ISC 4220
Algorithms 1

Interpolation

1. The gamma function for $x > 0$ is given by the integral:

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt.$$

For an integer argument n,

$$\Gamma(n) = (n-1)!$$

Let us interpolate through the following data points to get a polynomial approximation to the gamma function.

x_i	$f(x_i)$
1	1
2	1
3	2
4	6
5	24

- Use divided differences or Lagrange interpolation to determine a polynomial of degree 4 that interpolates through the 5 points.
- Use the in-built cubic spline (for example, `interp1` with splines) to interpolate the same data.
- Plot and compare the interpolated polynomials, and the intrinsic Matlab function `gamma`.

2. In a short time, we will study successive parabolic optimization. We will claim that the maximum of a quadratic polynomial $p_2(x)$ passing through the points (x_0, f_0), (x_1, f_1), and (x_2, f_2), will be given by:

$$x_{max} = \frac{f_0(x_1^2 - x_2^2) + f_1(x_2^2 - x_0^2) + f_2(x_0^2 - x_1^2)}{2f_0(x_1 - x_2) + 2f_1(x_2 - x_0) + 2f_2(x_0 - x_1)}.$$

Let us indirectly test this formula.

- Find the quadratic interpolating polynomial, $p_2(x)$, which passes through the three points (1,3), (2,5), (3,3).
- Find the maximum of $p_2(x)$ by solving for $p_2'(x) = 0$. How does this compare with the formula for x_{max} above?