

Linear Optimization

The Simplex Workbook

DRAFT

STUDENT VERSION

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Preface

A little explanation is in order for our choice of the title *Linear Optimization* (and corresponding terminology) for what has traditionally been called *Linear Programming*. We feel that the word *programming* in this context can be confusing and/or misleading to students. Linear programming problems are referred to as optimization problems but the general term linear programming remains. This can cause people unfamiliar with the subject to think that it is about programming in the sense of writing computer code. It isn't. This workbook is about the beautiful mathematics underlying the ideas of optimizing linear functions subject to linear constraints and the algorithms used to solve such problems. In particular, much of what we discuss is the mathematics of simplex algorithm for solving such problems, developed by George Dantzig in the late 1940's.

The word *program* in linear programming is a historical artifact. When George Dantzig first developed the Simplex method to solve what are now called linear programming problems his initial model was a class of resource allocation problems to be solved for the US Air Force. The decisions about the allocations were called 'Programs' by the Air Force, and hence the term. The article 'Linear Programming' by George Dantzig (Operations Research, 50 (2002), 42–47) is a fascinating description of the origins of this subject written by the person who originated many of the ideas. Included is a description of how T.J. Koopmans (who won a Nobel Prize in economics for his work in decision science) suggested shortening Dantzig's description 'programming in a linear structure' to 'linear programming' during a walk on the beach with Dantzig. Also included is a note that, at the time, 'code' was the word used for computer instructions and not 'program'.

To be clear to potential students and students in the class that this is a mathematics class requiring a background in writing proofs and not a computer coding class we prefer the terminology linear optimization. We do look at computer algorithms but focus on the underlying mathematics. A small amount of computer coding (for example, simple MAPLE) programs will be very useful, but writing code is not the central purpose of the course.

The shorthand LP has been used to refer to both the general subject of linear programming as well as specific instances of linear programs. To distinguish, in our notation, LO (linear optimization) refers the general class of optimizing linear functions subject to linear constraints while LOP (linear optimization problem) refers to specific instances of such problems. Furthermore, using the optimization term brings the subject in line with other, closely related fields that are increasingly called Optimization (Nonlinear, Quadratic, Convex, Integer, Combinatorial).