Algorithms for Science Applications

Modeling

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References

- S. Chapra and R. Canale, Numerical Methods for Engineers
- Wikipedia - Free Fall
- Explain That Stuff

Terminal velocity

Consider modeling the terminal velocity of a skydiver

We appeal to Newton's second law $F = ma$, and simplify some of the underlying physics

We assume that there are only two relevant forces:

(i) gravity pulling the person down

(ii) drag or friction force due to air

Skydiver

- We start with:

$$\begin{aligned} ma &= F \\ m\frac{dv}{dt} &= F_G + F_D \end{aligned}$$

- The gravitational force is $F_G = mg$
- The drag force is $F_D = -c_d v^2$, where c_d is the drag coefficient
- Thus we have,

$$\begin{aligned} m\frac{dv}{dt} &= mg - c_d v^2 \\ \frac{dv}{dt} &= g - \frac{c_d}{m} v^2 \end{aligned}$$

Skydiver

We can analytically solve the equation

$$\boxed{\frac{dv}{dt} = g - \frac{c_d}{m}v^2}$$

with the initial condition $v(t = 0) = 0$, to get

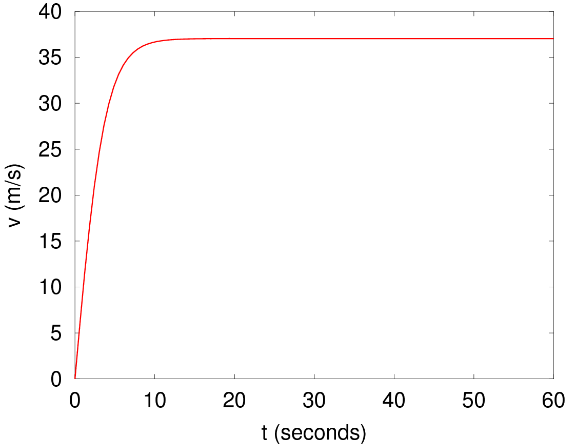
$$v(t) = \sqrt{\frac{mg}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}}t\right)$$

where

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

Skydiver

- With $m = 70$ kg, $c_d = 0.5$ kg/m, $g = 9.8$ m/s^2:



Applications of this model

- Penny dropping problem: A popular urban legend says that a penny dropped from the top of the Empire State Building would penetrate the skull of a person walking on the streets.
- Why driving fast lowers your mileage?
- Parachutes: air resistance dominated by the parachute allows for people of different weight to hold hands together and fall down together
- Galileo's thought experiment