



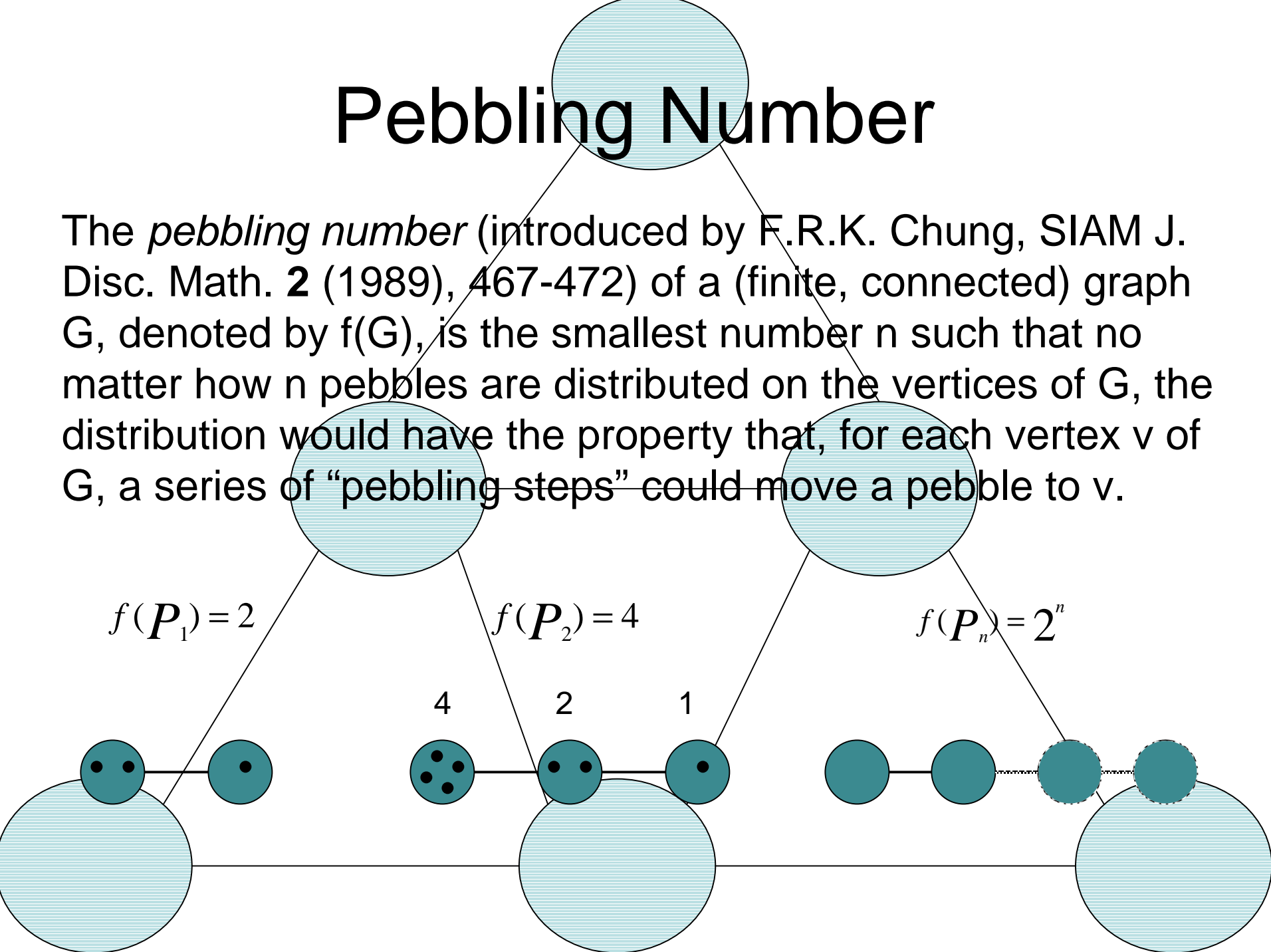
Deep Graphs

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The University of Louisiana at Monroe

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Emerging Scholars Program

Pebbling Number

The *pebbling number* (introduced by F.R.K. Chung, SIAM J. Disc. Math. **2** (1989), 467-472) of a (finite, connected) graph G , denoted by $f(G)$, is the smallest number n such that no matter how n pebbles are distributed on the vertices of G , the distribution would have the property that, for each vertex v of G , a series of “pebbling steps” could move a pebble to v .



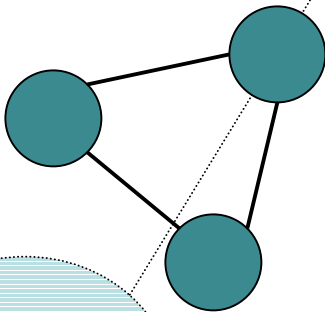
Deep Graph

A graph G is called *deep* (developed by Dr. Andrew J. Hetzel) if, for each positive integer $n \leq f(G)$, there exists a subgraph H of G such that $f(H) = n$.

- No path with more than 2 vertices is a deep graph.

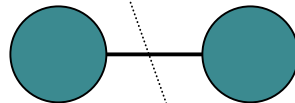
Deep Graph K_3

$$f(K_3) = 3$$



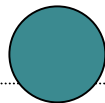
Subgraphs of K_3

P_1



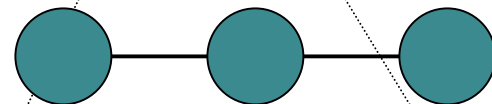
$$f(P_1) = 2$$

$$f(V) = 1$$



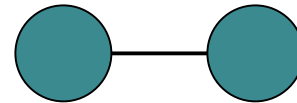
P_2 is not a deep graph

$$f(P_2) = 4$$

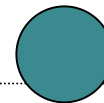


Subgraphs of P_2

$$f(P_1) = 2$$



$$f(V) = 1$$

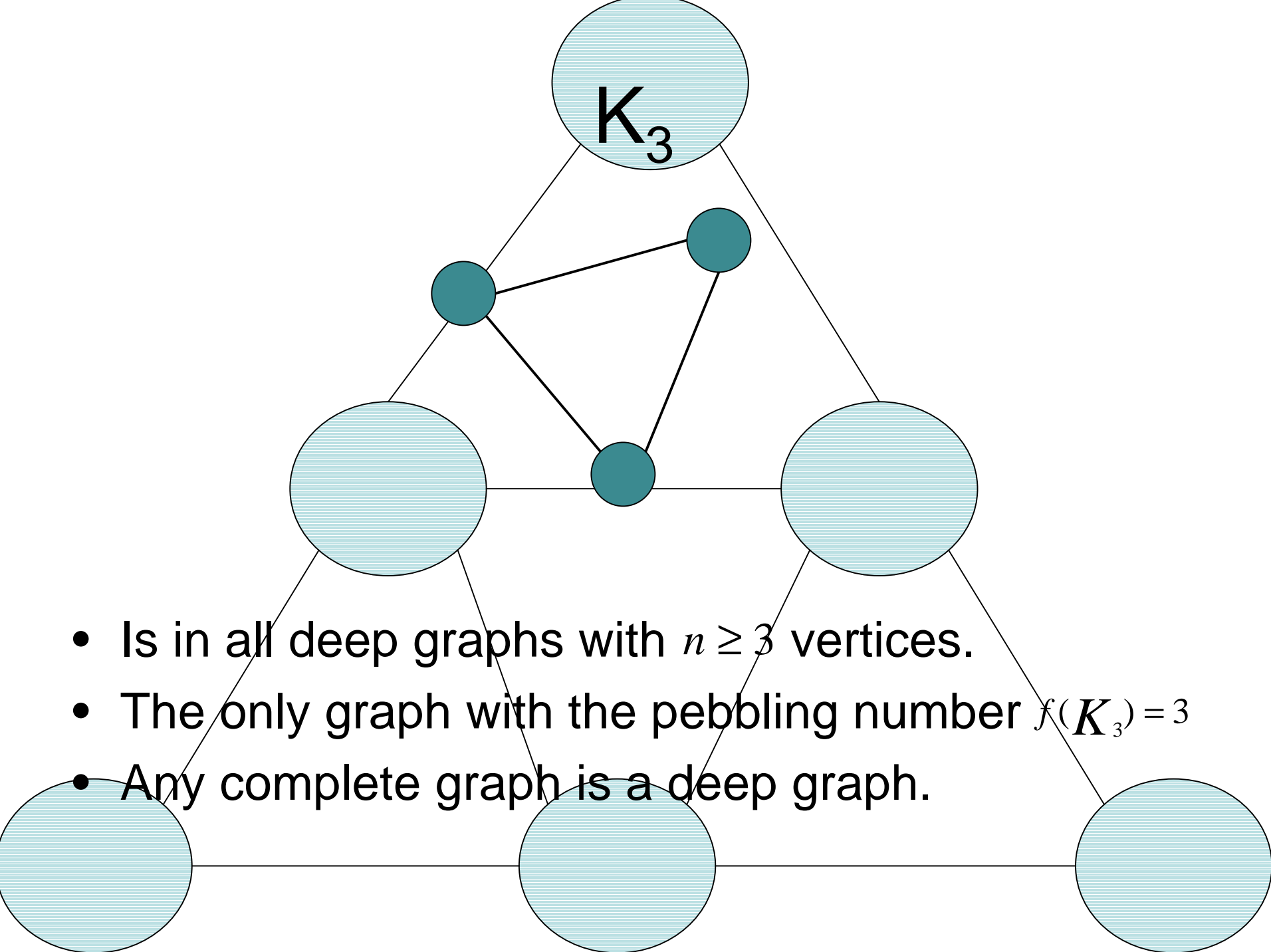


Applications



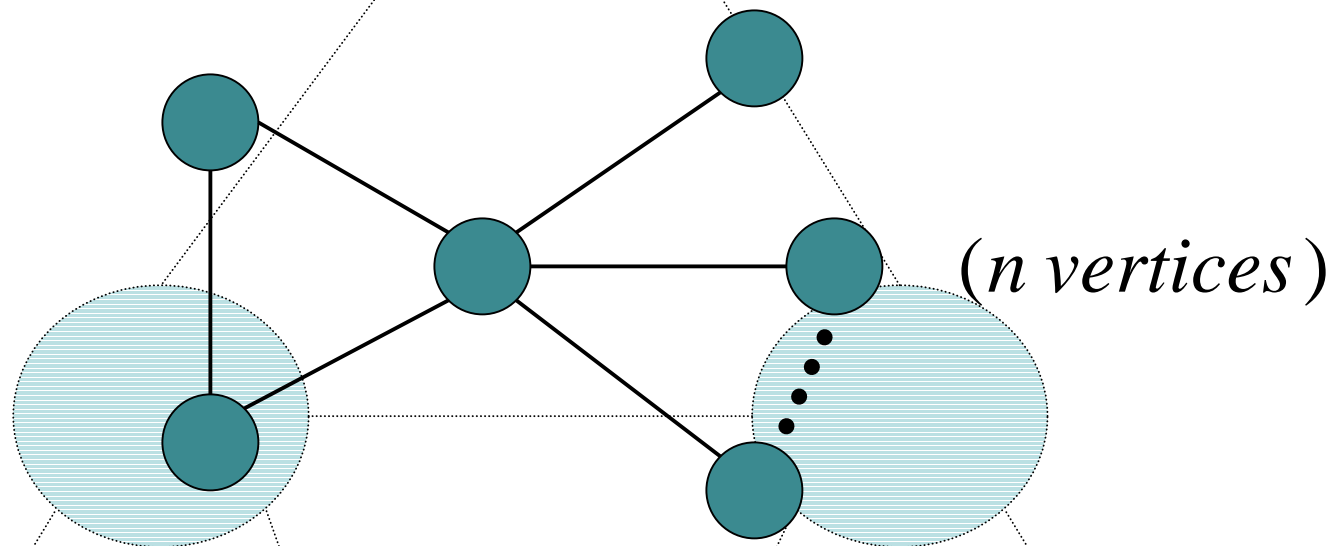
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graph TD; A((Applications)) --- B(( )); A --- C(( )); B --- D(( )); C --- E(( )); D --- F(( )); E --- G(( )); F --- H(( ))
```

- Networking
- Information Transfer
- Military



- Is in all deep graphs with $n \geq 3$ vertices.
- The only graph with the pebbling number $f(K_3) = 3$
- Any complete graph is a deep graph.

Graph G_n

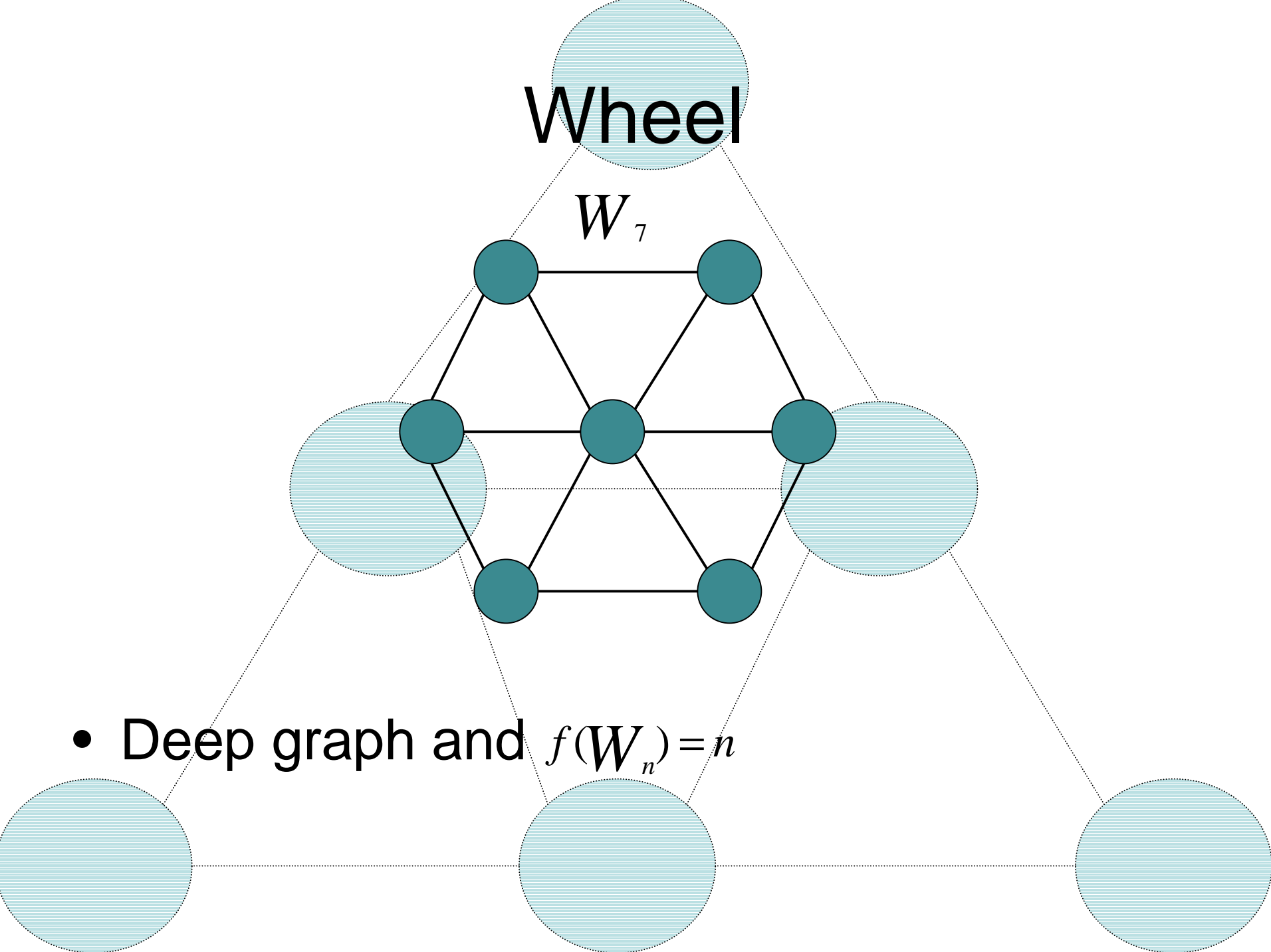


- Pebbling number $f(G_n) = n + 1, n \geq 4$
- G_n is a deep graph

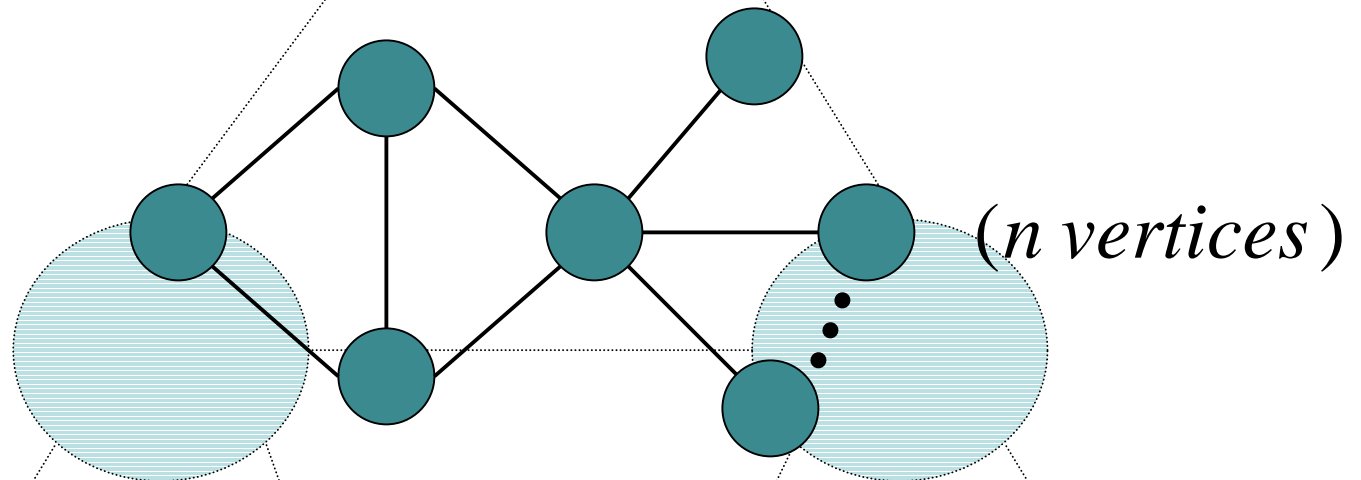
Wheel

W_7

- Deep graph and $f(W_n) = n$

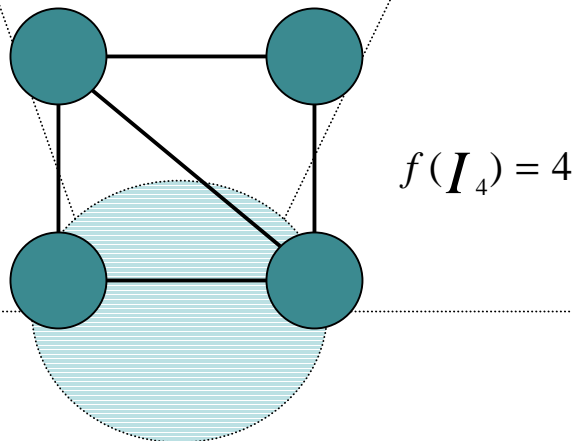
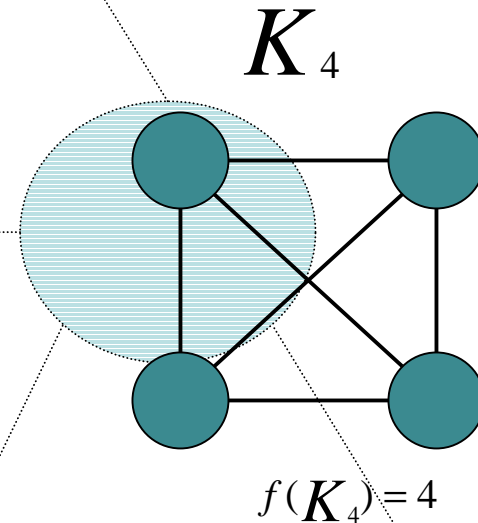
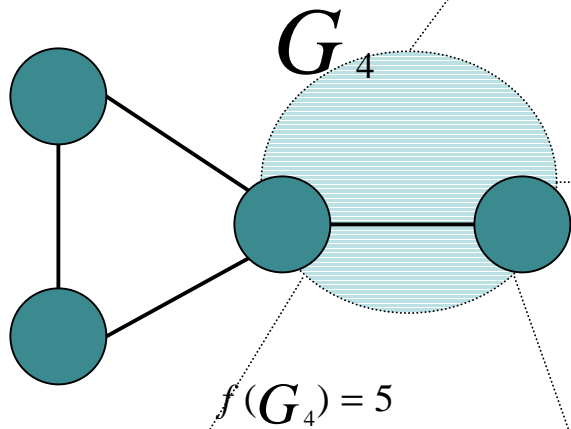


Graph H_n

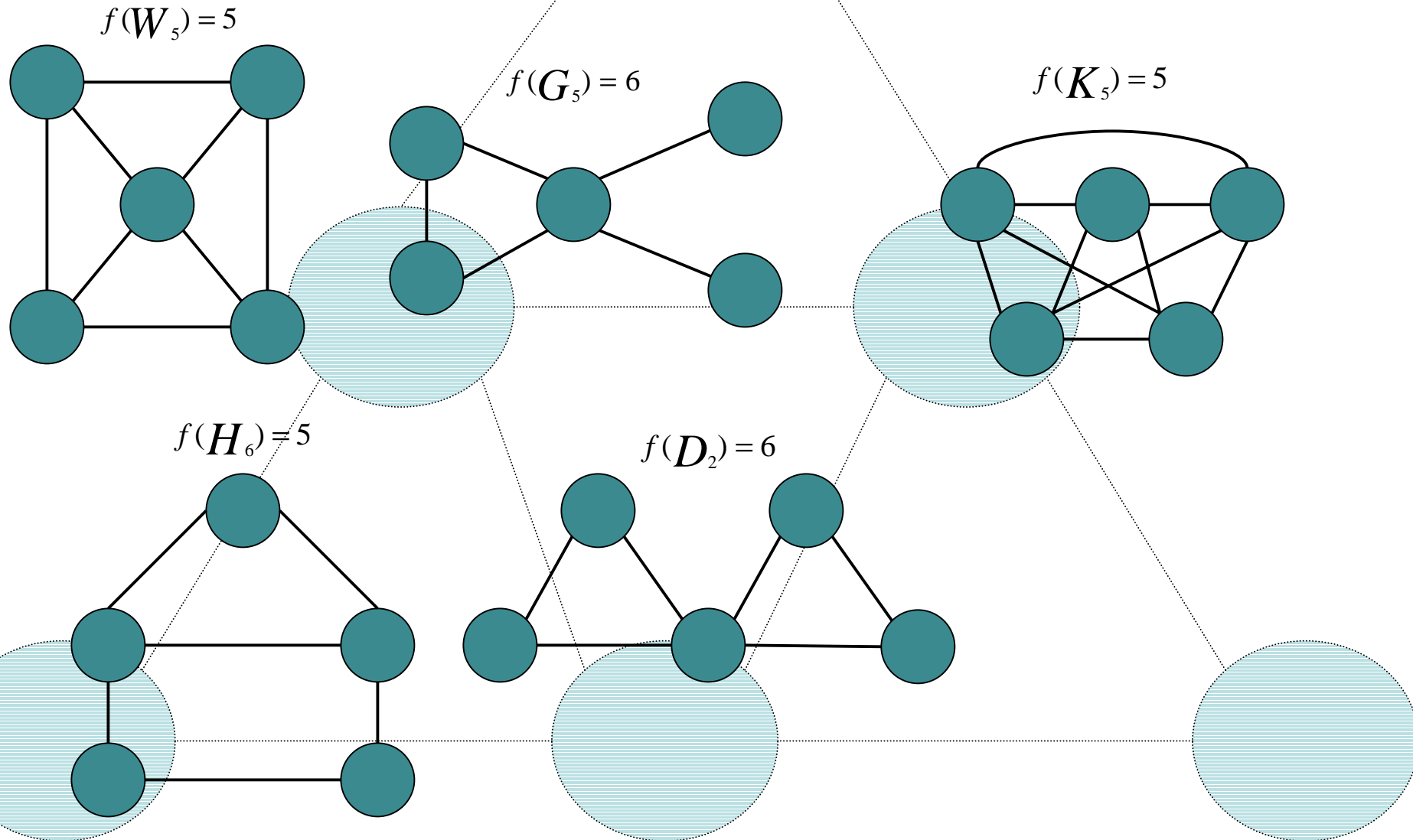


- Pebbling number $f(H_n) = n + 3$
- Graph H_n is a deep graph when $n \geq 7$
- Graph has a diameter of 3.

Classification of all deep graphs with 4 vertices

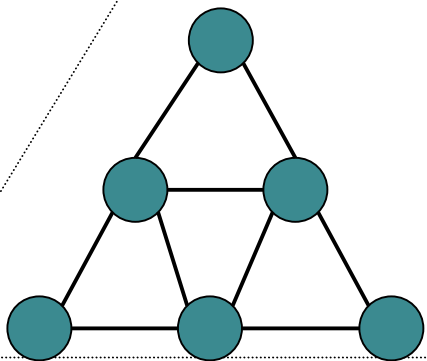


Some deep graphs with 5 vertices

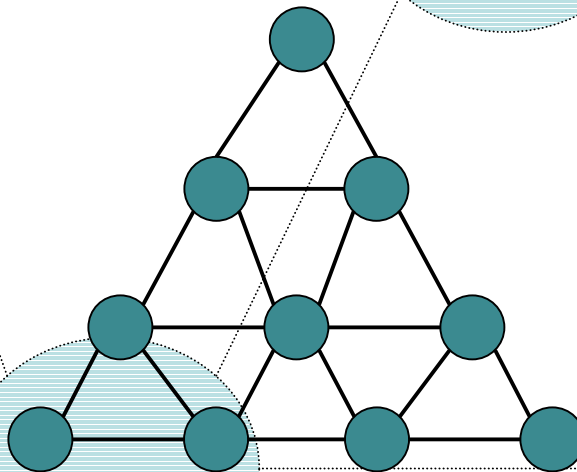


Open problems

- Deep Graphs with a diameter ≥ 4 ?
- What graphs other than K_3 must be in a deep graph?
- Pyramid with 6 vertices is a deep graph. Is a pyramid with more than 6 vertices a deep graph?



E_6 is deep.



Is E_{10} deep?