

Assignment 3 Solutions

2.1.1

$$\begin{array}{c}
 \frac{}{\{f: \text{int} \rightarrow \text{bool}, y: \text{int}\} \vdash y: \text{int}} \text{(var)} \\
 \frac{}{\{f: \text{int} \rightarrow \text{bool}\} \vdash 3: \text{int}} \text{(intLit)} \quad \frac{}{\{f: \text{int} \rightarrow \text{bool}, y: \text{int}\} \vdash y + y: \text{int}} \text{(var)} \quad \frac{}{\{f: \text{int} \rightarrow \text{bool}\} \vdash \text{let } y = 3 \text{ in } y + y: \text{int}} \text{(let)} \\
 \frac{}{\{f: \text{int} \rightarrow \text{bool}\} \vdash 3: \text{int}} \text{(intLit)} \quad \frac{}{\{f: \text{int} \rightarrow \text{bool}, y: \text{int}\} \vdash y + y: \text{int}} \text{(var)} \quad \frac{}{\{f: \text{int} \rightarrow \text{bool}, x: \text{int}\} \vdash f: \text{int} \rightarrow \text{bool}} \text{(var)} \quad \frac{}{\{f: \text{int} \rightarrow \text{bool}, x: \text{int}\} \vdash x: \text{int}} \text{(var)} \quad \frac{}{\{f: \text{int} \rightarrow \text{bool}, x: \text{int}\} \vdash f \ x: \text{bool}} \text{(app)} \\
 \frac{}{\{f: \text{int} \rightarrow \text{bool}\} \vdash \text{let } x = \text{let } y = 3 \text{ in } y + y \text{ in } f \ x: \text{bool}} \text{(let)}
 \end{array}$$

2.1.2

$$\begin{array}{c}
 \frac{\frac{\frac{}{(var)} \{x: int \rightarrow int, y: (int \rightarrow int) \rightarrow int\} \vdash y: (int \rightarrow int) \rightarrow int}{\{x: int \rightarrow int, y: (int \rightarrow int) \rightarrow int\} \vdash y\ x: int} \quad \frac{}{(var)} \{x: int \rightarrow int, y: (int \rightarrow int) \rightarrow int\} \vdash x: int}{\{x: int \rightarrow int, y: (int \rightarrow int) \rightarrow int\} \vdash y\ x: int} (app) \\
 \frac{\frac{\frac{}{(fun)} \{x: int \rightarrow int\} \vdash fun\ y \rightarrow y\ x: ((int \rightarrow int) \rightarrow int) \rightarrow int}{\{x: int \rightarrow int\} \vdash fun\ x \rightarrow fun\ y \rightarrow y\ x: (int \rightarrow int) \rightarrow ((int \rightarrow int) \rightarrow int) \rightarrow int} \quad \frac{\frac{}{(fun)} \{x: int\} \vdash x: int}{\{x: int\} \vdash fun\ x \rightarrow x: int \rightarrow int} (fun)}{\{x: int\} \vdash fun\ x \rightarrow fun\ y \rightarrow y\ x: (int \rightarrow int) \rightarrow ((int \rightarrow int) \rightarrow int) \rightarrow int} (app) \\
 \frac{\frac{\frac{}{(fun)} \{f: int \rightarrow int\} \vdash f: int \rightarrow int}{\{f: int \rightarrow int\} \vdash fun\ f \rightarrow f\ 1: int} \quad \frac{\frac{}{(var)} \{z\} \vdash 1: int}{\{z\} \vdash fun\ f \rightarrow f\ 1: int} (intLit)}{\{z\} \vdash fun\ f \rightarrow f\ 1: int} (app) \\
 \frac{\{x: int\} \vdash fun\ x \rightarrow fun\ y \rightarrow y\ x: (int \rightarrow int) \rightarrow ((int \rightarrow int) \rightarrow int) \rightarrow int \quad \{z\} \vdash fun\ f \rightarrow f\ 1: int}{\{z\} \vdash (fun\ x \rightarrow fun\ y \rightarrow y\ x) (fun\ x \rightarrow x) (fun\ f \rightarrow f\ 1): int} (app)
 \end{array}$$

2.2.1

$$\begin{array}{c}
 \frac{}{3 \Downarrow 3} \text{ (intLitE)} \quad \frac{\frac{}{3 \Downarrow 3} \text{ (intLitE)} \quad \frac{}{4 \Downarrow 4} \text{ (intLitE)}}{3 = 4 \Downarrow \perp} \text{ (eqE)} \quad \frac{\frac{}{3 \Downarrow 3} \text{ (intLitE)} \quad \frac{}{2 \Downarrow 2} \text{ (intLitE)}}{3 + 2 \Downarrow 5} \text{ (addIntE)} \\
 \hline
 \frac{}{3 \Downarrow 3} \text{ (intLitE)} \quad \frac{\text{if } 3 = 4 \text{ then } 3 + 1 \text{ else } 3 + 2 \Downarrow 5}{\text{let } x = 3 \text{ in if } x = 4 \text{ then } x + 1 \text{ else } x + 2 \Downarrow 5} \text{ (if False)} \\
 \hline
 \text{let } x = 3 \text{ in if } x = 4 \text{ then } x + 1 \text{ else } x + 2 \Downarrow 5 \text{ (letE)}
 \end{array}$$

2.2.2

$$\begin{array}{c}
 \frac{}{3 \Downarrow 3} \text{ (intLitE)} \quad \frac{}{3 \Downarrow 3} \text{ (intLitE)} \\
 \hline
 3 + 3 \Downarrow 6 \quad \frac{}{6 \Downarrow 6} \text{ (intLitE)} \\
 \hline
 \text{let } y = 3 + 3 \text{ in } y \quad \frac{}{6 \Downarrow 6} \text{ (intLitE)} \\
 \hline
 \text{let } x = \text{let } y = 3 + 3 \text{ in } y \text{ in } x \Downarrow 6 \text{ (letE)}
 \end{array}$$

2.3

$$\frac{\Gamma, f: \tau_1 \vdash e: \tau_1 \quad \Gamma, f: \tau_1 \vdash e: \tau}{\Gamma \vdash \text{let rec } f = e_1 \text{ in } e_2: \tau} \text{ (let Rec)}$$

$$\frac{\begin{array}{c} \text{(var)} \\ \frac{}{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash f : \text{int} \rightarrow \text{bool}} \end{array} \quad \frac{\begin{array}{c} \text{(var)} \\ \frac{}{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash x : \text{int}} \end{array} \quad \frac{\begin{array}{c} \text{(intLit)} \\ \frac{}{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash 1 : \text{int}} \end{array}}{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash x + 1 : \text{int}} \text{(addInt)} \\ \frac{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash f(x+1) : \text{bool}}{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash f(x+1) = \text{false} : \text{bool}} \text{(app)} \\ \frac{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash f(x+1) = \text{false} : \text{bool}}{\{f : \text{int} \rightarrow \text{bool}\} \vdash \text{fun } x \rightarrow f(x+1) = \text{false} : \text{int} \rightarrow \text{bool}} \text{(eq)} \\ \frac{\{f : \text{int} \rightarrow \text{bool}\} \vdash \text{fun } x \rightarrow f(x+1) = \text{false} : \text{int} \rightarrow \text{bool}}{\{f : \text{int} \rightarrow \text{bool}\} \vdash \text{let rec } f = \text{fun } x \rightarrow f(x+1) = \text{false} \text{ in } f : \text{int} \rightarrow \text{bool}} \text{(letRec)}$$