Nuclear Reactor Thermal-Hydraulics

NUGN520 - Homework

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CHAPTER 1

Reactor Heat Generation

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the fourth chapter of the book, covering the subject of the reactor heat generation.

1.1 Uranyl Sulphate

1.1.1 Problem

Calculate the number density of U^{235}, nuclei/cm^3, in a 10 percent enriched uranyl sulphate (UO_2SO_4) in a 50 percent by mass aqueous solution at $500F$. Assume density of solution to be equal to that of water.

1.1.2 Solution

We know that in order to obtain the fissionable fuel density N_{ff}, we can use Equation 1.1.

$$N_{ff} = \frac{N_A}{M_{ff}} \rho_{ff} \tag{1.1}$$

Where,

M_{ff} = Molecular mass of fissionable fuel used
N_A = Avogadro number
ρ_{ff} = Density of fissionable fuel used

In this equation, we know M_{ff} and N_A. We thus have to compute ρ_{ff}, using Equation 1.2.

$$\rho_{ff} = rf\rho_{fm} \tag{1.2}$$

Where,

f = Mass fraction of the fuel in the fuel material
r = Enrichment
ρ_{fm} = Density of fuel material

The mass fraction is given by Equation 1.3.

$$f = \frac{rM_{ff} + (1-r)M_{nf}}{rM_{ff} + (1-r)M_{nf} + 6 * M_O + M_S} = 0.650 \tag{1.3}$$

The density of the solution being taken equal to that of water, 1 $g.cm^{-3}$, and the solution being composed at 50% mass of the fuel material, knowing the density of water in our case would allow us to compute the density of the fuel material. Using some water tables, we can obtain the density of saturated water at 500F, $\rho = 0.784\ g.cm^{-3}$. Thus, this would give us a density of the fuel material equal to $\rho_{fm} = 1.216\ g.cm^{-3}$.

Consequently, we can calculate $N_{ff} = 2.02 \times 10^{20}\ cm^{-3}$.

1.2 Fission cross section

1.2.1 Problem

Calculate the effective fission cross section of 3 percent enriched UO_2 if it is used in an ordinary water-moderated thermal reactor. The core lattice arrangement is such that the fuel/moderator ratio is 1:1.9 by volume. Assume the moderator is at 500F and 2000 psia, and that the other reactor-core materials contribute 2 barns of 1/V absorption per atom of hydrogen.

1.2.2 Solution

The effective fission cross section is given by Equation 1.4.

$$\bar{\sigma_f} = 0.8862 f(T)\sigma_{f,0}\sqrt{\frac{T_0}{T}} \tag{1.4}$$

For $T = 500F = 533K$, $f(T) = 0.93$ and $\sigma_{f,0} = 577.1\ b$, we can calculate $\bar{\sigma_f} = 352.7\ b$.

1.3 Volumetric thermal source

1.3.1 Problem

Calculate the volumetric thermal source strength, $Btu.h^{-1}.ft^{-3}$ at a point 49.9% of the radial distance and halfway above the centerplane of a cylindrical, homogeneous bare reactor core containing enriched UO_2SO_4 in solution in H_2O. The density of the fuel is 0.255 $g.cm^{-3}$. The temperature of the solution is 533K. The enrichment is 10 percent. The maximum neutron flux in the core is 1×10^{14}.

1.3.2 Solution

N_{ff} can be calculated for the fuel at hand using Equations 1.1 and 1.2. Using the density of the fuel given ($\rho_f = 0.255\ g.cm^{-3}$), we can obtain Equation 1.5.

$$N_{ff} = \frac{N_A}{235.0439} e \rho_f = 6.533 \times 10^{19}\ cm^{-3} \tag{1.5}$$

The fission cross section for the fuel is given by problem 1.2. We obtain $\bar{\sigma_f} = 352.7\ b$. If we assume the energy per fission to be $G = 180\ MeV$, we can use Equation 1.6.

$$q''' = G N_{ff} \bar{\sigma_f} \phi(r_1, z_1) \tag{1.6}$$

The only unknown left in Equation 1.6 is the flux at the given point. It can be calculated using Equation 1.7, with $z_1 = H/4$ and $r_1 = 49.9 * R/100$.

$$\phi(r_1, z_1) = \phi_0 \cos(\frac{\pi}{4}) J_0(\frac{49.9}{100} 2.405) = \phi_0 * \frac{1}{sqrt2} * 0.671 = 4.75 \times 10^{13} \tag{1.7}$$

Using a conversion constant ($1\ MeV.s^{-1}.cm^{-3} = 1.5477 \times 10^{-8}\ Btu.h^{-1}.ft^{-3}$), this gives us $q''' = 3.049 \times 10^6\ Btu.h^{-1}.ft^{-3}$.

1.4 Total heat generation

1.4.1 Problem

A fast homogeneous fluid-fueled reactor uses uranium in solution in molten Bismuth so that the fuel concentration is $1 \times 10^{20}\ U^{235}\ cm^{-3}$. *The reactor core is 5 feet in diameter and 8 feet high. The extrapolated height is 12 feet. Because of strong radial reflection, the neutron flux may be considered uniform in the radial direction. The fast neutron flux at the core entrance (bottom) is* 1×10^{15}. *Take $\bar{\sigma_f}$ for fast neutrons as 5 barns. Calculate the total heat generated in the core.*

1.4.2 Solution

In a cylindrical core, we can write Equation 1.8.

$$q''' = GN_{ff}\bar{\sigma_f}\phi_0 \cos(\frac{\pi z}{H_e})J_0(\frac{2.405r}{R_e}) \tag{1.8}$$

The definition of a radially uniform flux eluding me, I will assume that it implies a constant radial flux $S = J_0(0) = 1$, so that Equation 1.8 becomes Equation 1.9.

$$q''' = GN_{ff}\bar{\sigma_f}\phi_0 \cos(\frac{\pi z}{H_e}) \tag{1.9}$$

The total heat generated in the core is given by $Q = \int \Delta Q$, where $\Delta Q = q'''\Delta V$, V the volume, as seen in Equation 1.10 and 1.11.

$$\Delta Q = GN_{ff}\bar{\sigma_f}\phi_0 \cos(\frac{\pi z}{H_e})2\pi r\Delta r\Delta z \tag{1.10}$$

$$Q = GN_{ff}\bar{\sigma_f}\phi_0 2\pi \int_0^R r dr \int_{-H/2}^{H/2} \cos(\frac{\pi z}{H_e})dz \tag{1.11}$$

Solving the integrals, we obtain Equation 1.12

$$Q = GN_{ff}\bar{\sigma_f}\phi_0 2\pi \frac{R^2}{2}\frac{H_e\sqrt{3}}{\pi} = GN_{ff}\bar{\sigma_f}\phi_0 R^2 H_e\sqrt{3} \tag{1.12}$$

Considering the following values:

$$\begin{aligned} G &= 180 \text{ MeV} \\ N_{ff} &= 1\times 10^{20}\ cm^{-3} \\ \phi_0 &= 1\times 10^{15}\ cm^{-2}.s^{-1} \\ \bar{\sigma_f} &= 5\times 10^{-24}\ cm^2 \end{aligned}$$

We can obtain the total heat generated $Q = 2.21\times 10^{20}\ MeV.s^{-1} = 1.21\times 10^{8}\ Btu.hr^{-1}$

1.5 Minimum critical volume

1.5.1 Problem

Find the minimum critical volume of a cylindrical bare reactor.

1.5.2 Solution

The buckling for a cylindrical core, with extrapolated radius and height R_e and H_e, was computed during the third homework. We recall Equation 1.13.

$$B^2 = \left(\frac{2.405}{R_e}\right)^2 + \left(\frac{\pi}{H_e}\right)^2 \tag{1.13}$$

In this case, we will assume that $R = R_e$ and $H = H_e$. Then, we get Equation 1.14.

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 \tag{1.14}$$

We can subsitute this into the volume, as seen in Equation 1.15.

$$V = \pi R^2 H = \pi 2.405^2 * \frac{H^3}{B^2H^2 - \pi^2} \tag{1.15}$$

The minimum critical volume is given when the derivative of V with respect to H is zero, Equation 1.18. In order to obtain Equation 1.18, one starts from Equation 1.16.

$$\frac{dV}{dH} = \pi 2.405^2 * \frac{H^3}{B^2H^2 - \pi^2} \tag{1.16}$$

Applying the multiplication rule, with $f(H) = \pi 2.405^2 * H^3$ and $g(H) = \frac{1}{B^2H^2-\pi^2}$, we can obtain Equation 1.17. This is done by knowing that $\frac{d(fg)}{dH} = f'g + fg'$, and by calculating $f'(H) = 3 * \pi 2.405^2 * H^2$. $g'(H)$ can be obtained through the quotient rule, stating that if $g(H) = \frac{i(H)}{h(H)}$, then $g'(H) = \frac{i'(H)h(H) - i(H)h'(H)}{h^2(H)}$. Consequently, posing $i(H) = 1$ and $h(H) = B^2H^2 - \pi^2$, we can finally obtain $g'(H) = \frac{-2HB^2}{B^2H^2-\pi^2}$.

$$\frac{dV}{dH} = \frac{3 * \pi 2.405^2 * H^2}{B^2H^2 - \pi^2} - \frac{2 * \pi 2.405^2 * H^4 B^2}{(B^2H^2 - \pi^2)^2} \tag{1.17}$$

And finally,

$$\frac{dV}{dH} = \frac{2.405^2 H^2 (\pi B^2 H^2 - 3\pi^3)}{(\pi^2 - B^2H^2)^2} = 0 \tag{1.18}$$

This is possible if and only if $H^2 = \frac{3\pi^3}{\pi B^2} = 3\frac{\pi^2}{B^2}$, and hence $H = \frac{\sqrt{3}\pi}{B}$. In that case, we can compute $R^2 = \frac{2.405^2}{B^2 - \frac{\pi^2 B^2}{3\pi^2}}$, and hence $R = \sqrt{\frac{3}{2}}\frac{2.405}{B}$.

Plugging this back into the volume equation (Equation 1.15), we obtain $V_{min} = \frac{148.3}{B^3}$.

The surface area is given by Equation 1.19.

$$A = 2\pi RH + 2\pi R^2 \tag{1.19}$$

Plugging in the values for R and H obtained earlier, we trivially compute the surface area corresponding to the minimum critical volume, $A = \frac{155.2}{B^2}$.

Bibliography

[1] M. M. El-Wakil, *Nuclear Heat Transport*, American Nuclear Society, 1993.

[2] G. Leinweber, D. Barry, R. Block, M. Rapp, J. Hoole, Y. Danon, R. Bahran, D. Williams, J. Geuther, and F. Saglime III, *Thermal total cross sections of europium from neutron capture and transmission measurements*, Transactions of the American Nuclear Society, 107 (2012), p. 1007.