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NE 250
Problem set 5
1. (10 points) Answer the following questions as true or false. Provide a one sentence justification for your answer.

a. For reactors, we often use the -eigenvalue formulation rather than the k-eigenvalue formulation for criticality calculations.
False. k-eigenvalue calculations are more well suited for reactors because of the geometry of the problem.
b. On a surface $\vec{r}_s$ with fixed (not necessarily vacuum) boundary conditions, the uncollided flux must satisfy $\psi_0 = 0$, $\hat{\Omega} \cdot \hat{n} < 0$.
True. This is the correct representation of the uncollided flux.
c. On a surface $\vec{r}_s$ with fixed (not necessarily vacuum) boundary conditions, the once collided flux must satisfy $\psi_1 = 0$, $\hat{\Omega} \cdot \hat{n} < 0$.
False. This is the second mode of the angular flux, but not the once collided flux.
d. The streaming operator in the transport equation is self-adjoint.
False. The streaming operator has opposite signs in the transport equation and the adjoint transport equation and is therefore not self-adjoint.
e. The source for the adjoint problem is always proportional to the source for the forward problem.
This is true because the adjoint equation is essentially a function to iterate between the forward and backward equations to solve the forward flux. Therefore the source for the adjoint problem must be proportional to the source for the forward problem in terms of solving for the forward flux at the point where the adjoint flux's source is.
f. Would you place a container of radioactive waste at point one or point two? Why?
I would place a container of waste at $\psi_1^\dagger$ because the large adjoint flux would indicate that there is decent shielding there since the source for the adjoint flux is scattered or reflected forward flux.

2. (20 points) Find the adjoint equation corresponding to:

$$[\hat{\Omega}\cdot\nabla+\Sigma(\vec{r},E)+\tfrac{\alpha}{v}]\psi(\vec{r},\hat{\Omega},E) = \int dE' \int d\hat{\Omega}'\Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi(\vec{r},\hat{\Omega},E')+\tfrac{\chi(E)}{4\pi}\int dE'\nu\Sigma_f(\vec{r},E')\int d\hat{\Omega}'\psi(\vec{r},\hat{\Omega}',E')$$

$$< \psi, \mathbf{L}\phi > \neq < \phi, \mathbf{L}\psi > \tag{1}$$

$$\mathbf{L}\psi = [-\hat{\Omega}\cdot\nabla - \Sigma(\vec{r},E) - \frac{\alpha}{v}]\psi(\vec{r},\hat{\Omega},E) + \int dE' \int d\hat{\Omega}'\Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi(\vec{r},\hat{\Omega},E') \tag{2}$$

$$< \psi^\dagger, \mathbf{L}\phi > = < \phi, \mathbf{L}^\dagger\psi^\dagger > \tag{3}$$

$$\mathbf{L}^\dagger\psi^\dagger = [\hat{\Omega}\cdot\nabla - \Sigma(\vec{r},E) - \frac{\alpha^\dagger}{v}]\psi^\dagger(\vec{r},\hat{\Omega},E) + \int dE \int d\hat{\Omega}\Sigma_s(\vec{r},E\to E',\hat{\Omega}\to\hat{\Omega}')\psi^\dagger(\vec{r},\hat{\Omega},E) \tag{4}$$

By (4) we get:

$$\begin{aligned}[-\hat{\Omega}\cdot\nabla + \Sigma(\vec{r},E) + \frac{\alpha^\dagger}{v}]\psi^\dagger(\vec{r},\hat{\Omega},E) = \int dE' \int d\hat{\Omega}'\Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi^\dagger(\vec{r},\hat{\Omega},E') \\ + \frac{\chi(E)}{4\pi}\int dE'\nu\Sigma_f(\vec{r},E')\int d\hat{\Omega}'\psi^\dagger(\vec{r},\hat{\Omega}',E')\end{aligned} \tag{5}$$

$$\frac{\alpha_l^\dagger}{v}\psi_l^\dagger = \mathbf{L}^\dagger\psi_l^\dagger \quad \text{(eq 6.27 Bell and Glasstone)} \tag{6}$$

$$\frac{\alpha_j}{v}\psi_j = \mathbf{L}\psi_j \quad \text{(eq 6.28 Bell and Glasstone)} \tag{7}$$

$$(\alpha_j - \alpha_l^\dagger) < \frac{1}{v}\psi_l^\dagger, \psi_j > = < \psi_l^\dagger, \mathbf{L}\psi_j > - < \psi_j, \mathbf{L}^\dagger\psi_l^\dagger > \tag{8}$$

By (3) this becomes:

$$(\alpha_j - \alpha_l^\dagger) < \frac{1}{v}\psi_l^\dagger, \psi_j >= 0 \tag{9}$$

Which is satisfied when $l = j = 0$ and:

$$\alpha_0 = \alpha_0^\dagger \tag{10}$$

Perturbation:

$$\Sigma^* = \Sigma + \Delta\Sigma \tag{11}$$
$$\psi^* = \psi + \Delta\psi \tag{12}$$

$$\begin{aligned}[\hat{\Omega} \cdot \nabla\psi^* + \Sigma\psi^* + \frac{\alpha^*}{v}\psi^* = \int dE' \int d\hat{\Omega}'\Sigma_s(\vec{r}, E' \to E, \hat{\Omega}' \to \hat{\Omega})\psi^*(\vec{r}, \hat{\Omega}, E') \\ + \frac{\chi(E)}{4\pi}\int dE'\nu\Sigma_f(\vec{r}, E')\int d\hat{\Omega}'\psi^*(\vec{r}, \hat{\Omega}', E')] \times \psi^\dagger\end{aligned} \tag{13}$$

$$(\alpha^* - \alpha^\dagger)\int dV \int d\hat{\Omega}\int dE\frac{1}{v}\psi^*\psi^\dagger \simeq \frac{\Delta\alpha}{v}\int dV \int d\hat{\Omega}\int dE\psi\psi^\dagger \tag{14}$$
$$\frac{\Delta\alpha}{v}\int dV \int d\hat{\Omega}\int dE\psi\psi^\dagger = -\int dV \int d\hat{\Omega}\Delta\sigma_s\psi\psi^\dagger + \int dV \int d\hat{\Omega}'\int \hat{\Omega}\Delta[\sigma_s f(\vec{r}, \hat{\Omega}' \to \hat{\Omega})]\psi\psi^\dagger \tag{15}$$

where:

$$\Sigma_s(\vec{r}, \hat{\Omega}' \to \hat{\Omega}) = \sigma_s f(\vec{r}, \hat{\Omega}' \to \hat{\Omega}) \tag{16}$$

3. (10 points) Suppose that, instead of detector response given by $R = V_d \int dE\Sigma_d(E)\phi(\vec{r}_d, E)$, we want to calculate the current:

$$J_n^+(\vec{r}_d, E) = \int_{\hat{\Omega}\cdot\hat{n}>0} d\hat{\Omega}\hat{\Omega}\cdot\hat{n}\psi(\vec{r}_d, \hat{\Omega}, E),\ \vec{r}_d \in \Gamma$$

the number of particles leaving V with boundary Γ at $\vec{r}_d$ in per cm^2 per unit energy. Derive an expression for $J_n^+(\vec{r}_d, E)$ in terms of the adjoint flux, $\psi^\dagger$
a. when the particles originate in V from some q_{ex}

Adjoint flux is essentially the flux at going in the opposite direction of the normal flux. At a boundary, the following holds:

$$\psi^\dagger(\vec{r}_d, \hat{\Omega}, E) \Longrightarrow \hat{\Omega}\cdot\hat{n} < 0 \tag{17}$$

so to get the current moving out of some volume, V, is as simple as just using the opposite of the adjoint flux moving into the system:

$$J_n^+(\vec{r}_d, E) = -\int_{\hat{\Omega}\cdot\hat{n}<0} d\hat{\Omega}\hat{\Omega}\cdot\hat{n}\psi^\dagger(\vec{r}_d, \hat{\Omega}, E) \tag{18}$$

where $\psi^\dagger$ is satisfied by:

$$[-\hat{\Omega}\cdot\nabla + \Sigma(\vec{r},E)]\psi^\dagger(\vec{r},\hat{\Omega},E) = q_{ex} \tag{19}$$

b. when the particles enter V across Γ as a known flux given by $\psi(\vec{r},\hat{\Omega},E) = \Psi(\vec{r},\hat{\Omega},E)$ with $\vec{r}\in\Gamma$ and $\hat{\Omega}\cdot\hat{n} < 0$
Normal vacuum boundary condition:

$$\psi^\dagger(\vec{r}_d,\hat{\Omega},E) = 0, \hat{\Omega}\cdot\hat{n} > 0 \tag{20}$$

For this system we also have:

$$\psi(\vec{r},\hat{\Omega},E) = \Psi(\vec{r},\hat{\Omega},E), \hat{\Omega}\cdot\hat{n} < 0 \tag{21}$$

This gives, at the surface Γ:

$$\psi^\dagger(\vec{r}_d,\hat{\Omega},E) = \Psi(\vec{r}_d,\hat{\Omega},E), \hat{\Omega}\cdot\hat{n} > 0 \tag{22}$$

This all becomes:

$$<\psi^\dagger, \mathbf{L}\psi> - <\psi, \mathbf{L}\psi^\dagger> = \int_{\hat{\Omega}\cdot\hat{n}>0} d\hat{\Omega}\hat{\Omega}\cdot\hat{n}\psi^\dagger(\vec{r}_d,\hat{\Omega},E)\Psi(\vec{r}_d,\hat{\Omega},E) \tag{23}$$

$$J_n^+(\vec{r}_d,E) = \int_{\hat{\Omega}\cdot\hat{n}>0} d\hat{\Omega}\hat{\Omega}\cdot\hat{n}\psi^\dagger(\vec{r}_d,\hat{\Omega},E)\Psi(\vec{r}_d,\hat{\Omega},E) \tag{24}$$

4.

$$\Sigma_{a_1} = \frac{\int_E^{E_0}\Sigma_a(E)\phi(E)dE}{\int_E^{E_0}\phi(E)dE} \tag{25}$$

$$\Sigma_{a_1} = \frac{\int_{10^7}^{1}\sigma(5MeV)/E\cdot 1/EdE}{\int_{10^7}^{1}1/EdE} \tag{26}$$

$$\Sigma_{a_1} = \frac{\sigma(5MeV)\int_{10^7}^{1}E^{-2}dE}{\int_{10^7}^{1}1/EdE} \tag{27}$$

$$\Sigma_{a_1} = \sigma(5MeV)\frac{1-10^{-7}}{\ln(10^{-7})} \tag{28}$$

5. Determine the neutron flux $\phi(E)$ resulting from a mono-energetic source at energy E_0 in an infinite hydrogenous medium if the scattering cross section is taken to be constant and the absorption cross section is inversely proportional to the neutron speed.

$$F_c(E) = \frac{\Sigma_s(E_0)S_0}{\Sigma_t(E_0)E}exp(-\int_E^{E_0}dE'\frac{\Sigma_a(E')}{\Sigma_t(E')E'}) \tag{29}$$

$$\phi(E) = \frac{F_c}{\Sigma_s(E_0)} = \frac{S_0}{\Sigma_t(E_0)E}exp(-\int_E^{E_0}dE'\frac{\Sigma_a(E')}{\Sigma_t(E')E'}) = \frac{S_0}{\Sigma_t(E_0)E}exp(-\frac{1}{\Sigma_t}\int_E^{E_0}dE'\frac{1}{E'^2}) \tag{30}$$

$$\phi(E) = \frac{S_0}{\Sigma_t(E_0)E}exp(\frac{1}{\Sigma_t}(\frac{1}{E_0}-\frac{1}{E})) \tag{31}$$

6.

$$\text{streaming: } \int_{E_g}^{E_{g+1}} dE\hat{\Omega}\nabla\psi(\vec{r},\hat{\Omega},E) = \hat{\Omega}\cdot\nabla\psi_g(\vec{r},\hat{\Omega}) \tag{32}$$

$$\text{external source: } \int_{E_g}^{E_{g+1}} dEq(\vec{r},\hat{\Omega},E) = q_g(\vec{r},\hat{\Omega}) \tag{33}$$

$$\text{fission source: } \int_{E_g}^{E_{g+1}} dE\chi/4\pi \int_0^\infty dE'\nu(E')\Sigma_f(E') \int_{4\pi} d\hat{\Omega}\psi(\vec{r},\hat{\Omega},E) = \chi_g/4\pi \sum_{g'=1}^{G} \nu\Sigma_{f,g'}\phi_{g'}(\vec{r}) \tag{34}$$

$$\text{scattering: } q_{s,g}(\hat{\Omega}) = \sum_{g'=1}^{G} \int_{4\pi} d\hat{\Omega}\Sigma_s(E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi_{g'}(\vec{r},\hat{\Omega}) \tag{35}$$

$$\text{total: } [\hat{\Omega}\cdot\nabla + \Sigma_{t,g}(\vec{r})]\psi_g(\vec{r},\hat{\Omega}) = q_g(\vec{r},\hat{\Omega}) + \chi_g/4\pi \sum_{g'=1}^{G} \nu\Sigma_{f,g'}\phi_{g'}(\vec{r}) + \sum_{g'=1}^{G} \int_{4\pi} d\hat{\Omega}\Sigma_s(E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi_{g'}(\vec{r},\hat{\Omega}) \tag{36}$$

7. (10 points) Answer the following short answer questions. If true or false, provide a one sentence justification for your answer.
a. Semi-true. The dot product takes away all angular dependence so it implies azimuthal and radial symmetry.
b. The first method has poor multigroup constants relative to a lengendre expansion technique.
c. The extended transport approximation is that scattering into a group is equal to scattering out of it for most energy groups.
d. 1. Uncertainties with a certain isotope are very high which can cause problems. 2. some isotopes only have a few measurements which can cast doubt on accuracy. 3. uncertainties can vary greatly between what data file is used (ENDF, JENDL, etc) causing a reproducibility error.
e. JENDL, JEFF, BROND