

Lab 3 Solutions

1.

$$\begin{array}{c}
 \frac{\frac{}{\{f: \text{int} \rightarrow \text{int}\} \vdash f: \text{int} \rightarrow \text{int}} \text{(var)} \quad \frac{\frac{}{\{f: \text{int} \rightarrow \text{int}\} \vdash 2: \text{int}} \text{(intLit)}}{\frac{}{\{f: \text{int} \rightarrow \text{int}\} \vdash f \ 2: \text{int}} \text{(app)}} \quad \frac{\frac{}{\{x: \text{int}\} \vdash x: \text{int}} \text{(var)} \quad \frac{\frac{}{\{x: \text{int}\} \vdash 2: \text{int}} \text{(intLit)}}{\frac{}{\{x: \text{int}\} \vdash x + 2: \text{int}} \text{(intAdd)}} \\
 \frac{}{\{ \} \vdash \text{fun } f \Rightarrow f \ 2: (\text{int} \rightarrow \text{int}) \rightarrow \text{int}} \text{(fun)} \quad \frac{}{\{ \} \vdash \text{fun } x \rightarrow x + 2: \text{int} \rightarrow \text{int}} \text{(fun)} \\
 \frac{}{\{ \} \vdash (\text{fun } f \Rightarrow f \ 2) (\text{fun } x \rightarrow x + 2): \text{int}} \text{(app)} \quad \frac{}{\{ \} \vdash 3: \text{int}} \text{(intLit)} \\
 \frac{}{\{ \} \vdash (\text{fun } f \Rightarrow f \ 2) (\text{fun } x \rightarrow x + 2) \ 3: \text{int}} \text{(mulInt)}
 \end{array}$$

2.

$$\textcircled{1} \quad [\lambda x. x+2 / f](f1) = (\text{fun } x \rightarrow x+2) 1$$

$$\textcircled{2} \quad [1 / x](x+2) = 1+2$$

Note: These are not necessary in your solutions.

They are included for clarification

