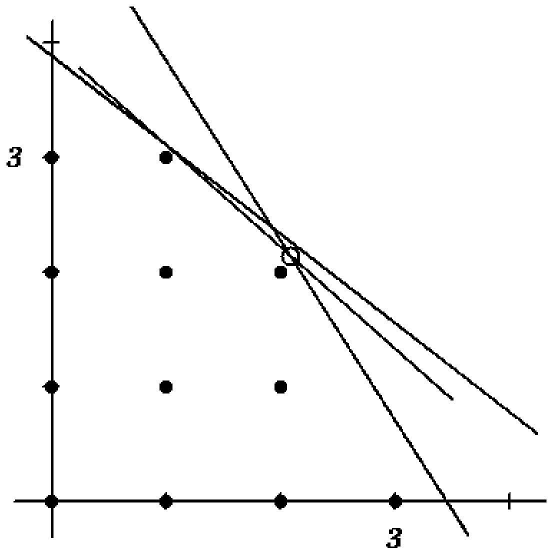

Figure 13.3: First cut for Problem 13.1.2

Since $x_3, x_4 \geq 0$, the right hand side of (13.1) is at most 47 and, in particular, strictly less than 50. Moreover, it is a multiple of 50, and so is at most 0. Hence we know that every feasible integer solution satisfies

$$-9x_3 - 43x_4 \;\leq\; -47\ , \tag{13.2}$$

and so we add this valid constraint to our system. Note that we do not need to solve the revised problem from scratch; let's use a hot start instead. We simply introduce a new slack variable x_5 (with the same basic coefficient of 50) into (13.2) and slide the resulting equality into Tableau 13.1.5 to obtain the following.

Tableau 13.1.6

$$\left[\begin{array}{rr|rrrr|r} 50 & 0 & 9 & -7 & 0 & 0 & 97 \\ 0 & 50 & -7 & 11 & 0 & 0 & 119 \\ 0 & 0 & -9 & -43 & 50 & 0 & -47 \\ \hline 0 & 0 & 34 & 18 & 0 & 50 & 1922 \end{array}\right]$$

Two pivots return the tableau to its initial, standard form in Tableau 13.1.7, revealing the new cut in terms of the decision variables (see Figure 13.3): $8x_1 + 9x_2 \leq 36$. We call this the **decision form** (as opposed to the **original form**) of the valid inequality or cutting plane.

decision/ original form

Tableau 13.1.7

$$\left[\begin{array}{rr|rrrr|r} 11 & 7 & 1 & 0 & 0 & 0 & 38 \\ 7 & 9 & 0 & 1 & 0 & 0 & 35 \\ 8 & 9 & 0 & 0 & 1 & 0 & 36 \\ \hline -10 & -8 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Of course, we don't want to pivot in this direction; we'd rather use the Dual Simplex algorithm (see Section 12.4) on Tableau 13.1.6. To pivot in the negative row, we consider the c-ratios $-34/9$

and $-18/43$ and choose the one closest to zero. The pivot $4 \mapsto 5$ results in the temporarily optimal (optimal for the relaxed problem) tableau below.

Tableau 13.1.8

$$\left[\begin{array}{rr|rrrr|r} 43 & 0 & 9 & 0 & -7 & 0 & 90 \\ 0 & 43 & -8 & 0 & 11 & 0 & 92 \\ 0 & 0 & 9 & 43 & -50 & 0 & 47 \\ \hline 0 & 0 & 26 & 0 & 18 & 43 & 1636 \end{array}\right]$$

Since the relaxed optimal solution $\mathbf{x} = (90, 92 \mid 0, 47, 0))^{\mathsf{T}}/43$ is again non-integral, we employ Gomory's trick again. We write $43x_2 - 8x_3 + 11x_5 = 92$ as $43(x_2 - x_3 - 2) = 6 - 35x_3 - 11x_5$, and derive the valid inequality

$$-35x_3 - 11x_5 \ \leq \ -6 \tag{13.3}$$

from the knowledge that $6 - 35x_3 - 11x_5$ is a multiple of 43 that is less than 43, and hence at most zero. As before, we slip this and the new slack into Tableau 13.1.8 (WebSim is really great for this — even better if you plan ahead by putting a handful of $0 \leq 0$ constraints in the original tableau) and follow Dual Simplex. Here we have Tableau 13.1.9, followed by Tableau 13.1.10.

Tableau 13.1.9

$$\left[\begin{array}{rr|rrrrr|r} 43 & 0 & 9 & 0 & -7 & 0 & 0 & 90 \\ 0 & 43 & -8 & 0 & 11 & 0 & 0 & 92 \\ 0 & 0 & 9 & 43 & -50 & 0 & 0 & 47 \\ 0 & 0 & -35 & 0 & -11 & 43 & 0 & -6 \\ \hline 0 & 0 & 26 & 0 & 18 & 0 & 43 & 1636 \end{array}\right]$$

Tableau 13.1.10

$$\left[\begin{array}{rr|rrrrr|r} 35 & 0 & 0 & 0 & -8 & 9 & 0 & 72 \\ 0 & 35 & 0 & 0 & 11 & -8 & 0 & 76 \\ 0 & 0 & 0 & 35 & -43 & 9 & 0 & 37 \\ 0 & 0 & 35 & 0 & 11 & -43 & 0 & 6 \\ \hline 0 & 0 & 0 & 0 & 8 & 26 & 35 & 1328 \end{array}\right]$$

Workout 13.1.11 *Confirm that the new cut is $11x_1 + 8x_2 \leq 40$, as shown in Figure 13.4.*

It's difficult to tell whether anything was shaved off, but indeed there was. It may take forever if we can barely see the cuts, but be patient, some cuts are big and some are small.

Workout 13.1.12 *Use MAPLE to draw the current feasible region.*

Once again the relaxed optimum is not integral, so we need to find another cut. Before doing so, however, we should pause to notice a pattern in the tableaux that could save us the trouble of thinking. Consider an entry $T_{r,j}$, other than the basic coefficient d, of the new row r in a tableau T that corresponds to a newly determined cut derived from row i of T. By the manner in which we discovered the cut, $T_{i,j} + T_{r,j}$ equals the greatest multiple of d that is at most $T_{i,j}$. That is, $T_{r,j} = -(T_{i,j} \mod d)$. (Careful — this is different from $-T_{i,j} \mod d$: e.g. $-(2 \mod 7) = -2$, while $-2 \mod 7 = 5$.)

We should also take note of some of the decisions we've made so far. As we found in defining pivotting rules (least subscript, etc.), we have choices here that may matter none in theory but may matter some in practice. For example, at present, x_k is non-integral for each $k \in \{1, 2, 3, 4\}$, so we