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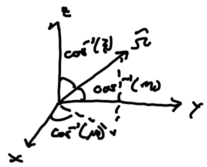
NE 250 HW 3

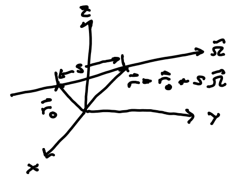
①

$$[\hat{\Omega}\cdot\nabla + \Sigma_t(\vec{r},E)]\,\psi(\vec{r},E,\hat{\Omega}) = \int_{4\pi} d\hat{\Omega}\int_0^\infty dE'\,\Sigma_s\psi + \frac{1}{k}\frac{\chi}{4\pi}\int_0^\infty dE'\,\nu\,\Sigma_f\int_{4\pi} d\hat{\Omega}\,\psi$$

$$\psi(\vec{r},E,\hat{\Omega}) = \psi_0(\hat{\Omega},E)\exp(i\hat{\Omega}\cdot\vec{B})$$

$$q = \int_{4\pi} d\hat{\Omega}\int_0^\infty dE'\,\Sigma_s\psi + \frac{1}{k}\frac{\chi}{4\pi}\int_0^\infty dE'\,\nu\,\Sigma_f\int_{4\pi} d\hat{\Omega}\,\psi$$





$$\frac{d\psi}{ds} = \hat{\Omega}\cdot\nabla\psi = \mu\frac{d\psi}{dx} + \eta\frac{d\psi}{dy} + \xi\frac{d\psi}{dz}$$

By expanding:

$$\frac{d\psi}{ds} + \Sigma_t\,\psi = q$$

2 $-\nabla \cdot D\nabla\phi(r) + \Sigma_a \phi(r) = \frac{1}{k} \nu \Sigma_f \phi(r)$

$$\Rightarrow \quad k = \frac{\int_V \nu \Sigma_f \phi(r)\, dV}{\int_V [\Sigma_a \phi(r) - D\nabla^2\phi(r)]\, dV}$$

for (n, xn) reactions ν is a function that incorporates all possible fission reactions dependent on the cross section for that reaction

$$\nu \Sigma_f \phi(r) \Rightarrow \nu(\sigma(E)) \Sigma_f(E) \phi(E, r)$$

$$\int_V \nu \Sigma_f \phi(\vec{r})\, dV \Rightarrow \int_V dV \int_0^\infty dE\, \nu(\sigma_f(E)) \Sigma_f(E) \phi(E, r)$$

$$K = \frac{\int_V dV \int_0^\infty dE\ \nu(\sigma_f(E))\Sigma_f(E)\phi(E,\vec{r})}{\int_V dV \int_0^\infty dE\left[\Sigma_a(E)\phi(E,\vec{r}) - D(E)\nabla^2\phi(E,\vec{r})\right]}$$

③ $E_{gain} = \frac{P_{total}}{P_{fusion}}$

$P(t) = w_f \nu \Sigma_f n(t)$

$\nabla^2\Psi_1 + B_1^2\Psi_1 = 0$

1 14 MeV $n_0' \rightarrow 0.8\ (n,2n) \rightarrow K_{sub}$

$$E_{gain} = \frac{0.8 \times 2 \times 200\ \text{MeV}}{1 \times 14\ \text{MeV}}$$

$$M = \frac{\phi_{total}}{\phi_{source}} = \sum_0^\infty K_{eff}^i = \frac{1}{1-K_{eff}}$$

$$n_{m-1\ generation} = n_0 \frac{1-K_{eff}^m}{1-K_{eff}},\quad m>0,\ K_{eff}<1$$

$m \rightarrow \infty$

$$n = n_0 \frac{1}{1-K_{eff}}$$

$$\frac{n}{n_0} = E_{gain} = \frac{1}{1-K_{eff}} = \frac{0.8 \times 2 \times 200}{1 \times 14}$$

$$K_{eff} = 0.96$$

④ ⓐ $A = \begin{bmatrix} 1 & 1 & -1 & 3 \\ 1 & 2 & -4 & -2 \\ 2 & 1 & 1 & 5 \\ -1 & 0 & 2 & -4 \end{bmatrix}$

column 1:

row 1: $\begin{vmatrix} 2 & -4 & -2 \\ 1 & 1 & 5 \\ 0 & 2 & -4 \end{vmatrix} = -48$ row 2: $\begin{vmatrix} 1 & -1 & 3 \\ 1 & 1 & 5 \\ 0 & 2 & -4 \end{vmatrix} = -12$ row 3: $\begin{vmatrix} 1 & -1 & 3 \\ 2 & -4 & -2 \\ 0 & -2 & -4 \end{vmatrix} = 24$

matrix of minors (following above):

$$\text{Minors}(A) = \begin{bmatrix} -48 & -36 & 0 & -12 \\ -12 & -2 & 2 & -4 \\ 24 & 8 & 4 & 4 \\ 12 & 18 & -6 & 0 \end{bmatrix}$$

matrix of cofactors:

$$\text{Cofactors}(A) = \begin{bmatrix} -48 & 36 & 0 & 12 \\ -12 & -2 & -2 & -4 \\ 24 & -8 & 4 & -4 \\ 12 & 18 & 6 & 0 \end{bmatrix}$$

$$\text{transpose}(C_A) = \begin{bmatrix} -48 & 12 & 24 & -12 \\ 36 & -2 & -8 & 18 \\ 0 & -2 & 4 & 6 \\ 12 & 4 & -4 & 0 \end{bmatrix}$$ *octave

$$A^{-1} = \frac{C_A^T}{\det(A)} = \begin{bmatrix} -2 & 1/2 & 1 & -1/2 \\ 3/2 & -1/12 & -1/3 & 3/4 \\ 0 & -1/12 & 1/6 & 1/4 \\ 1/2 & -1/6 & -1/6 & 0 \end{bmatrix}$$

ⓑ $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0 = (3-\lambda)^2 - 1$$

$$\lambda_{1,2} = 3 \pm 1 = (4, 2)$$

$$A\underline{v}_1 = 2I\underline{v}_1$$

$$\underline{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A\underline{v}_2 = 4I\underline{v}_2$$

$$\underline{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

⑤ $A = A^T$

$$A = \begin{bmatrix} 1 & 10 & 1 \\ 10 & -1 & 1 \\ 1 & 1 & 10 \end{bmatrix}$$

$\det(A) = -920$

$M_{11} = -11$ $\quad M_{21} = 99$ $\quad M_{31} = 11$

$M_{12} = 99$ $\quad M_{22} = 9$ $\quad M_{32} = -9$

$M_{13} = 11$ $\quad M_{23} = -9$ $\quad M_{33} = -101$

$$C_A = \begin{bmatrix} -11 & -99 & 11 \\ -99 & 9 & 9 \\ 11 & 9 & -101 \end{bmatrix}$$

$$C_A^T = C_A = \begin{bmatrix} -11 & -99 & 11 \\ -99 & 9 & 9 \\ 11 & 9 & -101 \end{bmatrix}$$

⑥ $L = \frac{d^2}{dx^2}$

$f(\pm a) = 0$

$f(x_1) = f(-x_n)$

$\nabla^2 f = \frac{d^2 f}{dx_1^2} + \cdots \frac{d^2 f}{dx_n^2}$

$\nabla^2 f = \nabla \cdot \nabla f$

$\langle \nabla^2 f_1, f_2 \rangle = \langle f_1, \nabla^2 f_2 \rangle$

$$\langle \nabla^2 f_1, f_2 \rangle = \int \underbrace{\nabla^2 f_1(x)}_{dv} \underbrace{\overline{f_2(x)}}_{u} dx_1$$

$IBP = uv - \int v\,du$

$v = \int \nabla^2 f_1(x)\,dx = \nabla f_1(x)$

$du = d(f_2(x)) = \frac{df}{dx} dx$

$$f_2(x)\nabla f_1(x) - \int \underbrace{\nabla f_1(x)}_{dv} \underbrace{\frac{df_2}{dx}}_{u} dx$$

$v = \int \nabla f_1(x)\,dx = f_1(x)$

$du = \frac{d^2 f}{dx^2} dx$

$$f_2(x)\nabla f_1(x) - \left(\nabla f_2(x) f_1(x) - \int f_1(x) \nabla^2 f_2(x)\,dx\right) = \int \nabla^2 f_1(x) f_2(x)\,dx$$

$$f_2(x)\nabla f_1(x) - f_1(x)\nabla f_2(x) + \langle f_1, \nabla^2 f_2 \rangle = \langle \nabla^2 f_1, f_2 \rangle$$

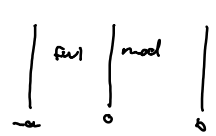
$=0$ $\qquad =0$

$f = \cos(n\pi x)$

$\nabla f = \sin(n\pi x)$

$\cos(n\pi x)\sin(m\pi x) = 0$

$$\Rightarrow \quad \langle f_1, \nabla^2 f_2 \rangle = \langle \nabla^2 f_1, f_2 \rangle$$

⑦ 

$s(x) = 0 \qquad -a \le x \le 0$

$= S_0 \qquad 0 < x \le b$

$$\nabla^2 \phi_{fuel} - \frac{1}{L^2}\phi_{fuel} = 0$$

$$\phi_{fuel} = C \cosh\frac{x+a}{L} + E\sinh\frac{x+a}{L}$$

$$J(x) = -D_{fuel}\frac{d\phi}{dx}$$

$$= -C\frac{D}{L}\sinh\frac{x+a}{L} - E\frac{D}{L}\cosh\frac{x+a}{L}$$

$$J(-\infty) = 0 \Rightarrow E = 0$$

$$\phi_{fuel}(x) = C\cosh\frac{x+a}{L}$$

Moderator:

$$\nabla^2\phi(x) - \frac{1}{L^2}\phi_{mod} = -\frac{S_0}{D}$$

homogeneous:

$$\phi_h(x) = -A\cosh\frac{b-x}{L} - B\sinh\frac{b-x}{L}$$

particular:

$$\phi_p(x) = \frac{S_0}{\Sigma_a}$$

general:

$$\phi_{mod}(x) = -A\cosh\frac{b-x}{L} - B\sinh\frac{b-x}{L} + \frac{S_0}{\Sigma_a}$$

$J(b) = 0$ * derivative is same as above

$$B = 0$$

$$\phi_{fuel}(0) = \phi_{mod}(0)$$

$$C\cosh\frac{a}{L_{fuel}} = \frac{S_0}{\Sigma_{a_{mod}}} - A\cosh\frac{b}{L_{mod}}$$

$$J_{fuel}(0) = J_{mod}(0)$$

$$-C\frac{D_{fuel}}{L_{fuel}}\sinh\frac{a}{L_{fuel}} = -A\frac{D_{mod}}{L_{mod}}\sinh\frac{b}{L_{mod}}$$

$$\begin{bmatrix} \cosh\frac{b}{L_{mod}} & \cosh\frac{a}{L_{fuel}} \\ \frac{D}{L_{mod}}\sinh\frac{b}{L_{mod}} & -\frac{D}{L_{fuel}}\sinh\frac{a}{L_{fuel}} \end{bmatrix}\begin{bmatrix} A \\ C \end{bmatrix} = \begin{bmatrix} \frac{S_0}{\Sigma_{a,mod}} \\ 0 \end{bmatrix}$$

* using maplesoft:

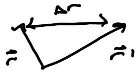
$$\phi_{fuel}(x) = \frac{S}{\Sigma_{a,mod}}\left(\frac{L_{fuel}D_{mod}\sinh\frac{b}{L_{mod}}\cosh\frac{x+a}{L_{fuel}}}{L_{mod}D_{fuel}\sinh\frac{a}{L_{fuel}}\cosh\frac{b}{L_{mod}} + L_{fuel}D_{mod}\sinh\frac{b}{L_{mod}}\cosh\frac{a}{L_{fuel}}}\right)$$

$$\phi_{mod}(x) = \frac{S_0}{\Sigma_{a,mod}}\left(1 - \frac{L_{mod}D_{fuel}\sinh\frac{a}{L_{fuel}}\cosh\frac{b-x}{L_{mod}}}{L_{mod}D_{fuel}\sinh\frac{a}{L_{fuel}}\cosh\frac{b}{L_{mod}} + L_{fuel}D_{mod}\sinh\frac{b}{L_{mod}}\cosh\frac{a}{L_{fuel}}}\right)$$

(8) $$G_{pt}(\vec{r},\vec{r}') = \frac{\exp\left(-\frac{|\vec{r}-\vec{r}'|}{L}\right)}{4\pi D|\vec{r}-\vec{r}'|}$$

$$G_{pl}(x,x') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{r},\vec{r}')\,dy\,dz$$

$$r = \sqrt{x^2+y^2+z^2}$$



* integrating over y and z leaves x and 2π factor

from wolfram:

$$G_{pl}(x,x') = \frac{2\pi L}{4\pi D}\exp\left(\frac{-|x-x'|}{L}\right)$$

$$= \frac{L}{2D}\exp\left(\frac{-|x-x'|}{L}\right)$$

9 (a) $-\frac{d^2\phi}{dx^2} + \frac{1}{L^2}\phi(x) = \frac{S_0}{D}$

$\phi(x) = c_1 e^{x/L} + c_2 e^{-x/L} + \frac{S_0}{\Sigma_a}$

$\phi(\pm\tilde{a}) = 0$

$c_1 = c_2 = -\frac{S_0}{\cosh(\frac{\tilde{a}}{L})\Sigma_a}$

$\phi(x) \simeq \left(1 - \frac{\cosh(\frac{x}{L})}{\cosh(\frac{\tilde{a}}{L})}\right)\frac{S_0}{\Sigma_a}$

(b) $\frac{1}{v}\frac{\partial\phi(x,t)}{\partial t} - D\frac{\partial^2\phi}{\partial x^2} + \Sigma_a\phi = \nu\Sigma_f\phi$

$\phi(x,t) = T(t)\psi(x)$

$\frac{1}{T}\frac{dT}{dt} = \frac{v}{\psi}\left[D\frac{d^2\psi}{dx^2} - (\Sigma_a - \nu\Sigma_f)\psi\right] = -\lambda$

$\frac{dT}{dt} = -\lambda T(t)$

$D\frac{d^2\psi}{dx^2} + (\nu\Sigma_f - \Sigma_a)\psi = -\frac{\lambda}{v}\psi$

$\frac{d^2\psi_n}{dx^2} + B_n^2\psi_n(x) = 0$

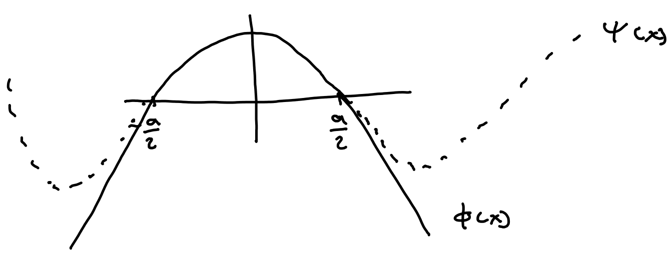
$\psi_n(x) = \cos(B_n x)$

$B_n^2 = \left(\frac{n\pi}{a}\right)^2$

for $-\frac{a}{2} \le x \le \frac{a}{2}$

$\psi(x) \simeq \phi(x)$

*neglecting amplitude scaling



10 sphere: $\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$

$\nabla^2\phi(r) + B^2\phi(r) = 0$

$\phi(r) = C_1\frac{\sin(Br)}{r} + C_2\frac{\cos(Br)}{r}$

$* \; C_2 = 0$

$\phi(R) = 0 = C_1\frac{\sin(BR)}{R}$

$B = \frac{n\pi}{R}$

$B_g^2 = \left(\frac{\pi}{R}\right)^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$

$R = \pi\left(\frac{\nu\Sigma_f - \Sigma_a}{D}\right)^{-1/2}$; $\phi(r) = \frac{C}{r}\sin\left(\frac{\pi r}{R}\right)$

cylinder:

$\frac{1}{r}\frac{d}{dr} r\frac{d\phi}{dr} + B^2\phi = 0$

$\phi(\tilde{R}) = 0$

$R(r) = AJ_0(\alpha r) + CY_0(\alpha r)$

$* \; C = 0$

$R(\tilde{R}) = 0 = AJ_0(\alpha\tilde{R})$

$\alpha\tilde{R} = \nu_0$

$R_n(r) = J_0\left(\frac{\nu_0 r}{\tilde{R}}\right)$

$\alpha^2 = \left(\frac{\nu_0}{\tilde{R}}\right)^2$

$B_g^2 = \left(\frac{\nu_0}{\tilde{R}}\right)^2$

$\phi(r) = J_0\left(\frac{\nu_0 r}{\tilde{R}}\right)$

Parallelpiped :

$$\frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2} + B_g^2\phi = 0$$

* 3D "slab" = $\prod_i \phi_i(x_i)$

$$\phi(x, y, z) \simeq A\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{b}}\right)\cos\left(\frac{\pi z}{\tilde{c}}\right)$$

$$B_g^2 = \sum_i B_{g,i}^2 = \left(\frac{\pi}{\tilde{a}}\right)^2 + \left(\frac{\pi}{\tilde{b}}\right)^2 + \left(\frac{\pi}{\tilde{c}}\right)^2$$

(11) Kord Smith Challenge

Model a core using only Monte Carlo

Core: 200 Fuel assemblies

100 axial planes

300 pins / assembly

10 depletion regions / pin

100 isotopes

$= 6 \times 10^9$ tallies to track

* needs 1% statistics on power peak

Challenge is to complete the above problem in less than 1 hour on a desktop computer.

Initial guess was possible by 2030.

In 2010 2 groups claimed to be approaching this goal in terms of accuracy but not in terms of time. Ke reported 95% of tallies at 3% st dev running 400 cores for 18 hours.

Given this context I truly believe there should be a field-wide attempt to develop a different type approach that gives accuracy of MC (or nearby accuracy) with faster implementation. MC was developed a long time ago and really isn't that accurate given large uncertainties in nuclear data. I think the computational side of nuclear engineering needs to develop a new approach and the experimental side needs to get more accurate data at a much larger energy range.