

---

---

# Nuclear Reactor Thermal-Hydraulics

*NUGN520 - Homework*

---

---

By

GUILLAUME L'HER



Department of Nuclear Engineering  
COLORADO SCHOOL OF MINES

Homework submitted for the Nuclear Reactor Thermal-Hydraulics class at the Colorado School of Mines.

SPRING 2017



## TABLE OF CONTENTS

	Page
<b>1 Atomic and nuclear structures and reactions</b>	<b>1</b>
1.1 Photodisintegration . . . . .	1
1.1.1 Problem . . . . .	1
1.1.2 Solution . . . . .	1
1.2 Age of the earth . . . . .	2
1.2.1 Problem . . . . .	2
1.2.2 Solution . . . . .	2
1.3 Positron decay . . . . .	2
1.3.1 Problem . . . . .	2
1.3.2 Solution . . . . .	2
1.4 Illuminators . . . . .	3
1.4.1 Problem . . . . .	3
1.4.2 Solution . . . . .	3
1.5 Food irradiator . . . . .	4
1.5.1 Problem . . . . .	4
1.5.2 Solution . . . . .	4
<b>2 Neutrons and their interactions</b>	<b>5</b>
2.1 Europium . . . . .	5
2.1.1 Problem . . . . .	5
2.1.2 Solution . . . . .	5
2.2 1/V absorber . . . . .	7
2.2.1 Problem . . . . .	7
2.2.2 Solution . . . . .	7
2.3 Boron . . . . .	8
2.3.1 Problem . . . . .	8
2.3.2 Solution . . . . .	8
2.4 Integral . . . . .	8
2.4.1 Problem . . . . .	8

## TABLE OF CONTENTS

---

2.4.2	Solution . . . . .	8
2.5	Root-mean-square . . . . .	9
2.5.1	Problem . . . . .	9
2.5.2	Solution . . . . .	9
<b>Bibliography</b>		<b>11</b>

## ATOMIC AND NUCLEAR STRUCTURES AND REACTIONS

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the first chapter of the book, covering the subject of atomic and nuclear structures and reactions.

## 1.1 Photodisintegration

### 1.1.1 Problem

*A relatively stationary Deuterium nucleus may be converted to a hydrogen nucleus by bombarding it with  $\gamma$  radiation, a process called photodisintegration. How much minimum  $\gamma$  energy is required for such conversion in Btu per gram of  $D_2$ ?*

### 1.1.2 Solution

The photodisintegration process can be described using the reaction presented in Equation 1.1.



Considering a stationary Deuterium nucleus, the energy in the system is carried by the various masses and the binding energy of the Deuterium. Hence, a photon being massless and using the principle of energy conservation, we can obtain Equation 1.2.

$$(1.2) \quad m({}^2_1\text{D}) = m_n + m({}^1_1\text{H}) - \frac{B}{c^2}$$

Where:

$B$  = Binding energy

$c$  = Speed of light

The following masses ( $1u = 1.66 \times 10^{-24}g$ ) are known.

$$m({}_1^2\text{D}) = 2.014102u$$

$$m({}_1^1\text{H}) = 1.007825u$$

$$m_n = 1.008665u$$

Using Equation 1.2, we can obtain the binding energy,  $B = 0.002388u$ . This corresponds to  $0.002388 * 931.5 = 2.224\text{MeV}$ , or  $3.377 \times 10^{-16}$  Btu.

The binding energy, equivalent to the minimum energy needed from the  $\gamma$  radiation, is equaled to  $3.377 \times 10^{-16}$  Btu for  $2.014102u$ , hence  $1.01 \times 10^8$  Btu/g.

## 1.2 Age of the earth

### 1.2.1 Problem

*Rutherford once assumed that when the earth was first formed it contained equal amount of  $U^{235}$  and  $U^{238}$ . From this he was able to determine the age of the earth, and the answer was not very different from that found from astronomical data. Find the Rutherford age of the earth.*

### 1.2.2 Solution

Let us assume a Uranium sample of 100 grams, containing equal amount of  $U^{235}$  and  $U^{238}$ . This simulates a sample from back when the earth was first formed, considering Rutherford's assumption. We know that today, a Uranium sample of 100 grams would contain approximately 0.7 grams of  $U^{235}$ .

The half-life of  $U^{235}$  is known,  $T_{1/2} = 7.1 \times 10^8$  years. In order to go from 50 grams to 0.7g, roughly 6 doubling period are needed. Hence, Rutherford's method allows us to calculate that the earth is approximately  $6 * 7.1 \times 10^8 = 4.3 \times 10^9$  years.

## 1.3 Positron decay

### 1.3.1 Problem

*Phosphorous-30 is a radioactive isotope undergoing positron decay. Calculate the energy in MeV/reaction.*

### 1.3.2 Solution

The positron decay process can be described using the reaction presented in Equation 1.3.

$$(1.3) \quad {}^{30}_{15}\text{P} \rightarrow {}^{30}_{14}\text{Si} + {}^0_1\text{e} + \nu_e$$

$\nu_e$  is an electron neutrino.

The conservation of energy indicates that the excess energy in the system is given by Equation 1.4.

$$(1.4) \quad Q = [m({}^{30}_{15}\text{P}) - m({}^{30}_{14}\text{Si}) - m_e - m_{\nu_e}] c^2$$

Knowing the various masses, we can calculate the energy.

$$m({}^{30}_{15}\text{P}) = 29.9783138u$$

$$m({}^{30}_{14}\text{Si}) = 29.97377017u$$

$$m_e, m_{\nu_e} = 0.00054858u$$

Thus,  $Q = 0.00344647u * c^2$ . We also have  $1u = 931.5 \text{ MeV}/c^2$ , hence, the energy per reaction is 3.21 MeV.

## 1.4 Illuminators

### 1.4.1 Problem

*The US Atomic Energy Commission allos, under license, the manufacture of illuminators for locks, watches, aircraft safety devices (switch plungers, control markers, exit signs, etc.) and other devices. The illuminators contain tritium plus a phosphor in the form of paint sealed in a plastic container. The low energy  $\beta$  radiation emitted from tritium is too weak to escape the container and no hazard is encountered. However, it acts upon the phosphor to provide luminosity. No more than 15 millicuries may be used in an automobile lock, or 4 curies in an aircraft safety device. In each case, find (a) the maximum amount of tritium in grams that may be used and (b) the percent decrease in luminosity after 5 years of operation.*

### 1.4.2 Solution

The half-life of tritium is 12.3 years. It is possible to calculate the number of atoms  $N$  needed to generate a given amount of Curies as a function of the decay constant  $\lambda = \frac{\ln(2)}{T_{1/2}}$ , using Equation 1.5.

$$(1.5) \quad N = C * \frac{3.7 \times 10^{10}}{\lambda}$$

$N_A$  being the Avogadro constant and  $m(\text{T})$  the mass of a mole of tritium, this number of atoms can then be translated into a mass as presented by Equation 1.6.

$$(1.6) \quad M = \frac{N}{N_A} * m(T)$$

We thus obtain a maximum mass of  $2.08 \times 10^{-6}$  grams in an automobile lock, and  $5.55 \times 10^{-4}$  grams in an aircraft safety device. The luminosity is directly correlated to the tritium decay. After a given time  $t$  of operation, the activity of the tritium is given by  $A = A_0 e^{-\lambda t}$ . Consequently, after 5 years of operation, the activity and thus the luminosity has changed by a factor  $e^{-5y*\lambda} = 0.755$ , thus exhibiting a decrease of 24.5%.

## 1.5 Food irradiator

### 1.5.1 Problem

*A "food irradiator" contains a Cesium-137 source of 170,000 Curies. (It is used to irradiate potatoes for sprout control, wheat flour for insect disinfection, etc.) Food is passed by the irradiator source at the rate of 300 lb/hr. Cesium-137 is a  $\beta$  emitter of 30 year half-life.*

*(a) What is the approximate mass of the source in grams?*

*(b) After 5 years of operation, what should be the rate of food processing in lb/hr if it were to receive the same dosage per lb?*

### 1.5.2 Solution

Knowing the curie level of the food irradiator, the mass and the half-life of the Cesium-137, we can use Equations 1.5 and 1.6 to compute the mass of the source. It is equaled to 1953.15 grams, or almost 2 kg.

After 5 years of operation, the beta emitter efficiency would have changed by a factor  $e^{-1/6} = 0.846$ . Consequently, the rate of food processing must be lowered by  $\frac{0.846-1}{0.846} = 18.2\%$  in order to compensate, resulting in a rate of 354.6 lb/hr.



## NEUTRONS AND THEIR INTERACTIONS

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the second chapter of the book, covering the subject of the neutrons and their interactions.

## 2.1 Europium

### 2.1.1 Problem

Naturally occurring Europium is subjected to a neutron flux of  $10 \times 10^{10}$  (thermal). Its density is  $5.24 \text{ g/cm}^3$ . Find (a) the rate of absorption reactions per  $\text{s.cm}^3$  and (b) the energy produced in absorption if all absorption reactions are of the  $(n, \gamma)$  type, in  $\text{MeV.s}^{-1}.\text{cm}^{-3}$ .

### 2.1.2 Solution

The reaction rate  $R$  is given by Equation 2.1.

$$(2.1) \quad R = \phi \Sigma = \phi N \sigma$$

Where:

$\phi$  = Flux ( $\text{n.s}^{-1}.\text{cm}^{-2}$ )

$\Sigma$  = Total macroscopic cross section ( $\text{cm}^{-1}$ )

$\sigma$  = Total microscopic cross section ( $\text{cm}^2$ )

$N$  = Nuclear density ( $\text{n.cm}^{-3}$ )

Naturally occurring Europium is composed of two isotopes,  $Eu^{151}$  and  $Eu^{153}$  (52.2%). The thermal absorption cross section is 9900 barn for  $Eu^{151}$  and 340 barn for  $Eu^{153}$  [2]. Knowing the sample density,  $\rho = 5.24 g.cm^{-3}$ , it is possible to calculate the nuclear density of  $Eu^{151}$  ( $N_{151}$ ) and of  $Eu^{153}$  ( $N_{153}$ ) in the sample, using Equation 2.2.

$$(2.2) \quad N_x = p_x * \rho * \frac{N_A}{A_t(x)}$$

Thus, the total macroscopic cross section for Europium,  $\Sigma_a$ , can be given by Equation 2.3.

$$(2.3) \quad \Sigma_a = N_{151} * \sigma_{a,151} + N_{153} * \sigma_{a,153}$$

Consequently:

$$\begin{aligned} N_{151} &= 9.994 \times 10^{21} n.cm^{-3} \\ N_{153} &= 1.077 \times 10^{22} n.cm^{-3} \\ \Sigma_a &= 102.606 cm^{-1} \end{aligned}$$

This gives us a total absorption reaction rate of  $1.03 \times 10^{12} r.s^{-1}.cm^{-3}$ .

The absorption reaction rate for  $Eu^{153}$  is specifically  $3.66 \times 10^{10} r.s^{-1}.cm^{-3}$ , while the reaction rate for  $Eu^{151}$  is  $9.89 \times 10^{11} r.s^{-1}.cm^{-3}$ . Assuming all reaction are of the type (n, $\gamma$ ), we have only the reactions given in Equation 2.4, for  $x \in \{151, 153\}$ .



The energy being conserved, we can write the energy balance equation, using Equation 2.5.

$$(2.5) \quad Q = (m({}^x_{63}Eu) + m({}^1_0n) - m({}^{x+1}_{63}Eu)) c^2$$

Using the method presented in section 1.3, we obtain  $Q(Eu^{153}) = 6.437 MeV$  and  $Q(Eu^{151}) = 6.169 MeV$ . Consequently, the energy produced is presented in Equation 2.6.

$$(2.6) \quad E_a = R_{153} * Q(Eu^{153}) + R_{151} * Q(Eu^{151})$$

This gives us  $E_a = 6.34 \times 10^{12} MeV.s^{-1}.cm^{-3}$

## 2.2 1/V absorber

### 2.2.1 Problem

The absorption mean free path for  $2200 \text{ m.s}^{-1}$  neutrons in a 1/V absorber is 1 cm. The corresponding reaction rate is  $1 \times 10^{12} \text{ s}^{-1}.\text{cm}^{-3}$ . The absorber has an atomic mass of 10 and a density of  $2.0 \text{ g.cm}^{-3}$ . Find (a) the  $2200 \text{ m.s}^{-1}$  flux and (b) the microscopic absorption cross section of 10-eV neutrons in barns.

### 2.2.2 Solution

The mean free path  $\lambda$  is the reciprocal of the macroscopic cross section,  $\lambda = \frac{1}{\Sigma}$ . Knowing that the neutrons with a speed of  $2200 \text{ m.s}^{-1}$ , equivalent to 0.0252 eV according to Equation 2.7, have a mean free path  $\lambda$  of 1 cm, with a reaction rate  $R$  of  $1 \times 10^{12} \text{ r.s}^{-1}.\text{cm}^{-3}$ , one can easily deduce the corresponding flux using Equation 2.8

$$(2.7) \quad E_n = \frac{1}{2} m V^2$$

$$(2.8) \quad \phi = \frac{R}{\Sigma} = R \lambda$$

The  $2200 \text{ m.s}^{-1}$  neutrons flux is thus  $1 \times 10^{12} \text{ n.s}^{-1}.\text{cm}^{-2}$ .

The neutron absorber has an atomic mass of 10, we can hence identify Boron-10. This nuclei has a wider 1/V region, up to 150 eV [1], encompassing the 10-eV neutrons. Consequently, we can use the fact that for the Boron, in the thermal region, the absorption cross section is inversely proportional to the square root of the neutron energy, as explicated in Equation 2.9.

$$(2.9) \quad \sigma_a = C_1 \sqrt{\frac{1}{E_n}}$$

Using the knowledge of the macroscopic cross section  $\Sigma = 1 \text{ cm}^{-1}$  for the neutrons of energy 0.025 eV, we can obtain the constant  $C_1$  with Equation 2.10.

$$(2.10) \quad C_1 = \frac{\sigma_a}{\sqrt{\frac{1}{E_n}}}$$

We can obtain  $C_1$  if we first compute  $\sigma_a$  as a function of  $\Sigma_a$ , which can be done by using Equation 2.11.

$$(2.11) \quad \sigma_a = \frac{\Sigma_a}{N} = \frac{A_t \Sigma_a}{\rho N_A}$$

Consequently,  $\sigma_a = 8.303 \times 10^{-24} \text{ cm}^2$  for  $2200 \text{ m.s}^{-1}$  neutrons, since  $\Sigma_a = 1 \text{ cm}^{-1}$ . This allows us to obtain  $C_1 = 1.318 \times 10^{-24}$ . Consequently, plugging this back into Equation 2.9 for 10-eV neutrons, we obtain  $\sigma_{a,10\text{eV}} = 0.417 \text{ barn}$ .

## 2.3 Boron

### 2.3.1 Problem

*Calculate the absorption macroscopic cross section in  $\text{cm}^{-1}$  of boron (density  $2.3 \text{ g.cm}^{-3}$  for (a)  $2200 \text{ m.s}^{-1}$ , and (b)  $10 \text{ eV}$  neutrons.*

### 2.3.2 Solution

Boron is composed of two isotopes,  $B^{10}$  and  $B^{11}$  (80.22%). However, due to the huge difference in absorption microscopic cross section between the two isotopes, respectively 3840 barns and 0.005 barns at thermal energy (0.025 eV), the contribution of  $B^{11}$  can be neglected.

The macroscopic cross section is given by Equation 2.12.

$$(2.12) \quad \Sigma_a = N_{10} * \sigma_{a,10} + N_{11} * \sigma_{a,11} \approx N_{10} * \sigma_{a,10}$$

Considering Equation 2.11, we can calculate  $N_{10} = 19.78\% * \frac{\rho N_A}{10}$ . We can thus obtain  $\Sigma_{a,0.025\text{eV}} = 105.067 \text{ cm}^{-1}$ .

Similarly to Problem 2.2, we can use Equation 2.9 after calculating  $C_1$  from Equation 2.10. We find that  $C_1 = 6.096 \times 10^{-22}$ , and consequently,  $\Sigma_{a,10\text{eV}} = N_{10} * C_1 \sqrt{\frac{1}{10\text{eV}}} = 5.274 \text{ cm}^{-1}$ .

## 2.4 Integral

### 2.4.1 Problem

*Evaluate:*

$$\int_0^\infty \frac{dx}{\cosh(x)}$$

### 2.4.2 Solution

$$(2.13) \quad \int_0^\infty \frac{dx}{\cosh(x)}$$

Knowing that  $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$ :

$$(2.14) \quad \int_0^\infty \frac{dx}{\cosh(x)} = \int_0^\infty \frac{2dx}{e^x + e^{-x}}$$

A change of variable can be done, by defining  $u = e^x$ . It ensues that  $1/u = e^{-x}$  and  $\frac{du}{dx} = u$ .

$$(2.15) \quad \int_0^\infty \frac{dx}{\cosh(x)} = 2 \int_1^\infty \frac{du}{u(u + \frac{1}{u})}$$

$$(2.16) \quad \int_0^\infty \frac{dx}{\cosh(x)} = 2 \int_1^\infty \frac{du}{(1 + u^2)}$$

This is an identity integral, and we can thus write:

$$(2.17) \quad \int_0^\infty \frac{dx}{\cosh(x)} = 2[\arctan(u) + C]_1^\infty$$

And finally:

$$(2.18) \quad \int_0^\infty \frac{dx}{\cosh(x)} = 2[\arctan(\infty) - \arctan(1)] = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

## 2.5 Root-mean-square

### 2.5.1 Problem

Suppose that the following nuclei are in thermal equilibrium:  $^{16}\text{O}$ ,  $^{14}\text{N}$ ,  $^{235}\text{U}$ , and  $^{238}\text{U}$ . Arrange these nuclei in order of increasing root-mean-square speed.

### 2.5.2 Solution

The root-mean-square speed is given by Equation 2.19.

$$(2.19) \quad V_{rms} = \sqrt{\frac{3RT}{M_m}}$$

Where:

$R$  = molar gas constant

$M_m$  = molar mass of the gas in kilograms per mole

$T$  = Temperature (K)

Assuming thermal equilibrium, we can obtain that  $\sqrt{3RT}$  is constant. The nuclei can thus be arranged as a function of their atomic mass. Consequently, the highest value of  $V_{rms}$  is obtained for  $^{14}\text{N}$ , and the lowest value is for  $^{238}\text{U}$ .



## BIBLIOGRAPHY

- [1] M. M. EL-WAKIL, *Nuclear Heat Transport*, American Nuclear Society, 1993.
- [2] G. LEINWEBER, D. BARRY, R. BLOCK, M. RAPP, J. HOOLE, Y. DANON, R. BAHRAN, D. WILLIAMS, J. GEUTHER, AND F. SAGLIME III, *Thermal total cross sections of europium from neutron capture and transmission measurements*, Transactions of the American Nuclear Society, 107 (2012), p. 1007.