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Preface

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1. Introduction

The Banff International Research Station workshop on "Generalizations of de Bruijn Cycles and Gray Codes" was held the 5–9 Dec. 2004. The meeting was an energetic success. We had many wonderful talks, open problem sessions and informal discussions. This workshop was a wonderful opportunity for many researchers in the breadth of this area to meet and exchange ideas. The chance to publish this special *Discrete Mathematics* journal issue dedicated to the workshop was an exciting opportunity.

We felt we would like to use the preface to list and thank the speakers at the workshop, thank all the paper contributers for this issue, and finally give a home to the wonderful pictures of the "landscape" of de Bruijn cycles and their relatives that Hal Fredricksen drew for and displayed at the meeting.

2. Workshop talks

Frank Ruskey, University of Victoria
Title: "Gray Codes, Polyominoes and Distributive Lattices"
Joint work with Stirling Chow.

Aaron Williams, University of Victoria
Title: "A Gray Code that Knuth Missed"

Robert Johnson, Queen Mary University of London
Title: "Long Cycles in the Middle Two Layers of the Cube"

Megan Dewar, Carleton University
Title: "Gray Codes of Block Designs"

Hal Fredricksen, West Point Naval School.
Title: "The Classical de Bruijn Sequence Problem"

Persi Diaconis, Stanford University, **Ronald Graham**, University of California, San Diego
Title: "Some Magic"

Brett Stevens, Carleton University
Title: "The Mathematics of Freedom and Constraint"

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Eduardo Moreno, University de Chile
Title: "de Bruijn Sequences for General Languages"

Anant Godbole,East Tennessee State University
Title: "Birthday problem with Dependence"

Glenn Hurlbert, Arizona State University
Title: "UCycles and Recent Results for $(n - k)$-Subsets of an n-Set"

Brendan McKay, Australian National University
Title: "untitled"

Karel Casteels, University of Waterloo
Title: "U Cycles of n-1 Partitions of n-Set" Joint work with Brett Stevens.

Brad Jackson, San Jose State University
Title: "A Recursive Construction for U Cycles of 2-Subspaces"
Joint work with: Joe Buhler and Ray Mayer.

Carla Savage, North Carolina State University
Title: "Enumeration of Sequences Constrained by the Ratio of Consecutive Parts"
Joint work with: Sylvie Corteel and Sunyoung Lee.

Mark Weston University of Victoria
Title: "Half-Simple Symmetric Venn Diagrams"
Joint work with: Charles E. Killian, Frank Ruskey and Carla D. Savage.

Kevin O'Bryant University of California, San Diego
Title: "The Density of the Outputs of Linear-Shift Registers"
Joint work with: Joshua N. Cooper and Dennis Eichhorn

Joshua Cooper, New York University
Title: "Cycles for Other Shapes of Sliding"

Brett Stevens, Carleton University
Title: "Literary Orderings"

Also attending the workshop were Joe Buhler (Reed College), Tom Roby (California State University) and Julian West (Malaspina University-College).

3. Hal Fredricksen's de Bruijn cycle "Mind Map"

Hal Fredricksen gave a wonderful survey and history of the de Bruijn sequence problem, and his talk included two "mind map" drawings he did of the associations of these problems to other areas of mathematics. The participants all enjoyed the pictures so much that we felt that we would like to give them a home in this special issue. What follows are Hal's pictures and the annotations that he wrote for them.

In Fig. 1 we present a mind map of the relationships between the de Bruijn graph, de Bruijn cycles (also called de Bruijn sequences) which are Hamilton cycles in the graph, and the decomposition of cycles by the weight of the shift register truth tables that provide for their generation. In Fig. 2 we present a more detailed mind map for just the de Bruijn cycles (sequences) with their relationship to complexity, motivation, randomness, algorithms and their history. We give a partial annotation of the mind map:

There are 3 areas of study in de Bruijn Graphs/Cycles shown in Fig. 1:

1. de Bruijn Graphs
2. de Bruijn Cycles (Sequences)
3. The Weight-Cycle Diagram.

The mind map in Fig. 1 shows connections between these topics:

1. Shift registers and Combs (multiple Eulerian Cycles) connect 1 & 2.
2. Cycles in the de Bruijn graph connect 1 & 3
3. de Bruijn sequences form the left boundary of 3
4. Maximum cycle decompositions of 2 from the right boundary of 3.

The graphs (1) are the superposition of adjacency quadruples that connect vertices in the graph via the shift register connections.

There exist many ways to draw the graph and since it rapidly becomes non-planar and complex, these methods (recursive techniques, Best diagram, m-pires, etc) become more important for the graph's study. Graphs are also a representation as a Trellis for Trellis Coding, Viterbi Algorithm, etc. and also provide a nice model of a Markov Process.

Other authors have considered questions of a representation on a higher genus surface (Hales & Butler) and have numbered the graph (Hales & Jewett, Harper). The graph has many cycle decompositions and decompositions into equal length cycles which provide the impetus for several parlor games and puzzles. Other authors have considered the questions of covers and packings of the graph by maximum independent sets.

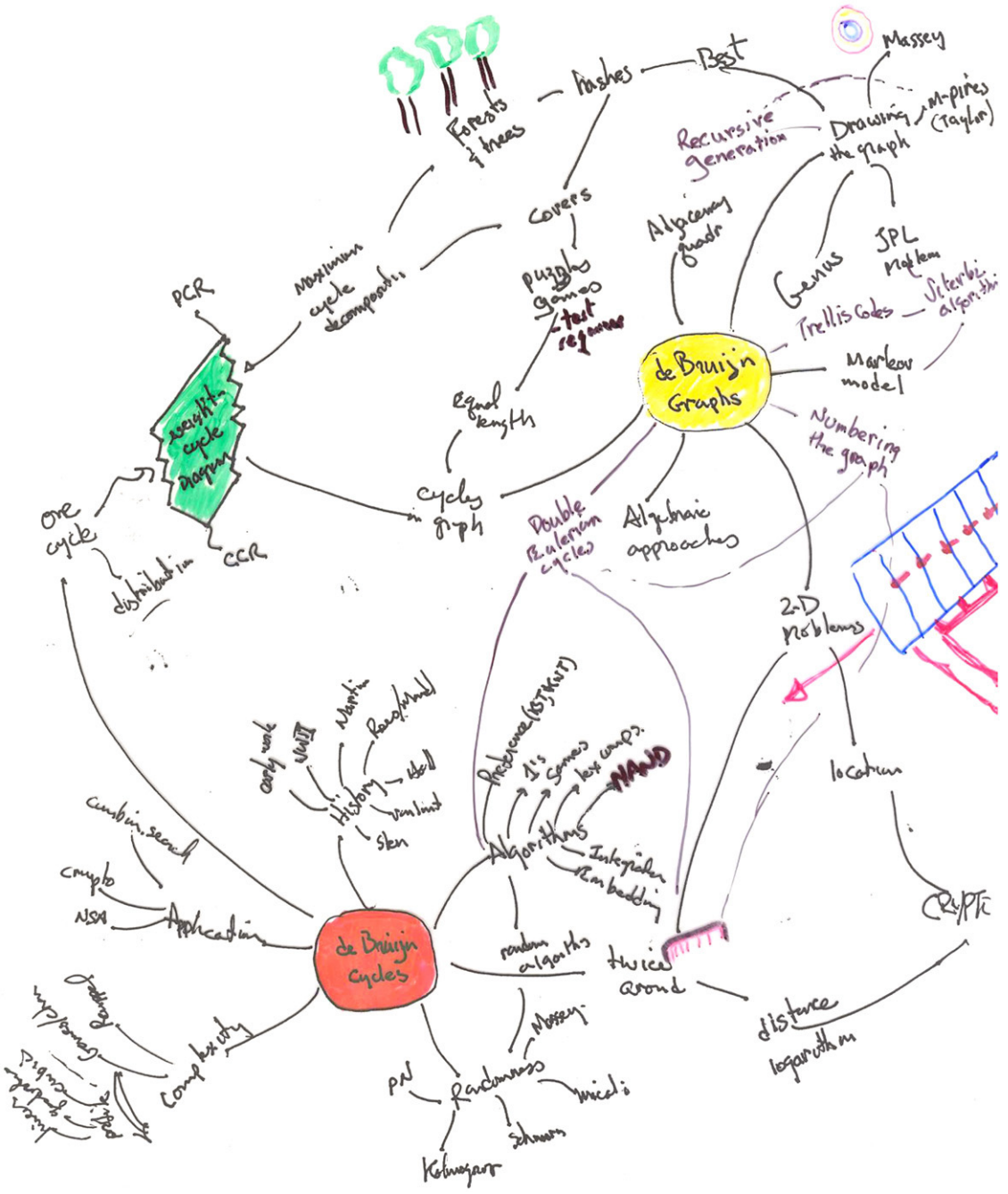

Fig. 1. A mind map of de Bruijn graphs and cycles and cycle weights. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The de Bruijn Cycles (2) have a rich and growing history and is shown in greater detail in the mind map of Fig. 2.

A number of elegant algorithms for their generation have been discovered, dissected and improved, primarily the Greedy (Prefer 1s algorithm) and the Lexicographic Compositions (Prefer Sames algorithm). Some algorithms relate to generation by shift registers (a connection between 1 & 2) and various cycle joining algorithms, e.g. Roth. Some are elegant combinatorial algorithms — Greedy, prefer 1s, prefer sames, lexicographic compositions, and those of Ralston and Lempel & Etzion as well as generalizations such as the key sequence theorem of Golomb & Welch.

Properties of randomness make the de Bruijn sequences attractive especially their pseudo-random property and the complexity measures on sequences studied by Kolmogorov/Chaitin, Schnorr, Micali, Maurer, etc. Complexity of the sequences has been studied by a number of authors including Ziv-Lempel, Reuppel, Games, Chan & Key, and the k-ary complexity profile offers an intriguing possibility for the analysis of these sequences

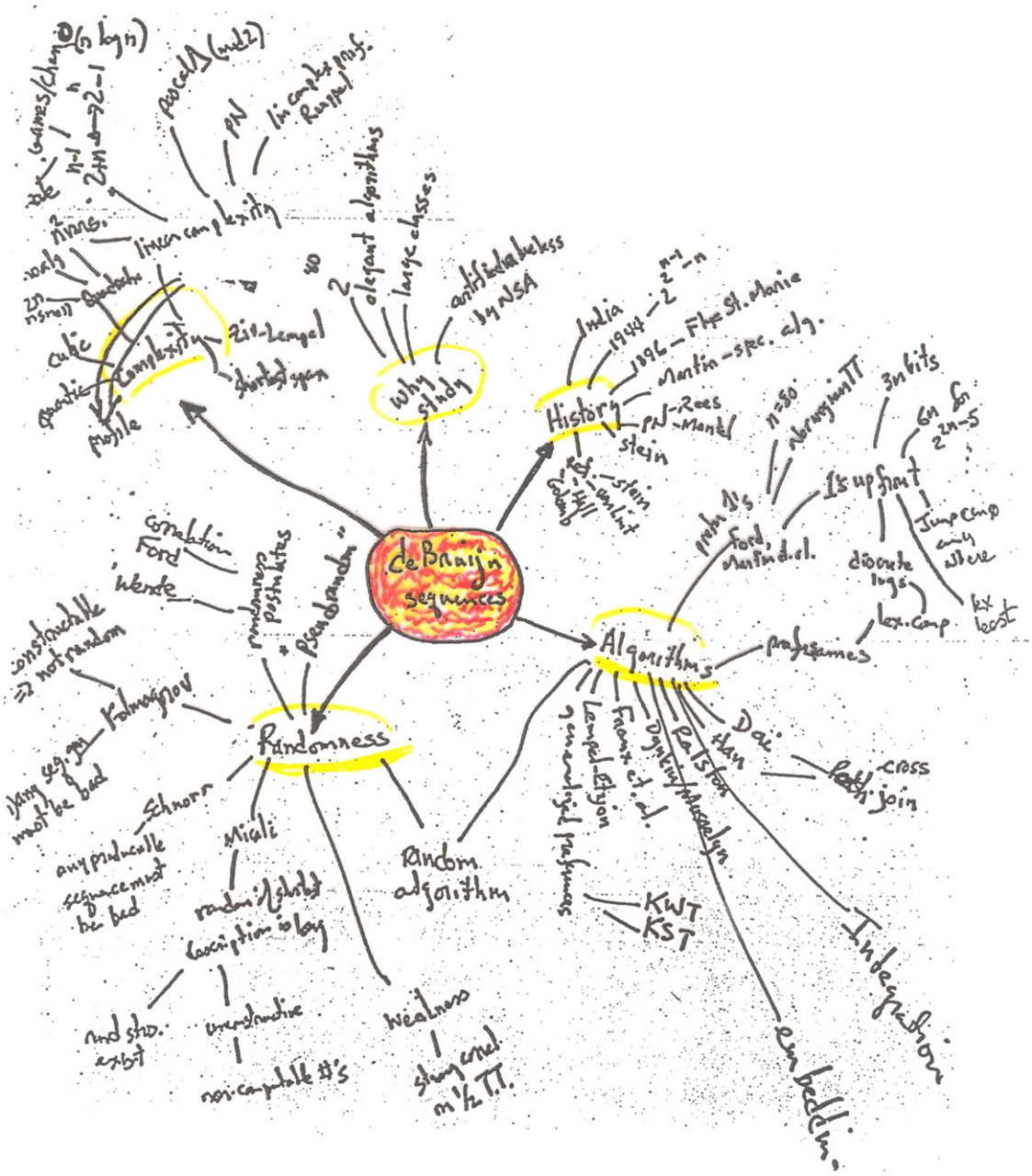

Fig. 2. A mind map of de Bruijn cycles (Sequences). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4. Conclusion

This issue contains some wonderful and diverse articles and retains a strong connection to the workshop. Several of the papers are the final, polished, products of research presented at the workshop (Casteels and Stevens; Chow and Ruskey; Hurlbert, Johnson and Zahl; Jackson, Buhler and Mayer; and, Ruskey and Williams.). These papers also include new results inspired by the discussion and talks at the workshop: an elegant solution of a seventeen year old conjecture (Johnson) and a paper related to Venn diagrams (Jiang and Savage); and an update on the middle-levels problem (Shields, Shields and Savage).

We would like to thank the Banff International Research Station for Mathematical Innovation and Discovery and its' sponsors, NSERC, NSF, CONACYT, Alberta Innovation and Science, PIMS, MSRI, MITACS and IM UNAM for hosting and funding the workshop and for making the excellent BIRS program happen. We thank all the authors who submitted manuscripts to this issue. We wish to thank Elsevier, *Discrete Mathematics* and especially editors Wayne Goddard and Douglas West for helping to make this special issue into a reality. We would also like to remember Prof. Peter Hammer.