Adam Glick NE 250 HW 4

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① (a) $\underbrace{\frac{\mu}{r^2}\frac{\partial(r^2\psi)}{\partial r}} + \frac{\partial}{\partial\mu}\left[\frac{(1-\mu^2)\psi}{r}\right]$

$$\frac{\mu}{r^2}\left[\psi\frac{\partial r^2}{\partial r} + r^2\frac{\partial}{\partial r}\psi\right] + \frac{r\left(\frac{\partial(1-\mu^2)\psi}{\partial\mu}\right) - (1-\mu^2)\psi\cancel{\frac{\partial r}{\partial\mu}}}{r^2} \quad \rightarrow 0,\ r \not\propto \mu$$

$$\frac{2\psi\mu}{r} + \mu\frac{\partial\psi}{\partial r} + \frac{1}{r}\left(-2\mu\psi + (1-\mu^2)\frac{\partial\psi}{\partial\mu}\right)$$

$$\mu\frac{\partial\psi}{\partial r} + \frac{(1-\mu^2)}{r}\frac{\partial\psi}{\partial\mu} + \frac{2\psi\mu}{r} - \frac{2\psi\mu}{r} = \mu\frac{\partial\psi}{\partial r} + \left(\frac{1-\mu^2}{r}\right)\frac{\partial\psi}{\partial\mu} \checkmark$$

(b) $$\int_{-1}^{1}\frac{\mu}{r^2}\frac{\partial(r^2\psi)}{\partial r}d\mu + \int_{-1}^{1}\frac{\partial}{\partial\mu}\left[\frac{(1-\mu^2)\psi}{r}\right]d\mu = \int_{-1}^{1}\mu\frac{\partial\psi}{\partial r}d\mu + \underbrace{\frac{(1-\mu^2)}{r}\psi\Big|_{-1}^{1}}_{=0}$$

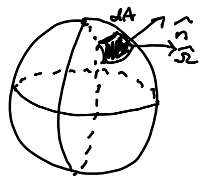
$$= \int_{-1}^{1}\mu\frac{\partial\psi}{\partial r}d\mu$$

$$\psi \simeq \frac{1}{4\pi}\left(\phi(r) - \underbrace{\frac{1}{\Sigma_{tr}}\frac{d\phi}{dr}}_{=3J(\vec{r})}\right)$$

$$= \int_{-1}^{1}d\mu\,\mu\frac{\partial}{\partial r}\left(\frac{1}{4\pi}(\phi(\vec{r}) + 3J(\vec{r})\right)$$

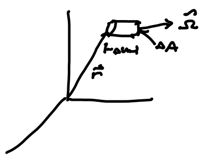
(c) $$\int_V dV = 4\pi\int_{r_1}^{r_2}dr\left[\int_{-1}^{1}d\mu\,\mu\frac{d}{dr}\left(\frac{1}{4\pi}(\phi(\vec{r}) + 3J(\vec{r}))\right)\right]$$

② ⓐ



$N(\vec r,\hat\Omega,E,t)\,v\,dt\,\hat n\cdot\hat\Omega\,dA\,dE = \hat n\cdot\hat\Omega\,\Psi(\vec r,\hat\Omega,E,t)\,dA\,dt\,dE$

$\int d\hat\Omega\,\hat n\cdot\hat\Omega\,\Psi(\vec r,\hat\Omega,E,t) = \hat n\cdot J(\vec r,E,t)$



$N(\vec r+\Delta u\hat\Omega,\hat\Omega,E,t)\,\Delta A\,v\,\Delta t\,d\Omega\,dE - N(\vec r,\hat\Omega,E,t)\,\Delta A\,v\,\Delta t\,d\Omega\,dE$ = net part

Gains = Losses

$\Rightarrow [N(\vec r,\hat\Omega,E,t+\Delta t)-N(\vec r,\hat\Omega,E,t)]\,\Delta u\,\Delta A\,dE\,d\Omega$

$= -[N(\vec r+\Delta u\hat\Omega,\hat\Omega,E,t)-N(\vec r,\hat\Omega,E,t)]\,\Delta A\,v\,\Delta t\,dE\,d\Omega$

$-\sigma(\vec r,E)\,N(\vec r,\hat\Omega,E,t)\,\Delta u\,\Delta A\,v\,\Delta t\,dE\,d\Omega$

$+q(\vec r,\hat\Omega,E,t)\,\Delta u\,\Delta A\,\Delta t\,d\Omega\,dE$

dividing by $\Delta u\Delta A\Delta t\,d\Omega\,dE$ and rearranging:

$\frac{1}{v}\frac{\partial}{\partial t}\Psi(\vec r,\hat\Omega,E,t) + \frac{d}{du}\Psi(\vec r,\hat\Omega,E,t) + \sigma(\vec r,E)\Psi(\vec r,\hat\Omega,E,t)$
$= q(\vec r,\hat\Omega,E,t)$

$\frac{d}{du} = \frac{\mu}{\rho}\frac{\partial}{\partial\rho}\rho\frac{d\rho}{du} - \frac{1}{\rho}\frac{\partial}{\partial\omega}\eta\frac{d\omega}{du}$

$\mu = \sqrt{1-\xi^2}\cos\omega$

$\frac{d\rho}{du} = \hat\Omega\cdot\hat\rho$ $\qquad \frac{d\omega}{du} = \hat\Omega\cdot\hat\omega$

$\frac{d}{du} = \hat\Omega\cdot\hat\rho\,\frac{\mu}{\rho}\frac{\partial}{\partial\rho}\rho - \hat\Omega\cdot\hat\omega\,\frac{1}{\rho}\frac{\partial}{\partial\omega}\eta$

$= \hat\Omega\cdot\vec\nabla$

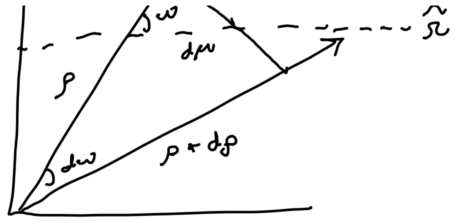
ⓑ $\left(\frac{\sqrt{1-\xi^2}}{\rho}\frac{\partial}{\partial\rho}\rho - \frac{1}{\rho}\frac{\partial}{\partial\omega}\right)\Psi + \Sigma_t\Psi = q$ * table for $\hat\Omega\cdot\vec\nabla$ for 1D cylinder

$\frac{1}{v}\frac{\partial\Psi(\vec r,E,\hat\Omega,t)}{\partial t} = S(\vec r,E,\hat\Omega,t) + \int_0^\infty\int_{4\pi}\Sigma_s(E'\to E,\hat\Omega'\to\hat\Omega)\Psi\,d\hat\Omega'\,dE' - \Sigma_t(\vec r,E)\Psi - \hat\Omega\cdot\nabla\Psi$

integrate $\int_{-1}^{1}d\mu\int_0^{2\pi}d\omega$

* $\mu = \Omega\hat z$ bell + glasstone
  $\mu = \cos\omega\sqrt{1-\xi^2}$
* integrate each term
* all
* don't integrate over μ b/c it is just a projection and not an

$d\rho$ $-\rho\,d\omega$



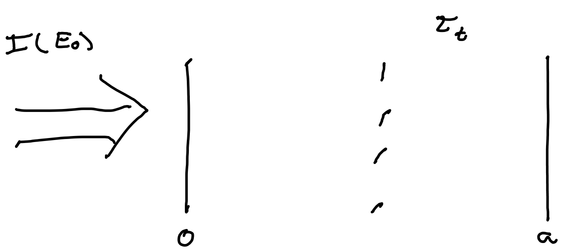
$$\int_{2\pi} \frac{1}{v} \frac{\partial \psi}{\partial t} d\omega = \int_{2\pi} S\, d\omega + \int_{2\pi}\int_0^\infty\int_{4\pi} \Sigma_s \psi\, d\hat{\Omega}'\, dE'\, d\omega - \int_{2\pi} \Sigma_t \psi\, d\omega - \int_{2\pi} \hat{\Omega}\cdot\nabla\psi\, d\omega$$

$$* \int_{2\pi} \hat{\Omega}\cdot\nabla\psi\, d\omega = \int_{2\pi} \frac{\sqrt{1-\xi^2}}{\rho} \frac{\partial}{\partial \rho} \rho \psi\, d\omega - \int_{2\pi} \frac{1}{\rho} \frac{\partial}{\partial \omega} \psi\, d\omega$$

$$= \frac{\sqrt{1-\xi^2}}{\rho} \frac{\partial}{\partial \rho} \rho \left[\psi(2\pi) - \psi(0)\right] - \frac{1}{\rho}\left[\psi(\rho, 2\pi, E) - \psi(\rho, 0, E)\right]$$

* all other terms get multiplied by 2π

③



bc: $\psi(\tilde{a}, \mu, E) = 0$

$\psi_0(0, \mu, E_0) = I(E_0)$

$$\mu \frac{d\psi}{dx} + \Sigma_t \psi = 0$$

$$\frac{d\psi}{dx} + \frac{\Sigma_t}{\mu}\psi = 0$$

$$\psi(x) = C_1 e^{-\frac{\Sigma_t}{\mu}x}$$

$$\psi(0) = I = C_1$$

$$\Rightarrow \psi(x) = I e^{-\frac{\Sigma_t}{\mu}x}$$

(4) first collision $= \int_0^x x\Psi(x)\,dx$

$$\mu \frac{d\Psi_1}{dx} + \Sigma_t \Psi_1 = \int_0^x x\Psi_0(x)\,dx$$

$$\mu \frac{d\Psi_1}{dx} + \Sigma_t \Psi_1 = \int_0^x dx\, x\, I e^{-\frac{\Sigma_t}{\mu}x}$$

(5) From class:

$$\phi(\vec{r}, E) = \int_V d^3r' \left[\frac{e^{-\tau(E,\vec{r}\to\vec{r}')}}{4\pi|\vec{r}-\vec{r}'|^2}\left(S(\vec{r}, E) + \frac{\chi(E)}{4\pi}\int_0^\infty dE'\,\nu(E)\,\Sigma_f(\vec{r}, E')\,\phi(\vec{r}, E') + \int_0^\infty dE'\,\Sigma_s(\vec{r}', E' \to\right.\right.$$

$S(E)\,S(\vec{r}) = S(\vec{r}, E)$ $\quad \Sigma_t(E) = \Sigma_t$

* infinite, homogeneous, poorly absorbing

let $R = |\vec{r}-\vec{r}'|$

$$\phi(\vec{r}, E) = \int_V d^3r' \left[\frac{e^{-\Sigma_t R}}{4\pi R^2} S(E)\,S(\vec{r})\right]$$

(6) $-D\frac{d^2\phi}{dx^2} + [\nu\Sigma_f - (\Sigma_a^F + \Sigma_a^m)]\phi = 0$

$P = \omega_f \Sigma_f \phi(x)$

$$-D\frac{d^2\phi}{dx^2} + \underbrace{\nu\sigma_f N_F(x)\phi(x)}_{const = P} - \Sigma_a\phi = 0$$

$\phi(x) = Ae^{x/L} + Be^{-x/L} + \frac{P}{D}$

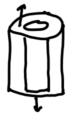
$A = 0$

$\sigma(x) = \frac{DB}{L} + P = 0$

$\Rightarrow B = -\frac{P}{LD}$

$\phi(x) = P - \frac{P}{LD}e^{-x/L}$

$$N_F(x) = \frac{\Sigma_a(P - \frac{P}{LD})e^{-x/L} + \frac{P}{L^2}e^{-x/L}}{\nu\sigma_f(P - \frac{P}{LD})e^{-x/L}}$$

(7) 

| core | Ref | Vac

Bare:

$\nabla^2\phi + \left(\frac{\nu\Sigma_f - \Sigma_a}{D}\right)\phi(r) = 0$

$\nabla^2\Psi_n + B_n^2\Psi_n(r) = 0$

Gen sol: $\phi(\vec{r}) = A_1 J_0\left(\frac{\nu_0}{R}\vec{r}\right)$, $\quad B_g = \frac{\nu_0}{R}$

Reflected:

Core: Gen Sol: $\phi^c(\vec{r}) = A_1^c J_0^c(B_m^c r)$

Reflector Gen Sol: $\phi^R(\vec{r}) = c_1 I_0 \frac{B_m^R r}{L^2} + c_2 K_0 \frac{B_m^R r}{L^2}$

$$L^R = \sqrt{\frac{D^R}{\Sigma_a^R}}$$

$$\phi^R(r) = C_2 K \frac{B_m^R r}{L^R}$$

Interface Boundary Condition:

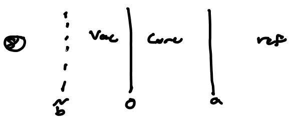
$$A^c J_0^c B_m^c R = C^R K_0 \frac{B_m^R R}{L^R}$$

$$-D^c B_m^c J_1^c(B_m^c R) = -\frac{1}{L^R} D^R C^R B_m^R K_1 R \frac{B_m^c}{L^R}$$

Criticality condition:

$$\frac{1}{D B_m^c} \frac{J_0^c(B_m^c R)}{J_1^c(B_m^c R)} = \frac{K_0(B_m^R/L^R)R}{D^R L^R B_m^R K_1(\frac{B_m^R}{L^R} R)}$$

②



Core:

$$-D^c \frac{d\phi^c}{dx^c} + (\Sigma_a^c - \nu\Sigma_f^c)\phi(x) = 0 \qquad BC: \phi^c(a) = \phi^R(a)$$

$$0 \le x \le a \qquad J^c(a) = J^R(a)$$

Reflector:

$$-D \frac{d\phi^R}{dx^R} + \Sigma_a^R \phi(x) = \frac{1}{\Sigma_a^R} S_0 \quad a \le x$$

Gen solution

$$\phi^c = A_1 \cos(B_m^c x) + A_2 \sin(B_m^c x)$$

$$\phi^c(x) = A_1 \cos(B_m^c x)$$

$$B_m^{c2} = \frac{\nu\Sigma_f^c - \Sigma_a}{D^c}$$

$$\frac{A_1}{4}\cos(B_m^c \tilde{b}) + \frac{D}{2} B_m^c \sin(B_m^c \tilde{b}) = 0$$

$$A_1 = 2D B_m^c \tan(B_m^c \tilde{b})$$

$$\phi^c(x) = 2D B_m^c \tan(B_m^c \tilde{b}) \cos(B_m^c x)$$

$$\phi^R(x) = A_3 e^{x/L} + A_4 e^{-x/L} + \frac{S_0}{\Sigma_a}$$

$$\phi^R(x) = A_4 e^{-x/L} + \frac{S_0}{\Sigma_a}$$

$$2D B_m^c \tan(B_m^c \tilde{b}) \cos(B_m^c a) = A_4 e^{-a/L} + \frac{S_0}{\Sigma_a}$$

$$\phi^R(x) = \left(2e^{a/L} D B_m^c \tan(B_m^c \tilde{b}) \cos(B_m^c a) - e^{a/L}\frac{S_0}{\Sigma_a}\right) e^{-x/L} + \frac{S_0}{\Sigma_a}$$

① $\sigma_t^{U235} = 698.9$ b

$\sigma_f = 585.1$ b

$\sigma_c = 98.71$ b

* taken from JENDL 4.0

$$\Sigma_{tr} = 19.1\ g/cm^3 \left(\frac{6.022 \times 10^{23}\ atoms/mol}{235\ g/mol}\right)(698.9 - 98.71 - 585.1) \times 10^{-24} = 10.4\ cm^{-1}$$

$\Sigma_a = 33.47\ cm^{-1}$

$\Sigma_f = 28.64\ cm^{-1}$

② $B_g^2 = B_m^2 \Rightarrow \left(\frac{\pi}{\tilde{R}}\right)^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$

$$\tilde{R} = \pi(3\Sigma_{tr}(\nu\Sigma_f - \Sigma_a))^{-1/2}$$

$$\Sigma_{tr}^{D_2O} = \frac{1}{3D} \approx \frac{1}{3 \times 0.87} = 0.383\ cm^{-1}$$

For homogeneous reactor:

$$\Sigma_{tr} = \frac{1}{2001}\Sigma_{tr}^U + \frac{2000}{2001}\Sigma_{tr}^{D_2O} = 0.388\ cm^{-1}$$

$$\Sigma_f = \frac{1}{2001}\Sigma_f^U + 0 = 0.014\ cm^{-1}$$

$$\Sigma_a = 0.0167\ cm^{-1}$$

$\tilde{R} \simeq 26.43$ cm

(b) $\bar{\lambda} = \frac{1}{\Sigma_{tot}} = \frac{1}{\Sigma_{tr} + \Sigma_a} \simeq 2.47$ cm

mean # of scattering events $\simeq \frac{26.43}{2.47} = 10.7$