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# Nuclear Reactor Thermal-Hydraulics

*NUGN520 - Homework*

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By

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## REACTOR HEAT GENERATION

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the seventh and eighth chapter of the book, covering the subject of heat conduction in reactor elements.

## 1.1 [7-6] - Rectangular fin

### 1.1.1 Problem

*A very long fin is rectangular in cross-section  $0.48 * 0.24$  in. It generates  $2 \times 10^6 \text{ Btu.h}^{-1}.\text{ft}^{-3}$ . The fin base is at  $1000^\circ\text{F}$ . It is cooled by a gas at  $600^\circ\text{F}$  with a uniform heat transfer coefficient  $100 \text{ Btu.h}^{-1}.\text{ft}^{-2}.\text{°F}^{-1}$ . Using a network with  $\Delta x = 0.12$  in., write the necessary set of finite difference equations for the nodal points and solve by any one of the techniques at your command.  $k$  for the fin material  $= 10 \text{ Btu.h}^{-1}.\text{ft}^{-1}.\text{°F}^{-1}$*

### 1.1.2 Solution

First, one can note that the problem is missing a graph. The considered geometry is given in Figure 1.1.

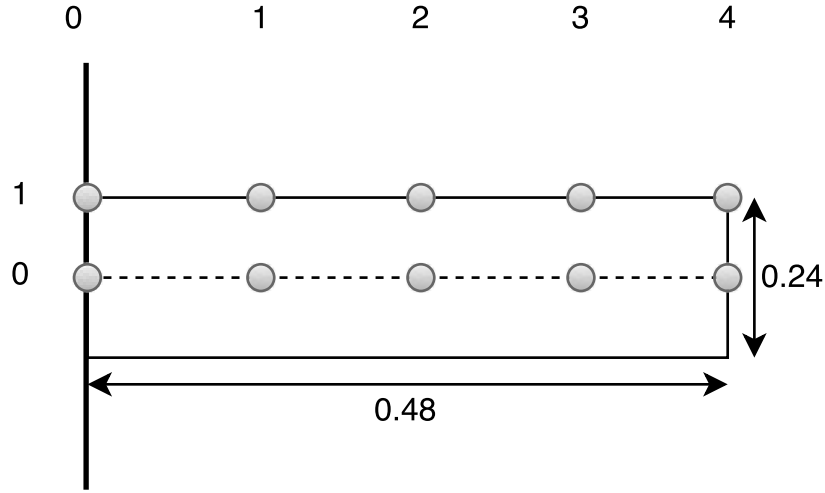


Figure 1.1: Representation of the problem geometry

We can identify four types of nodes: inner, y-bound (next to the upper boundary), x-bound (next to the right boundary) and corner.

The inner node temperature  $T_n$  is given by Equation 1.1.

$$(1.1) \quad T_n = \frac{T_{n-\Delta x} + T_{n+\Delta x} + T_{n-\Delta y} + T_{n+\Delta y}}{4} + \frac{\Delta T_g}{2}$$

In our case, we can see that  $T_{n-\Delta y} = T_{n+\Delta y}$  by symmetry for the inner nodes. Consequently, we obtain Equation 1.2.

$$(1.2) \quad T_n = \frac{T_{n-\Delta x} + T_{n+\Delta x} + 2T_{n+\Delta y}}{4} + \frac{\Delta T_g}{2}$$

For the x-bound nodes, we can write Equation 1.3.

$$(1.3) \quad T_n = \frac{T_{n-\Delta x} + T_{n+\Delta y} + Bi_{\Delta x} T_f}{2 + 2Bi_{\Delta x}} + \frac{\Delta T_g}{2 + Bi_{\Delta x}}$$

For the y-bound nodes, we can write Equation 1.4

$$(1.4) \quad T_n = \frac{T_{n-\Delta x} + T_{n+\Delta x} + 2T_{n+\Delta y} + 2Bi_{\Delta x} T_f}{4 + 2Bi_{\Delta x}} + \frac{\Delta T_g}{2 + Bi_{\Delta x}}$$

And finally, for the corner node, we can write (as seen in problem 7.1 previously) Equation 1.5.

$$(1.5) \quad T_n = \frac{T_{n-\Delta x} + T_{n+\Delta y} + 2Bi_{\Delta x} T_f}{2 + 2Bi_{\Delta x}} + \frac{\Delta T_g}{2 + 2Bi_{\Delta x}}$$

A few exceptions can be noted. Indeed, the two leftmost points, on the base of the fin, are given to be at  $1000^\circ F$ .

We can consequently write the matrix  $A$ .

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.25 & 0 & 1 & -0.5 & -0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{2Bi_{\Delta x}+4} & \frac{-1}{Bi_{\Delta x}+2} & 1 & 0 & \frac{-1}{2Bi_{\Delta x}+4} & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.25 & 0 & 1 & -0.5 & -0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{2Bi_{\Delta x}+4} & \frac{-1}{Bi_{\Delta x}+2} & 1 & 0 & \frac{-1}{2Bi_{\Delta x}+4} & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.25 & 0 & 1 & -0.5 & -0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2Bi_{\Delta x}+4} & \frac{-1}{Bi_{\Delta x}+2} & 1 & 0 & \frac{-1}{2Bi_{\Delta x}+4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{Bi_{\Delta x}+2} & 0 & 1 & \frac{-1}{Bi_{\Delta x}+2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{2Bi_{\Delta x}+2} & \frac{-1}{2Bi_{\Delta x}+2} & 1 \end{bmatrix}$$

$\frac{\Delta T_g}{2}$  can be calculated using  $\Delta T_g = \frac{(\Delta x)^2 q'''}{2k}$ .  $T_f$ , the temperature of the gas, is known. We can thus obtain  $b$ .

$$b = \begin{bmatrix} 1000 \\ 1000 \\ 7.2 \\ 229.5 \\ 7.2 \\ 229.5 \\ 7.2 \\ 229.5 \\ 229.5 \\ 330.5 \end{bmatrix}$$

The equation  $A.x = b$  can thus be solved, using Python. The python script is given in 1.1.2.1. We obtain the following results (Equation 1.6):

$$(1.6) \quad \begin{aligned} T(0,0) &= 1000.0 \\ T(0,1) &= 1000.0 \\ T(1,0) &= 804.0 \\ T(1,1) &= 740.9 \\ T(2,0) &= 705.3 \\ T(2,1) &= 665.0 \\ T(3,0) &= 658.5 \\ T(3,1) &= 635.7 \\ T(4,0) &= 628.3 \\ T(4,1) &= 617.8 \end{aligned}$$

### 1.1.2.1 Python script

```
import numpy as np

m = 10      # size of system is m x m

dx = 0.12
deltax, k, qtriple, h = (dx/10)**2, 10., 2.e6, 100.    # BG units
tb = 1000.
ts = 600.
dtg = (0.5*deltax)*qtriple/k
bi = h * dx / k

amat = np.zeros((m,m))
b = np.zeros(m)

amat[0,0] = 1.
amat[1,1] = 1.
b[0] = tb
b[1] = tb

for i in range(2, m, 2):
    try:
        amat[i,i-2] = -0.25
        amat[i,i] = 1.
        amat[i,i+1] = -0.5
        amat[i,i+2] = -0.25
        b[i] = dtg/2.
    except IndexError:
        break

for i in range(3, m, 2):
    try:
        amat[i,i-2] = -1/(2*bi+4)
        amat[i,i-1] = -1/(bi+2)
        amat[i,i] = 1.
        amat[i,i+2] = -1/(2*bi+4)
        b[i] = (bi*ts + dtg)/(bi+2)
    except IndexError:
        break

amat[m-1,m-1] = 1.
amat[m-1,m-2] = -1/(2*bi+2)
amat[m-1,m-3] = -1/(2*bi+2)
b[m-1] = (2*bi*ts + dtg)/(2*bi+2)

amat[m-2,m-2] = 1.
amat[m-2,m-1] = -1/(bi+2)
amat[m-2,m-4] = -1/(bi+2)
b[m-2] = (bi*ts + dtg)/(bi+2)
print(b)
ysol = np.linalg.solve(amat,b)
```



```
print('Temperature solution:')

temp = []
for i in range(5):
    for j in range(2):
        temp.append("T(%s,%s)" % (i, j))

for i,t in enumerate(temp):
    print("%s = %.1f" % (t, ysol[i]))

# check of solution:
if not np.allclose(np.dot(amat, ysol), b):
    print("Solution does not match!")
```

## 1.2 [7-13] - Heat flux

### 1.2.1 Problem

A long fuel element has a rectangular cross-section  $1 \times 2$  in. It generates  $1 \times 10^6 \text{ Btu.h}^{-1}.\text{ft}^{-3}$  and has a thermal conductivity of  $1.085 \text{ Btu.h}^{-1}.\text{ft}^{-1}.\text{°F}^{-1}$ . All surfaces are held at  $1000^\circ\text{F}$ . Find the heat flux,  $\text{Btu.h}^{-1}.\text{ft}^{-2}$  at the center point of each side using the approximate analytical solution of section 7-13.

### 1.2.2 Solution

The book [1] gives us a solution for a long rectangular fuel element (Equations 7-34 to 7-44). It is given here by Equation 1.7.

$$(1.7) \quad T(x, y) = \frac{3}{4} \frac{q'''}{k(a^2 + b^2)} (a^2 - x^2)(b^2 - y^2)$$

However, this solution is obtained for different boundary conditions ( $T_s = 0^\circ\text{F}$ ). In our case,  $T_s = 1000^\circ\text{F}$ . Then, we can rewrite Equation 1.7 to Equation 1.8 to account for this different boundary condition.

$$(1.8) \quad T(x, y) = \frac{3}{4} \frac{q'''}{k(a^2 + b^2)} (a^2 - x^2)(b^2 - y^2) + 1000 = C(a^2 - x^2)(b^2 - y^2) + 1000$$

The heat flux is given in the x-direction by Equation 1.9 and in the y-direction by Equation 1.10.

$$(1.9) \quad q''_x = -k \frac{\partial T}{\partial x}$$

$$(1.10) \quad q''_y = -k \frac{\partial T}{\partial y}$$

Consequently, we can write Equations 1.11 and 1.12.

$$(1.11) \quad q''_x = -2kCx(y^2 - b^2)$$

$$(1.12) \quad q''_y = -2kCy(x^2 - a^2)$$

At the center point of each side, we have Equations 1.13 and 1.14.

$$(1.13) \quad q''_x \Big|_{x=\alpha, y=0} = 2kCab^2$$

$$(1.14) \quad q''_y \Big|_{x=0, y=b} = 2kCba^2$$

So, we obtain  $q''_x = 2.5 \times 10^4 \text{ Btu.h}^{-1}.\text{ft}^{-2}$  and  $q''_y = 5.0 \times 10^4 \text{ Btu.h}^{-1}.\text{ft}^{-2}$ .

### 1.3 [8-1] - Cool down time

#### 1.3.1 Problem

A 2 in. diam. steel ball initially at a uniform  $850^\circ\text{F}$ , is suddenly subjected to an environment at  $200^\circ\text{F}$ . The natural convection heat transfer coefficient is  $2 \text{ Btu.h}^{-1}.\text{ft}^{-2}.\text{F}^{-1}$ . Find the time necessary for the ball to cool down to  $300^\circ\text{F}$ .

#### 1.3.2 Solution

The book [1] gives us the relationship between the temperature in a two-bodies problem, Equation 1.15.

$$(1.15) \quad \frac{T_f - T(t)}{T_f - T_i} = e^{-t/\tau}$$

Where:

$$\tau = \frac{c_1 \rho_1 V_1}{h A_1}$$

1 = index relative to the cooling body

Knowing that the cooling body is a steel ball, we can obtain its density and specific heat capacity. Knowing it is a sphere, we can obtain its surface area and volume. The heat transfer coefficient being known, we can thus calculate  $\tau = \frac{0.12 * 490 * \frac{4\pi * (0.0833)^3}{3}}{2 * 4\pi * (0.0833)^2} = 0.816$ . Consequently, we have Equation 1.16.

$$(1.16) \quad \frac{200 - 300}{200 - 850} = 0.154 = e^{-t/\tau}$$

$$(1.17) \quad t = -\tau \ln(0.154) = 1.528 \text{ h}$$

The ball will cool down to  $300^\circ\text{F}$  in 1.528 h, or roughly an hour and a half.

## 1.4 [8-4] -

### 1.4.1 Problem

*A flat-plate fuel element 1.25 \* 0.25 in. in cross section is initially at a uniform 1000°F. Suddenly heat was generated at the rate of  $1.5 \times 10^7 \text{ Btu.h}^{-1}.\text{ft}^{-3}$ . All surfaces were maintained at 1000°F. Find by a numerical technique the time it takes the element to reach a maximum temperature 99.5% of the way to maximum steady-state temperature.  $k = 1.085 \text{ Btu.h}^{-1}.\text{ft}^{-1}.\text{°F}^{-1}$ .  $c = 0.06 \text{ Btu.lb}^{-1}.\text{°F}^{-1}$ .  $\rho = 740 \text{ lb.ft}^{-3}$ .*

### 1.4.2 Solution

## BIBLIOGRAPHY

- [1] M. M. EL-WAKIL, *Nuclear Heat Transport*, American Nuclear Society, 1993.