Dilution of precision in angle-of-arrival positioning systems

Article *in* Electronics Letters · April 2006

Impact Factor: 0.93 · DOI: 10.1049/el:20064410 · Source: IEEE Xplore

1 author:

Andrew G Dempster
University of New South Wales

Dilution of precision in angle-of-arrival positioning systems

A.G. Dempster

Dilution of precision relates position accuracy to measurement accuracy. An expression for positions determined by measuring the angle-of-arrival of radio signals is evaluated.

Introduction: Angle-of-arrival (AOA) techniques were for some time candidates for positioning mobile telephones, but time-of-arrival (TOA) and time-difference-of-arrival (TDOA) schemes were eventually favoured, despite AOA having comparable accuracy [1]. However, as new mobile communications systems emerge that use smart antennas to spatially separate signals [2], once again AOA can be considered. Recently, AOA has been deployed in ad hoc positioning systems [3].

The best-known TOA system is GPS, which can also be used for mobile phone positioning if assistance from the base is provided. For both TOA and TDOA systems, expressions that relate the measurement error to the position error have been defined, using an expression for *dilution of precision*, or DOP. This Letter defines the counterpart DOP expression for AOA systems.

Dilution of precision for TOA and TDOA: TOA systems measure the time-of-flight of a signal from several known points b_i. Converting these times to ranges r_i, position can be found by intersecting circles (2D) or spheres (3D) with centres b_i and radii r_i. This solution process can be linearised by using a truncated Taylor series around a 'guess' position and solving for the correction to that guess. For GPS, if four ranges are measured [4]:

$$\Delta\mathbf{x} = \mathbf{H}^{-1}\Delta\rho \tag{1}$$

where $\Delta\mathbf{x}$ is a vector of position (and receiver clock) corrections, $\Delta\rho$ is the difference between the real range measurements and those expected at the guess position, and $\mathbf{H}$ is a 4×4 matrix the rows of which are unit vectors between the guess and each satellite, and a fourth term ($=1$) corresponding to the clock correction. Iterating (1) leads to an accurate position solution. Where more than four satellites are used in an over-determined solution, $\mathbf{H}$ is non-square and a generalised inverse is used:

$$\Delta\mathbf{x} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\Delta\rho \tag{2}$$

The expected value of the positional error can be related to expected range error via:

$$\sigma_a = \mathrm{DOP}\,\sigma_r \tag{3}$$

where σ_r is the range s.d., σ_a is the position s.d. and DOP is dilution of precision. If σ_a was the vertical error, then the DOP term would be VDOP. Similarly, HDOP and PDOP represent horizontal and 3D position DOP, respectively. GDOP (geometric DOP) also contains a term reflecting the receiver time offset. This can be evaluated as follows [4]:

$$\begin{aligned} E\left(\mathbf{dx}\,\mathbf{dx}^T\right) &= E[(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\,\mathbf{d}\boldsymbol{\rho}\,\mathbf{d}\boldsymbol{\rho}^T\mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}] \\ &= (\mathbf{H}^T\mathbf{H})^{-1}\sigma_r^2 \end{aligned} \tag{4}$$

The last step is possible if the assumption is made that the covariance matrix of range measurements $E(\mathbf{d}\boldsymbol{\rho}\,\mathbf{d}\boldsymbol{\rho}^T)$ is simply a diagonal with identical elements σ_r^2, i.e. the measurements are uncorrelated with equal variance. The effects of geometry are reflected in $(\mathbf{H}^T\mathbf{H})^{-1}$ and GDOP can be defined:

$$\mathrm{GDOP} = \sqrt{\mathrm{trace}\left(\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\right)} \tag{5}$$

If $\mathbf{H}$ is constructed using a local co-ordinate ENU system, then similar to GDOP, PDOP can be defined by the first three diagonal elements, HDOP by the first two, and VDOP by the third.

For TDOA systems, the measurements are the differences in arrival time at the receiver of signals from different base stations. Rather than intersecting circles or spheres as in TOA, this leads to the intersection of hyperbolae and hyperboloids. The calculations are very similar to those for TOA, with the $\mathbf{H}$ matrix no longer containing a receiver time offset term, and each row contains the difference between the unit vectors to the bases used for the corresponding measurement [5].

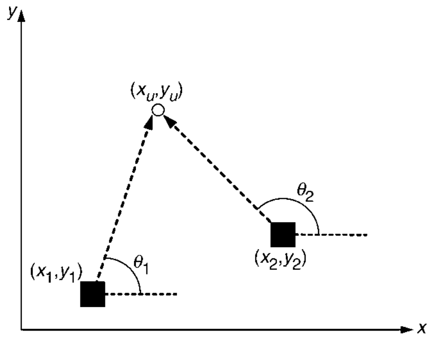

Fig. 1 *Two base stations and user and angles measured to find user position*

Positioning using AOA: AOA positioning is relatively simple. In a mobile phone scenario, it is unlikely that the receiver will have the ability to measure angle of arrival, so the base stations would perform this function. The base stations can measure this angle with respect to an absolute reference (such as north). The mobile would not have this luxury and would need to perform the relatively more difficult positioning task of 'resection' (see e.g. [6]). Because absolute angles can be measured, the problem can be illustrated as in Fig. 1. Because the base station locations (x_i, y_i) are known, the user position (x_u, y_u) can be simply calculated by intersecting two lines passing through the base stations with the measured angles:

$$\frac{y_u - y_i}{x_u - x_i} = \tan(\theta_i) \tag{6}$$

So, for two stations,

$$\begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} \tan\theta_1 & -1 \\ \tan\theta_2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} x_1\tan\theta_1 - y_1 \\ x_2\tan\theta_2 - y_2 \end{bmatrix} \tag{7}$$

Over-determined solutions can use a pseudo-inverse for least-squares approximation, or a centroid of the line intersections [1].

Angle of arrival dilution of precision (AOA DOP): The dilution of precision takes the form

$$\sigma_p = \mathrm{DOP}\,\sigma_\theta \tag{8}$$

where σ_p is the position s.d. and σ_θ is the AOA measurement s.d. Starting with (6) and examining dy_u first,

$$\begin{aligned} \frac{d}{dy_u}\left(\frac{y_u - y_i}{x_u - x_i}\right) &= \frac{d}{dy_u}\left(\tan(\theta_i)\right) \\ \frac{1}{x_u - x_i} &= \sec^2(\theta_i)\frac{d\theta_i}{dy_u} \\ dy_u &= \frac{r^2}{x_u - x_i}d\theta_i \end{aligned} \tag{9}$$

where $r = \sqrt{((x_u - x_i)^2 + (y_u - y_i)^2)}$ is the user-station i range, and $\cos(\theta) = (x_u - x_i)/r$. Similarly, $dx_u = -r^2/(y_u - y_i)d\theta_i$ so:

$$d\theta_i = \frac{1}{r^2}\left(-\left(y_u - y_i\right)dx_u + \left(x_u - x_i\right)dy_u\right) \tag{10}$$

or in matrix terms for two stations:

$$\begin{bmatrix} d\theta_1 \\ d\theta_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-(y_u - y_1)}{r^2} & \dfrac{(x_u - x_1)}{r^2} \\ \dfrac{-(y_u - y_2)}{r^2} & \dfrac{(x_u - x_2)}{r^2} \end{bmatrix} \begin{bmatrix} dx_u \\ dy_u \end{bmatrix} = \mathbf{H}\begin{bmatrix} dx_u \\ dy_u \end{bmatrix} \tag{11}$$

which is readily modified for an over-determined solution. As in other positioning systems, we can assume that the AOA measurements are uncorrelated and have equal variance, so by using $\mathbf{H}$ from (11), DOP can thus be evaluated from (4).

Application to an example: Two stations were placed at $(0, 0)$ and $(1, 0)$ and the DOP evaluated in the region $0 < x < 1$, $0 < y < 2$. The result is illustrated in Fig. 2. It can be seen that DOP grows near the line connecting the stations. This is due to high XDOP, i.e. in this region there is no ability to measure position along the x-axis because the vectors to both stations approach parallel with the x-axis. Similarly, DOP grows for large x, because YDOP is high, i.e. a small error in the angle produces a large error in y for both stations, again because the vectors to both stations approach parallel with the y-axis. The boundary between these high XDOP and high YDOP regions is shown in Fig. 3. Also shown is the minimum DOP that occurs along lines of constant x in Fig. 2. From both these plots, it is clear that there is a region of good positioning away from the x-axis, but not too far away.

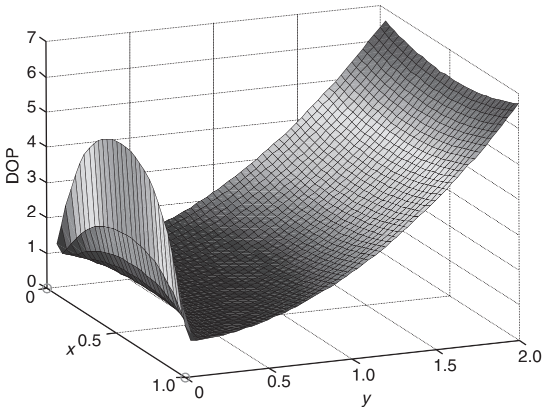

Fig. 2 *DOP surface for two stations at (0, 0) and (1, 0)*

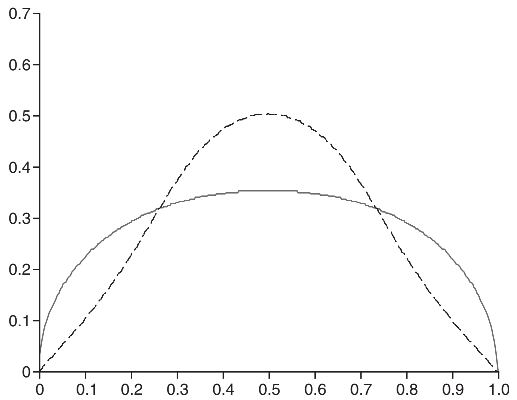

Fig. 3 *For two stations at (0, 0) and (1, 0), points where DOP is minimum and where XDOP = YDOP*

—— DOP is minimum
------ XDOP = YDOP

Conclusion: An expression for dilution of precision has been derived for angle-of-arrival positioning systems which allows the quality of an AOA position to be determined.

21 December 2005
Electronics Letters online no: 20064410
doi: 10.1049/el:20064410

A.G. Dempster (*School of Surveying and Spatial Information Systems, University of New South Wales, Sydney 2052, Australia*)

E-mail: a.dempster@unsw.edu.au

References

1 Caffery, J., Jr., and Stuber, G.L.: 'Subscriber location in CDMA cellular networks', *IEEE Trans. Veh. Technol.*, 1998, **47**, (2), pp. 406–416
2 Mannion, P.: 'Smart basestations maximize capacity', *Communication Systems Design*, 16 February 2003
3 Niculescu, D., and Nath, B.: 'Ad hoc positioning system (APS) using AOA'. Proc. IEEE INFOCOM'03, pp. 1734–1743
4 Kaplan, E.D.: 'Understanding GPS' (Artech House, 1996)
5 Shin, D.-H., and Sung, T.-K.: 'Comparisons of error characteristics between TOA and TDOA positioning', *IEEE Trans. Aerosp. Electron. Syst.*, 2002, **38**, (1), pp. 307–311
6 Uren, J., and Price, W.F.: 'Surveying for Engineers'. 4th ed., Palgrave Macmillan, 2006, pp. 236–240