## **Termination**, exercises

M2IF – Automated deduction

X. Urbain — M2IF CR16

$$x - 0 \rightarrow x$$

$$s(x) - s(y) \rightarrow x - y$$

$$0 \div s(y) \rightarrow 0$$

$$s(x) \div s(y) \rightarrow s((x - y) \div s(y)))$$

$$\begin{array}{cccc} 0 \leq y & \rightarrow & \mathsf{true} \\ s(x) \leq 0 & \rightarrow & \mathsf{false} \\ s(x) \leq s(y) & \rightarrow & x \leq y \\ 0 - y & \rightarrow & 0 \\ s(x) - y & \rightarrow & ifte(s(x) \leq y), s(x), y) \\ ifte(\mathsf{true}, s(x), y) & \rightarrow & 0 \\ ifte(\mathsf{false}, s(x), y) & \rightarrow & s(x - y) \\ 0 \div s(y) & \rightarrow & 0 \\ s(x) \div s(y) & \rightarrow & s((x - y) \div s(y)) \end{array}$$

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\begin{array}{ccc} \mathsf{half}(0) & \to & 0 \\ \mathsf{half}(s(s(x))) & \to & s(\mathsf{half}(x)) \\ \log(s(0)) & \to & 0 \\ \log(s(s(x))) & \to & s(\log(s(\mathsf{half}(x)))) \end{array}
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X. Urbain — M2IF CR16

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\begin{array}{cccc} \operatorname{concat}(\operatorname{cons}(u,v),y) & \to & \operatorname{cons}(u,\operatorname{concat}(v,y)) \\ & \operatorname{Fewer}\bot(x,\bot) & \to & \operatorname{false} \\ & \operatorname{Fewer}\bot(\bot,\operatorname{cons}(w,z)) & \to & \operatorname{true} \\ & \operatorname{Fewer}\bot(\operatorname{cons}(u,v),\operatorname{cons}(w,z)) & \to & \operatorname{Fewer}\bot(\operatorname{concat}(u,v),\operatorname{concat}(w,z)) \end{array}
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$$0+y \rightarrow y$$

$$s(x)+0 \rightarrow s(x)$$

$$s(x)+s(y) \rightarrow s(s(x)+(y+0))$$

**DP** Dershowitz

$$\begin{array}{cccc} x \times (y+1) & \rightarrow & (x \times (y+(1 \times 0))) + x \\ & x \times 1 & \rightarrow & x \\ & x+0 & \rightarrow & x \\ & x \times 0 & \rightarrow & 0 \end{array}$$

**DP** Walther

$$f(0) \rightarrow s(0)$$

$$f(s(0)) \rightarrow s(0)$$

$$f(s(s(x))) \rightarrow f(f(s(x)))$$