Nuclear Reactor Thermal-Hydraulics

NUGN520 - Homework

By

GUILLAUME L'HER

Department of Nuclear Engineering
COLORADO SCHOOL OF MINES

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CHAPTER 1

Atomic and nuclear structures and reactions

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the first chapter of the book, covering the subject of atomic and nuclear structures and reactions.

1.1 Photodisintegration

1.1.1 Problem

A relatively stationary Deuterium nucleus may be converted to a hydrogen nucleus by bombarding it with γ radiation, a process called photodisintegration. How much minimum γ energy is required for such conversion in Btu per gram of D_2?

1.1.2 Solution

The photodisintegration process can be described using the reaction presented in Equation 1.1.

$$ {}^{2}_{1}\mathrm{D} + {}^{0}_{0}\gamma \rightarrow {}^{1}_{1}\mathrm{H} + {}^{1}_{0}\mathrm{n} \tag{1.1}$$

Considering a stationary Deuterium nucleus, the energy in the system is carried by the various masses and the binding energy of the Deuterium. Hence, a photon being massless and using the principle of energy conservation, we can obtain Equation 1.2.

$$ m({}^{2}_{1}\mathrm{D}) = m_n + m({}^{1}_{1}\mathrm{H}) - \frac{B}{c^2} \tag{1.2}$$

Where:

B = Binding energy
c = Speed of light

The following masses ($1u = 1.66 \times 10^{-24} g$) are known.

$m({}^2_1\text{D}) = 2.014102u$
$m({}^1_1\text{H}) = 1.007825u$
$m_n \quad = 1.008665u$

Using Equation 1.2, we can obtain the binding energy, $B = 0.002388u$. This corresponds to $0.002388 * 931.5 = 2.224$MeV, or 3.377×10^{-16} Btu.

The binding energy, equivalent to the minimum energy needed from the γ radiation, is equaled to 3.377×10^{-16} Btu for $2.014102u$, hence 1.01×10^8 Btu/g.

1.2 Age of the earth

1.2.1 Problem

Rutherford once assumed that when the earth was first formed it contained equal amount of U^{235} and U^{238}. From this he was able to determine the age of the earth, and the answer was not very different from that found from astronomical data. Find the Rutherford age of the earth.

1.2.2 Solution

Let us assume a Uranium sample of 100 grams, containing equal amount of U^{235} and U^{238}. This simulates a sample from back when the earth was first formed, considering Rutherford's assumption. We know that today, a Uranium sample of 100 grams would contain approximately 0.7 grams of U^{235}.

The half-life of U^{235} is known, $T_{1/2} = 7.1 \times 10^8$ years. In order to go from 50 grams to 0.7g, roughly 6 doubling period are needed. Hence, Rutherford's method allows us to calculate that the earth is approximately $6 * 7.1 \times 10^8 = 4.3 \times 10^9$ years.

1.3 Positron decay

1.3.1 Problem

Phosphorous-30 is a radioactive isotope undergoing positron decay. Calculate the energy in $MeV/reaction$.

1.3.2 Solution

The positron decay process can be described using the reaction presented in Equation 1.3.

$$ {}^{30}_{15}\mathrm{P} \rightarrow {}^{30}_{14}\mathrm{Si} + {}^{0}_{1}\mathrm{e} + \nu_e \tag{1.3} $$

ν_e is an electron neutrino.

The conservation of energy indicates that the excess energy in the system is given by Equation 1.4.

$$ Q = \left[m({}^{30}_{15}\mathrm{P}) - m({}^{30}_{14}\mathrm{Si}) - m_e - m_{\nu_e} \right] c^2 \tag{1.4} $$

Knowing the various masses, we can calculate the energy.

$$ \begin{aligned} m({}^{30}_{15}\mathrm{P}) &= 29.9783138u \\ m({}^{30}_{14}\mathrm{Si}) &= 29.97377017u \\ m_e, m_{\nu_e} &= 0.00054858u \end{aligned} $$

Thus, $Q = 0.00344647u * c^2$. We also have $1u = 931.5MeV/c^2$, hence, the energy per reaction is 3.21 MeV.

1.4 Illuminators

1.4.1 Problem

The US Atomic Energy Commission allos, under license, the manufacture of illuminators for locks, watches, aircraft safety devices (switch plungers, control markers, exit signs, etc.) and other devices. The illuminators contain tritium plus a phosphor in the form of paint sealed in a plastic container. The low energy β radiation emitted from tritium is too weak to escape the container and no hazard is encountered. However, it acts upon the phosphor to provide luminosity. No more than 15 millicuries may be used in an automobile lock, or 4 curies in an aircraft safety device. In each case, find (a) the maximum amount of tritium in grams that may be used and (b) the percent decrease in luminosity after 5 years of operation.

1.4.2 Solution

The half-life of tritium is 12.3 years. It is possible to calculate the number of atoms N needed to generate a given amount of Curies as a function of the decay constant $\lambda = \frac{ln(2)}{T_{1/2}}$, using Equation 1.5.

$$ N = C * \frac{3.7 \times 10^{10}}{\lambda} \tag{1.5} $$

N_A being the Avogadro constant and m(T) the mass of a mole of tritium, this number of atoms can then be translated into a mass as presented by Equation 1.6.

$$M = \frac{N}{N_A} * m(T) \tag{1.6}$$

We thus obtain a maximum mass of 2.08×10^{-6} grams in an automobile lock, and 5.55×10^{-4} grams in an aircraft safety device. The luminosity is directly correlated to the tritium decay. After a given time t of operation, the activity of the tritium is given by $A = A_0 e^{-\lambda t}$. Consequently, after 5 years of operation, the activity and thus the luminosity has changed by a factor $e^{-5y*\lambda} = 0.755$, thus exhibiting a decrease of 24.5%.

1.5 Food irradiator

1.5.1 Problem

A "food irradiator" contains a Cesium-137 source of 170,000 Curies. (It is used to irradiate potatoes for sprout control, wheat flour for insect disinfection, etc.) Food is passed by the irradiator source at the rate of 300 lb/hr. Cesium-137 is a β emitter of 30 year half-life.

(a) What is the approximate mass of the source in grams?

(b) After 5 years of operation, what should be the rate of food processing in lb/hr if it were to receive the same dosage per lb?

1.5.2 Solution

Knowing the curie level of the food irradiator, the mass and the half-life of the Cesium-137, we can use Equations 1.5 and 1.6 to compute the mass of the source. It is equaled to 1953.15 grams, or almost 2 kg.

After 5 years of operation, the beta emitter efficiency would have changed by a factor $e^{-1/6} = 0.846$. Consequently, the rate of food processing must be lowered by $\frac{0.846-1}{0.846} = 18.2\%$ in order to compensate, resulting in a rate of 354.6 lb/hr.

CHAPTER 2

NEUTRONS AND THEIR INTERACTIONS

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the second chapter of the book, covering the subject of the neutrons and their interactions.

2.1 Europium

2.1.1 Problem

Naturally occuring Europium is subjected to a neutron flux of 10×10^{10} *(thermal). Its density is* $5.24 g/cm^3$*. Find (a) the rate of absorption reactions per* $s.cm^3$ *and (b) the energy produced in absorption if all absorption reactions are of the (n,* γ*) type, in* $MeV.s^{-1}.cm^{-3}$.

2.1.2 Solution

The reaction rate R is given by Equation 2.1.

$$R = \phi\Sigma = \phi N\sigma \tag{2.1}$$

Where:

ϕ = Flux ($n.s^{-1}.cm^{-2}$)
Σ = Total macroscopic cross section (cm^{-1})
σ = Total microscopic cross section (cm^2)
N = Nuclear density ($n.cm^{-3}$)

Naturally occurring Europium is composed of two isotopes, Eu^{151} and Eu^{153} (52.2%). The thermal absorption cross section is 9900 barn for Eu^{151} and 340 barn for Eu^{153} [2]. Knowing the sample density, $\rho = 5.24 g.cm^{-3}$, it is possible to calculate the nuclear density of Eu^{151} (N_{151}) and of Eu^{153} (N_{153}) in the sample, using Equation 2.2.

$$N_x = p_x * \rho * \frac{N_A}{A_t(x)} \tag{2.2}$$

Thus, the total macroscopic cross section for Europium, Σ_a, can be given by Equation 2.3.

$$\Sigma_a = N_{151} * \sigma_{a,151} + N_{153} * \sigma_{a,153} \tag{2.3}$$

Consequently:

$$\begin{aligned} N_{151} &= 9.994 \times 10^{21}\ n.cm^{-3} \\ N_{153} &= 1.077 \times 10^{22}\ n.cm^{-3} \\ \Sigma_a &= 102.606\ cm^{-1} \end{aligned}$$

This gives us a total absorption reaction rate of $1.03 \times 10^{12}\ r.s^{-1}.cm^{-3}$.

The absorption reaction rate for Eu^{153} is specifically $3.66 \times 10^{10}\ r.s^{-1}.cm^{-3}$, while the reaction rate for Eu^{151} is $9.89 \times 10^{11}\ r.s^{-1}.cm^{-3}$. Assuming all reaction are of the type (n,γ), we have only the reactions given in Equation 2.4, for $x \in \{151, 153\}$.

$${}^{x}_{63}\mathrm{Eu} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{x+1}_{63}\mathrm{Eu} + \gamma \tag{2.4}$$

The energy being conserved, we can write the ernergy balance equation, using Equation 2.5.

$$Q = \left(m({}^{x}_{63}\mathrm{Eu}) + m({}^{1}_{0}\mathrm{n}) - m({}^{x+1}_{63}\mathrm{Eu})\right) c^2 \tag{2.5}$$

Using the method presented in section 1.3, we obtain $Q(Eu^{153}) = 6.437\ MeV$ and $Q(Eu^{151}) = 6.169\ MeV$. Consequently, the energy produced is presented in Equation 2.6.

$$E_a = R_{153} * Q(Eu^{153}) + R_{151} * Q(Eu^{151}) \tag{2.6}$$

This gives us $E_a = 6.34 \times 10^{12}\ MeV.s^{-1}.cm^{-3}$

2.2 1/V absorber

2.2.1 Problem

The absorption mean free path for 2200 $m.s^{-1}$ *neutrons in a 1/V absorber is 1 cm. The corresponding reaction rate is* 1×10^{12} $s^{-1}.cm^{-3}$. *The absorber has an atomic mass of 10 and a density of* 2.0 $g.cm^{-3}$. *Find (a) the* 2200 $m.s^{-1}$ *flux and (b) the microscopic absorption cross section of 10-eV neutrons in barns.*

2.2.2 Solution

The mean free path λ is the reciprocal of the macroscopic cross section, $\lambda = \frac{1}{\Sigma}$. Knowing that the neutrons with a speed of 2200 $m.s^{-1}$, equivalent to 0.0252 eV according to Equation 2.7, have a mean free path λ of 1 cm, with a reaction rate R of 1×10^{12} $r.s^{-1}.cm^{-3}$, one can easily deduce the corresponding flux using Equation 2.8

$$E_n = \frac{1}{2} m V^2 \tag{2.7}$$

$$\phi = \frac{R}{\Sigma} = R\lambda \tag{2.8}$$

The 2200 $m.s^{-1}$ neutrons flux is thus 1×10^{12} $n.s^{-1}.cm^{-2}$.

The neutron absorber has an atomic mass of 10, we can hence identify Boron-10. This nuclei has a wider 1/V region, up to 150 eV [1], encompassing the 10-eV neutrons. Consequently, we can use the fact that for the Boron, in the thermal region, the absorption cross section is inversely proportional to the square root of the neutron energy, as explicited in Equation 2.9.

$$\sigma_a = C_1 \sqrt{\frac{1}{E_n}} \tag{2.9}$$

Using the knowledge of the macroscopic cross section $\Sigma = 1$ cm^{-1} for the neutrons of energy 0.025 eV, we can obtain the constant C_1 with Equation 2.10.

$$C_1 = \frac{\sigma_a}{\sqrt{\frac{1}{E_n}}} \tag{2.10}$$

We can obtain C_1 if we first compute σ_a as a function of Σ_a, which can be done by using Equation 2.11.

$$\sigma_a = \frac{\Sigma_a}{N} = \frac{A_t \Sigma_a}{\rho N_A} \tag{2.11}$$

Consequently, $\sigma_a = 8.303 \times 10^{-24}\ cm^2$ for 2200 $m.s^{-1}$ neutrons, since $\Sigma_a = 1\ cm^{-1}$. This allows us to obtain $C_1 = 1.318 \times 10^{-24}$. Consequently, plugging this back into Equation 2.9 for 10-eV neutrons, we obtain $\sigma_{a,10eV} = 0.417\ barn$.

2.3 Boron

2.3.1 Problem

Calculate the absorption macroscopic cross section in cm^{-1} of boron (density 2.3 $g.cm^{-3}$ for (a) 2200 $m.s^{-1}$, and (b) 10 eV neutrons.

2.3.2 Solution

Boron is composed of two isotopes, B^{10} and B^{11} (80.22%). However, due to the huge difference in absorption microscopic cross section between the two isotopes, respectively 3840 barns and 0.005 barns at thermal energy (0.025 eV), the contribution of B^{11} can be neglected.

The macroscopic cross section is given by Equation 2.12.

$$\Sigma_a = N_{10} * \sigma_{a,10} + N_{11} * \sigma_{a,11} \approx N_{10} * \sigma_{a,10} \tag{2.12}$$

Considering Equation 2.11, we can calculate $N_{10} = 19.78\% * \frac{\rho N_A}{10}$. We can thus obtain $\Sigma_{a,0.025eV} = 105.067\ cm^{-1}$.

Similarly to Problem 2.2, we can use Equation 2.9 after calculating C_1 from Equation 2.10. We find that $C_1 = 6.096 \times 10^{-22}$, and consequently, $\Sigma_{a,10eV} = N_{10} * C_1\sqrt{\frac{1}{10eV}} = 5.274\ cm^{-1}$.

2.4 Integral

2.4.1 Problem

Evaluate:

$$\int_0^{\infty} \frac{dx}{cosh(x)}$$

2.4.2 Solution

$$\int_0^{\infty} \frac{dx}{cosh(x)} \tag{2.13}$$

Knowing that $cosh(x) = \frac{1}{2}(e^x + e^{-x})$:

$$\int_0^{\infty} \frac{dx}{cosh(x)} = \int_0^{\infty} \frac{2dx}{e^x + e^{-x}} \tag{2.14}$$

A change of variable can be done, by defining $u = e^x$. It ensues that $1/u = e^{-x}$ and $\frac{du}{dx} = u$.

$$\int_0^\infty \frac{dx}{cosh(x)} = 2\int_1^\infty \frac{du}{u(u+\frac{1}{u})} \tag{2.15}$$

$$\int_0^\infty \frac{dx}{cosh(x)} = 2\int_1^\infty \frac{du}{(1+u^2)} \tag{2.16}$$

This is an identity integral, and we can thus write:

$$\int_0^\infty \frac{dx}{cosh(x)} = 2[arctan(u) + C]_1^\infty \tag{2.17}$$

And finally:

$$\int_0^\infty \frac{dx}{cosh(x)} = 2[arctan(\infty) - arctan(1)] = \pi - \frac{\pi}{2} = \frac{\pi}{2} \tag{2.18}$$

2.5 Root-mean-square

2.5.1 Problem

Suppose that the following nuclei are in thermal equilibrium: ^{16}O, ^{14}N, ^{235}U, and ^{238}U. Arrange these nuclei in order of increasing root-mean-square speed.

2.5.2 Solution

The root-mean-square speed is given by Equation 2.19.

$$V_{rms} = \sqrt{\frac{3RT}{M_m}} \tag{2.19}$$

Where:

R = molar gas constant
M_m = molar mass of the gas in kilograms per mole
T = Temperature (K)

Assuming thermal equilibrium, we can obtain that $\sqrt{3RT}$ is constant. The nuclei can thus be arranged as a function of their atomic mass. Consequently, the highest value of V_{rms} is obtained for ^{14}N, and the lowest value is for ^{238}U.

CHAPTER 3

FLUX IN THE CORE

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the second chapter of the book, covering the subject of the neutrons and their interactions.

3.1 Finite cylinder

3.1.1 Problem

Find the flux in a finite cylinder of radius R and height H using the diffusion theory.

3.1.2 Solution

The diffusion equation can be written following Equation 3.1.

$$\nabla^2\phi + B^2\phi = 0 \tag{3.1}$$

In a cylindrical coordinates system, the Laplacian ∇ can be explicited according to Equation 3.2.

$$\nabla^2\phi = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{3.2}$$

Consequently, we can rewrite Equation 3.1 to obtain Equation 3.3.

$$\frac{\partial^2\phi(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial\phi(r,z)}{\partial r} + \frac{\partial^2\phi(r,z)}{\partial z^2} + B^2\phi(r,z) = 0 \tag{3.3}$$

In order to solve this differential equation, one assumes that the axial flux component is independent from the radial flux component. In this case, the variables can be separated, using Definition 3.4.

$$\phi(r,z) = \rho(r)\zeta(z) \tag{3.4}$$

This implies Equation 3.5.

$$\zeta(z)\frac{\partial^2\rho(r)}{\partial r^2} + \frac{\zeta(z)}{r}\frac{\partial\rho(r)}{\partial r} + \rho(r)\frac{\partial^2\zeta(z)}{\partial z^2} + B^2\rho(r)\zeta(z) = 0 \tag{3.5}$$

Now, it is useful to divide the equation by $\rho(r)\zeta(z)$ in order to isolate the r-component from the z-component, as seen in Equation 3.6.

$$\frac{1}{\rho(r)}\frac{\partial^2\rho(r)}{\partial r^2} + \frac{1}{r\rho(r)}\frac{\partial\rho(r)}{\partial r} + \frac{1}{\zeta(z)}\frac{\partial^2\zeta(z)}{\partial z^2} = -B^2 \tag{3.6}$$

Now, we have an equation of the form $f(x)+g(y) = cst$. This equation can only be verified if $f(x) = cst$ and $g(y) = cst$. Consequently, we have the following system of equations 3.7 and 3.8 to solve.

$$\frac{1}{\rho(r)}\frac{\partial^2\rho(r)}{\partial r^2} + \frac{1}{r\rho(r)}\frac{\partial\rho(r)}{\partial r} = -\alpha \tag{3.7}$$

$$\frac{1}{\zeta(z)}\frac{\partial^2\zeta(z)}{\partial z^2} = -\beta \tag{3.8}$$

Where:

$\alpha + \beta = B^2$

Focusing on Equation 3.7 first, one can multiply both sides by $\rho(r)$, obtaining Equation 3.9.

$$\frac{\partial^2\rho(r)}{\partial r^2} + \frac{1}{r}\frac{\partial\rho(r)}{\partial r} + \alpha\rho(r) = 0 \tag{3.9}$$

Equation 3.9 is known as a Bessel equation. The solutions to this equation are the first and second kind of Bessel functions, $J_0(\sqrt{\alpha}r)$ (Figure 3.1) and $Y_0(\sqrt{\alpha}r)$ (Figure 3.2).

Consequently, the solution will be of the form $\rho(r) = AJ_0(\sqrt{\alpha}r) + CY_0(\sqrt{\alpha}r)$, A and C being constants.

We can now use boundary conditions. We know that the radial component of the flux has to be greater or equal to 0 $\forall r$. In our system, r = 0 at the center of the cylinder. However, $\lim_{r\to 0} Y_0(r) = -\infty$, implying that $C = 0$.

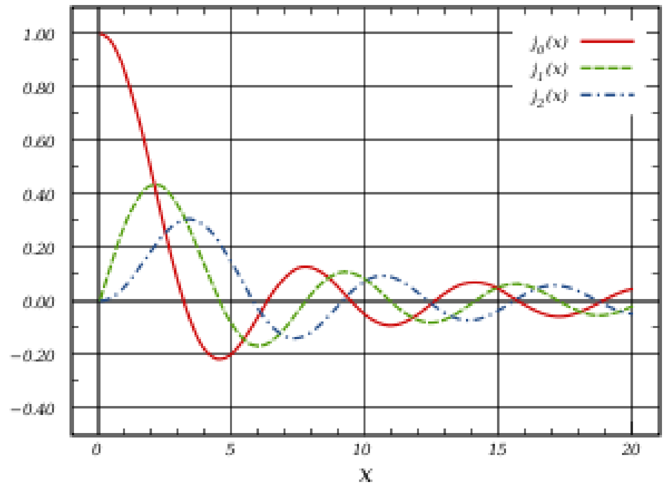

FIGURE 3.1. Bessel function of the first kind - J.

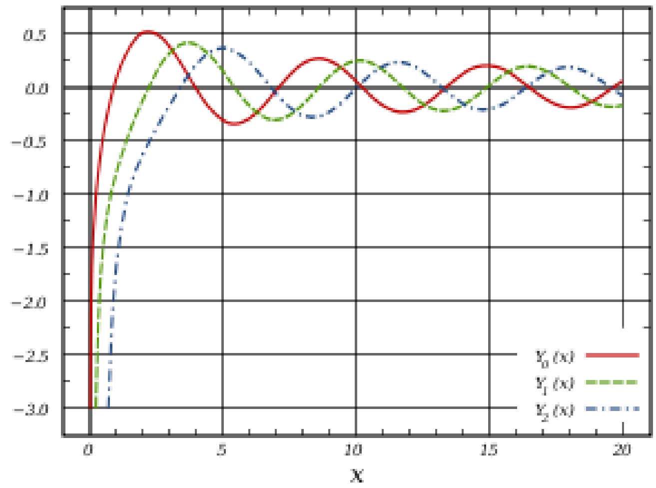

FIGURE 3.2. Bessel function of the second kind - Y.

We also know that at a radius $R_e = R + \delta_e$, the flux is to be zero. Thus, $\phi(R_e) = 0$. So, we have to solve Equation 3.10.

$$AJ_0(\sqrt{\alpha}R_e) = 0 \tag{3.10}$$

This can be solved by A = 0, in which case we would obtain the trivial solution $\rho(r) = 0$, of no interest. $J_0(x) = 0$ can be verified for several x. However, since the flux has to be positive, the solution has to be the first zero of the function, happening for $x = 2.405$. Hence, Equation 3.10 is verified for $\sqrt{\alpha}R_e = 2.405$.

Thus, we have the solution of the radial component of the flux, presented in Equation 3.11.

$$\rho(r) = AJ_0(\frac{2.405}{R_e}r) \tag{3.11}$$

Now, we can focus on the axial component of the flux, shown in Equation 3.8. It is interesting to recognize in this equation the equation for an infinite slab geometry, for which the solution is known, explicited in Equation 3.13.

$$\zeta(z) = D\cos(\frac{\pi}{H_e}z) \tag{3.12}$$

Finally, one can compute the flux $\phi(r,z)$, according to Equation 3.4.

$$\phi(r,z) = \phi_0 * J_0(\frac{2.405}{R_e}r)\cos(\frac{\pi}{H_e}z) \tag{3.13}$$

Where $\phi_0 = A * D$.

We can also deduce the buckling $B^2 = \alpha + \beta = \left(\frac{2.405}{R_e}\right)^2 + \left(\frac{\pi}{H_e}\right)^2$.

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