

Lab 3 Solutions

1.

$$\frac{\frac{\frac{\frac{\{f: \text{int} \rightarrow \text{int}\} + f: \text{int} \rightarrow \text{int}}}{\{f: \text{int} \rightarrow \text{int}\} + 2: \text{int}} \text{ (var)}}{\{x: \text{int}\} + x: \text{int}} \text{ (intLit)}}{\{x: \text{int}\} + 2: \text{int}} \text{ (var)}$$
$$\frac{\{x: \text{int}\} + 2: \text{int}}{\{x: \text{int}\} + x + 2: \text{int}} \text{ (intLit)}$$
$$\frac{\{x: \text{int}\} + x + 2: \text{int}}{\{x: \text{int}\} + x + 2: \text{int}} \text{ (intAdd)}$$
$$\frac{\frac{\{f: \text{int} \rightarrow \text{int}\} + f 2: \text{int}}{\{f: \text{int} \rightarrow \text{int}\} + f z: (\text{int} \rightarrow \text{int}) \rightarrow \text{int}} \text{ (fun)}}{\{\} + \text{fun } f \rightarrow f z: (\text{int} \rightarrow \text{int}) \rightarrow \text{int}} \text{ (fun)}$$
$$\frac{\{\} + \text{fun } x \rightarrow x + z: \text{int} \rightarrow \text{int}}{\{\} + \text{fun } x \rightarrow x + z: \text{int} \rightarrow \text{int}} \text{ (fun)}$$
$$\frac{\{\} + \text{fun } f \rightarrow f z)(\text{fun } x \rightarrow x + z): \text{int}}{\{\} + (\text{fun } f \rightarrow f z)(\text{fun } x \rightarrow x + z) * 3: \text{int}} \text{ (app)}$$
$$\frac{\{\} + (\text{fun } f \rightarrow f z)(\text{fun } x \rightarrow x + z) * 3: \text{int}}{\{\} + 3: \text{int}} \text{ (intLit)}$$
$$\frac{\{\} + 3: \text{int}}{\{\} + 3: \text{int}} \text{ (mulInt)}$$

3.

$$\textcircled{1} \quad [\lambda x. x+2 / f](f1) = (fun x \rightarrow x+2) 1$$

$$\textcircled{2} \quad [1/x](x+z) = 1+z$$

Note: These are not necessary in your solutions.

They are included for clarification

$$\frac{\frac{\frac{(fun f \rightarrow f 1) \Downarrow \lambda f. f 1}{(fun x \rightarrow x + 2) \Downarrow \lambda x. x + 2} \quad (fun E)}{(fun x \rightarrow x + 2) \Downarrow \lambda x. x + 2 \quad 1 \Downarrow 1} \quad (fun E)}{(fun x \rightarrow x + 2) \Downarrow \lambda x. x + 2 \quad 1 \Downarrow 1} \quad (app E)$$