

# Pricing Modeling Notes

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## 0.1 Preliminary definitions

### 0.1.1 Default and Prepayment probabilities:

$\forall t \in \{1, 2, \dots, T\}$

$p_p(t)$  : Probability that the loan will prepay at time  $t$  given that it has survived to that point

$p_d(t)$  : Probability that the loan will default at time  $t$  given that it has survived to that point

### 0.1.2 Survival function:

$S(t)$  : Probability that a loan survives until period  $t$

$$S(t) = \prod_{s=1}^t (1 - p_d(s) - p_p(s)) \quad (1)$$

$$= (1 - p_d(1) - p_p(1)) \times (1 - p_d(2) - p_p(2)) \times \dots \times (1 - p_d(t) - p_p(t)) \quad (2)$$

### 0.1.3 Balance function:

The Current Balance function  $B(t)$  is the remaining balance left at time  $t - 1$  for a loan with principal  $B=B(1)$ . For non conventional loan payments this function might not have a closed form solution.

### 0.1.4 Constant installments:

The remaining balance at time  $t$  for a loan with principal (Balance at  $t=0$ )  $B$  is given by

$$B(t) = B \frac{(1+r)^T - (1+r)^{t-1}}{(1+r)^T - 1} \quad (3)$$

$$I(t) = r \times B(t) \quad (4)$$

The acute reader will notice that the definition of  $B(t)$ , in terms of the remaining balance left at time  $t - 1$ , was given so that we can state such a simple equation for  $I(t)$ .

### 0.1.5 Constant amortization:

The remaining balance at time  $t$  for a loan with principal (Balance at  $t=0$ )  $B$  is given by:

$$B(t) = B \times \left(1 - \frac{t-1}{T}\right) \quad (5)$$

## 0.2 Terms included in the incremental profit

### 0.2.1 Interest on loans:

$$LI(t) = S(t)B(t)r \quad (6)$$

### 0.2.2 Cost of Funds:

$$COF(t) = S_c(t)B_c(t)r_c \quad (7)$$

Where:

$$S_c(t) = \Pi_{s=0}^t [1 - p_p(s) - (1 - LGD(s))p_d(s)] \quad (8)$$

### 0.2.3 Equity Benefit (Capital Rebate):

$$EB(t) = \alpha S(t)B(t)r_c \quad (9)$$

### 0.2.4 Fees Additional Source of revenue:

$$F(t) = fS(t) \quad (10)$$

### 0.2.5 Servicing Costs:

$$SC(t) = \sigma S(t) \quad (11)$$

### 0.2.6 Loss from Default:

$$EL(t) = p_d(t)LGD(t)S(t)B(t) \quad (12)$$

### 0.2.7 Recovery costs

$$C(t) = c \times p_d(t)S(t) \quad (13)$$

### 0.2.8 Equity Capital Charge:

$$EC(t) = \alpha S(t)B(t)r_e \quad (14)$$

### 0.3 Incremental Profit Definition:

The net present value is given by:

$$NPV(x(t), r, T) = \sum_{t=1}^T \frac{x(t)}{(1+r)^t} \quad (15)$$

Element	Notation	Calculation
Lending Interest	$LI$	$NVP(LI(t), r_d, T)$
Cost of Funds	$COF$	$NVP(COF(t), r_d, T)$
Equity benefit	$EB$	$NVP(EB(t), r_d, T)$
Fees	$LI$	$NVP(F(t), r_d, T)$
Ancillary profit	$A$	—
Origination cost	$OC$	—
Commision	$COM$	—
Servicing Costs	$SC$	$NVP(SC(t), r_d, T)$
Expected Loss	$EL$	$NVP(EL(t), r_d, T)$
Collection costs	$C$	$NVP(C(t), r_d, T)$
Equity charge	$EC$	$NVP(EC(t), r_d, T)$

Element	Notation	Calculation
Net Interest Income	$NII$	$LI - COF + EB$
Total Income	$TI$	$NII + A + F$
Net Income before tax	$NIBT$	$TI - OC - COM - SC - LD - C$
Net Income after tax	$NIAT$	$(1 - \tau) \times NIBT$
Incremental profit	$IP$	$NIAT - EC$