Nuclear Fuel Cycle

NUGN506 - Homework

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Table of Contents

Page

CHAPTER 1

NUCLEAR FUEL FABRICATION

Several problems related to the reactor flux and power are considered in this homework.

1.1 Problem 5-1

1.1.1 Problem

A cylindrical thermal reactor with a radius of 1.8 m and a height of 3.8 m operates at a power of 1100 MWe with a thermal conversion efficiency (MWth –> MWe) of 33%. The reactor is fueled with 95 tons of uranium enriched to 3.0 wt% U-235. Assume 200 MeV released per fission and a U-235 fission cross section of 470 b for the neutron spectrum in this reactor. What is the average neutron flux?

1.1.2 Solution

We can first obtain the number of fission needed to get 1 joule or 1 Watt-second, considering $E_f = 200\ MeV$. One fission releases $200 * 1.602177 \times 10^{-13} = 3.2044 \times 10^{-11}$ Watt-seconds. That is, you need $1/3.2044 \times 10^{-11} = 3.1207 \times 10^{10}$ fission events to generate 1 Watt-second.

We can then link the reactor power P to the flux ϕ using:

$$P = \frac{\phi * \Sigma_f * V}{3.1207 \times 10^{10}} \quad (1.1)$$

And so, the flux is given by:

$$\phi = \frac{P * 3.1207 \times 10^{10}}{\Sigma_f * V} \quad (1.2)$$

The volume is $V = 3.868 \times 10^7\ cm^3$. $P = 3.3 \times 10^9\ Wth$.

The microscopic cross-section is $\sigma_f = 470\ b$. We can obtain the macroscopic cross-section using $\Sigma_f = \sigma_f * \rho$, where ρ is the atomic density in the fuel. We will consider here a classical reactor with a pin radius of 0.505 cm, a cladding with a width of 0.04 cm and a lattice size of 1.4 cm. Considering a 3% enriched UO2 fuel with a density of 10.1 $g.cm^{-3}$, water-moderated (0.8 $g.cm^{-3}$), HT9 cladding reactor (7.874 $g.cm^{-3}$), we can homogenize the reactor core (neglecting the absence of fuel in the control rods and some other geometric effects) and compute that we have $4.44 \times 10^{22}\ atoms.cm^{-3}$ (full calculation details available upon request). Consequently, $\Sigma_f = 470 \times 10^{-24} * 4.44 \times 10^{22} = 20.86 cm^{-1}$.

Plugging all this in the previous equation, we can obtain:

$$\phi = \frac{3.3 \times 10^9 * 3.1207 \times 10^{10}}{20.86 * 3.868 \times 10^7} = 1.276 \times 10^{11}\ n.cm^{-2}.s^{-1} \tag{1.3}$$

I could also (probably should) have used the amount of UO2 present (95 tons) to compute the atomic density of UO2 within the given volume:

The atomic weight of UO2 is 270 g/mol. The density of the fuel in the active reactor core is $95 \times 10^6 / 3.868 \times 10^7 = 2.456\ g.cm^{-3}$. Consequently, the atomic density in the reactor is $\frac{\rho N_A}{M} = \frac{2.456 * 6.022 \times 10^{23}}{270} = 5.478 \times 10^{21}\ atoms.cm^{-3}$. Using this value, we can obtain $\Sigma_f = 470 \times 10^{-24} * 5.478 \times 10^{21} = 2.57 cm^{-1}$, which eventually gives us a flux of $\phi = 1.03 \times 10^{12}\ n.cm^{-2}.s^{-1}$.

1.2 Problem 5-2

1.2.1 Problem

Calculate the macroscopic (n,γ) cross section (Σ_γ) of natural uranium in the form of uranium metal (density = 18.95 g/cm 3) for reactions in the resonance region of the neutron spectrum.

1.2.2 Solution

We'll neglect the U-234 presence in the uranium metal, and consider natural uranium (0.711% U-235, 99.289% U-238). In the resonance region, $\sigma_{\gamma,U8} = 277\ b$, while $\sigma_{\gamma,U5} = 140\ b$.

We can calculate the total macroscopic cross-section using:

$$\Sigma_\gamma = \sigma_{\gamma,U8} * N_{U8} + \sigma_{\gamma,U5} * N_{U5} \tag{1.4}$$

N_{U8} and N_{U5} can be obtained from the metal density:

$$N_{U5} = \frac{e * \rho N_A}{M} = \frac{e * \rho N_A}{(1-e) * 238.0289 + e * 235.0439} \tag{1.5}$$

$$N_{U8} = \frac{(1-e)*\rho N_A}{M} = \frac{(1-e)*\rho N_A}{(1-e)*238.0289+e*235.0439} \tag{1.6}$$

With a natural enrichement, $M = (1-e)*238.0289+e*235.0439 = 238.0077\ g/mol$

This gives us $N_{U8} = \frac{0.99289*18.95*N_A}{238.0077} = 4.76\times10^{22}\ atoms.cm^{-3}$ and $N_{U5} = \frac{0.00711*18.95*N_A}{238.0077} = 3.41\times10^{20}\ atoms.cm^{-3}$.

And consequently, $\Sigma_\gamma = 277\times10^{-24}*4.76\times10^{22}+140\times10^{-24}*3.41\times10^{20} = 13.23\ cm^{-1}$

1.2.3 Problem

Calculate the mass of Pu-239 produced in a reactor operating for one year with an average flux of $1.2\times10^{12}\ n.cm^{-2}.s^{-1}$*, loaded with 90 tons of uranium enriched to 3 wt% U-235. For cross sections, assume that for U-238* $\sigma_c = 2.1\ b$ *and* $\sigma_a = 2.3\ b$ *and for Pu-239* $\sigma_a = 600b$ *and the half-life of Pu-239 is 24,400 years.*

1.2.4 Solution

Pu-239 is created by capture of a neutron on U-238 and beta decay of the resulting U-239 with a short half-life. Consequently, we can consider that every neutron capture by U-238 results in Pu-239. The reaction rate for the neutron capture by U-238 is $R = \phi*\sigma_c = 1.2\times10^{12}*2.1\times10^{-24} = 2.52\times10^{-12}s^{-1}$. We can multiply by the number of U-238 atoms present in the fuel to compute the number of Pu-239 created per second.

$$N_{U8} = \frac{(1-e)*\rho V N_A}{M} = \frac{(1-e)*m*N_A}{(1-e)*238.0289+e*235.0439} = \frac{0.97*90\times10^6*N_A}{237.94} = 2.21\times10^{29} \tag{1.7}$$

Consequently, every second, we will have created $N_{U8}*R = 5.57\times10^{17}$ atoms of Pu-239 in the reactor. However, the Pu-239 would decay, with a half-life of 24,400 years, and it would also absorb neutrons.

These losses are given by $(\lambda+\sigma_{a,Pu239}*\phi)*N_{Pu-239}$. We can calculate the losses to be $(\frac{\ln(2)}{T_{1/2}}+\sigma_{a,Pu239}*\phi)*N_{Pu-239} = 4\times10^8$, the impact of the natural radioactive decay is negligible.

Consequently, the losses to capture and radioactive decay are negligible compared to the production of Pu-239, and we create 1.76×10^{25} atoms of Pu-239 after one year, that is $1.76\times10^{25}*239.05216*1.66054e-24\ g = 6.99\ kg$.

BIBLIOGRAPHY