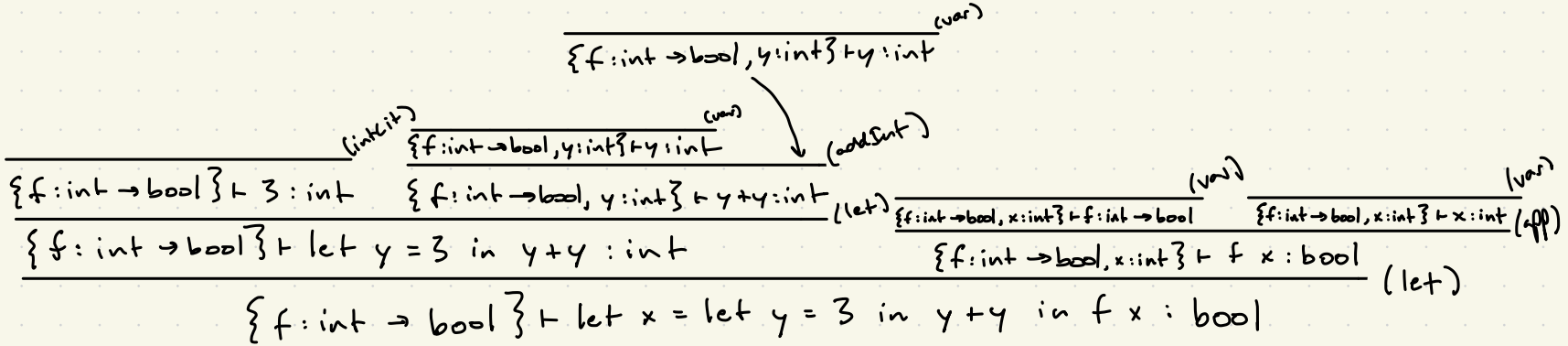
# Assignment 3 Solutions

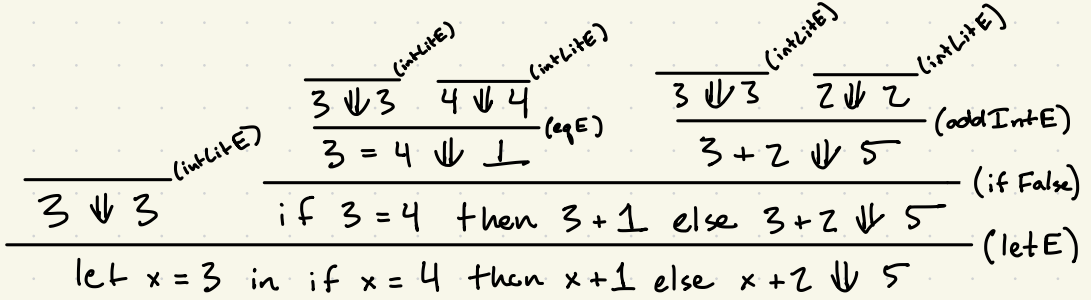
2.1.1



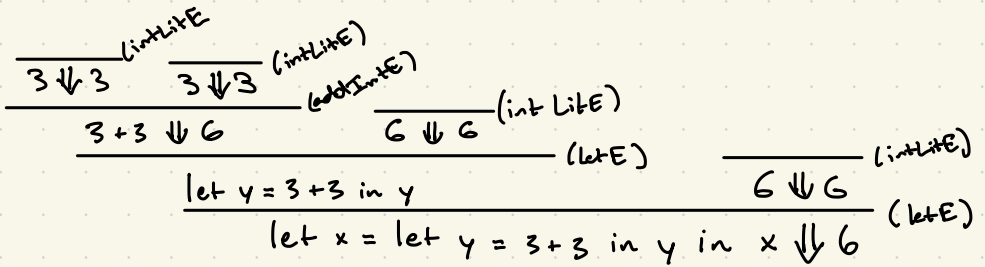
2.1.2

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\{x : int \to int,\ y : (int \to int) \to int\} \vdash y : (int \to int) \to int}(var) \quad \dfrac{}{\{x : int \to int,\ y : (int \to int) \to int\} \vdash x : int}(var)}{\{x : int \to int,\ y : (int \to int) \to int\} \vdash y\ x : int}(app)}{\{x : int \to int\} \vdash fun\ y \to y\ x : ((int \to int) \to int) \to int}(fun)}{\{\} \vdash fun\ x \to fun\ y \to y\ x : (int \to int) \to ((int \to int) \to int) \to int}(fun) \quad \dfrac{\dfrac{}{\{x : int\} \vdash x : int}(var)}{\{\} \vdash fun\ x \to x : int \to int}(fun)}{\{\} \vdash (fun\ x \to fun\ y \to y\ x)(fun\ x \to x) : ((int \to int) \to int) \to int}(app) \quad \dfrac{\dfrac{\dfrac{}{\{f : int \to int\} \vdash f : int \to int}(var) \quad \dfrac{}{\{\} \vdash 1 : int}(int\text{-}lit)}{\{f : int \to int\} \vdash f\ 1 : int}(app)}{\{\} \vdash fun\ f \to f\ 1 : (int \to int) \to int}(fun)}{\{\} \vdash (fun\ x \to fun\ y \to y\ x)(fun\ x \to x)(fun\ f \to f\ 1) : int}(app)$$

2.2.1


3 ⇓ 3 (intLitE)
3 ⇓ 3 (intLitE)
4 ⇓ 4 (intLitE)
3 = 4 ⇓ 1 (eqE)
3 ⇓ 3 (intLitE)
2 ⇓ 2 (intLitE)
3 + 2 ⇓ 5 (oddIntE)
if 3 = 4 then 3 + 1 else 3 + 2 ⇓ 5 (ifFalse)
let x = 3 in if x = 4 then x + 1 else x + 2 ⇓ 5 (letE)

2.2.2


3 ⇓ 3 (intLitE)
3 ⇓ 3 (intLitE)
3 + 3 ⇓ 6 (oddIntE)
6 ⇓ 6 (intLitE)
let y = 3 + 3 in y (letE)
6 ⇓ 6 (intLitE)
let x = let y = 3 + 3 in y in x ⇓ 6 (letE)

2.3

$$\frac{\Gamma, f : \tau_1 \vdash e : \tau_1 \qquad \Gamma, f : \tau_1 \vdash e : \tau}{\Gamma \vdash \text{let rec } f = e_1 \text{ in } e_2 : \tau}\ (\text{letRec})$$

$$\dfrac{\dfrac{}{\{f : int \to bool, x : int\} \vdash f : int \to bool}\ (var) \quad \dfrac{\dfrac{}{\{f : int \to bool, x : int\} \vdash x : int}\ (var) \quad \dfrac{}{\{f : int \to bool, x : int\} \vdash 1 : int}\ (intLit)}{\{f : int \to bool, x : int\} \vdash x + 1 : int}\ (addInt)}{\{f : int \to bool, x : int\} \vdash f\ (x+1) : bool}\ (app) \quad \dfrac{}{\{f : int \to bool, x : int\} \vdash false : bool}\ (false)$$

$$\dfrac{\{f : int \to bool, x : int\} \vdash f\ (x+1) = false : bool}{\{f : int \to bool\} \vdash fun\ x \to f\ (x+1) = false : int \to bool}\ (fun) \quad \dfrac{}{\{f : int \to bool\} \vdash f : int \to bool}\ (var)$$

$$\{\} \vdash \text{let rec } f = fun\ x \to f\ (x+1) = false \text{ in } f : int \to bool\ (letRec)$$