
Control rod calibration

Calibration of the transient rod

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ABSTRACT

The differential and integral worth of the control rods must be measured and calibrated from time to time, to check for the rods efficiency. Several limits must be respected, such as the maximum core reactivity, the maximum pulse reactivity, the maximum reactivity insertion rate and, for research reactors, the maximum experiment reactivity. This is done by using the reactivity values of the control rods, hence the utmost importance of having precise values for these parameters. In this project, the positive period method is used, consisting of measuring the reactivity brought by moving up the calibrated control rod in discrete steps, until it is completely out of the core.

In power reactors, various methods are used. For example, an Exchange-Dilution method can sometimes be used for reactor with boron water.

The procedure for this experiment will be to get to a critical, stable state with the control rod to calibrate in its down position. Then, it will be taken up a discrete amount of steps, and the period will be to get a twenty-fold increase in power (from 20W to 400W) will be measured. Using the delayed neutron fractions and the period in the Inhour equation, the reactivity will be calculated.

The results show that the transient control rod is relatively well calibrated, vertically centered in the core. It also demonstrates that the licensing limits for this reactor are comfortably respected.

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THEORY

The differential and integral worth of the control rods must be measured and calibrated from time to time, to check for the rods efficiency. Several limits must be respected, such as the maximum core reactivity, the maximum pulse reactivity, the maximum reactivity insertion rate and, for research reactors, the maximum experiment reactivity. This is done by using the reactivity values of the control rods, hence the utmost importance of having precise values for these parameters. The procedure is issued from handouts from the USGS-Reactor Lab course at the Colorado School of Mines [1].

1.1 Reactor period

The reactor period is the time required for the reactor power to change by a factor of e , Euler's number. Its unit is the second. The relationship between power and period in a nuclear reactor is:

$$(1.1) \quad P(t) = P(t = 0) * e^{\frac{t}{\tau}}$$

$P(t)$ = Reactor power at time t

τ = Reactor period, in seconds

t = Time interval, in seconds

The smaller the value of τ , the more rapid the change in reactor power. A positive period means that power is increasing. A period of less than 3 seconds is too short for an operator to be able to act, so a rod withdrawal interlock exists on this parameter. Some reactors even have an automatic SCRAM if they pass below this threshold.

Hence, we can obtain the period:

$$(1.2) \quad \tau = \frac{t}{\ln\left(\frac{P(t)}{P(t=0)}\right)}$$

1.2 Delayed neutrons

The link between the period and the reactivity of the core is made by looking at the change in prompt and delayed neutron populations. For delayed critical operations, the stable period is controlled by delayed neutrons only.

A little digression about the importance of delayed neutrons in a nuclear reactor is given in appendix A.

Delayed neutrons precursors are usually categorized into six groups, arranged by half-life. Table 1.1 presents the different group characteristics.

Group	Half-life $T_{1/2}$ (s)	Mean life τ_m (s)	Decay constant $\lambda(s^{-1})$	Fraction of total thermal neutrons f
1	55.7	80.2	0.0124	0.000231
2	22.7	32.7	0.0305	0.00153
3	6.2	8.9	0.111	0.00137
4	2.3	3.3	0.301	0.00277
5	0.61	0.88	1.14	0.000805
6	0.23	0.33	3.01	0.000294

Table 1.1: Precursors groups data

One can note that the sum of the groups fractions adds to the effective delayed neutron fraction.

1.3 Reactivity inference

Using the reactor period and the delayed neutrons groups characteristics, it is possible to compute the reactivity, thanks to the Inhour equation:

$$(1.3) \quad \rho = \frac{I}{\tau} + \sum_{i=1,6} \frac{f_i}{1 + \lambda_i \tau}$$

I = Prompt neutron lifetime ($\approx 39\mu s$)

τ = Reactor period, in seconds

f_i = Precursor group fraction

λ_i = Precursor group decay constant

One can divide this equation by the effective delayed neutron fraction β_{eff} to obtain the reactivity in dollars. It is important to note that the Inhour equation is valid if and only if the period is stable. Thus, when measuring reactivity for control rod calibrations, the reactor must be in a stable critical state.

However, the transient effect can be approximated in case the period is not stable, using:

$$(1.4) \quad \tau = \frac{l^*}{\rho} + \frac{\beta_{eff} - \rho}{\lambda_{eff}\rho + \dot{\rho}}$$

l^* = Prompt neutron lifetime ($\approx 39\mu s$)

τ = Reactor period, in seconds

β_{eff} = Effective delayed neutron fraction

ρ = Reactivity

λ_{eff} = Effective delayed neutron precursor decay constant

$\dot{\rho}$ = Rate of change in reactivity

The effective delayed neutron precursor decay constant represents the fraction of precursor atoms decaying in a second. It depends on the core critical state.

1.4 Procedure

The procedure for this experiment will be to get to a critical, stable state with the control rod to calibrate in its down position. Then, it will be taken up a discrete amount of steps, and the period will be to get a twenty-fold increase in power (from 20W to 400W) will be measured. Using the delayed neutron fractions and the period in the Inhour equation, the reactivity will be calculated.

Using the three rods that are not being calibrated, the power will be brought to 2W. In Manual mode, since the rod positions needs to be stable, the reactor will be stabilized. This takes at least five minutes, time after which the delayed neutron population will have caught up with the prompt neutrons and stabilize. In order to use the Inhour equation, we saw that stability was primordial. Once that is done, a position for the transient rod will be chosen to add roughly 25 cents worth of reactivity in the core, and the transient rod will be moved up. The timers starts when the power hits 20W, and stops when it hits 400W. At that moment, the operator will lower the three rods not being calibrated in order to go back to a stable critical state at 2W. Repeat the procedure until the transient rod is fully up.

At each step, the time taken to go up from 20W to 400W will give the period. This can then be used, in correlation with the delayed neutron information, to compute the reactivity brought in by the transient rod motion. The operator has to be mindful of the fact that the differential rod worth will be smaller when the rod is nearing the top of the core, and thus plan the steps carefully so as to avoid meaningless data for the end of the curves.

RESULTS

This chapter presents the results obtained during the experiment performed on September 28th, 2016. It presents the differential and integral rod worth for the transient rod, data obtained during the rod calibration. It also presents the transit time values and the maximum reactivity insertion rate of the transient rod.

The fitting curve of figure 2.1, given by equation 2.1, shows a perfect fit ($R^2 = 1$, more details on R^2 calculation in annexe C) and shows an inflexion point at around 426 steps on the integral values. The fitting curve of figure 2.2, on the differential points, is given by equation 2.2. It demonstrates a R^2 of 98% and finds the actual inflexion point to be at around 523 steps. This goes to show that the transient rod is relatively well vertically-centered in the core, considering the uncertainties of the measurements. Indeed, the real steps in the GSTR core go from 10 to 993, hence a mid-position of 502 in the core.

$$(2.1) \quad y = ax^4 + bx^3 + cx^2 + dx + e$$

Where:

$$a = 3.22810^{-12}$$

$$b = -1.24210^{-8}$$

$$c = 1.23510^{-5}$$

$$d = -9.53610^{-4}$$

$$e = 8.74410^{-3}$$

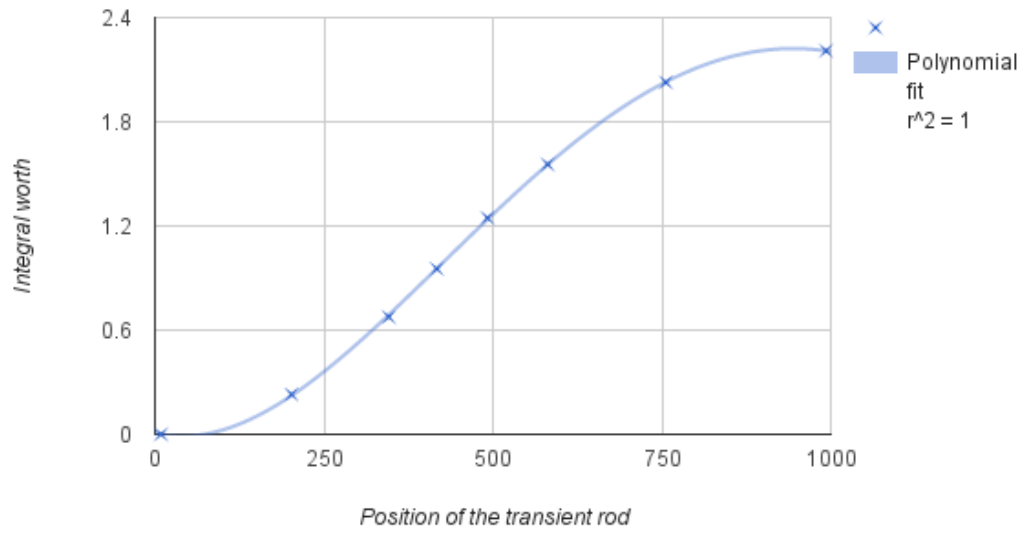


FIGURE 2.1. Integral transient rod worth.

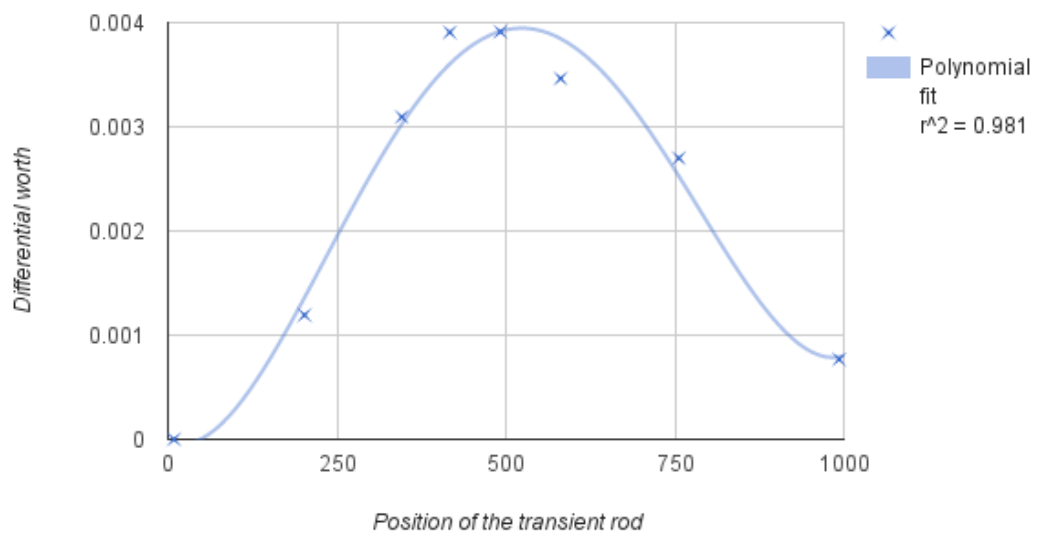


FIGURE 2.2. Differential transient rod worth.

$$(2.2) \quad y = ax^4 + bx^3 + cx^2 + dx + e$$

Where:

$$\begin{aligned} a &= 6.65210^{-14} \\ b &= -1.35510^{-10} \\ c &= 7.27310^{-8} \\ d &= -2.91410^{-6} \\ e &= -1.26410^{-5} \end{aligned}$$

The transient rod can be taken from its down position to its up position in 37 seconds, meaning that in those 37 seconds, the pulse rod can introduce 2.21\$ worth of reactivity to the core. This value is below the maximum pulse reactivity for which the reactor is licensed (3\$). Moreover, this means that the rod has a reactivity insertion rate of $2.21\$/37s = 0.06\$.s^{-1}$, well below the licensed limit of $0.28\$.s^{-1}$.

The fact that the measured value are so far from the licensed one comes mostly from the core itself. Indeed, the fuel elements have been irradiated for a long time, and are much less energetic now that they were at the reactor's first start-up.

This experiment was performed with a "clean" core, free of Xenon poisoning. However, had it not been the case, the curves obtained would have been much different. Indeed, the Xenon would have had the effect of moving the neutron flux peak vertically in the core, moving the "center" of the core. A large Xenon poisoning, say 1.5\$, might prevent the rod calibration from being done, the differential worth being potentially negligible for some steps near the axial offset caused by the Xenon peak.

2.1 Uncertainties

Several sources of uncertainties are to be considered in this experiment. The time measurements, used to calculate the reactor period, are approximative, and probably only good within 1 second at best. The period is thus considered to be:

$$(2.3) \quad \tau \equiv \tau \pm \frac{1}{\ln(20)}s = \tau \pm 0.33s$$

This translates to an approximation on the inflexion point of the integral reactivity worth of the transient rod of roughly ± 2 steps. There is also an inherent uncertainty to the rod position in the core, considered equaled to one step, which means a two steps uncertainty for the differential rod worth. This translates to $\pm 1\%$ on the differential reactivity, approximately 5 steps. Thus, the updated global uncertainty on the centered position of the transient rod is 523 ± 7 steps, keeping

in mind the fact that R^2 is only 98.2%. Moreover, the stability of the nuclear power when the transient rod was pulled up was estimated using visual confirmation, introducing an uncertainty.

Other sources of uncertainties include the actual rod position measurements, the six precursor groups approximations, the β_{eff} value, the prompt neutron lifetime, potential Xenon presence,...

All things considered, the transient rod might be very slightly off-centered, skewed toward the top of the core.

CONCLUSION

A rod calibration was performed on the USGS TRIGA reactor core, for the pulse (or transient) rod. Only the pulse rod was moved up, while the other three rods were used to stabilize the reactor in a critical state after each steps. The neutron delayed fraction and characteristics, along with the reactor period, were used to compute the differential and thus integral reactivity worth of the calibrated rod. This allowed the operator to see that the transient rod could be considered well vertically-centered in the core, if potentially slightly higher than nominal. It can be noted that the experiment was facilitated by the absence of Xenon poison in the core.

Moreover, it showed that the licensing limits were still respected, notably the maximum pulse reactivity and the maximum reactivity insertion rate.



DELAYED NEUTRONS

This appendix presents the importance of the delayed neutrons in a nuclear reactor. It presents, without going into detailed mathematics, the impact of their non-existence of a nuclear chain reaction.

A.1 Neutron lifetime

First, let's talk about the neutron lifetime. In a PWR reactor, it is 10^{-5} seconds (10^{-7} seconds in a BWR). This means that after this time on average, the neutron will have disappeared (absorbed, absorbed to induce fission, or leaked out of the reactor).

A.2 Multiplication factor

The multiplication factor k leads to the critical, subcritical and supercritical states. $k < 1$: Subcritical, the chain reaction dies $k = 1$: Critical, the chain reaction is nice, the power is constant $k > 1$: Supercritical, the neutron population increases with each generation, the power increases. No good.

A.3 A world without delayed neutrons

If we do not consider the delayed neutrons into the kinetics equation, and a multiplication factor equals to 1.001 (quite close to 1, we all agree) then we have the following evolution of the neutron population with time:

$$(A.1) \quad n(t) = N * e^{k-1} * e^{\frac{t}{X}}$$

Where X is the mean lifetime of the neutrons inside the reactor. So, in a PWR for example, and considering $k = 1.001$ (difficult to get closer to criticality in real operations):

$$(A.2) \quad n(t) = N * e^{k-1} * e^{\frac{t}{10^{-5}}}$$

This gives us :

$$(A.3) \quad n(t) = N * e^{100t}$$

This means that the neutron population is **not at all** under control. After 1 second, we have the original population (N) multiplied by e^{100} (gigantic number). So the power in the reactor would increase very quickly, even though the multiplication factor is as close as possible to 1. So, what are we missing ? The *delayed neutrons*.

A.4 The real world, with delayed neutrons

Where do they come from ? To answer that, we must be sure to understand where the neutrons come from in a reactor. Several possibilities :

1. Fission induced neutrons (mainly uranium-235 gets hits by a low energy neutron, and produce on average around 2 neutrons)
2. External sources
3. Different decay reactions (instead of undergoing fission, an atom, uranium-235 for example, would absorb one neutron and release two neutrons, and some other reactions like that)

So, what are we missing ? Well, the fission of an atom creates two smaller atoms. Those are the ones (called precursors) that will release neutrons later, by decaying. Thus *delayed neutrons*. Indeed, the mean lifetime of a precursor neutron is 13 seconds roughly (compared to the 10^{-5} seconds in a PWR). Delayed neutrons represents around 700 pcm (0.7%) of the whole neutrons "produced" during a generation. This very small difference is what actually allow us to control the chain reaction in a nuclear reactor.

APPENDIX B

DETAILED DATA TABLES

This appendix presents the raw data from the experiment in table B.1, that is the time measured as necessary to cover the power delta between 20W and 400W. It then explicits the various calulcation steps in table B.2.

Position	Start Power	Stop Power	Time 1 (s)	Time 2 (s)	Avg time (s)
9					
202	20	400	88.1	87.14	87.62
346	20	400	23	23.28	23.14
417	20	400	63.13	63	63.065
492	20	400	56.75	56.79	56.77
581	20	400	51.58	51.7	51.64
756	20	400	19.95	20	19.975
993	20	400	128.32	127.4	127.86

Table B.1: Raw data - Time to reach power differential

Position	Period	$\sum_{i=1,6} \frac{f_i}{1+\lambda_i T}$	Differential worth	Integral worth	Differential worth per step
9			0.00	0.00	0.00
202	29.25	1.61E-03	0.23	0.23	1.19E-03
346	7.72	3.11E-03	0.45	0.68	3.09E-03
417	21.05	1.94E-03	0.28	0.95	3.91E-03
492	18.95	2.05E-03	0.29	1.25	3.91E-03
581	17.24	2.16E-03	0.31	1.55	3.46E-03
756	6.67	3.30E-03	0.47	2.03	2.70E-03
993	42.68	1.27E-03	0.18	2.21	7.68E-04

Table B.2: Data - Period, differeital and integral reactivity worth



COEFFICIENT OF DETERMINATION R^2

This appendix presents the mathematics behind the confidence factor, also known as the R^2 value. The R^2 value, representing the fit quality, can be found using the mean (\bar{y}), the total sum of squares (SS_{tot}), and the residual sum of squares (SS_{res}).

Each is defined as:

$$(C.1) \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$(C.2) \quad SS_{tot} = \sum_i (y_i - \bar{y})^2$$

$$(C.3) \quad SS_{res} = \sum_i (y_i - f_i)^2$$

$$(C.4) \quad R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

where:

f_i = Fitting function value at point x_i

BIBLIOGRAPHY

- [1] T. DEBEY, *USGS reactor lab: Control rod calibration*.
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