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NE 250
Problem set 5
1. (10 points) Calculate the mean, variance, and the cumulative distribution function for each of the following probability density functions:

a. f(x) = $\begin{cases} \frac{1}{a} & 0 \leq x \leq a \\ 0 & x < 0, x > a \end{cases}$
mean = $\int_{-\infty}^{\infty} xf(x)dx = \frac{1}{a}\int_0^a xdx = \frac{1}{2a}a^2 = \frac{a}{2}$
Var[x] = E[x^2] - E[x]
E[x^2] = $\int_{-\infty}^{\infty} x^2 f(x)dx = \frac{1}{a}\int_0^a x^2 dx = \frac{1}{3a}a^3 = \frac{a^2}{3}$
E[x]^2 = $\frac{a^2}{4}$
Var[x]^2 = $\frac{a^2}{12}$
for x<0 F(x) = 0 = P(x)
for x>a F(x) = 1 = P(x)
for $0 \leq x \leq a$:

F(x) = P(x) = $\int_{-\infty}^{x} f(t)dt = \frac{1}{a}\int_0^x dt = \frac{x}{a}$
F(x) = $\begin{cases} \frac{x}{a} & 0 \leq x \leq a \\ 0 & x < 0 \\ 1 & x > a \end{cases}$
b. f(x) = $\lambda e^{-\lambda x}, x > 0$
mean = $\int_0^{\infty} x\lambda e^{-\lambda x}dx = \frac{1}{\lambda}$
variance = $\int_0^{infty} x^2\lambda e^{-\lambda x} - \frac{1}{\lambda^2} = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$
CDF:
F(x) = $\int_0^x \lambda e^{-\lambda x}dt = x\lambda e^{-\lambda x}$
F(x) = $\begin{cases} x\lambda e^{-\lambda x} & x > 0 \\ 0 & x < 0 \\ 1 & x \to \infty \end{cases}$