Nuclear Reactor Thermal-Hydraulics

NUGN520 - Homework

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CHAPTER 1

HEAT TRANSFER AND FLUID FLOW, NONMETALLIC COOLANTS

Several exercises from the book written by M. M. El Wakil [1] are tackled in this homework. The problems in this section relate to the ninth chapter of the book, covering the subject of heat transfer and fluid flow, for nonmetallic coolants.

1.1 [9-1] - Heat transfer and neutron flux

1.1.1 Problem

A research reactor core is cubical, 20 ft on the side. The fuel elements are 1-in.-diameter solid natural uranium metal rods, placed horizontally in the center of 3-in.-diameter graphite holes. Air is the coolant, forced at atmospheric pressure and an initial velovity of 15 fps. For the centermost fuel element, air enters at $80°F$ *and leaves at* $190°F$*. Its surface temperature averages* $425°F$*. Calculate (a) the heat-transfer coefficient, and (b) the maximum neutron flux in the core, neglecting cladding and extrapolation lengths.*

1.1.2 Solution

The heat transfer coefficient is given by Equation 1.1.

$$h = \frac{q}{A_s(T_w - T_f)} \tag{1.1}$$

Where:

A_s = Surface area
T_w = Surface temperature
T_f = Fluid temperature

The surface temperature is known, $T_w = 425°F$. The fluid temperature changes along the x-direction. An average along this dimension can approximate this value, and so $T_f = \frac{190+80}{2} = 135°F$. The surface area is given by $A_s = L * P = 2\pi r_i * L$, r_i being the radius of the fuel rod, and L the length of the rod.

Consequently, the only unknown is q, which can be obtained by using Equation 1.2.

$$q = \dot{m} c_p (T_{f,f} - T_{f,i}) \tag{1.2}$$

Where:

$T_{f,f}$ = Final fluid temperature
$T_{f,i}$ = Initial fluid temperature
$\dot{m}$ = Mass flow rate

The mass flow rate is given by Equation 1.3. In this case, $v = 15\ fps = 54000\ fph$, and the density of air at an average of $135°F$ can be taken to be $\rho = 0.07\ lb.ft^{-3}$. The cross section area is $A_c = \pi(R^2 - r_i^2)$, where R is the radius of the graphite hole.

$$\dot{m} = \rho v A_c = 0.07 * 54000 * \pi(0.125^2 - 0.042^2) = 164.6\ lb.h^{-1} \tag{1.3}$$

At the average fluid temperature, $c_p = 0.24\ Btu.lb^{-1}.°F^{-1}$. We can thus write:

$$q = 164.6 * 0.24 * (190 - 80) = 4345.4\ Btu.h^{-1} \tag{1.4}$$

$$h = \frac{4345.4}{2\pi * 0.042 * 20 * (425 - 135)} = 2.84\ Btu.h^{-1}.ft^{-2}.°F^{-1} \tag{1.5}$$

Now, in order to calculate the neutron flux, we can use Equation 1.6.

$$\phi = \frac{q'''}{GN\sigma c} \tag{1.6}$$

G is the energy per fission, taken to be 190 MeV, c is a conversion factor ($c = 1.5477 \times 10^{-8}$).

The cross-section can be calculated using the correlation $\sigma = 0.8862 * \sigma_0 * (\frac{T_0}{T})^{0.5} = 0.8862 * 577.1 \times 10^{-24} * (\frac{293}{T})^{0.5}$.

N is the number density, $N = \frac{N_A e \rho}{M_f}$, in which ρ depends (slightly) on the temperature.

Hence, in order to compute σ and N, the maximum fuel temperature, at $r = 0$, must be obtained (to then compute the maximum neutron flux). This is done using Equation 1.7 (equation 5.27 in [1].

$$T_m = T_s + \frac{q'''}{2k_f}s^2 \tag{1.7}$$

k_f is equal to 15.9 $Btu.h^{-1}.ft^{-1}.^{\circ}F^{-1}$ in our case, and $s = 0.042\ ft$. So, we obtain $T_m = 1202\ ^{\circ}F^{-1}$. The density of uranium at this temperature is $\rho = 18.33\ g.cm^{-3}$, and the cross-section is calculated to be $\sigma = 252.5\ b$. The number density can thus be derived, and $N = 3.29 \times 10^{20}$.

Finally, we can calculate q''' from q:

$$q''' = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi r_i^3} = 1.4 \times 10^7\ Btu.h^{-1}.ft^{-3} \tag{1.8}$$

And now, we can compute the maximum neutron flux, $\phi = 5.74 \times 10^{13}\ s^{-1}.cm^{-2}$.

1.2 [9-2] - Heat transfer and pumping power

1.2.1 Problem

Compare the heat transfer coefficients and the pumping power (hp) per 1000-ft length of 1-in.-ID smooth-drawn tubing of the following coolant: air at 10 atm, 100 fps, and $400°F$ *; and water at 20 fps and* $400°F$.

1.2.2 Solution

We can use the Dittus-Boelter correlation to obtain h by calculating the Nusselt number, Equation 1.9

$$Nu = 0.023Re^{0.8}Pr^{0.4} = \frac{hD_e}{k} \tag{1.9}$$

All the unknown can be obtained in tables. Finding the data for air at 10 atm is however very challenging and cumbersome, especially in imperial units. The following data [**?**] was used:

$$\begin{aligned}
k &= 0.9\ cal.cm^{-1}.s^{-1}.K^{-1} = 217.7\ Btu.h^{-1}.ft^{-1}.^{\circ}F^{-1}\\
D_e &= 1\ in = 0.083\ ft = 2.54\ cm\\
v &= 100\ fps = 360000\ fph = 3048\ cm.s^{-1}\\
\rho &= 0.5\ lb.ft^{-3} = 0.008 g.cm^{-3}\\
\mu &= 2.6\ g.cm^{-1}.s^{-1}\\
c_p &= 0.246\ cal.g^{-1}.K^{-1}
\end{aligned}$$

This allows us to obtain $Re = 12.6$ and $Pr = 0.705$, and consequently, $h = 398.2\ Btu.h^{-1}.ft^{-2}.^{\circ}F^{-1}$. The pumping power is given by Equation 1.10.

$$W = \Delta p A_c v = \frac{f}{8g_c} L \rho D_e v^3 \tag{1.10}$$

$g_c = 4.17 \times 10^8\ lb_m.ft.lb_f^{-1}.h^{-2}$. The Reynolds number corresponds to a laminar flow, hence a Moody friction factor of $f = \frac{Re}{64} = 0.197$. This gives us a pumping power $W = 1.2 \times 10^8\ ft.lb.h^{-1} = 58.5\ hp$.

For the water at $400°F$, we have:

$$\begin{aligned}
k &= 0.3809\ Btu.h^{-1}.ft^{-1}.°F^{-1}\\
D_e &= 1\ in = 0.083\ ft\\
v &= 20\ fps = 72000\ fph\\
\rho &= 53.648\ lb.ft^{-3}\\
\mu &= 0.327\ lb.h^{-1}.ft^{-1}\\
c_p &= 1.0794\ Btu.lb^{-1}.°F^{-1}
\end{aligned}$$

This gives us, using Equation 1.9, $Re = 9.8 \times 10^5$ and $h = 6354\ Btu.h^{-1}.ft^{-2}.°F^{-1}$. Hence, $f = 0.012$ from the Moody chart, and $W = 6.0 \times 10^6\ ft.lb.h^{-1} = 3.1\ hp$.

1.3 [I1] - Eucken formula

1.3.1 Problem

By using the Eucken formula, estimate the thermal conductivity of argon at 600 and 1200 K.

1.3.2 Solution

The Eucken formula states that:

$$k = \left(c_p + \frac{5}{4}\frac{R}{M}\right)\mu(T) \tag{1.11}$$

For a monatomic gas, we have $c_p = \frac{5}{2}\frac{R}{M}$, and consequently, the Eucken formula reduces to Equation 1.12.

$$k = \frac{15}{4}\frac{R}{M}\mu(T) = 3.75\frac{R}{M}\mu(T) \tag{1.12}$$

R is the gas constant, $R = 8.314\ J.K^{-1}.mol^{-1}$, and M is the molecular mass of Argon, $M = 39.948\ g.mol^{-1} = 39.948 \times 10^{-3}\ kg.mol^{-1}$. A table gave the viscosity $\mu = 3.9 \times 10^{-5}\ Pa.s$ at 600K.

$$k = \frac{3.75 * 8.314 * 3.9 \times 10^{-5}}{39.948 \times 10^{-3}} = 0.03\ W.m^{-1}.K^{-1} \tag{1.13}$$

No data was found for the viscosity of Argon at 1200K, but the value of the thermal conductivity could be obtained in the exact same way by replacing the value of $\mu(T)$.

1.4 [I2] - Euler equation

1.4.1 Problem

Show that we can also write the Euler equation for inviscid fluid flow as $\frac{\partial}{\partial t}\nabla \times \vec{V} = \nabla \times (\vec{V} \times \nabla \times \vec{V})$.

1.4.2 Solution

The Navier-Stokes equation can be written:

$$\mu \nabla^2 \vec{v} + \rho \vec{g} - \nabla p = \rho \frac{d\vec{v}}{dt} \tag{1.14}$$

Euler equation is obtained when $\mu = 0$:

$$\rho \vec{g} - \nabla p = \rho \frac{d\vec{v}}{dt} \tag{1.15}$$

We can take the curl of this equation. The curl of a gradient, and of a constant, is zero, so $\nabla \times (\rho \vec{g}) = 0$ and $\nabla \times (\nabla p) = 0$. Consequently:

$$\nabla \times \left(\rho \frac{d\vec{v}}{dt} \right) = 0 \tag{1.16}$$

We know that:

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \tag{1.17}$$

So, Equation 1.16 can be written, with $\rho = cst$:

$$\nabla \times \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = 0 \tag{1.18}$$

By seeing that $\nabla \times \left(\frac{\partial \vec{v}}{\partial t} \right) = \frac{\partial \nabla \vec{v}}{\partial t}$, and developing the expression, we can write:

$$\frac{\partial}{\partial t} \nabla \times \vec{v} + \nabla \times (\vec{v} \times \nabla \times \vec{v}) = 0 \tag{1.19}$$

We can now use the relations 1.20 and 1.21 to replace the expression in Equation 1.19 and obtain Equation 1.22.

$$\frac{1}{2}\nabla v^2 = \vec{v} \times (\nabla \times \vec{v}) + (\vec{v} \cdot \nabla)\vec{v} \tag{1.20}$$

$$(\vec{v} \cdot \nabla)\vec{v} = \frac{1}{2}\nabla v^2 - \vec{v} \times (\nabla \times \vec{v}) \tag{1.21}$$

$$\nabla \times \left(\frac{\partial \vec{v}}{\partial t} + \frac{1}{2}\nabla v^2 - \vec{v} \times (\nabla \times \vec{v})\right) = 0 \tag{1.22}$$

Knowing that $\nabla \times \left(\frac{1}{2}\nabla v^2\right) = 0$, we can obtain:

$$\nabla \times \left(\frac{\partial \vec{v}}{\partial t} - \vec{v} \times (\nabla \times \vec{v})\right) = 0 \tag{1.23}$$

And so:

$$\frac{\partial \vec{v}}{\partial t} = \vec{v} \times (\nabla \times \vec{v}) \tag{1.24}$$

1.5 [I3] - Parallel flat plates

1.5.1 Problem

Consider laminar fluid flow between two flat plates of infinite width. Take the plates as located at $y = 0$ and y_0. (a) Develop, or otherwise write, the parabolic solution for the x-component of velocity $v_x(y)$. (b) Calculate the average speed v_{avg} by integrating from $y = 0$ to $y = y_0$. Then rewrite the result of part (a) for $v_x(y)$ by using v_{avg}.

1.5.2 Solution

The following equation needs to be solved:

$$\rho \frac{d\vec{v}}{dt} = -\nabla P + \mu \nabla^2 \vec{v} \tag{1.25}$$

In this equation, $P = p + \rho\psi$. We can consider a uniform P gradient in the x-direction, so that $\frac{dP}{dx} = -G$. Moreover, $\frac{d\vec{v}}{dt} = 0$. Consequently, Equation 1.25 becomes:

$$\nabla^2 \vec{v} = \frac{\nabla P}{\mu} \tag{1.26}$$

And, for the x-direction,

$$\frac{\partial^2 v_x}{\partial y^2} = \frac{-G}{\mu} \tag{1.27}$$

Integrating once, we obtain:

$$\frac{\partial v_x}{\partial y} = \frac{-G}{\mu} y + C_1 \tag{1.28}$$

Knowing that the speed profile will be maximum at $y = \frac{y_0}{2}$, we have $\left.\frac{\partial v_x}{\partial y}\right|_{y=y_0/2} = 0$. Consequently, $C_1 = \frac{G y_0}{2\mu}$, and we can write:

$$\frac{\partial v_x}{\partial y} = \frac{G}{\mu}\left(\frac{y_0}{2} - y\right) \tag{1.29}$$

Integrating a second time,

$$v_x = \frac{G}{\mu}\frac{y_0}{2} y - \frac{G}{2\mu} y^2 + C_2 \tag{1.30}$$

Knowing that $v_x(0) = v_x(y_0) = 0$, we can obtain $C_2 = 0$, and finally write:

$$v_x = \frac{G}{2\mu} y(y_0 - y) \tag{1.31}$$

The average velocity can be obtained by using:

$$v_{avg} = \frac{\int_0^{y_0} v_x(y)dy}{\int_0^{y_0} dy} = \frac{1}{y_0}\int_0^{y_0} \frac{G}{2\mu} y(y_0 - y)dy \tag{1.32}$$

$$v_{avg} = \frac{G}{2\mu y_0}\left[\frac{y_0 y^2}{2} - \frac{y^3}{3}\right]_0^{y_0} = \frac{G}{12\mu} y_0^2 \tag{1.33}$$

Thus, we can write:

$$v_x = \frac{6 v_{avg}}{y_0^2} y(y_0 - y) \tag{1.34}$$

BIBLIOGRAPHY

[1] M. M. EL-WAKIL, *Nuclear Heat Transport*, American Nuclear Society, 1993.