# ISC 4220
# Algorithms 1
## Assignment 5

## LU Decomposition and Gauss-Siedel

**1.** From an algorithmic standpoint, it is advisable to avoid computing the inverse of a matrix. However, LU decomposition can be used if you *have* to compute $\mathbf{A}^{-1}$, by using the property $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$. For a $n \times n$ matrix $\mathbf{A}$, one can set up the following system:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$$
$$\mathbf{A}\begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & ... & \mathbf{x}_n \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & ... & \mathbf{b}_n \end{bmatrix}$$
$$\mathbf{A}\mathbf{X} = \mathbf{B}$$

where $\mathbf{x}_i$ and $\mathbf{b}_i$ are $n \times 1$ vectors, $\mathbf{b}_i$ is the $i^{\text{th}}$ column of the $n \times n$ identity matrix, and $\mathbf{A}^{-1} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & ... & \mathbf{x}_n \end{bmatrix}$. Thus, we have to repeatedly solve for $\mathbf{A}\mathbf{x}_i = \mathbf{b}_i$.

Use the Matlab intrinsic function `[L, U, P] = lu(A)` to compute the matrix inverse of

$$\mathbf{A} = \begin{bmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix}.$$

Verify your answer by computing $\mathbf{A}\mathbf{A}^{-1}$. *Hint*: Note, $\mathbf{P}\mathbf{A}\mathbf{X} = \mathbf{P}\mathbf{B} \implies \mathbf{L}\mathbf{U}\mathbf{X} = \mathbf{P}\mathbf{B}$.

**2.** Use Gauss-Siedel to solve the following tridiagonal system until the 2-norm of the residual is less than 0.01. Compare it with the "true" solution obtained by using Matlab's backslash operator.

$$\begin{bmatrix} 0.80 & -0.40 & 0.00 \\ -0.40 & 0.80 & -0.40 \\ 0.00 & -0.40 & 0.80 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

*Hint*: Look up the documentation on the Matlab functions `tril` and `triu` to extract the upper and lower triangular parts of a matrix.

**3.** The Hilbert matrix is defined by $a_{ij} = 1/(i + j - 1)$.

- Construct a $4 \times 4$ Hilbert matrix, and find its condition number using the row-sum norm.

- How many digits of precision will be lost due to ill-conditioning?

- Scale each row of the matrix by dividing each row by its largest element. Repeat the steps above for this scaled matrix.