Nuclear Reactor Thermal-Hydraulics

NUGN520 - Homework

By

GUILLAUME L'HER

Department of Nuclear Engineering
COLORADO SCHOOL OF MINES

Homework submitted for the Nuclear Reactor Thermal-Hydraulics class at the Colorado School of Mines.

SPRING 2017

Table of Contents

Page

1 Heat transfer and fluid flow, nonmetallic coolants **1**

1.1 [9-3] - Nitrogen-cooled reactor . . . 1

1.1.1 Problem . . . 1

1.1.2 Solution . . . 1

1.2 [9-6] - W'/q . . . 3

1.2.1 Problem . . . 3

1.2.2 Solution . . . 3

1.3 [J1] - Temperature distribution . . . 4

1.3.1 Problem . . . 4

1.3.2 Solution . . . 4

1.4 [J2] - Triangular pipe . . . 5

1.4.1 Problem . . . 5

1.4.2 Solution . . . 6

1.5 [J3] - Scaling of the Navier-Stokes equation . . . 7

1.5.1 Problem . . . 7

1.5.2 Solution . . . 7

Bibliography **9**

CHAPTER 1

HEAT TRANSFER AND FLUID FLOW, NONMETALLIC COOLANTS

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the ninth chapter of the book, covering the subject of heat transfer and fluid flow, for nonmetallic coolants.

1.1 [9-3] - Nitrogen-cooled reactor

1.1.1 Problem

A nitrogen-cooled reactor uses UO_2 fuel. The fuel elements are $0.5\ in$ in diameter and are encased in a graphite can $0.1\ in$ thick. At a particular cross section the coolant pressure and bulk temperature are 6 atm and $1000°F$. The stream velocity is 375 fps. At the same section the volumetric thermal source strength is $2 \times 10^6\ Btu.h^{-1}.ft^{-3}$ and the maximum fuel temperature is $5000°F$. Determine (a) the heat-transfer coefficient and (b) the necessary equivalent diameter of the coolant channel. Neglect contact resistance between the fuel and the graphite. Use $k_g = 35\ Btu.h^{-1}.ft^{-1}.°F^{-1}$ (aged).

1.1.2 Solution

We can use Equation 5.51a from the book [1], which states, for a cylindrical fuel element with cladding and coolant:

$$T_{f,m} - T_b = \frac{q''' r_f^2}{4k_f} + \frac{q''' r_f^2}{2}\left(\frac{1}{k_g}\ln\left(\frac{r_f + r_w}{r_f}\right) + \frac{1}{h(r_f + r_w)}\right) \tag{1.1}$$

Where:

$T_{f,m}$ = Maximum temperature in the fuel
T_b = Coolant bulk temperature
r_f = Fuel radius position
r_w = Wall radius position
k_g = Aged thermal conductivity of graphite

We can reorganize the terms to express h. Doing so gives us:

$$h = \frac{1}{\left(\left(5000 - 1000 - \frac{2\times10^6 * (0.25/12)^2}{4*1.1}\right) * \frac{2}{2\times10^6 * (0.25/12)^2} - \left(\frac{1}{35}\ln\left(\frac{0.35/12}{0.25/12}\right)\right)\right) * \frac{0.35}{12}} \tag{1.2}$$

And so, $h = 3.9\ Btu.h^{-1}.ft^{-2}.^{\circ}F^{-1}$.

We can now use the Dittus-Boelter equation, $Nu = 0.023Re^{0.8}Pr^{0.4}$, to compute the equivalent diameter needed. The Nitrogen bulk properties are used in this correlation, except for the fact that the free-stream temperature must be replaced by the adiabatic wall temperature T_{fa}.

This equation translates to:

$$\frac{hD_e}{k} = 0.023\left(\frac{D_e v\rho}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{0.4} \tag{1.3}$$

This gives:

$$D_e = \left(0.023\left(\frac{k}{h}\frac{v\rho}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{0.4}\right)^5 \tag{1.4}$$

The adiabiatic wall temperature can be obtained using the Mach number M and the recovery factor F_R. Indeed, Equation 9-41 of the book [1] gives:

$$T_{fa} = F_R(T_{fs} - T_f) + T_f \tag{1.5}$$

Equation 9-40 gives:

$$T_{fs} = T_f\left(1 + \frac{\gamma - 1}{2}M\right) \tag{1.6}$$

And the Mach number is given by Equation 9-37:

$$M = \frac{v}{\sqrt{\gamma R g_c T_f}} \tag{1.7}$$

Where:

γ = 1.4 for Nitrogen
R = 54.99 $ft.lb_f/lb_m$
T_f = 1459.7 °R
g_c = 4.17×10^8
v = 1.35×10^6 fph

We thus obtain $M = 0.197$, $T_{fs} = 1007.8\ °F$ and, using $F_R = 0.89$, $T_{fa} = 1006.9\ °F$.

Using data from tables of Nitrogen at 6 atm and 1006.9°F, we get values extremely similar to the ones at 1000°F:

k = 32.3 $Btu.h^{-1}.ft^{-1}.°F^{-1}$
v = 375 $fps = 1.35 \times 10^6\ fph$
ρ = 0.16 $lb.ft^{-3}$
μ = 8.76 $lb.ft^{-1}.h^{-1}$
c_p = 0.27 $Btu.lb^{-1}.°F^{-1}$
h = 3.9 $Btu.h^{-1}.ft^{-2}.°F^{-1}$

This data, when plugged into Equation 1.4 gives an erroneous number for the equivalent diameter, with the very high velocity causing the diameter to be way too high to get to the given heat transfer coefficient.

1.2 [9-6] - W'/q

1.2.1 Problem

Ligth liquid water is used as a reactor coolant. In a particular channel, the average bulk temperature is 300°F. *Determine the percent change in* W'/q *if it were to be used in the same channel, with the same mass-flow rate, the same mean temperature between cladding and coolant, but at average bulk temperature of* 200°F *and* 400°F. *Assume saturated conditions in all cases.*

1.2.2 Solution

Equation 9.9 from the book [1] can be used. It states:

$$\frac{W'}{q} = 7.07 * 10^{-14} \left(\frac{1}{D_e^{0.2}}\right) \left(\frac{v^{2.8}}{h\Delta T_m}\right) \rho^{0.8} \mu^{0.2} = \alpha \rho^{0.8} \mu^{0.2} \tag{1.8}$$

α can be taken as a constant given the data. The only variable is the change in bulk temperature, implying a change in density and viscosity. The table from the appendix gives the following data:

$\rho(200°F) = 60.1\ lb.ft^{-3}$
$\rho(300°F) = 57.3\ lb.ft^{-3}$
$\rho(400°F) = 53.6\ lb.ft^{-3}$
$\mu(200°F) = 0.74\ lb.ft^{-1}.h^{-1}$
$\mu(300°F) = 0.45\ lb.ft^{-1}.h^{-1}$
$\mu(400°F) = 0.33\ lb.ft^{-1}.h^{-1}$

So, we get :

$$\left.\frac{W'}{q}\right|_{T_b=300°F} = 21.7\alpha \tag{1.9}$$

$$\left.\frac{W'}{q}\right|_{T_b=200°F} = 24.9\alpha = 11.1\% \left.\frac{W'}{q}\right|_{T_b=300°F} \tag{1.10}$$

$$\left.\frac{W'}{q}\right|_{T_b=400°F} = 19.4\alpha = -10.6\% \left.\frac{W'}{q}\right|_{T_b=300°F} \tag{1.11}$$

1.3 [J1] - Temperature distribution

1.3.1 Problem

For pressure driven laminar flow between parallel plates of separation h, the velocity components are $u = U(1 - y^2/h^2)$, $v = w = 0$, where U is the centerline velocity. Similarly to the example done in class, use an energy equation to find the temperature distribution $T(y)$ for a constant wall temperature T_w.

1.3.2 Solution

We can write the energy equation:

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{v}.\nabla T\right) = k\nabla^2 T + \mu\Phi + \dot{q} \tag{1.12}$$

In a steady state, with constant heat generation and no dissipation terms, we can write:

$$\rho c_p (\vec{v}.\nabla T) = k\nabla^2 T \tag{1.13}$$

In 2-D cartesian coordinates:

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{1.14}$$

Noting that $\alpha = \frac{k}{\rho c_p}$, and that $v = 0$, we can write:

$$\frac{u}{\alpha}\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \tag{1.15}$$

We can assume that the rate of change of temperature in the x-direction will be negligible compared to the rate of change in temperature in the y-direction, thus, $\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$. We can now sustitute the velocity distribution $u(y)$:

$$\frac{U}{\alpha}\left(1 - \frac{y^2}{h^2}\right)\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial y^2} \tag{1.16}$$

Integration with respect to y, we obtain:

$$\frac{\partial T}{\partial y} = \frac{U}{\alpha}\frac{\partial T}{\partial x}\left(y - \frac{y^3}{3h^2}\right) + C_1 \tag{1.17}$$

Knowing the symmetry boundary condition $\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0$, we can set $C_1 = 0$. Integrating a second time:

$$T(y) = \frac{U}{2\alpha}\frac{\partial T}{\partial x}\left(y^2 - \frac{y^4}{6h^2}\right) + C_2 \tag{1.18}$$

Knowing the boundary condition $T(y = 0) = T_m$, we can set $C_2 = T_m$. However, this implies knowing T_m. Another boundary condition states that $T(y = h/2) = T_w$, T_w being the temperature at the plate surface. In that case, $C_2 = T_m = T_w - \frac{23}{192}\frac{U}{\alpha}\frac{\partial T}{\partial x}h^2$.

And so,

$$T(y) = \frac{U}{2\alpha}\frac{\partial T}{\partial x}\left(y^2 - \frac{y^4}{6h^2}\right) + T_w - \frac{23}{192}\frac{U}{\alpha}\frac{\partial T}{\partial x}h^2 \tag{1.19}$$

1.4 [J2] - Triangular pipe

1.4.1 Problem

Consider steady laminar fluid flow in a fixed duct of equilateral triangular cross section. Take the side length as s and the center of the triangle at the origin, with one side parallel to the x axis. (a) Write the equations of the line segments defining the sides of the triangle. (b) Using from (a) a product form for the velocity of the fluid, develop a solution of the Navier-Stokes equation. Gravity may be ignored.

1.4.2 Solution

The equation defining the segments of the triangle are:

$$y = -\frac{s}{2\sqrt{3}} \tag{1.20}$$

$$y = x\sqrt{3} + \frac{s}{\sqrt{3}} \tag{1.21}$$

$$y = -x\sqrt{3} + \frac{s}{\sqrt{3}} \tag{1.22}$$

We can then use the three equations above to guess a solution that automatically satisfies all three zero velocity boundary conditions at the three wall. The solution would thus be of the form:

$$v_z = C_1 S(x,y) = C_1\left(y + \frac{s}{2\sqrt{3}}\right)\left(y + x\sqrt{3} - \frac{s}{\sqrt{3}}\right)\left(y - x\sqrt{3} - \frac{s}{\sqrt{3}}\right) \tag{1.23}$$

The Navier-Stokes equation can be written:

$$\mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2}\right) = \frac{\partial p}{\partial z} \tag{1.24}$$

And so:

$$\frac{\partial^2 v_z}{\partial x^2} = C_1(-6y - s\sqrt{3}) \tag{1.25}$$

$$\frac{\partial^2 v_z}{\partial y^2} = C_1(6y - s\sqrt{3}) \tag{1.26}$$

$$\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} = -2C_1 s\sqrt{3} \tag{1.27}$$

Substituting this equation into the Navier-Stokes equation:

$$-2C_1 s\sqrt{3} = \frac{1}{\mu}\frac{\partial p}{\partial z} \implies C_1 = \frac{1}{2\mu s\sqrt{3}}\frac{\partial p}{\partial z} \tag{1.28}$$

And consequently,

$$v_z = \frac{1}{2\mu s\sqrt{3}}\frac{\partial p}{\partial z}\left(y + \frac{s}{2\sqrt{3}}\right)\left(y + x\sqrt{3} - \frac{s}{\sqrt{3}}\right)\left(y - x\sqrt{3} - \frac{s}{\sqrt{3}}\right) \tag{1.29}$$

1.5 [J3] - Scaling of the Navier-Stokes equation

1.5.1 Problem

Scaling for the Navier-Stokes (NS) equation. Consider the NS equation with gravity neglected, $\rho\frac{d\vec{V}}{dt} = -\nabla p + \mu\nabla^2\vec{V}$. *Suppose that* $p(\vec{x},t)$ *and* $\vec{V}(\vec{x},t)$ *solve these equations. There are scaling relationships,* $\vec{V}_\tau = \tau^\alpha\vec{V}(\tau^\beta\vec{x},\tau^\gamma t)$ *and* $p_\tau = \tau^\delta p(\tau^\beta\vec{x},\tau^\gamma t)$ *for any positive* τ*, such that* p_τ *and* $\vec{V}_\tau$ *also solve the NS equation. (a) What are the relations between the exponents* α,β,γ*, and* δ *for this scaling to hold? (b) The equation of continuity also holds,* $\nabla\cdot\vec{V}_\tau = 0$*. Does this impose any further condition on the four exponents? Why or why not?*

1.5.2 Solution

The Navier-Stokes equation, with gravity neglected, is:

$$\rho\left(\frac{\partial\vec{v}}{\partial t} + \vec{v}.\nabla\vec{v}\right) = -\nabla p + \mu\nabla^2\vec{v} \tag{1.30}$$

We have various scaling propositions:

$$\begin{aligned}
\vec{v_\tau} &= \tau^\alpha\vec{v}\\
p_\tau &= \tau^\delta p\\
\vec{x_\tau} &= \tau^\beta\vec{x}\\
t_\tau &= \tau^\gamma t\\
\nabla_\tau &= \tau^{-\beta}\nabla
\end{aligned}$$

We can now rewrite the Navier-Stokes equation:

$$\rho\tau^{\gamma-\alpha}\left(\frac{\partial\vec{v_\tau}}{\partial t_\tau}\right) + \rho\tau^{\beta-2\alpha}\left(\vec{v_\tau}.\nabla_\tau\vec{v_\tau}\right) = -\tau^{\beta-\delta}\nabla_\tau p_\tau + \tau^{2\beta-\alpha}\mu\nabla_\tau^2\vec{v_\tau} \tag{1.31}$$

Multiplying both sides by $\frac{\tau^{2\alpha-\beta}}{\rho}$, we obtain the dimensionless:

$$\tau^{\alpha+\gamma-\beta}\left(\frac{\partial\vec{v_\tau}}{\partial t_\tau}\right) + \vec{v_\tau}.\nabla_\tau\vec{v_\tau} = -\frac{1}{\rho}\tau^{2\alpha-\delta}\nabla_\tau p_\tau + \tau^{\alpha+\beta}\frac{\mu}{\rho}\nabla_\tau^2\vec{v_\tau} \tag{1.32}$$

Knowing that $\vec{v_\tau}$ and p_τ verifies the Navier-Stokes equation, we can deduce that:

$$\begin{cases}\tau^{\alpha+\gamma-\beta} &= 1\\ \tau^{2\alpha-\delta} &= 1\\ \tau^{\alpha+\beta} &= 1\end{cases} \implies \begin{cases}(\alpha+\gamma-\beta)*\ln(\tau) &= 1\\ (2\alpha-\delta)*\ln(\tau) &= 1\\ (\alpha+\beta)*\ln(\tau) &= 1\end{cases} \implies \begin{cases}\tau = 1 & \text{or:}\\ \alpha+\gamma-\beta &= 0\\ 2\alpha-\delta &= 0\\ \alpha+\beta &= 0\end{cases} \implies \begin{cases}\gamma &= -2\alpha\\ \delta &= 2\alpha\\ \beta &= -\alpha\end{cases}$$

We also know that the continuity equation (for an incompressible flow in this case) holds:

$$\nabla \cdot \vec{v_\tau} = \nabla \cdot \vec{v} = 0 \tag{1.33}$$

Considering only the x-direction:

$$\nabla \cdot \vec{v_\tau} = \tau^{-\alpha} \tau^{-\beta} \frac{\partial \vec{v_{x,\tau}}}{\partial x_\tau} = 0 \tag{1.34}$$

The term $\tau - \alpha - \beta$ simplifies out of the equation. Consequently, we haven't gained any insight on the scaling parameters.

BIBLIOGRAPHY

[1] M. M. EL-WAKIL, *Nuclear Heat Transport*, American Nuclear Society, 1993.