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# Approach to criticality

*Initial fuel loading of the reactor*

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A report submitted for the Nuclear Reactor Laboratory class  
at the Colorado School of Mines.

SEPTEMBER 28, 2016



## ABSTRACT

Criticality is usually derived from historical records of the fuel loading pattern. In some cases, one might not know when the criticality occurs. Typically, this is the case when refueling the reactor, either with a different loading pattern or with changes in some materials in the core area.

It can be dangerous to reach criticality before expected, thus one needs to know when to expect it. In order to do so, neutron counts from a detector can be used to determine the subcritical multiplication factor, which gives a good indication of the criticality state of the reactor.

After taking 15 fuel assemblies from the core, they were put back in a few at a time. The subcritical multiplication factor was computed, allowing for a good prediction of the number of assemblies needed in the core to go critical. At the GSTR facility, a full core minus one fuel rod was enough to get to criticality.

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## THEORY

When refueling a reactor, either with a different loading pattern or with changes in some materials in the core area, the operator does not know when criticality might occur. For example, especially at nuclear reactor, testing of highly reactive fuel rods could happen, in which case a full core's reactivity might exceed the control rods worth. A predictive way of getting the criticality information is to compute the subcritical multiplication factor, loading the fuel rods a few at a time, and updating the prediction at each step.

The procedure is issued from handouts from the USGS-Reactor Lab course at the Colorado School of Mines [? ].

## 1.1 Source neutrons

Source neutrons can be produced by different means. They could appear from natural decay of fissile material, in the fuel, from gamma-sources (a  $(\gamma - n)$  reaction) or from a discrete neutron source placed in the core. In the case of the latter, which is installed at the GSTR facility, it can mask the core criticality by emitting neutrons, and absorbing neutrons depending on the neutron flux.

In one generation, the number of neutrons produced by the discrete source is given by:

$$(1.1) \quad N = S k_{eff}$$

where:

$N$  = Total neutron population

$S$  = Number of source neutrons

$k_{eff}$  = Effective multiplication factor ( $< 1$  for a subcritical core)

Consequently, after say two generations, the neutron population is given by:

$$(1.2) \quad N = S + S * (k_{eff}) + (S k_{eff}) * k_{eff}$$

This series can be written for a number n of generations as:

$$(1.3) \quad N = S \frac{1}{1 - k_{eff}}$$

One can also define the subcritical multiplication factor  $M$ , as:

$$(1.4) \quad M = \frac{N}{S} = \frac{1}{1 - k_{eff}}$$

And its inverse, which is more convenient and easy to use in this case study:

$$(1.5) \quad \frac{1}{M} = 1 - k_{eff}$$

It is important to note that in a subcritical core, the fission reaction cannot sustain itself. It will die down. One might think that it means that all your neutrons will disappear from your core. But the neutron source keeps emitting neutrons into the core, and at each generation, more source-fission neutrons are added to the system ?? . Eventually, the power level, i.e. the neutron population in the core, stabilize. As can be seen on figure 1.1, the number of generations (time-dependent) needed for the stabilization to occur strongly depends on the effective multiplication factor value, doing so quickly for  $k_{eff} = 0.5$  but showing an almost linear correlation for  $k_{eff} = 0.99$ .

## 1.2 Neutron count rate

Sometimes, the reactivity  $\rho$  cannot be directly measured, and must be computed from other available data. The reactivity is directly correlated to the number of neutrons in the core, and as such to the count rate.

$$(1.6) \quad \rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$(1.7) \quad \frac{1}{M} = 1 - k_{eff,c} = \frac{\rho}{\rho - 1} = \frac{CR_i}{CR_c}$$



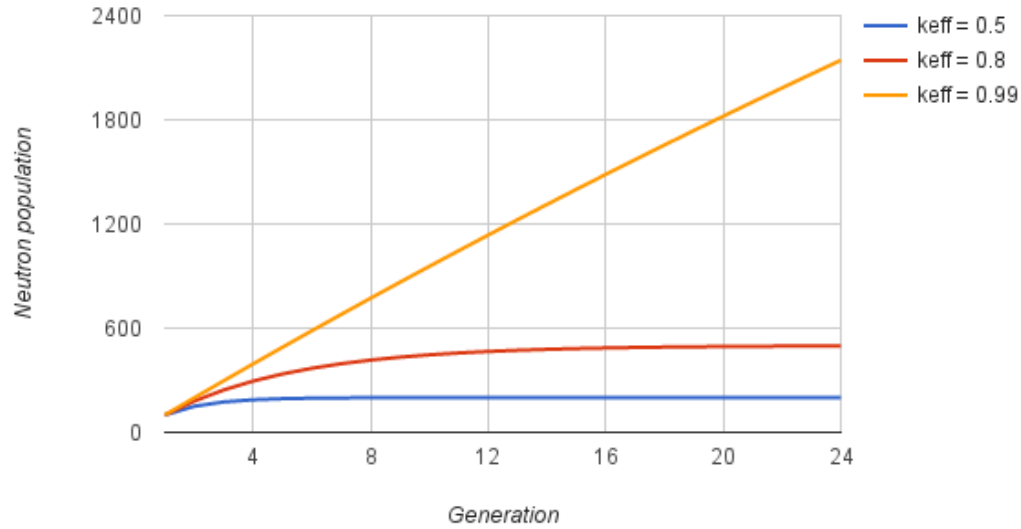


FIGURE 1.1. Neutron population with a fixed source at different  $k_{eff}$  values.

$CR_i$  = Initial count rate

$CR_c$  = Current count rate

$k_{eff,c}$  = Current effective multiplication factor

Plotting  $\frac{1}{M}$ , i.e. the change in the count rate, as a function of the number of fuel elements in the core, will allow prediction of the number of elements needed to reach criticality.

However, in order for the measurements to be correct, it is necessary to place the source in a position where it won't mask the neutron flux from this addition to the neutron detector. A more detailed point will be made about this in ??.

## 1.3 Procedure

In preparation of this approach to criticality experiment, fifteen fuel elements were taken from the core. They are identified by serial numbers written on them, but without high quality camera, this is difficult to read. So, careful care is observed when moving the fuel rods in and out of the core, and their positions before and after being moved is traced in the Reactor Operations Logbook and in the Fuel Element History Logbook. When moving several fuel rods in one sitting, the movements must be planned ahead.

Depending on the movements plan to be followed, the neutron source should be moved to a position where it would not interfere with the detector reading, as explicated by figure 1.2.

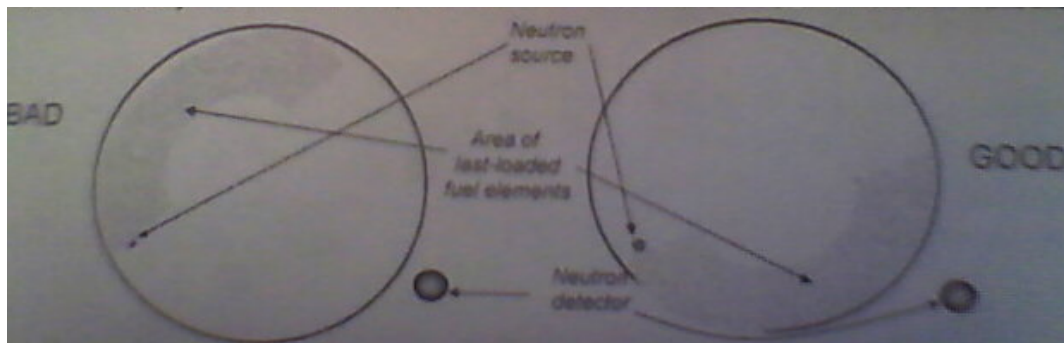


FIGURE 1.2. Correct neutron source positioning in the core.

To physically move the fuel rods, a manual fuel-handling tool is used. Two persons are usually needed to operate it: one to guide it, and the other to latch or unlatch on its head.

The reactor is in a shutdown state (all control rods down) during any fuel-handling operations.

After a few fuel rods have been inserted back into the core, the control rods are taken to the top, and the relative subcritical multiplication factor can be computed using data from the NM1000. This data is then used to predict the number of fuel elements to be inserted before reaching criticality without control rods. A conservative and step-by-step (not too many fuel rods at once) approach should be adopted, since the first few data points can be misleading.

This chapter presents the results obtained during the experiment performed on September 21st, 2016. It presents the subcritical multiplication factor measurements and deduces the number of fuel rods needed to reach criticality.

## 2.1 Subcritical multiplication factor

The subcritical multiplication factor,  $M$ , is calculated using the NM1000 raw measurements. Since the neutron flux varies during the time of the measurement, a comprehensive chunk of data is recorded and averaged. Some measurements have been done before the reactor was fully stabilized, but it falls within the calculation uncertainties. Tables ?? and ?? presents the raw results from the NM1000, and table 2.1 calculates the subcritical multiplication factor and the updated criticality prediction.

Core inventory (# fuel rods)	Average count rate	$\frac{1}{M}$	Predictive critical core inventory
112	0.76	0.20	
115	1.10	0.14	122
118	1.85	0.08	122
120	2.62	0.06	123
122	3.49	0.04	124
123	4.87	0.03	124
124	7.28	0.02	125
125	12.18	0.01	125

Table 2.1:  $\frac{1}{M}$  calculation and criticality prediction

Fitting this data linearly should give us, at the intersection with the x-axis – that is, when  $M$

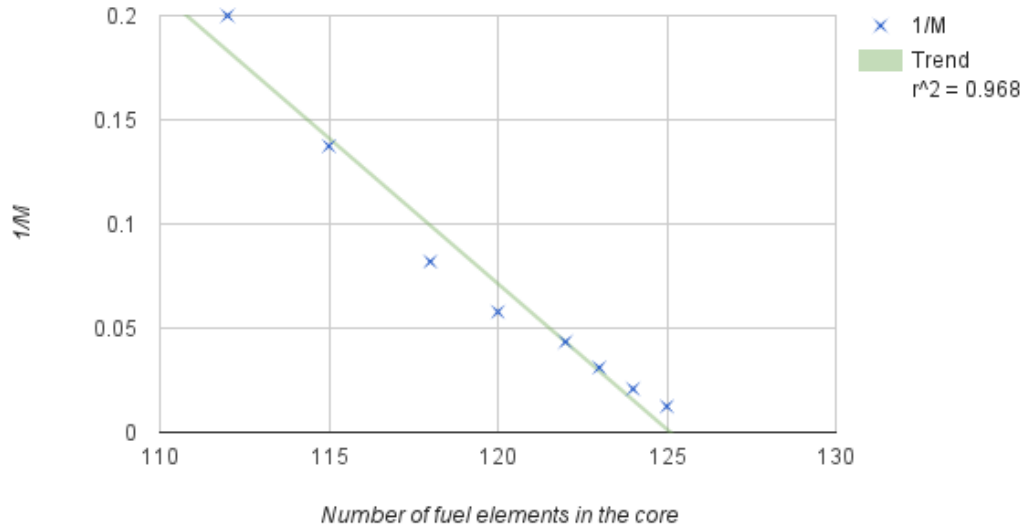


FIGURE 2.1.  $\frac{1}{M}$  as a function of the number of fuel rods in the core.

is infinite and thus the core is critical – the amount of fuel rods needed in the core. One can see on figure ?? that the linearity is not ideal, with an  $R^2$  of only 97%. As a reminder, appendix ?? goes into more details for the calculation of  $R^2$ . This imperfect linearity is expected, since not all the fuel rods have the same worth, some being more irradiated than other, due to their position or time spent in the core, or their initial uranium mass for example. Moreover, the source positioning impacts the reading.

## 2.2 Criticality prediction

One can see the evolution of the prediction as a function of the number of fuel rods in the core on figure ?. Keeping in mind that the actual criticality was achieved with 126 fuel rods compared to a full core of 127 rods, one can see that the estimates were lower. *That is good*. Indeed, this means that the approach to criticality was made, by design, in a very conservative way. The method told the operators that less fissile material was needed to achieve criticality, which allowed for a slower reload and no unexpected criticality incident.

The reactor became critical at approximately 4 watts with one fuel element still out of the core and control rods at the following positions:

1. Transient: full up,
2. Shim 1: full up,

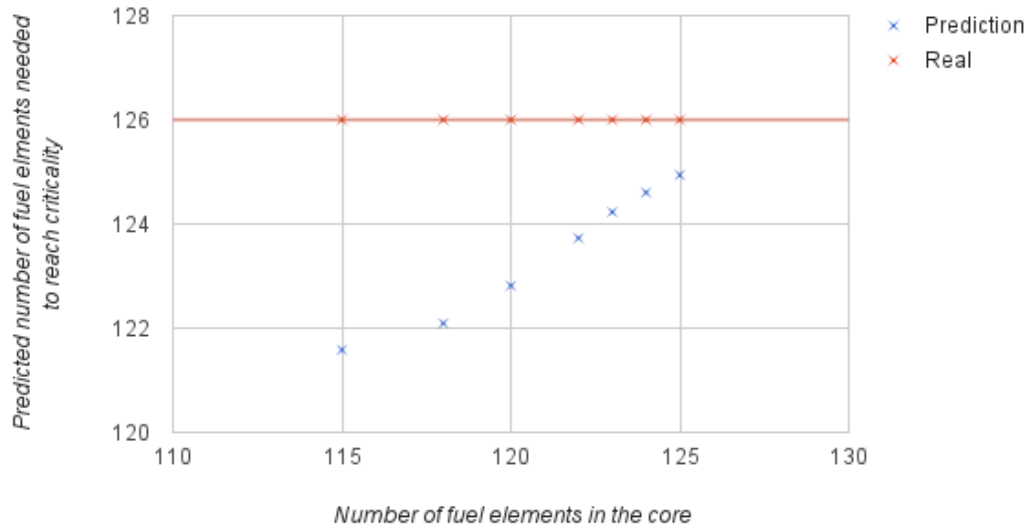


FIGURE 2.2. Evolution of the criticality prediction.

### 3. Shim 2: 897 units.

From historical data, the operators can know that the insertion of Shim 2 rod by a hundred unit injected 4 cents of reactivity to the core. The reactivity excess for the entire core (all fuel elements in) was measured at \$3.88 that day, and the reactivity of the regulating rod, which was left fully down, was \$3.088.

The reactivity missing from the last element (E-16) not being inserted and the reactor being exactly critical is, in dollars:

$$(2.1) \quad \rho = 3.88 - 3.088 - 0.04 = 0.752$$



## CONCLUSION

The USGS TRIGA research nuclear reactor (GSTR) have been used to irradiate a sample in order to determine its isotopic composition. While this method can also give the quantities (mass) of each nucleide in the sample, only an energy calibration of the spectrometer was performed, rendering this information unavailable. Instead, the elements in the sample have been identified, using the activated sample activity and a spectrometry.

The sample was found to have a 36 minutes half-life. The spectrometry generated low confidence data, due to potential ongoing work on the library. Nonetheless, if the results from the measurements are considered correct within the usual 1 keV uncertainty, it has been derived that the sample contained Hafnium and Gadolinium mostly, with an accumulation of Tantalum probably caused by the activation of Hafnium.





## NEUTRON POPULATION IN A SUBCRITICAL SYSTEM

This appendix presents the stabilization of the neutron population in a subcritical system, for different values of  $k_{eff}$ , after ten generations. It illustrates the fact that the neutron population will not die down to nothing in presence of a neutron source.

Generation	1	2	3	4	5	6	7	8	9	10
Population	100	99	98.01	97.03	96.06	95.10	94.15	93.21	92.27	91.35
		100	99	98.01	97.03	96.06	95.10	94.15	93.21	92.27
			100	99	98.01	97.03	96.06	95.10	94.15	93.21
				100	99	98.01	97.03	96.06	95.10	94.15
					100	99	98.01	97.03	96.06	95.10
						100	99	98.01	97.03	96.06
							100	99	98.01	97.03
								100	99	98.01
									100	99
										100
Neutron population	100	199	297.01	394.04	490.10	585.20	679.35	772.55	864.83	956.18

Table A.1: Neutron population from a 100-neutrons source, considering  $k_{eff} = 0.99$

APPENDIX A. NEUTRON POPULATION IN A SUBCRITICAL SYSTEM

Generation	1	2	3	4	5	6	7	8	9	10
Population	100	80	64	51.20	40.96	32.77	26.21	20.97	16.78	13.42
		100	80	64	51.20	40.96	32.77	26.21	20.97	16.78
			100	80	64	51.20	40.96	32.77	26.21	20.97
				100	80	64	51.20	40.96	32.77	26.21
					100	80	64	51.20	40.96	32.77
						100	80	64	51.20	40.96
							100	80	64	51.20
								100	80	64
									100	80
										100
Neutron population	100	180	244	295.20	336.16	368.93	395.14	416.11	432.89	446.31

Table A.2: Neutron population from a 100-neutrons source, considering  $k_{eff} = 0.8$

Generation	1	2	3	4	5	6	7	8	9	10
Population	100	50	25	12.50	6.25	3.13	1.56	0.78	0.39	0.20
		100	50	25	12.50	6.25	3.13	1.56	0.78	0.39
			100	50	25	12.50	6.25	3.13	1.56	0.78
				100	50	25	12.50	6.25	3.13	1.56
					100	50	25	12.50	6.25	3.13
						100	50	25	12.50	6.25
							100	50	25	12.50
								100	50	25
									100	50
										100
Neutron population	100	150	175	187.50	193.75	196.88	198.44	199.22	199.61	199.80

Table A.3: Neutron population from a 100-neutrons source, considering  $k_{eff} = 0.5$

**DETAILED DATA TABLES**

**T**his appendix presents the count rate read on the NM1000, before the signal had been processed by the detector and fed to the control room as a power percentage. It shows the measured mean count rate for various number of fuel rods inserted in the core. As a reminder, the GSTR core can hold up to 127 fuel assemblies.

Core inventory						Raw data														
112	0.70	0.73	0.73	0.72	0.73	0.70	0.72	0.70	0.72	0.77	0.77	0.76	0.78	0.80	0.81	0.84	0.81	0.80	0.75	0.78
115	1.20	1.14	1.11	1.05	1.06	1.05	1.08	1.09	1.11	1.05	1.09	1.11	1.11	1.13	1.11	1.14	1.14	1.13	1.08	1.05
118	1.91	1.92	1.86	1.78	1.75	1.72	1.72	1.69	1.69	1.77	1.81	1.81	1.84	1.83	1.92	1.98	1.97	1.98	2.00	2.00
120	2.84	2.83	2.83	2.91	2.91	2.88	2.80	2.69	2.70	2.64	2.56	2.58	2.56	2.53	2.42	2.36	2.38	2.36	2.31	2.28
122	3.31	3.27	3.30	3.34	3.41	3.52	3.56	3.50	3.53	3.56	3.63	3.64	3.63	3.56	3.55	3.52	3.52	3.47	3.47	3.42
123	5.13	5.20	5.08	5.06	5.03	4.94	4.98	4.81	4.92	4.89	4.89	4.83	4.70	4.73	4.80	4.72	4.75	4.69	4.67	4.67
124	7.13	6.91	6.66	6.88	7.13	6.94	7.09	7.22	7.53	7.53	7.66	7.59	7.55	7.63	7.69	7.25	7.34	7.38	7.03	7.38
125	11.20	11.30	12.60	12.80	12.80	12.00	11.60	11.60	11.90	12.00	12.00	11.90	12.20	11.90	12.90	12.60	12.80	12.60	12.60	12.20

Table B.1: NM1000 count rate - raw data

Core inventory (# fuel rods)	Average count rate
112	0.76
115	1.10
118	1.85
120	2.62
122	3.49
123	4.87
124	7.28
125	12.18

Table B.2: NM1000 count rate - average data

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## POWER INSTRUMENTATION

**T**his appendix presents the three different detectors used at the GSTR facility to compute the reactor thermal power output. In this project, only the NM1000 is used.

### C.1 NM1000

The NM1000 is a wide range, digital instrument. It is a fission chamber which can compute power from 0.0001 W to 1.2 MW. It is not linked to a SCRAM system, but contains a couple of interlocks.

### C.2 NP/NPP1000

The NP1000 and NPP1000 are similar instruments. They are analog ionization chambers, with a range of 10 kW to 2 GW, so they can follow reactor pulses but are mostly useless at startup. They are connected to two types a SCRAM, high power and loss of high voltage.



FIGURE C.1. NM1000.



FIGURE C.2. NP/NPP1000.



## COEFFICIENT OF DETERMINATION $R^2$

This appendix presents the mathematics behind the confidence factor, also known as the  $R^2$  value. The  $R^2$  value, representing the fit quality, can be found using the mean ( $\bar{y}$ ), the total sum of squares ( $SS_{tot}$ ), and the residual sum of squares ( $SS_{res}$ ).

Each is defined as:

$$(D.1) \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$(D.2) \quad SS_{tot} = \sum_i (y_i - \bar{y})^2$$

$$(D.3) \quad SS_{res} = \sum_i (y_i - f_i)^2$$

$$(D.4) \quad R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

where:

$f_i$  = Fitting function value at point  $x_i$



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