

1. VOEVODSKY'S CONNECTIVITY THEOREM FOR \mathbb{P}^1 -SPECTRA

The following is from an email from Aravind to me concerning the proof of the connectivity theorem for \mathbb{P}^1 spectra.

Most of the proof appears in Morel's Trieste notes (in the section on the homotopy t-structure for P^1 -spectra). First you prove a connectivity result for P^1 -stable homotopy sheaves by using Morel's S^1 -stable connectivity theorem and studying what happens under G_m loops and G_m -suspension: suspension preserves connectivity, and Morel shows that taking G_m -loops has the effect of making a "contraction". Then, you globalize this.

Our goal is to prove theorem 4.14 of [Voev98], which should be restated in terms of \mathbb{P}^1 -spectra.

Theorem 1.1. Let X be a pointed smooth scheme over $\mathrm{Spec}(k)$ where k is an infinite field. Let \mathcal{Y} be a pointed space. Then for any $n > \dim(X)$, and any integer m

$$\mathcal{SH}(k)(\Sigma^\infty X, S^n \wedge \mathbb{G}_m^m \wedge \Sigma^\infty \mathcal{Y}) = 0.$$

Remark 1. We can formulate the theorem by using homotopy sheaves. Indeed, if we can show that for any pointed space \mathcal{Y} the homotopy sheaves $\pi_{n+\alpha m}^{\mathbb{A}^1}(\mathcal{Y})$ are -1 connected, the result will follow, as

$$\begin{aligned} \mathcal{SH}(k)(\Sigma^\infty X, S^n \wedge \mathbb{G}_m^m \wedge \Sigma^\infty \mathcal{Y}) &= \mathcal{SH}(k)(S^{-n} \wedge \mathbb{G}_m^{-m} \wedge \Sigma^\infty X, \Sigma^\infty \mathcal{Y}) \\ &= \pi_{-n-m\alpha}^{\mathbb{A}^1}(X). \end{aligned}$$

and by the connectivity theorem, this will vanish whenever $-n < 0$, i.e., whenever $n > 0$. This is a stronger statement than [?, theorem 4.14].

The line of attack is then to show that for a pointed space \mathcal{Y} , the S^1 suspensions spectrum $\Sigma_s^\infty \mathcal{Y}$ in the stable model category is -1 connected. Then we establish the first connectivity theorem which ensures the \mathbb{A}^1 localization of $\Sigma_s^\infty \mathcal{Y}$ is -1 connected. Finally, we show that inverting \mathbb{G}_m does not affect the connectivity, i.e., $\Sigma_m^\infty \Sigma_s^\infty \mathcal{Y}$ is again -1 connected.

2. ASSUMPTIONS FROM PREVIOUS LECTURES

2.1. Facts about Nisnevich topology. Points and neighborhoods. Distinguished squares determine the topology. Relation to étale, zariski, and fpqc topology.

Proposition 2.1. [Mor04, 2.4.1] Let M be a sheaf of abelian groups on Sm/k , and let $X \in \mathrm{Sm}/k$ with Krull dimension d . Then whenever $n > d$, $H_{Nis}^n(X; M) = 0$.

Proposition 2.2. [Mor04, 2.4.1] For any $X \in \mathrm{Sm}/k$, and for any $x \in X(k)$, there is an isomorphism of pointed sheaves of sets in the Nisnevich topology

$$X/(X - \{x\}) \cong \mathbb{A}^n/(\mathbb{A}^n - \{0\}).$$

2.2. Unstable model category $\Delta^{op}\mathrm{Shv}(\mathrm{Sm}/k, Nis)$.

- (1) Don't forget the adjunction for sheafification $a_{Nis} \dashv U$. i.e., to give a map $a_{Nis}\mathcal{X} \rightarrow \mathcal{Y}$, of sheaves, it is equivalent to just give a map $\mathcal{X} \rightarrow U\mathcal{Y}$.
- (2) Give the unstable model category the injective model structure to start. (This is Morel's choice, so we stick with it)
- (3) Weak equivalences: $\mathcal{X} \rightarrow \mathcal{Y}$ iff for all U the map $\mathcal{X}(U) \rightarrow \mathcal{Y}(U)$ a simplicial w.e.

- (4) Cofibs: $\mathcal{X} \rightarrow \mathcal{Y}$ is a cofib. iff for any $U \in \text{Sm}/k$ the map $\mathcal{X}(U) \rightarrow \mathcal{Y}(U)$ is a monomorphism.
- (5) Fibrations: what they need to be.
- (6) The ass. htpy. cat. to the injective model category structure is denoted $\mathcal{H}_s(k)$. The pointed htpy. cat. is denoted $\mathcal{H}_{s,\bullet}(k)$.
- (7) Important properties/constructions in these categories: Cartesian closed, i.e., have smash/tensor \times and \wedge , with right adjoints $\underline{\text{Hom}}$ and $\underline{\text{Hom}}_\bullet$. Representable sheaf functor and constant simplicial set functor.

Definition 2.3. Internal hom. (Ref: P. Pelaez)

Let $U \in \text{Sm}/k$. Let Δ_U^n denote the simplicial sheaf given by

$$(V, m) \in \text{Sm}/k \times \Delta^{op} \rightarrow \text{Sm}/k(V, U) \times \Delta_m^n.$$

In other words, $\Delta_U^n = (rU) \times c\Delta^n$.

Let \mathcal{X} and \mathcal{Y} be spaces. The internal hom in the unpointed category is given by the formula

$$(U, m) \in \text{Sm}/k \times \Delta \rightarrow \text{Hom}_{\Delta^{op}\text{Shv}}(X \times \Delta_U^m, Y).$$

How to describe the adjunction?

$$\text{Hom}(\mathcal{X} \times \mathcal{Y}, \mathcal{Z}) \cong \text{Hom}(\mathcal{Y}, \underline{\text{Hom}}(\mathcal{X}, \mathcal{Z}))$$

Certainly we may send a map $g \in \text{Hom}(\mathcal{Y}, \underline{\text{Hom}}(\mathcal{X}, \mathcal{Z}))$ to the map $\eta(g)$ given by

$$\begin{aligned} \eta(g)(U, n) : \mathcal{X}_n(U) \times \mathcal{Y}_n(U) &\rightarrow \mathcal{Z}_n(U) \\ (a, b) &\rightarrow g(U, n)(U, n)(a, \text{id}, \text{id}). \end{aligned}$$

Why is this a bijection?

Definition 2.4. Internal hom in the pointed category. Consider \mathcal{X} and \mathcal{Y} pointed spaces. For a point $x \in \mathcal{X}$, there is an evaluation map $ev_x : \underline{\text{Hom}}(\mathcal{X}, \mathcal{Y}) \rightarrow \mathcal{Y}$, where at $(U, n) \in (\text{Sm}/k \times \Delta)^{op}$ we send $g : \mathcal{X} \times \Delta_U^n \rightarrow \mathcal{Y}$ to $g(U, n)(x, \text{id}, \text{id}) \in \mathcal{Y}_n(U)$. This makes sense as we have the map

$$g(U, n) : \mathcal{X}_n(U) \times \text{Sm}/k(U, U) \times \Delta_n^n \rightarrow \mathcal{Y}_n(U).$$

The pointed internal hom $\underline{\text{Hom}}_\bullet(\mathcal{X}, \mathcal{Y})$ is the fiber $ev_x^{-1}(y)$.

2.3. \mathbb{A}^1 localization.

- (1) See [?, Prop 2.3.3.] for details on the various properties of fibrant objects in the unstable motivic category.
- (2)

Definition 2.5. A space \mathcal{X} is called \mathbb{A}^1 local if for any smooth scheme U , the canonical map

$$\text{Hom}(rU, \mathcal{X}) \rightarrow \text{Hom}(rU \times \mathbb{A}^1, \mathcal{X})$$

is a bijection.

- (3)

Definition 2.6. A map $f : \mathcal{X} \rightarrow \mathcal{Y}$ is an \mathbb{A}^1 weak equivalence if

$$\text{Hom}(\mathcal{Y}, \mathcal{Z}) \rightarrow \text{Hom}(\mathcal{X}, \mathcal{Z})$$

is a bijection for every \mathbb{A}^1 local space \mathcal{Z} .

- (4) The unstable motivic homotopy category is obtained by left Bousfield localization of the injective model category structure on spaces with respect to the class of maps $W = W_{\mathbb{A}^1} = \{U \times \mathbb{A}^1 \rightarrow U \mid U \in \mathbf{Sm}/k\}$.
- (5) The general theory of Bousfield localization then gives a localization functor $L_{\mathbb{A}^1} : \mathcal{H}_s(k) \rightarrow L_W \mathcal{H}_s(k)$; denote the category $L_W \mathcal{H}_s(k)$ by $\mathcal{H}(k)$. The localization functor sends \mathbb{A}^1 weak equivalences to isomorphisms.

See [?, Definition 3.3.1]. Keep the same underlying category, the weak equivalences are the \mathbb{A}^1 -local weak equivalences, do not change the cofibrations from the ones in the injective model structure on $\Delta^{op}\mathbf{Shv}(\mathbf{Sm}/k, \mathbf{Nis})$, and let the fibrations be what they need to be.

- (6) Morel writes $L_{\mathbb{A}^1}$ for the (derived) functor which sends a space (without base point) \mathcal{X} to an \mathbb{A}^1 -localization.
- (7) Morel gives an explicit construction which takes a pointed space \mathcal{Y} and produces a space $L^\infty \mathcal{Y}$ which is \mathbb{A}^1 local and a map $\mathcal{Y} \rightarrow L^\infty \mathcal{Y}$ which is an \mathbb{A}^1 weak equivalence.

Let \mathcal{Y}_f be the functorial fibrant replacement of \mathcal{Y} with respect to the injective model structure. Consider \mathbb{A}^1 to be pointed at 0. Morel defines $L^{(1)}(\mathcal{Y})$ to be the cone of the map $ev_1 : \underline{Hom}_\bullet(\mathbb{A}^1, \mathcal{Y}_f) \rightarrow \mathcal{Y}_f$. So there is a map $\mathcal{Y} \rightarrow L^{(1)}(\mathcal{Y})$ obtained from the trivial cofibration $\mathcal{Y} \rightarrow \mathcal{Y}_f$ and the defining map $\mathcal{Y}_f \rightarrow L^{(1)}(\mathcal{Y})$.

So $L^{(1)}(-)$ is a functor with a natural transformation $\eta : \text{id} \rightarrow L^{(1)}(-)$. Define by induction $L^{(n)}(\mathcal{Y}) = L^{(1)}(L^{(n-1)}(\mathcal{Y}))$. There is thus a directed system $L^{(n-1)}(\mathcal{Y}) \rightarrow L^{(n)}(\mathcal{Y})$. Denote the direct limit/hocolim of this directed system by $L^\infty(\mathcal{Y})$.

Proposition 2.7. The natural morphism $\mathcal{Y} \rightarrow L^\infty(\mathcal{Y})$ is an \mathbb{A}^1 weak equivalence, and $L^\infty(\mathcal{Y})$ is \mathbb{A}^1 local.

- (8) So we can use the functor L^∞ as an \mathbb{A}^1 localization functor. To be precise, we let $\mathbf{Spt}_s^{\mathbb{A}^1 loc}(k)$ be the subcategory of $\mathbf{Spt}_s(k)$ consisting of the \mathbb{A}^1 local spectra. We may equip $\mathbf{Spt}_s^{\mathbb{A}^1 loc}(k)$ with a model structure with weak equivalences the \mathbb{A}^1 weak equivalences, the cofibrations are the stable cofibrations, and the fibrations are what they have to be.

The functor $L^\infty : \mathbf{Spt}_s(k) \rightarrow \mathbf{Spt}_s^{\mathbb{A}^1 loc}(k)$ is left adjoint to the inclusion functor, and is a left Quillen functor. The homotopy category of $\mathbf{Spt}_s^{\mathbb{A}^1 loc}(k)$ is categorically equivalent to $\mathcal{SH}^{S^1}(k)$.

(9)

Definition 2.8. Let \mathcal{X} be a space. Define $\pi_0(\mathcal{X})$ to be the sheaf on \mathbf{Sm}/k associated to $U \rightarrow \pi_0(\mathcal{X}(U))$. A space \mathcal{X} is called 0-connected if and only if $\pi_0(\mathcal{X})$ is the trivial sheaf.

Let (\mathcal{X}, x) be a pointed space. Define $\pi_n(\mathcal{X})$ to be the sheafification of the presheaf on \mathbf{Sm}/k given by

$$U \rightarrow \pi_n(\mathcal{X}(U)).$$

A pointed space \mathcal{X} is called n -connected if it is 0-connected and for all $i \leq n$, the sheaves $\pi_i(\mathcal{X})$ are trivial.

(10)

Proposition 2.9. Let \mathcal{X} be a 0-connected simplicial sheaf. Then $L^\infty \mathcal{X}$ is also 0-connected.

Proof. M.V. IHES paper? Sketch of argument? It shouldn't be too hard by chasing components around. \square

- (11) For a sheaf of abelian groups M on \mathbf{Sm}/k and a natural number n , a Dold-Kan construction gives a simplifical presheaf $K(M, n)$. It is called the Eilenberg-MacLane spectrum of type (M, n) and has homotopy sheaves as expected:

$$\pi_m(K(M, n)) = \begin{cases} 0 & \text{if } m \neq n \\ M & \text{if } m = n \end{cases}.$$

Important equation for this construction. For $X \in \mathbf{Sm}/k$, M a sheaf of Abelian groups,

$$\mathcal{H}_s(k)(rX, K(M, n)) \cong H_{Nis}^n(X; M).$$

- (12) Use square brackets to denote the maps in the unstable pointed (motivic) homotopy category, i.e., $[\mathcal{X}, \mathcal{Y}] = \mathcal{H}_\bullet(k)(\mathcal{X}, \mathcal{Y})$.

Use $\pi_n^{\mathbb{A}^1}(\mathcal{X})$ for the sheaf of homotopy groups in the motivic category, i.e., $\pi_n^{\mathbb{A}^1}(\mathcal{X}) = \pi_n(L^\infty \mathcal{X})$. This is also obtained by sheafifying the presheaf given by

$$U \in \mathbf{Sm}/k \mapsto [S^n \wedge U_+, \mathcal{X}].$$

2.4. Homotopy purity, connectedness calculations.

Proposition 2.10. Weak n -connectedness is equivalent to n -connectedness

Proof. Details? \square

Proposition 2.11. If V is an irreducible, smooth k -scheme, and $U \subseteq V$ is a dense open subset, the space $L_{\mathbb{A}^1}(V/U)$ is 0-connected.

Proof. Seems to use that local structure of spaces is given by $\mathbb{A}^n/(\mathbb{A}^n - \{0\}) \wedge L_+$. How do all the reductions work? Also uses some results specific to working over perfect fields. \square

2.5. S^1 -spectra.

Definition 2.12. Let $\mathbf{Spt}_s(k)$ denote the category of S^1 -spectra of spaces $\Delta^{op}\mathbf{Shv}(\mathbf{Sm}/k)$. We first endow this category with the projective model structure, i.e., a map $f : E \rightarrow F$ is a weak equivalence iff for any n the map $f_n : E_n \rightarrow F_n$ is a w.e.; a map $f : E \rightarrow F$ is a fibration iff for all n the map $f_n : E_n \rightarrow F_n$ is a fibration. The cofibrations are characterized by the property that $f : E \rightarrow F$ is a cofib iff $f_0 : E_0 \rightarrow F_0$ is a cofib and for any $n \geq 1$

$$\begin{array}{ccc} S^1 \wedge E_{n-1} & \xrightarrow{\sigma_{n-1}} & E_n \\ \downarrow S^1 \wedge f_{n-1} & & \downarrow \\ S^1 \wedge F_{n-1} & \longrightarrow & P.O. \\ & \searrow \sigma_{n-1} & \downarrow f_n \\ & & F_n \end{array}$$

This model structure does not actually invert $S^1 \wedge -$. To accomplish this, we must localize with respect to the stable equivalences.

Definition 2.13. A map $f : E \rightarrow F$ of S^1 -spectra is a stable equivalence iff for any $n \in \mathbb{Z}$ the induced map of homotopy sheaves $\pi_n(f) : \pi_n(E) \rightarrow \pi_n(F)$ is an isomorphism.

The stable model category structure on $\text{Spt}_s(k)$ is given by declaring the weak equivalences to be the stable weak equivalences, and the cofibrations to be the same as those for the projective model structure. This is indeed a left Bousfield localization, but we will not describe it further as such.

Definition 2.14. Let $\text{Spt}_s^{\mathbb{A}^1}(k)$ denote the category of S^1 spectra endowed with the stable model category structure localized at the collection of maps $\{\Sigma^\infty U_+ \wedge \mathbb{A}^1 \rightarrow \Sigma^\infty U_+ \mid U \in \text{Sm}/k\}$.

Let $\mathcal{SH}^{S^1}(k)$ denote the homotopy category associated to $\text{Spt}_s^{\mathbb{A}^1}(k)$. We will use $\mathcal{H}_s^{S^1}(k)$ to denote the homotopy category of $\text{Spt}_s(k)$.

Definition 2.15. Let E be an S^1 spectrum of spaces. Let π_n denote the sheaf obtained by taking the colimit of the directed system $\pi_{n+r}(E_r)$ in $\text{Ab}(\text{Sm}/k, \text{Nis})$. That is,

$$\pi_n(E) = \text{colim}_r \pi_{n+r}(E_r).$$

In particular, for a $U \in \text{Sm}/k$, we have

$$\pi_n(E)(U) = \text{colim}_r \pi_{n+r}(E_r)(U).$$

Definition 2.16. An S^1 -spectrum E is said to be n -connected if for any $m \leq n$, the homotopy sheaves $\pi_m(E)$ are trivial.

Definition 2.17. There is a left Quillen functor $\Sigma_s^\infty : \text{Spc}_\bullet \rightarrow \text{Spt}_s(k)$ given by $(\Sigma^\infty \mathcal{Y})_n = (S^1)^{\wedge n} \wedge \mathcal{Y}$ with the evident bonding maps. The right adjoint to this functor is given by “evaluation at 0”, i.e., $\Omega^\infty(E) = E_0$.

Remark 2. The right derived functor $R\Omega^\infty : \mathcal{SH}_s^{S^1}(k) \rightarrow \mathcal{H}_\bullet(k)$ is given by the formula

$$R\Omega^\infty(E) = \text{colim}_i \Omega_s^i E_i.$$

This comes from the fact that fibrant S^1 spectra are exactly the Ω spectra, and the description of the fibrant replacement functor.

Remark 3. We also get a left Quillen functor $\Sigma_s^\infty : \text{Spc}_\bullet^{\mathbb{A}^1}(k) \rightarrow \text{Spt}_s^{\mathbb{A}^1}(k)$ given by the same formula as above.

What is the fibrant replacement functor in $\text{Spt}_s^{\mathbb{A}^1}(k)$?

Remark 4. The stable homotopy category is symmetric monoidal, with smash product \wedge and internal hom $\underline{\text{Hom}}$. Using symmetric spectra, one can give these constructions on the category of spectra.

2.6. t structure.

Definition 2.18. Let \mathfrak{C} be a triangulated category. A t -structure on \mathfrak{C} is a pair of full subcategories $(\mathfrak{C}_{\geq 0}, \mathfrak{C}_{\leq 0})$ which satisfies

- (1) For any $X \in \mathfrak{C}_{\geq 0}$ and any $Y \in \mathfrak{C}_{\leq 0}$, $\text{Hom}_{\mathfrak{C}}(X, Y[-1]) = 0$.
- (2) $\mathfrak{C}_{\geq 0}[1] \subseteq \mathfrak{C}_{\geq 0}$ and $\mathfrak{C}_{\leq 0}[-1] \subseteq \mathfrak{C}_{\leq 0}$
- (3) for any $X \in \mathfrak{C}$ there exists a distinguished triangle

$$Y \rightarrow X \rightarrow Z \rightarrow Y[1]$$

for which $Y \in \mathfrak{C}_{\geq 0}$, $Z \in \mathfrak{C}_{\leq 0}[-1]$.

The heart of a t -structure is the full subcategory given by $\mathfrak{C}_{\geq 0} \cap \mathfrak{C}_{\leq 0}$.

Definition 2.19 (t -structure on $\mathcal{SH}_s^{S^1}(k)$). Define $\mathcal{SH}_s^{S^1}(k)_{\geq 0}$ to be the full subcategory of $\mathcal{SH}_s^{S^1}(k)$ consisting of objects E such that $\pi_n(E) = 0$ whenever $n < 0$.

Define $\mathcal{SH}_s^{S^1}(k)_{\leq 0}$ to be the full subcategory of $\mathcal{SH}_s^{S^1}(k)$ consisting of objects E such that $\pi_n(E) = 0$ whenever $n > 0$.

Theorem 2.20. The triple $(\mathcal{SH}_s^{S^1}(k), \mathcal{SH}_s^{S^1}(k)_{\geq 0}, \mathcal{SH}_s^{S^1}(k)_{\leq 0})$ is a t -structure on $\mathcal{SH}_s^{S^1}(k)$.

Proposition 2.21. [Mor03, Lemma 4.2.4] The functor $L^\infty : \mathrm{Spt}_s^{S^1}(k) \rightarrow \mathrm{Spt}_{s, \mathbb{A}^1}^{S^1}(k)$ identifies the \mathbb{A}^1 -localized S^1 stable homotopy category with the homotopy category of \mathbb{A}^1 -local S^1 spectra.

Theorem 2.22 (S^1 stable connectivity theorem). Let $E \in \mathcal{SH}_s^{S^1}(k)$, and suppose that whenever $n < 0$ the sheaf $\pi_n E = 0$. Then for all $n < 0$, $\pi_n L_{\mathbb{A}^1} E = 0$.

Theorem 2.23. The pair $(\mathcal{SH}_{\geq 0}^{S^1}(k), \mathcal{SH}_{\leq 0}^{S^1}(k))$ is a t -structure on the category $\mathcal{SH}^{S^1}(k)$.

Definition 2.24. Strictly \mathbb{A}^1 invariant sheaf of Abelian groups.

If M is strictly \mathbb{A}^1 invariant sheaf of groups, define the Eilenberg-MacLane spectrum HM associated to it.

Proposition 2.25. HM is \mathbb{A}^1 local iff M is strictly \mathbb{A}^1 invariant.

Proposition 2.26. The heart of the homotopy t structure is equivalent to the category of strictly \mathbb{A}^1 invariant sheaves.

3. INVERTING \mathbb{G}_m ; \mathbb{P}^1 SPECTRA

3.1. \mathbb{G}_m suspension and loops.

3.2. Homotopy sheaves of $\underline{Hom}(\mathbb{G}_m, E)$.

Definition 3.1. Contraction of a sheaf of pointed sets G (or abelian groups G). G_{-1} Introduce notation G_{con} to help clear up index confusion.

Theorem 3.2. [Mor03, Lemma 4.3.11] $\pi_n(\underline{Hom}(\mathbb{G}_m, E)) \rightarrow \pi_n(E)_{-1}$ is iso.

Lemma 3.3. If M is a strictly \mathbb{A}^1 invariant sheaf of abelian groups, then

$$\underline{Hom}(\mathbb{G}_m, HM) \cong H(M_{-1})$$

Lemma 3.4. When $n \neq 0$,

$$(1) \quad [\Sigma^\infty \mathbb{G}_m, HM[n]]_s^{S^1} = 0.$$

The following map is an iso.

$$(2) \quad [\Sigma^\infty \mathbb{G}_m, HM]_s^{S^1} \rightarrow [S^0, H(M_{-1})]_s^{S^1}$$

REFERENCES

- [A-1974] Adams, J.F., *Stable Homotopy and Generalized Homology*. Chicago Lectures in Mathematics, (1974).
- [B] Blander, Benjamin, *Local Projective Model Structures on Simplicial Presheaves*. K-Theory, **24** (2001) 283–301.
- [DHI] Dugger, Dan; Hollander, Sharon; Isaksen, Dan, *Hypercovers and simplicial presheaves*.
- [DLØRV] Dundas, B.; Levine, M.; Østvær, P.; Röndigs, O.; Voevodsky, V., *Motivic Homotopy Theory*. Springer (2000).
- [Hir] Phillip, Hirschhorn, *Model Categories and Their Localization*. AMS (2003).
- [H-Mod] Hovey, Mark, *Model Categories*. online preprint (1991).
- [H-Spt] Hovey, Mark, *Spectra and symmetric spectra in general model categories*. journal? (2001).
- [J] Jardine, J.F., *Simplicial presheaves*. Journal of Pure and Applied Algebra, **47** (1987) 35–87.
- [Mor03] Morel, Fabien, *An introduction to \mathbb{A}^1 homotopy theory*.
- [Mor04] Morel, Fabien, *On the motivic π_0 of the sphere spectrum*. NATO science series.
- [Mor05] Morel, Fabien, *The stable \mathbb{A}^1 connectivity theorems*. preprint (2004).
- [Voev98] Voevodsky, Vladimir. *\mathbb{A}^1 -Homotopy Theory*. Doc. Math. J., (1998) pp. 579–604.