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NE 250

Problem set 5

1. (10 points) Calculate the mean, variance, and the cumulative distribution function for each of the following probability density functions:

$$\text{a. } f(x) = \begin{cases} \frac{1}{a} & 0 \leq x \leq a \\ 0 & x < 0, x > a \end{cases}$$

$$\text{mean} = \int_{-\infty}^{\infty} x f(x) dx = \frac{1}{a} \int_0^a x dx = \frac{1}{2a} a^2 = \frac{a}{2}$$

$$\text{Var}[x] = E[x^2] - E[x]^2$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x) dx = \frac{1}{a} \int_0^a x^2 dx = \frac{1}{3a} a^3 = \frac{a^2}{3}$$

$$E[x]^2 = \left(\frac{a}{2}\right)^2 = \frac{a^2}{4}$$

$$\text{Var}[x] = \frac{a^2}{3} - \frac{a^2}{4} = \frac{a^2}{12}$$

$$\text{for } x < 0 \quad F(x) = 0 = P(x)$$

$$\text{for } x > a \quad F(x) = 1 = P(x)$$

$$\text{for } 0 \leq x \leq a:$$

$$F(x) = P(x) = \int_{-\infty}^x f(t) dt = \frac{1}{a} \int_0^x dt = \frac{x}{a}$$

$$F(x) = \begin{cases} \frac{x}{a} & 0 \leq x \leq a \\ 0 & x < 0 \\ 1 & x > a \end{cases}$$

$$\text{b. } f(x) = \lambda e^{-\lambda x}, x > 0$$

$$\text{mean} = \int_0^{\infty} x \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$$

$$\text{variance} = \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx - \frac{1}{\lambda^2} = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

CDF:

$$F(x) = \int_0^x \lambda e^{-\lambda t} dt = x \lambda e^{-\lambda x}$$

$$F(x) = \begin{cases} x \lambda e^{-\lambda x} & x > 0 \\ 0 & x < 0 \\ 1 & x \rightarrow \infty \end{cases}$$

4. Answers for 4 parts a and b are in python file in folder, NE250HW6P.py