Nuclear Reactor Thermal-Hydraulics

NUGN520 - Homework

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CHAPTER 1

FLUX IN THE CORE

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the second chapter of the book, covering the subject of the neutrons and their interactions.

1.1 Finite cylinder

1.1.1 Problem

Find the flux in a finite cylinder of radius R and height H using the diffusion theory.

1.1.2 Solution

The diffusion equation can be written following Equation 1.1.

$$\nabla^2\phi + B^2\phi = 0 \tag{1.1}$$

In a cylindrical coordinates system, the Laplacian ∇ can be explicited according to Equation 1.2.

$$\nabla^2\phi = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{1.2}$$

Consequently, we can rewrite Equation 1.1 to obtain Equation 1.3.

$$\frac{\partial^2\phi(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial\phi(r,z)}{\partial r} + \frac{\partial^2\phi(r,z)}{\partial z^2} + B^2\phi(r,z) = 0 \tag{1.3}$$

In order to solve this differential equation, one assumes that the axial flux component is independent from the radial flux component. In this case, the variables can be separated, using Definition 1.4.

$$\phi(r,z) = \rho(r)\zeta(z) \tag{1.4}$$

This implies Equation 1.5.

$$\zeta(z)\frac{\partial^2\rho(r)}{\partial r^2} + \frac{\zeta(z)}{r}\frac{\partial\rho(r)}{\partial r} + \rho(r)\frac{\partial^2\zeta(z)}{\partial z^2} + B^2\rho(r)\zeta(z) = 0 \tag{1.5}$$

Now, it is useful to divide the equation by $\rho(r)\zeta(z)$ in order to isolate the r-component from the z-component, as seen in Equation 1.6.

$$\frac{1}{\rho(r)}\frac{\partial^2\rho(r)}{\partial r^2} + \frac{1}{r\rho(r)}\frac{\partial\rho(r)}{\partial r} + \frac{1}{\zeta(z)}\frac{\partial^2\zeta(z)}{\partial z^2} = -B^2 \tag{1.6}$$

Now, we have an equation of the form $f(x) + g(y) = cst$. This equation can only be verified if $f(x) = cst$ and $g(y) = cst$. Consequently, we have the following system of equations 1.7 and 1.8 to solve.

$$\frac{1}{\rho(r)}\frac{\partial^2\rho(r)}{\partial r^2} + \frac{1}{r\rho(r)}\frac{\partial\rho(r)}{\partial r} = -\alpha \tag{1.7}$$

$$\frac{1}{\zeta(z)}\frac{\partial^2\zeta(z)}{\partial z^2} = -\beta \tag{1.8}$$

Where:

$\alpha + \beta = B^2$

Focusing on Equation 1.7 first, one can multiply both sides by $\rho(r)$, obtaining Equation 1.9.

$$\frac{\partial^2\rho(r)}{\partial r^2} + \frac{1}{r}\frac{\partial\rho(r)}{\partial r} + \alpha\rho(r) = 0 \tag{1.9}$$

Equation 1.9 is known as a Bessel equation. The solutions to this equation are the first and second kind of Bessel functions, $J_0(\sqrt{\alpha}r)$ (Figure 1.1) and $Y_0(\sqrt{\alpha}r)$ (Figure 1.2).

Consequently, the solution will be of the form $\rho(r) = AJ_0(\sqrt{\alpha}r) + CY_0(\sqrt{\alpha}r)$, A and C being constants.

We can now use boundary conditions. We know that the radial component of the flux has to be greater or equal to 0 $\forall r$. In our system, r = 0 at the center of the cylinder. However, $\lim_{r\to 0} Y_0(r) = -\infty$, implying that $C = 0$.

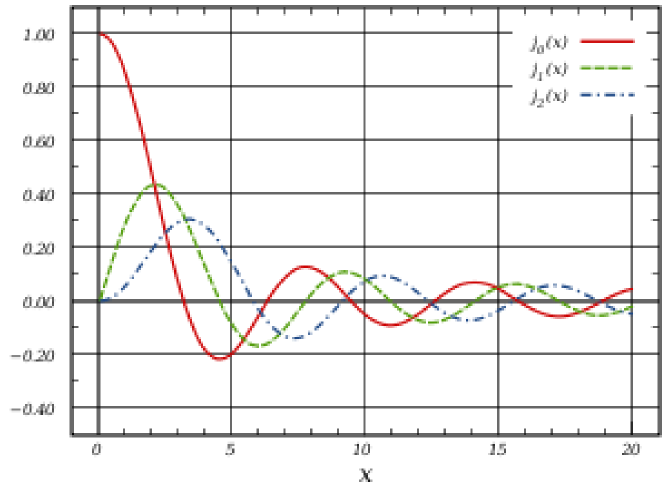

FIGURE 1.1. Bessel function of the first kind - J.

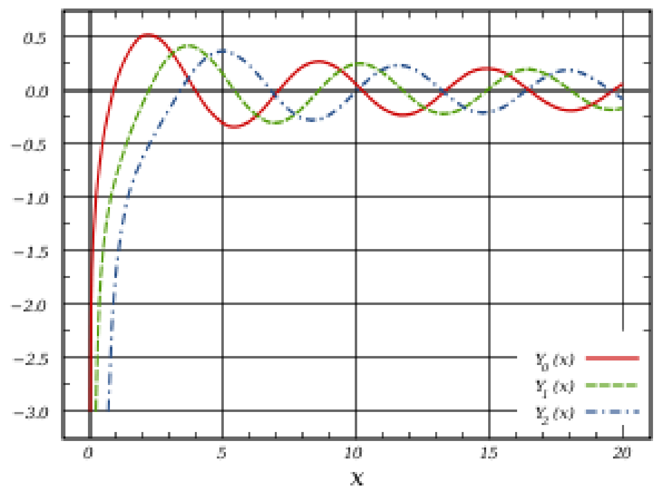

FIGURE 1.2. Bessel function of the second kind - Y.

We also know that at a radius $R_e = R + \delta_e$, the flux is to be zero. Thus, $\phi(R_e) = 0$. So, we have to solve Equation 1.10.

$$AJ_0(\sqrt{\alpha}R_e) = 0 \tag{1.10}$$

This can be solved by A = 0, in which case we would obtain the trivial solution $\rho(r) = 0$, of no interest. $J_0(x) = 0$ can be verified for several x. However, since the flux has to be positive, the solution has to be the first zero of the function, happening for $x = 2.405$. Hence, Equation 1.10 is verified for $\sqrt{\alpha}R_e = 2.405$.

Thus, we have the solution of the radial component of the flux, presented in Equation 1.11.

$$\rho(r) = AJ_0(\frac{2.405}{R_e}r) \tag{1.11}$$

Now, we can focus on the axial component of the flux, shown in Equation 1.8. It is interesting to recognize in this equation the equation for an infinite slab geometry, for which the solution is known, explicited in Equation 1.13.

$$\zeta(z) = D\cos(\frac{\pi}{H_e}z) \tag{1.12}$$

Finally, one can compute the flux $\phi(r,z)$, according to Equation 1.4.

$$\phi(r,z) = \phi_0 * J_0(\frac{2.405}{R_e}r)\cos(\frac{\pi}{H_e}z) \tag{1.13}$$

Where $\phi_0 = A * D$.

We can also deduce the buckling $B^2 = \alpha + \beta = \left(\frac{2.405}{R_e}\right)^2 + \left(\frac{\pi}{H_e}\right)^2$.

Bibliography

[1] M. M. El-Wakil, *Nuclear Heat Transport*, American Nuclear Society, 1993.

[2] G. Leinweber, D. Barry, R. Block, M. Rapp, J. Hoole, Y. Danon, R. Bahran, D. Williams, J. Geuther, and F. Saglime III, *Thermal total cross sections of europium from neutron capture and transmission measurements*, Transactions of the American Nuclear Society, 107 (2012), p. 1007.