Nuclear Reactor Thermal-Hydraulics

NUGN520 - Homework

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CHAPTER 1

NEUTRONS AND THEIR INTERACTIONS

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the second chapter of the book, covering the subject of the neutrons and their interactions.

1.1 Europium

1.1.1 Problem

Naturally occuring Europium is subjected to a neutron flux of 10×10^{10} *(thermal). Its density is* $5.24 g/cm^3$*. Find (a) the rate of absorption reactions per* $s.cm^3$ *and (b) the energy produced in absorption if all absorption reactions are of the (n,* γ*) type, in* $MeV.s^{-1}.cm^{-3}$*.*

1.1.2 Solution

The reaction rate R is given by Equation 1.1.

$$R = \phi\Sigma = \phi N \sigma \tag{1.1}$$

Where:

ϕ = Flux ($n.s^{-1}.cm^{-2}$)
Σ = Total macroscopic cross section (cm^{-1})
σ = Total microscopic cross section (cm^2)
N = Nuclear density ($n.cm^{-3}$)

Naturally occurring Europium is composed of two isotopes, Eu^{151} and Eu^{153} (52.2%). The thermal absorption cross section is 9900 barn for Eu^{151} and 340 barn for Eu^{153} [2]. Knowing the sample density, $\rho = 5.24 g.cm^{-3}$, it is possible to calculate the nuclear density of Eu^{151} (N_{151}) and of Eu^{153} (N_{153}) in the sample, using Equation 1.2.

$$N_x = p_x * \rho * \frac{N_A}{A_t(x)} \tag{1.2}$$

Thus, the total macroscopic cross section for Europium, Σ_a, can be given by Equation 1.3.

$$\Sigma_a = N_{151} * \sigma_{a,151} + N_{153} * \sigma_{a,153} \tag{1.3}$$

Consequently:

$$\begin{aligned} N_{151} &= 9.994 \times 10^{21}\ n.cm^{-3} \\ N_{153} &= 1.077 \times 10^{22}\ n.cm^{-3} \\ \Sigma_a &= 102.606\ cm^{-1} \end{aligned}$$

This gives us a total absorption reaction rate of $1.03 \times 10^{12}\ r.s^{-1}.cm^{-3}$.

The absorption reaction rate for Eu^{153} is specifically $3.66 \times 10^{10}\ r.s^{-1}.cm^{-3}$, while the reaction rate for Eu^{151} is $9.89 \times 10^{11}\ r.s^{-1}.cm^{-3}$. Assuming all reaction are of the type (n,γ), we have only the reactions given in Equation 1.4, for $x \in \{151, 153\}$.

$${}^{x}_{63}\mathrm{Eu} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{x+1}_{63}\mathrm{Eu} + \gamma \tag{1.4}$$

The energy being conserved, we can write the ernergy balance equation, using Equation 1.5.

$$Q = \left(m({}^{x}_{63}\mathrm{Eu}) + m({}^{1}_{0}\mathrm{n}) - m({}^{x+1}_{63}\mathrm{Eu})\right) c^2 \tag{1.5}$$

Using the method presented in section **??**, we obtain $Q(Eu^{153}) = 6.437\ MeV$ and $Q(Eu^{151}) = 6.169\ MeV$. Consequently, the energy produced is presented in Equation 1.6.

$$E_a = R_{153} * Q(Eu^{153}) + R_{151} * Q(Eu^{151}) \tag{1.6}$$

This gives us $E_a = 6.34 \times 10^{12}\ MeV.s^{-1}.cm^{-3}$

1.2 1/V absorber

1.2.1 Problem

The absorption mean free path for 2200 $m.s^{-1}$ *neutrons in a 1/V absorber is 1 cm. The corresponding reaction rate is* 1×10^{12} $s^{-1}.cm^{-3}$. *The absorber has an atomic mass of 10 and a density of* 2.0 $g.cm^{-3}$. *Find (a) the* 2200 $m.s^{-1}$ *flux and (b) the microscopic absorption cross section of 10-eV neutrons in barns.*

1.2.2 Solution

The mean free path λ is the reciprocal of the macroscopic cross section, $\lambda = \frac{1}{\Sigma}$. Knowing that the neutrons with a speed of 2200 $m.s^{-1}$, equivalent to 0.0252 eV according to Equation 1.7, have a mean free path λ of 1 cm, with a reaction rate R of 1×10^{12} $r.s^{-1}.cm^{-3}$, one can easily deduce the corresponding flux using Equation 1.8

$$E_n = \frac{1}{2} m V^2 \tag{1.7}$$

$$\phi = \frac{R}{\Sigma} = R\lambda \tag{1.8}$$

The 2200 $m.s^{-1}$ neutrons flux is thus 1×10^{12} $n.s^{-1}.cm^{-2}$.

The neutron absorber has an atomic mass of 10, we can hence identify Boron-10. This nuclei has a wider 1/V region, up to 150 eV [1], encompassing the 10-eV neutrons. Consequently, we can use the fact that for the Boron, in the thermal region, the absorption cross section is inversely proportional to the square root of the neutron energy, as explicited in Equation 1.9.

$$\sigma_a = C_1 \sqrt{\frac{1}{E_n}} \tag{1.9}$$

Using the knowledge of the macroscopic cross section $\Sigma = 1$ cm^{-1} for the neutrons of energy 0.025 eV, we can obtain the constant C_1 with Equation 1.10.

$$C_1 = \frac{\sigma_a}{\sqrt{\frac{1}{E_n}}} \tag{1.10}$$

We can obtain C_1 if we first compute σ_a as a function of Σ_a, which can be done by using Equation 1.11.

$$\sigma_a = \frac{\Sigma_a}{N} = \frac{A_t \Sigma_a}{\rho N_A} \tag{1.11}$$

Consequently, $\sigma_a = 8.303 \times 10^{-24}\ cm^2$ for 2200 $m.s^{-1}$ neutrons, since $\Sigma_a = 1\ cm^{-1}$. This allows us to obtain $C_1 = 1.318 \times 10^{-24}$. Consequently, plugging this back into Equation 1.9 for 10-eV neutrons, we obtain $\sigma_{a,10eV} = 0.417\ barn$.

1.3 Boron

1.3.1 Problem

Calculate the absorption macroscopic cross section in cm^{-1} *of boron (density* $2.3\ g.cm^{-3}$ *for (a)* 2200 $m.s^{-1}$*, and (b) 10 eV neutrons.*

1.3.2 Solution

Boron is composed of two isotopes, B^{10} and B^{11} (80.22%). However, due to the huge difference in absorption microscopic cross section between the two isotopes, respectively 3840 barns and 0.005 barns at thermal energy (0.025 eV), the contribution of B^{11} can be neglected.

The macroscopic cross section is given by Equation 1.12.

$$\Sigma_a = N_{10} * \sigma_{a,10} + N_{11} * \sigma_{a,11} \approx N_{10} * \sigma_{a,10} \tag{1.12}$$

Considering Equation 1.11, we can calculate $N_{10} = 19.78\% * \frac{\rho N_A}{10}$. We can thus obtain $\Sigma_{a,0.025eV} = 105.067\ cm^{-1}$.

Similarly to Problem 1.2, we can use Equation 1.9 after calculating C_1 from Equation 1.10. We find that $C_1 = 6.096 \times 10^{-22}$, and consequently, $\Sigma_{a,10eV} = N_{10} * C_1\sqrt{\frac{1}{10eV}} = 5.274\ cm^{-1}$.

1.4 Integral

1.4.1 Problem

Evaluate:

$$\int_0^{\infty} \frac{dx}{cosh(x)}$$

1.4.2 Solution

$$\int_0^{\infty} \frac{dx}{cosh(x)} \tag{1.13}$$

Knowing that $cosh(x) = \frac{1}{2}(e^x + e^{-x})$:

$$\int_0^{\infty} \frac{dx}{cosh(x)} = \int_0^{\infty} \frac{2dx}{e^x + e^{-x}} \tag{1.14}$$

A change of variable can be done, by defining $u = e^x$. It ensues that $1/u = e^{-x}$ and $\frac{du}{dx} = u$.

$$\int_0^{\infty} \frac{dx}{cosh(x)} = 2\int_1^{\infty} \frac{du}{u(u+\frac{1}{u})} \tag{1.15}$$

$$\int_0^{\infty} \frac{dx}{cosh(x)} = 2\int_1^{\infty} \frac{du}{(1+u^2)} \tag{1.16}$$

This is an identity integral, and we can thus write:

$$\int_0^{\infty} \frac{dx}{cosh(x)} = 2[arctan(u)+C]_1^{\infty} \tag{1.17}$$

And finally:

$$\int_0^{\infty} \frac{dx}{cosh(x)} = 2[arctan(\infty)-arctan(1)] = \pi - \frac{\pi}{2} = \frac{\pi}{2} \tag{1.18}$$

1.5 Root-mean-square

1.5.1 Problem

Suppose that the following nuclei are in thermal equilibrium: ^{16}O, ^{14}N, ^{235}U, and ^{238}U. Arrange these nuclei in order of increasing root-mean-square speed.

1.5.2 Solution

The root-mean-square speed is given by Equation 1.19.

$$V_{rms} = \sqrt{\frac{3RT}{M_m}} \tag{1.19}$$

Where:

R = molar gas constant
M_m = molar mass of the gas in kilograms per mole
T = Temperature (K)

Assuming thermal equilibrium, we can obtain that $\sqrt{3RT}$ is constant. The nuclei can thus be arranged as a function of their atomic mass. Consequently, the highest value of V_{rms} is obtained for ^{14}N, and the lowest value is for ^{238}U.

Bibliography

[1] M. M. El-Wakil, *Nuclear Heat Transport*, American Nuclear Society, 1993.

[2] G. Leinweber, D. Barry, R. Block, M. Rapp, J. Hoole, Y. Danon, R. Bahran, D. Williams, J. Geuther, and F. Saglime III, *Thermal total cross sections of europium from neutron capture and transmission measurements*, Transactions of the American Nuclear Society, 107 (2012), p. 1007.