Nuclear Reactor Thermal-Hydraulics

NUGN520 - Homework

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Chapter 1

Reactor Heat Generation

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the fifth chapter of the book, covering the subject of heat conduction in reactor elements.

1.1

1.1.1 Problem

A 0.5-in.-diameter fuel element is made of 3 percent enriched UO_2. It is surrounded by a 0.003-in.-thick helium layer and a 0.03-in.-thick Zircaloy 2 cladding. A certain section of the element operates in boiling light water at 1000 psia. The boiling heat transfer coefficient is $10000\ Btu.h^{-1}.ft^{-2}.F$, and the temperature drop in the boiling film at the section is $30.4\ F$. Calculate the maximum fuel temperature at that section. $k_{He} = 0.16$, $k_{clad} = 8\ Btu.h^{-1}.ft^{-2}.F$.

1.1.2 Solution

The following assumption are made:

1. T_∞ is the temperature a certain distance from the cladding
2. $T_\infty = 70\ F$.
3. The radius of the fuel element is r_h
4. The helium layer thickness is c_h
5. The full radius of the element is r_z

6. The conductivity of UO_2 is taken at 800 F, $k_f = 2.5\ Btu.h^{-1}.ft^{-1}.F^{-1}$

Let us first compute the temperature distribution in the fuel. We know that Equation 1.1 describes the heat conduction within our usual assumptions.

$$q'''2\pi r\Delta r L = q_{r+\Delta r} - q_r \tag{1.1}$$

We also know the relations 1.2 and 1.3.

$$q_r = -k_f A\frac{dT}{dr} = -2\pi L k_f r\frac{dT}{dr} \tag{1.2}$$

$$q_{r+\Delta r} = q_r + \frac{dq_r}{dr}\Delta r = -2\pi L k_f r\frac{dT}{dr} - 2\pi L k_f\left(r\frac{d^2T}{dr^2} + \frac{dT}{dr}\right)\Delta r \tag{1.3}$$

Consequently, we can combine Equations 1.2 and 1.3 to obtain Equation 1.4.

$$-\frac{q'''}{k_f} = \left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right) \tag{1.4}$$

This equation can be written following Equation 1.5.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{q'''}{k_f} = 0 \tag{1.5}$$

Multiplying both sides by r and integrating, we obtain Equation 1.6.

$$r\frac{dT}{dr} = -\frac{q'''r^2}{2k_f} + C_1 = 0 \tag{1.6}$$

Now dividing both sides by r and integrating again, we can compute Equation 1.7.

$$T(r) = -\frac{q'''r^2}{4k_f} + C_1\ln(r) + C_2 = 0 \tag{1.7}$$

Using the boundary conditions $\left.\frac{dT}{dr}\right|_{r=0} = 0$ and $T(r=0) = T_m$, we can respectively define $C_1 = 0$ and $C_2 = T_m$.

Thus, we have Equation 1.8.

$$T_m - T(r_h) = \frac{q'''r_h^2}{4k_f} \tag{1.8}$$

Now, we can solve for the temperature distribution in the Helium layer. In this layer, we can assume that no heat is generated, thus using Equation 1.9.

$$\frac{d}{dr}\left(rk_h\frac{dT}{dr}\right)=0 \tag{1.9}$$

Integrating this Equation from r_h to r_h+c_h, we can write Equation 1.10.

$$\int_{r_h}^{r_h+c_h}\frac{d}{dr}\left(rk_h\frac{dT}{dr}\right)=0 \tag{1.10}$$

Knowing Equation 1.6, we can see that $r_hk_hd\frac{dT}{dr}=-\frac{q'''r_h^2}{2}$, and we can consequently write Equation 1.11.

$$rk_h\frac{dT}{dr}+\frac{q'''r_h^2}{2}=0 \tag{1.11}$$

Dividing by rk_h and integrating, we obtain Equation 1.12.

$$T(r_h+c_h)-T(r_h)=\frac{q'''r_h^2}{2k_h}\ln\left(\frac{r_h}{r_h+c_h}\right) \tag{1.12}$$

In the cladding Zr_2, the solution is identical. We can thus write Equation 1.13.

$$T(r_z)-T(r_h+c_h)=\frac{q'''(r_h+c_h)^2}{2k_z}\ln\left(\frac{r_h+c_h}{r_z}\right) \tag{1.13}$$

Anf finally, we can express the heat flow to the coolant. We know that $q''(r_z)=-k_w\frac{dT}{dr}\Big|_{r_z}=h_w(T(r_z)-T_\infty)$. We know, using Equation 1.6 that we can write Equation 1.14.

$$\frac{q'''(r_h+c_h)^2}{2}=h_wr_z(T(r_z)-T_\infty) \tag{1.14}$$

And finally, we can write Equation 1.15.

$$T(r_z)-T_\infty=\frac{q'''(r_h+c_h)^2}{2h_wr_z} \tag{1.15}$$

We are given the temperature drop in the coolant, 30.4 F, as well as the boiling heat transfer coefficient $h_w=10000\ Btu.h^{-1}.ft^{-2}.F^{-1}$. Converting the radii to feet, we can calculate q''' from Equation 1.15. We obtain $q'''=3.309\times10^7\ Btu.h^{-1}.ft^{-3}$. We can now plug this value back into Equations **??** ($T(r_z)-T(r_h+c_h)=-121.8\ F$, 1.12 ($T(r_h+c_h)-T(r_h)=-640.6\ F$) and 1.8

$(T_m - T(r_h) = 1431.6\ F)$ to find the temperature T_m, which will be the maximum temperature at the section.

Consequently, $T_m = T_\infty + 1431.6 + 640.6 + 121.8 + 30.4 = T_\infty + 2224.4\ F$. Assuming $T_\infty = 70$, we obtain $T_m = 2294.4\ F$.

Nota bene: We did not use the fuel enrichment, nor did we use the water pressure information in our calculations. We could have used this information to compute the q''', however the flux is unknown. In that regards, I am unsure as to the usefulness of this data.

Bibliography

[1] M. M. El-Wakil, *Nuclear Heat Transport*, American Nuclear Society, 1993.

[2] G. Leinweber, D. Barry, R. Block, M. Rapp, J. Hoole, Y. Danon, R. Bahran, D. Williams, J. Geuther, and F. Saglime III, *Thermal total cross sections of europium from neutron capture and transmission measurements*, Transactions of the American Nuclear Society, 107 (2012), p. 1007.