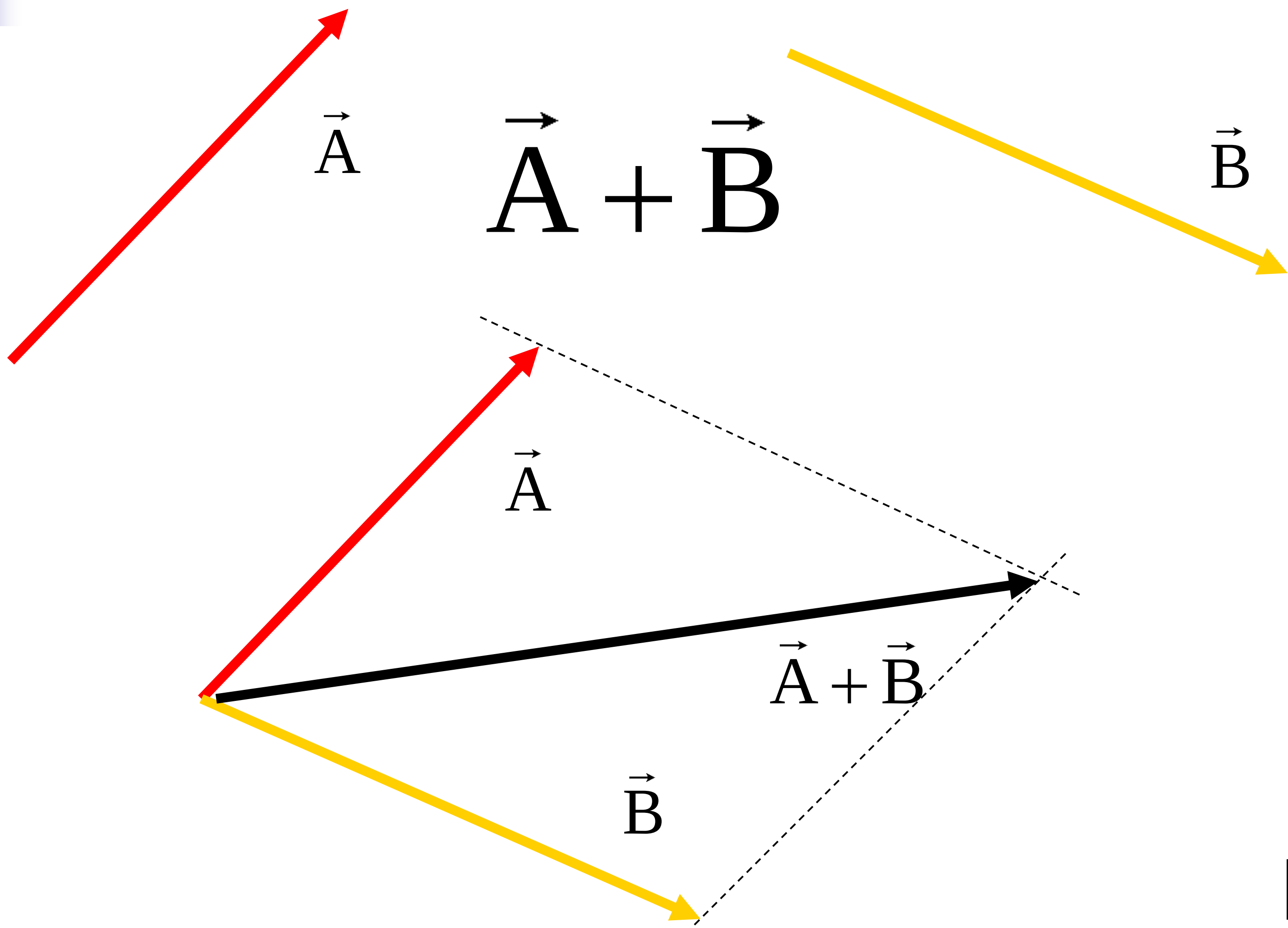


Vector addition
Associative Law

$$\vec{V}_1 + \left(\vec{V}_2 + \vec{V}_3\right) = \left(\vec{V}_1 + \vec{V}_2\right) + \vec{V}_3$$

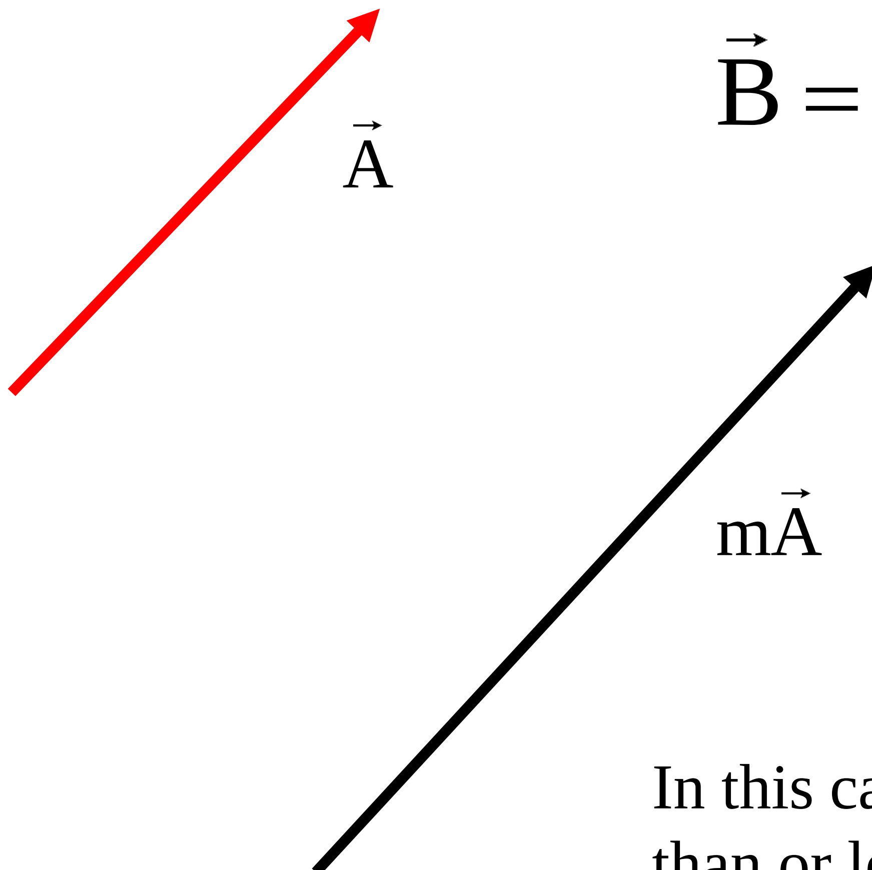


Vector addition
Parallelogram method



Multiplication of a vector by a scalar

$$\vec{B} = m\vec{A}$$

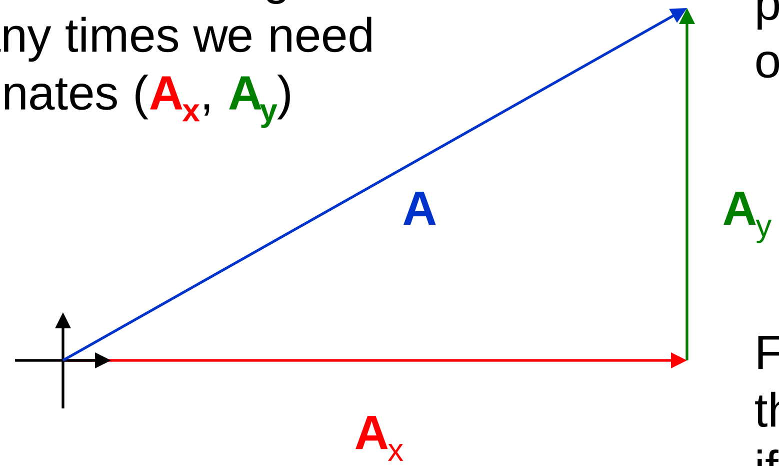


In this case is m greater than or less than 1?

Decomposing a Vector : Part I

Earlier we saw that a vector can be written as a magnitude and an angle (A, θ_A). However many times we need it in Cartesian Coordinates (**A_x**, **A_y**)

Similarly, the y coordinate of the vector (**A_y**) is the projection of the vector (**A**) onto the x-axis.



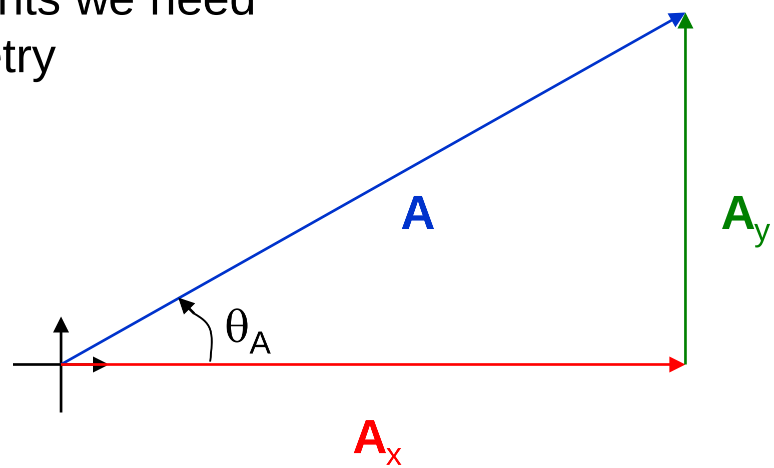
The x coordinate of the vector (**A_x**) is the projection of the vector (**A**) onto the x-axis.

Finally, we can see that the x and y coordinates if added together give the original vector.

$$A_x + A_y = A$$

Decomposing a Vector: Part II

To get the algebraic values for the two components we need to use Trigonometry



By using SOHCAHTOA and knowing that we have a right triangle we know that:

$A_x = A \text{ Cos } (\theta_A)$ $A_y = A \text{ Sin } (\theta_A)$

Finally to fill out the list we know that:

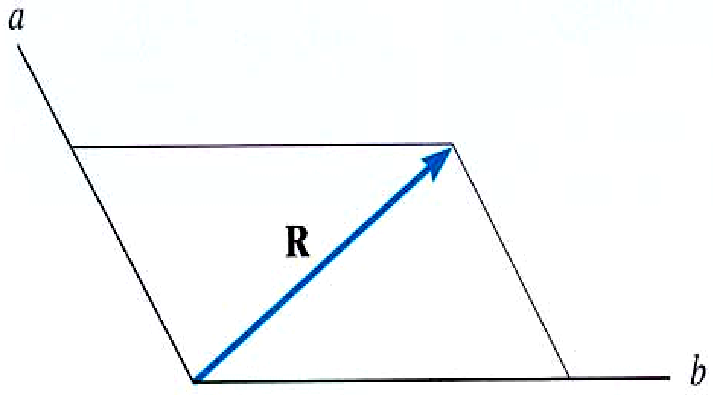
$A_x^2 + A_y^2 = A^2$

And Pythagorean's theorem states that:

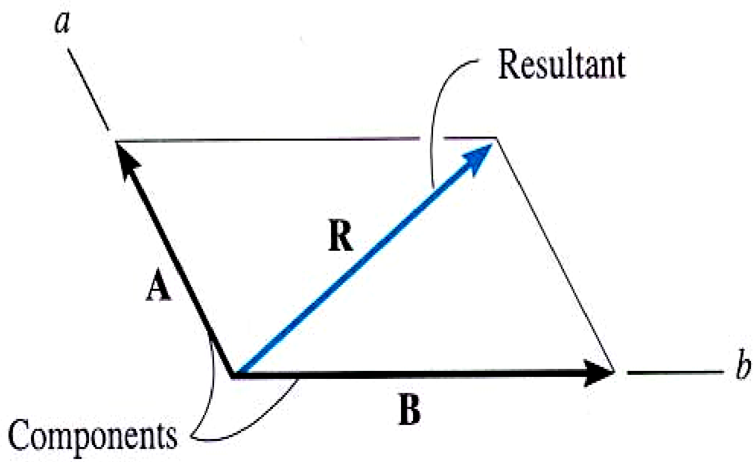
$$\frac{A_y}{A_x} = \text{Tan } (\theta_A)$$

RESOLUTION OF A VECTOR

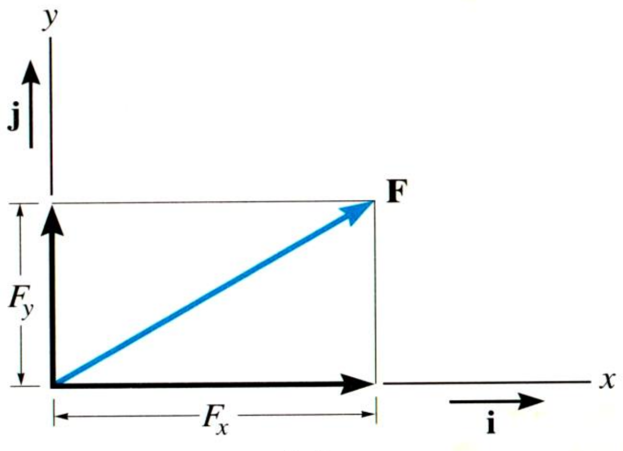
"Resolution" of a vector is breaking up a vector into components. It is kind of like using the parallelogram law in reverse.



Extend parallel lines from the head of **R** to form components



CARTESIAN VECTOR NOTATION



(a)

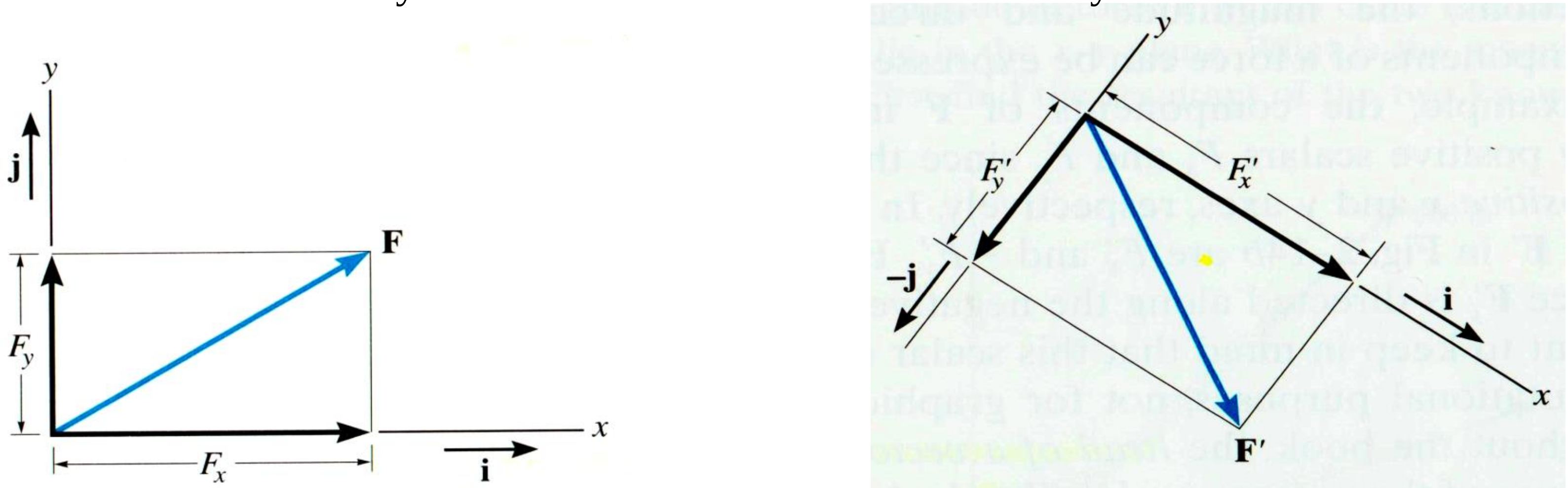
- We ‘ resolve’ vectors into components using the x and y axes system
- Each component of the vector is shown as a magnitude and a direction.

- The directions are based on the x and y axes. We use the “unit vectors” ***i*** and ***j*** to designate the x and y axes.

CARTESIAN VECTOR NOTATION

For example,

$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$ ***or*** $\boldsymbol{F'} = F'_x\,\boldsymbol{i} + F'_y\,\boldsymbol{j}$



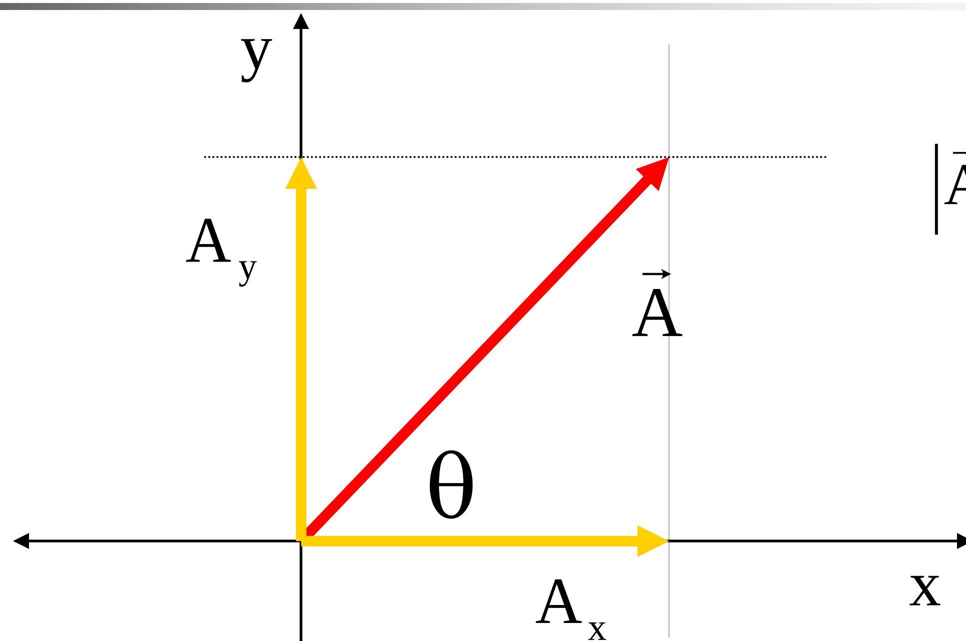
(a) (b)

The x and y axes are always perpendicular to each other. Together,they can be directed at any inclination.

Components of a vector

$$A_x = |\vec{A}|\cos\theta$$

$$A_y = |\vec{A}|\sin\theta$$

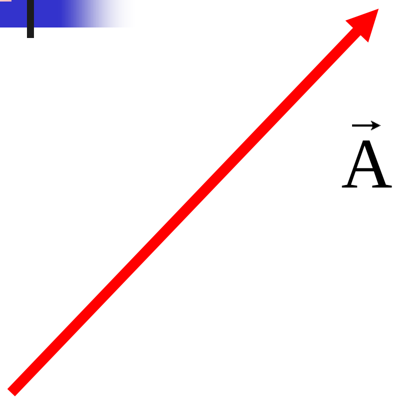


$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

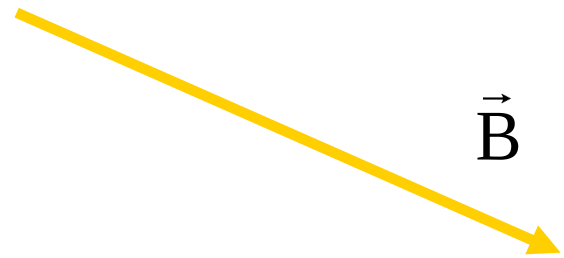
$$\tan\theta = \frac{A_y}{A_x}$$

Using unit vectors: $\vec{A} = A_x\hat{i} + A_y\hat{j}$

Vector Addition using components



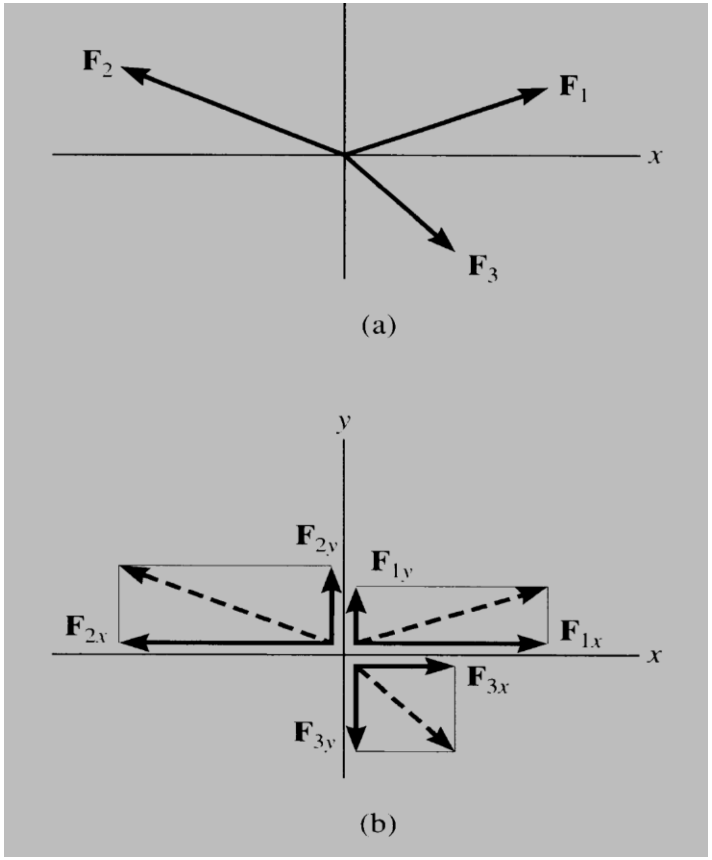
$$\vec{A} + \vec{B}$$



$$\vec{A} = A_x\hat{i} + A_y\hat{j} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j}$$

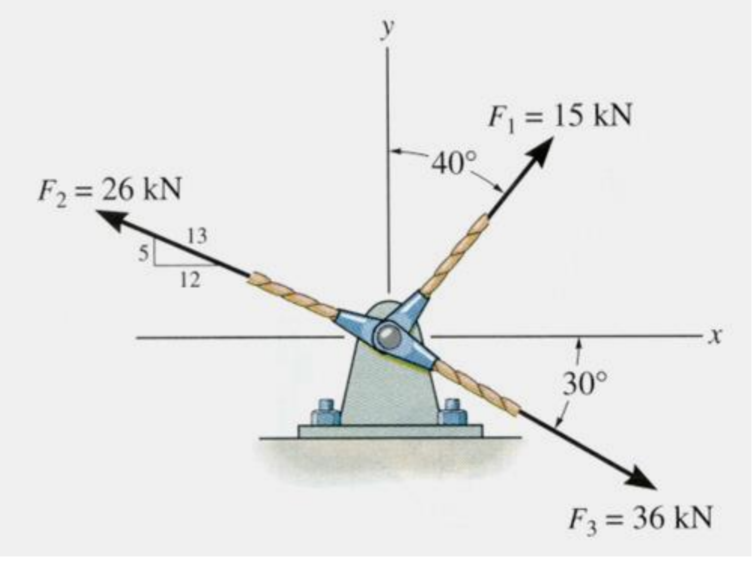
$$\vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

ADDITION OF SEVERAL VECTORS



- Step 1 is to resolve each vector into its components
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

EXAMPLE



Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force.

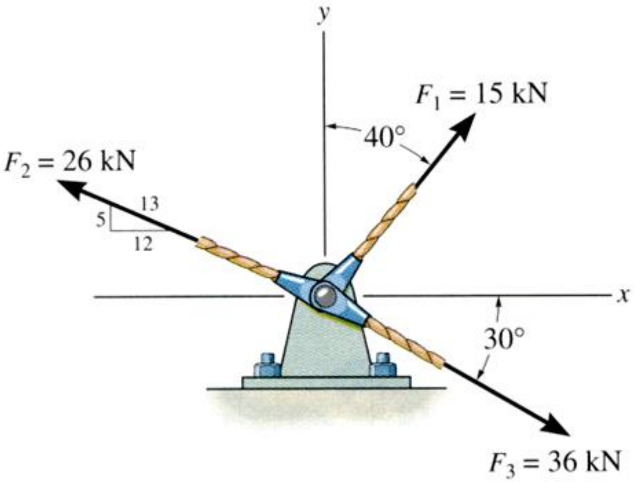
<u>Plan:</u>

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

EXAMPLE (continued)



$\boldsymbol{F_1} = \{ 15 \sin 40° \, \boldsymbol{i} + 15 \cos 40° \, \boldsymbol{j} \}$ kN

$= \{ 9.642 \, \boldsymbol{i} + 11.49 \, \boldsymbol{j} \}$ kN

$\boldsymbol{F_2} = \{ -(12/13)26 \, \boldsymbol{i} + (5/13)26 \, \boldsymbol{j} \}$ kN

$= \{ -24 \, \boldsymbol{i} + 10 \, \boldsymbol{j} \}$ kN

$\boldsymbol{F_3} = \{ 36 \cos 30° \, \boldsymbol{i} - 36 \sin 30° \, \boldsymbol{j} \}$ kN

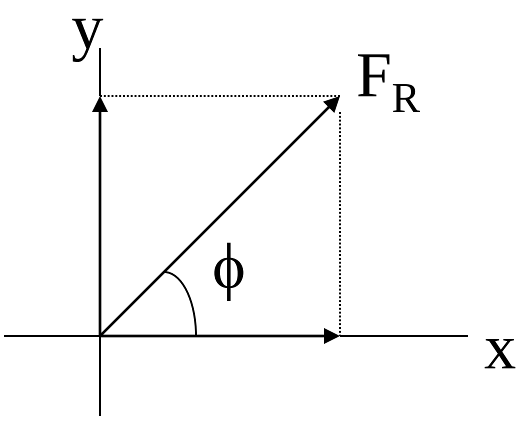
$= \{ 31.18 \, \boldsymbol{i} - 18 \, \boldsymbol{j} \}$ kN

EXAMPLE (continued)

Summing up all the $\boldsymbol{i}$ and $\boldsymbol{j}$ components respectively, we get,

$$\boldsymbol{F_R} = \{ (9.642 – 24 + 31.18)\, \boldsymbol{i} + (11.49 + 10 – 18)\, \boldsymbol{j} \} \text{ kN}$$

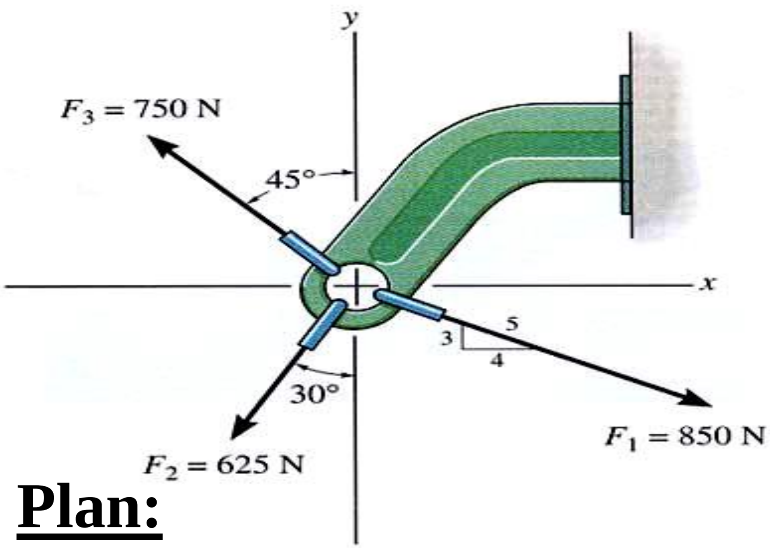
$$= \{ 16.82\, \boldsymbol{i} + 3.49\, \boldsymbol{j} \} \text{ kN}$$



$$F_R = ((16.82)^2 + (3.49)^2)^{1/2} = 17.2 \text{ kN}$$

$$\phi = \tan^{-1}(3.49/16.82) = 11.7°$$

Example



Given: Three concurrent forces acting on a bracket

Find: The magnitude and angle of the resultant force.

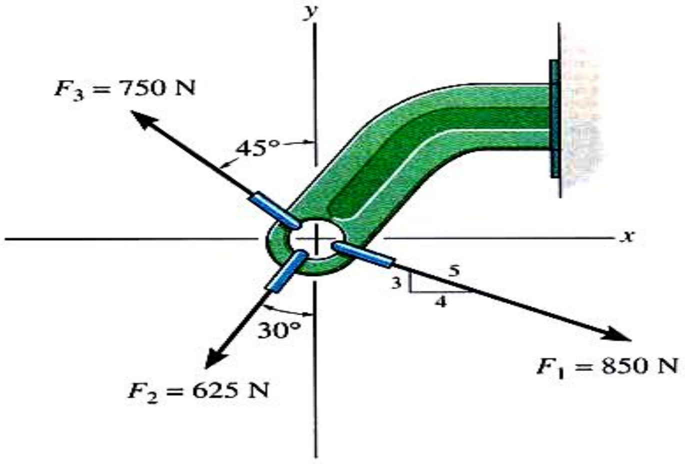
Plan:

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

Example (continued)



$\boldsymbol{F1} = \{ (4/5)\, 850\, \boldsymbol{i} - (3/5)\, 850\, \boldsymbol{j} \}$ N

$= \{ 680\, \boldsymbol{i} - 510\, \boldsymbol{j} \}$ N

$\boldsymbol{F_2} = \{ -625 \sin(30°)\, \boldsymbol{i} - 625 \cos(30°)\, \boldsymbol{j}$

$= \{ -312.5\, \boldsymbol{i} - 541.3\, \boldsymbol{j} \}$ N

$\boldsymbol{F_3} = \{ -750 \sin(45°)\, \boldsymbol{i} + 750 \cos(45°)\, \boldsymbol{j} \}$ N

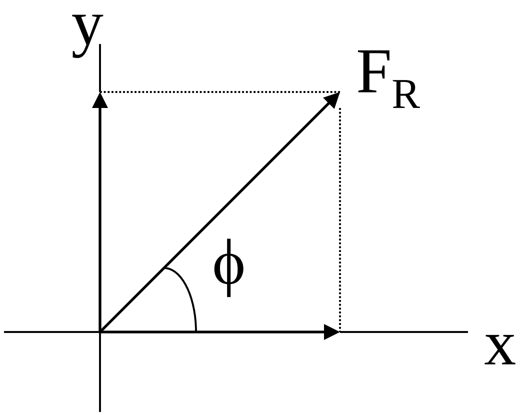
$\{ -530.3\, \boldsymbol{i} + 530.3\, \boldsymbol{j} \}$ N

EXAMPLE (continued)

Summing up all the ***i*** and ***j*** components respectively, we get,

$\boldsymbol{F_R}$ = { ((4/5) 850 – 312.5 + 530.3) ***i*** + (-(3/5) 850 – 541.3 + 530.3) ***j*** } N

= { 897.8 ***i*** – 521 ***j*** } N

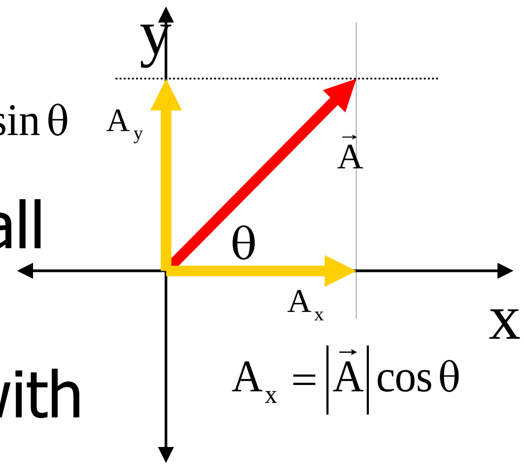


$F_R = ((897.8)^2 + (-521)^2)^{1/2} = 1038\text{ N}$

$\phi = \tan^{-1}(-521/897.8) = -30.1°$

Steps for vector addition

- Select a coordinate system
- Draw the vectors
- Find the x and y coordinates of all vectors
- Find the resultant components with addition and subtraction
- Use the Pythagorean theorem to find the magnitude of the resulting vector
- Use a suitable trig function to find the angle with respect to the x axis



$$A_y = \left|\vec{A}\right| \sin\theta$$

$$A_x = \left|\vec{A}\right| \cos\theta$$

$$\vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$

SCALARS AND VECTORS

	Scalars	Vectors
Examples:	mass, volume	force, velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a "carrot"