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NE 250
Problem set 5
1. (10 points) Calculate the mean, variance, and the cumulative distribution function for each of the following probability density functions:

a. f(x) = $\begin{cases} \frac{1}{a} & 0 \leq x \leq a \\ 0 & x < 0, x > a \end{cases}$
mean = $\int_{-\infty}^{\infty} xf(x)dx = \frac{1}{a}\int_0^a xdx = \frac{1}{2a}a^2 = \frac{a}{2}$
Var[x] = E[x^2] - E[x]
E[x^2] = $\int_{-\infty}^{\infty} x^2 f(x)dx = \frac{1}{a}\int_0^a x^2 dx = \frac{1}{3a}a^3 = \frac{a^2}{3}$
E[x]^2 = $\frac{a^2}{4}$
Var[x]^2 = $\frac{a^2}{12}$
for x<0 F(x) = 0 = P(x)
for x>a F(x) = 1 = P(x)
for $0 \leq x \leq a$:

F(x) = P(x) = $\int_{-\infty}^{x} f(t)dt = \frac{1}{a}\int_0^x dt = \frac{x}{a}$
F(x) = $\begin{cases} \frac{x}{a} & 0 \leq x \leq a \\ 0 & x < 0 \\ 1 & x > a \end{cases}$
b. f(x) = $\lambda e^{-\lambda x}, x > 0$
mean = $\int_0^{\infty} x\lambda e^{-\lambda x}dx = \frac{1}{\lambda}$
variance = $\int_0^{infty} x^2\lambda e^{-\lambda x} - \frac{1}{\lambda^2} = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$
CDF:
F(x) = $\int_0^x \lambda e^{-\lambda x}dt = x\lambda e^{-\lambda x}$
F(x) = $\begin{cases} x\lambda e^{-\lambda x} & x > 0 \\ 0 & x < 0 \\ 1 & x \to \infty \end{cases}$

3. (15 points) Answer the following short answer questions. If true or false, provide a one sentence justification for your answer.
a. True or False: we do not worry about normalizing PDFs for Monte Carlo, we can just sample from them directly.
True, but in practice it is necessary to preserve the physics of most MC calculations.

b. Compare and contrast: list three strengths and three weaknesses each for Monte Carlo and Deterministic methods.
MC strengths: easy parallelization on CPU because you track individual particles, number of particles governs accuracy, continuous in energy.
Deterministic strengths: fast, works well in high absorption and shielding materials, gives global solutions.
MC weaknesses: slow, possibly memory intensive, essentially needs variance reduction.
Deterministic weaknesses: needs acceleration methods, complicated parallelization, discretized geometry and rest of parameters can limit accuracy.

c. In analog Monte Carlo, the number of particles simulated, N , governs the solution accuracy. What is the relationship between N and relative error?
Relative error R is related to particles sampled by:
$R = \sqrt{\sum_{i=1}^{N}\chi_i^2/(\sum_{i=1}^{N}\chi_i)^2 - \frac{1}{N}}$

d. What theorem allows us define confidence intervals about the expected value of our PDF using the sample mean and sample standard deviation? What is a crucial condition of that theorem?
Central limit theorem. Needs IID requirement.

e. Why do we have to stop particles and boundary crossing when tracking their move- ment?
Stopping at boundary crossings is due to the movement out of one region with one set of conditions such as cross sections, into a new region with a different set of conditions which changes the interaction probabilities.

f. When would you expect a collision estimator to be more accurate? When would you expect a track length estimator to be more accurate? Why?
Collision estimator is more accurate for parameters integrated across the wholegeometry and track length works for finer parameters such as in a single fuel pin. This is due to the differences in the methods for calculating integral reaction rates.

g. What is the consistent part in the CADIS method?
The consistency is from the birth weights matching the target weights.

4. Answers for 4 parts a and b are in python file in folder, NE250HW6P.py