Neutron Waveguide Transport

Modeling Transport for Neutron Radiation Therapy

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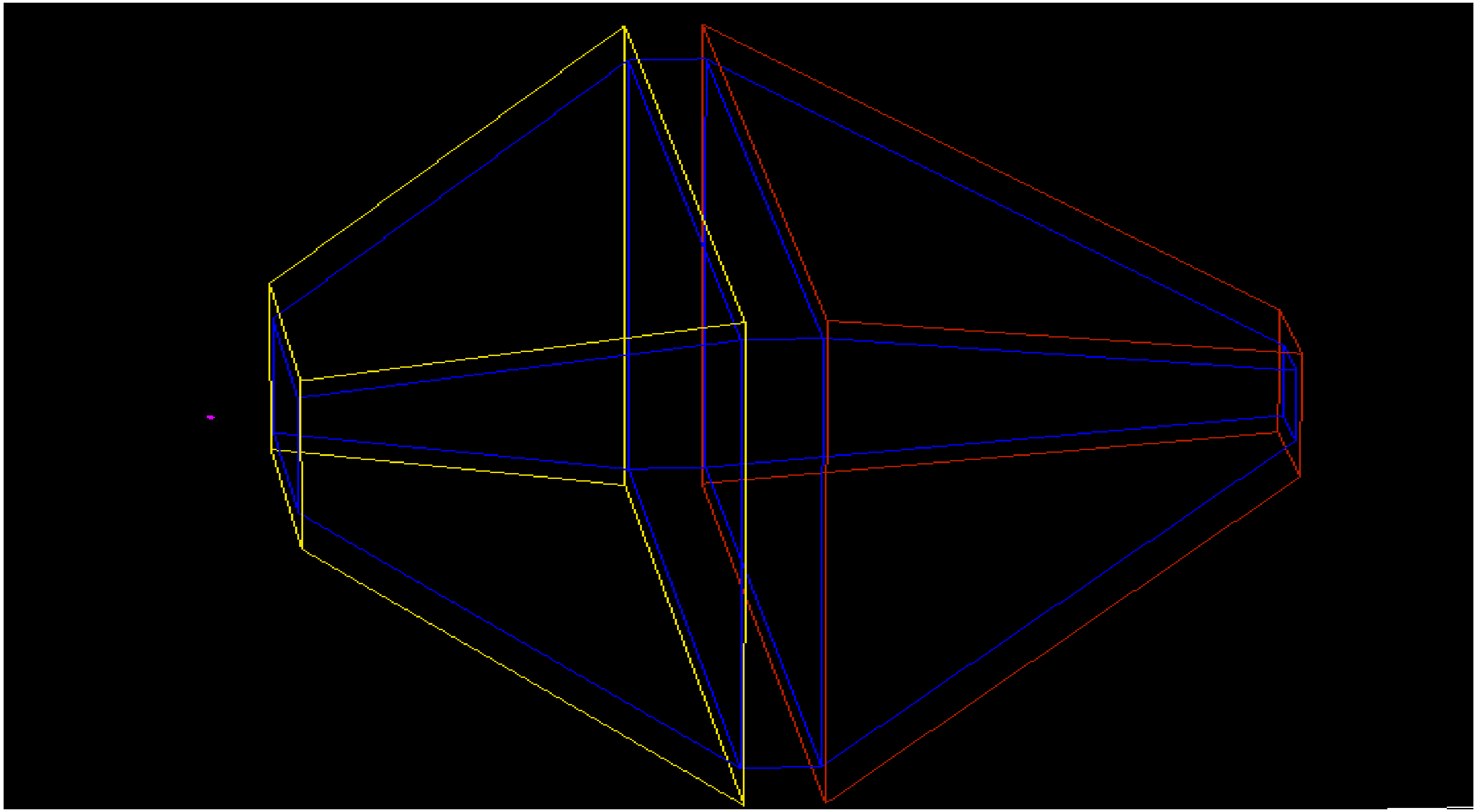
Waveguide - Purpose



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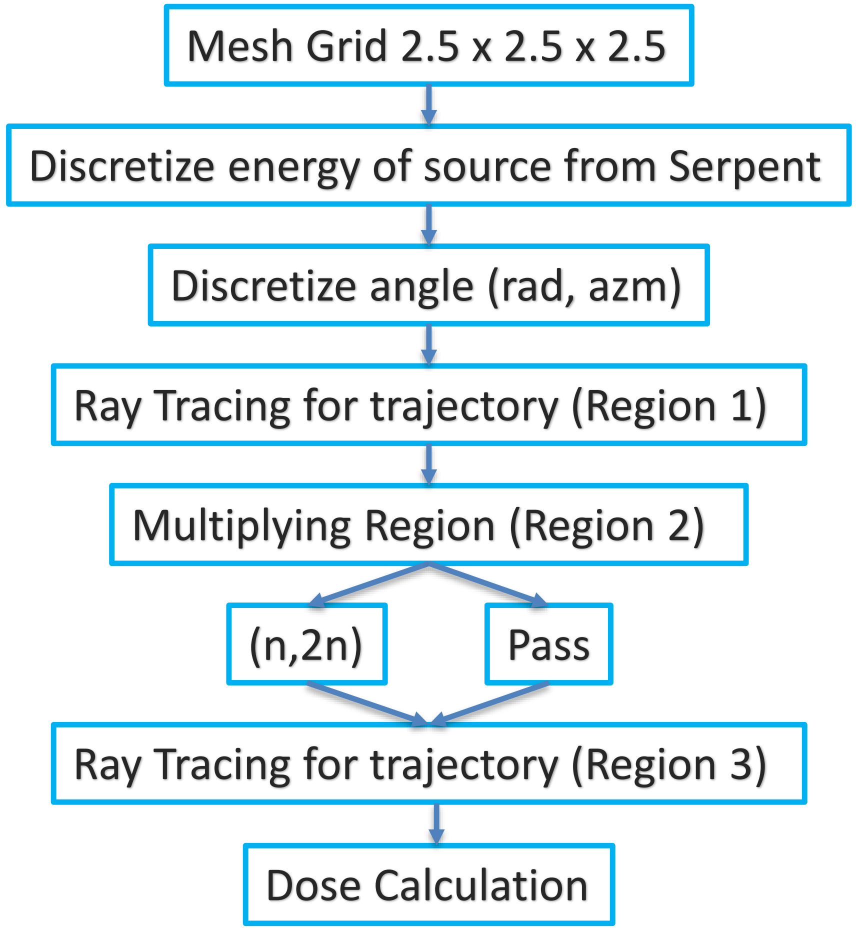
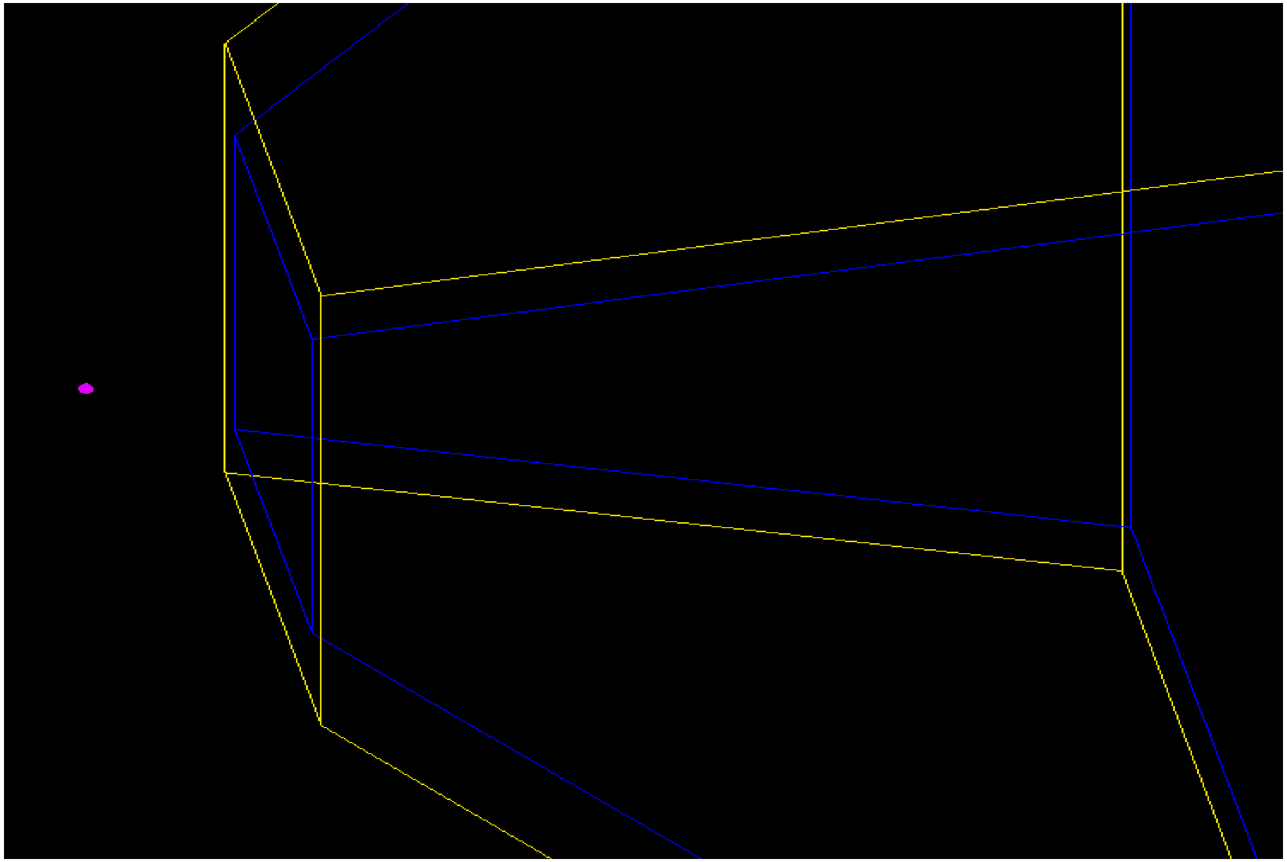
Flow Chart

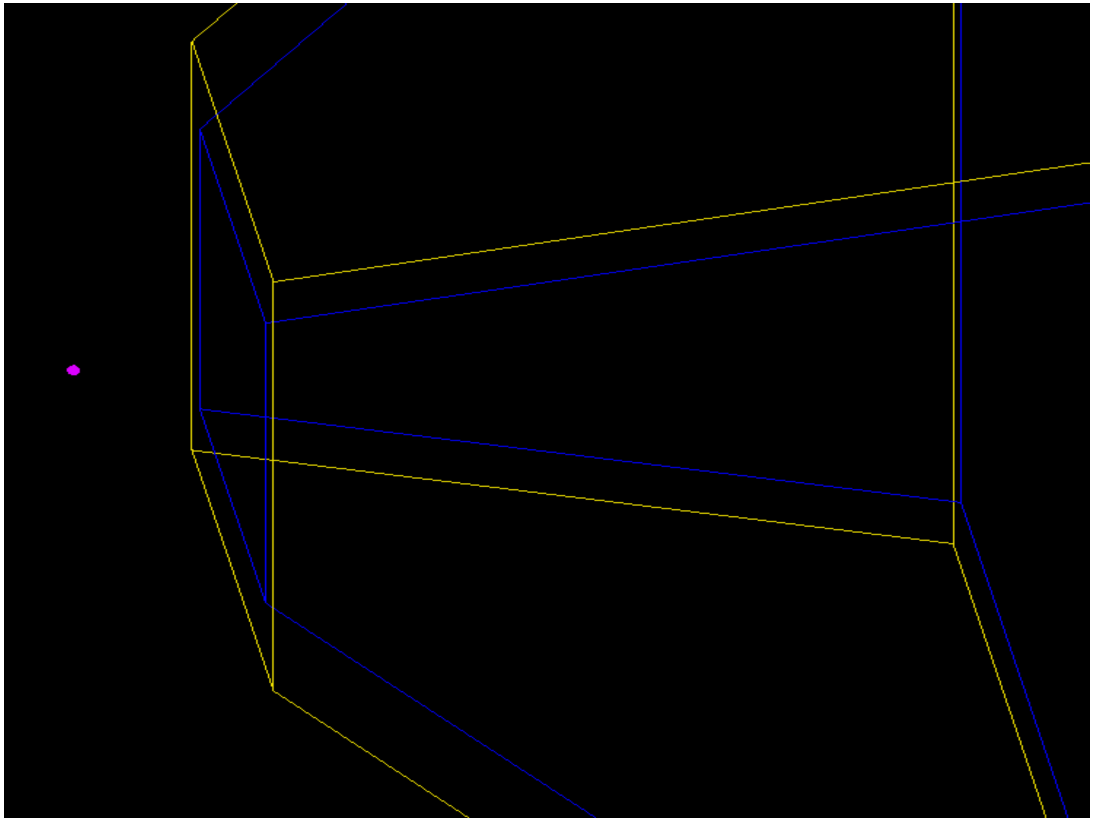

Mesh Grid 2.5 x 2.5 x 2.5
Discretize energy of source from Serpent
Discretize angle (rad, azm)
Ray Tracing for trajectory (Region 1)
Multiplying Region (Region 2)
(n,2n)
Pass
Ray Tracing for trajectory (Region 3)
Dose Calculation

Capture Section



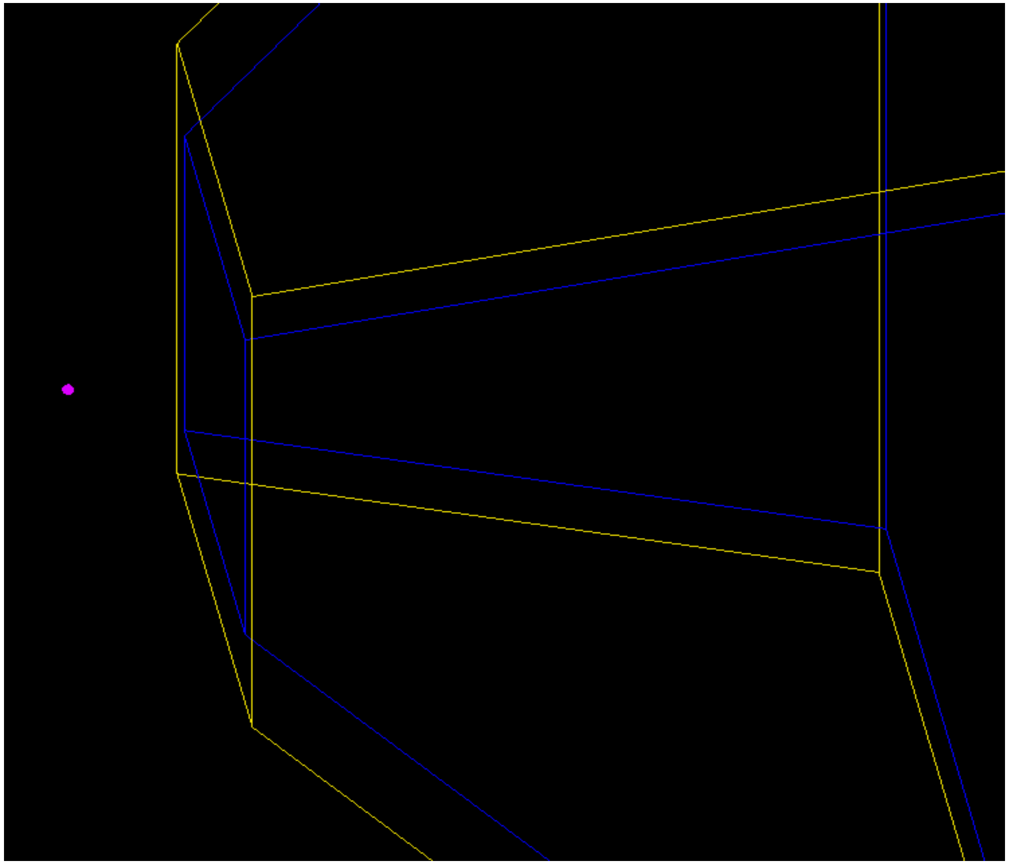
Capture Section

- Beryllium reflecting walls 30 cm thick on every side
- ZrH_2 is inside the Beryllium walls
- Walls diverge at 30^o radially by 47.7^o azimuthally
- Orthogonal length is 1.5 meters which is equivalent to radius of reactor vessel



Capture Section

- Point source travels into the waveguide isotropically
- ZrH_2 attenuates the fluence so diffusion equation is solved in this region
    - DIFFUSION!



Diffusion

- Ficks Law
  - $J = -D\frac{d\emptyset}{dx} = -D\nabla\emptyset$
- Change Rate = Production Rate - Absorption Rate - Leakage Rate
- $\frac{dn}{dt} = s - \Sigma_a\emptyset - \nabla J$
- $D\nabla^2\emptyset - \Sigma_a + S = \frac{dn}{dt}$
- In our case time-independent:
  - $D\nabla^2\emptyset - \Sigma_a + S = 0$
- Dividing by D we get:
  - $\nabla^2\emptyset - \frac{\emptyset}{L^2} + \frac{S}{D} = 0$
  - where: $L^2 = \frac{D}{\Sigma_a}$

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Diffusion

- $\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$
- Spherical coordinate it becomes:
  - $\frac{d^2\emptyset}{dr^2} + \frac{2}{r}\frac{d\emptyset}{dr} - \frac{1}{L^2}\emptyset = 0$
  - $\emptyset(r) = A\frac{e^{-\frac{r}{L}}}{r} + C\frac{e^{\frac{r}{L}}}{r}$, since neutron density flux must remain finite, C = 0.
  - $\frac{d\emptyset}{dr} = A(\frac{1}{rL} + \frac{1}{r^2})e^{-\frac{r}{L}}$
  - $J = \frac{S}{4\pi r^2}$
  - $J = -D\frac{d\emptyset}{dr} = DA(\frac{1}{rL} + \frac{1}{r^2})e^{-\frac{r}{L}}$
  - $S = \lim_{r\to 0} 4\pi DA\left(\frac{r}{L} + 1\right)e^{-\frac{r}{L}} = 4\pi DA$ $\Rightarrow$ $A = \frac{S}{4\pi D}$
- $\emptyset(r) = \frac{Se^{-\frac{r}{L}}}{4\pi Dr}$

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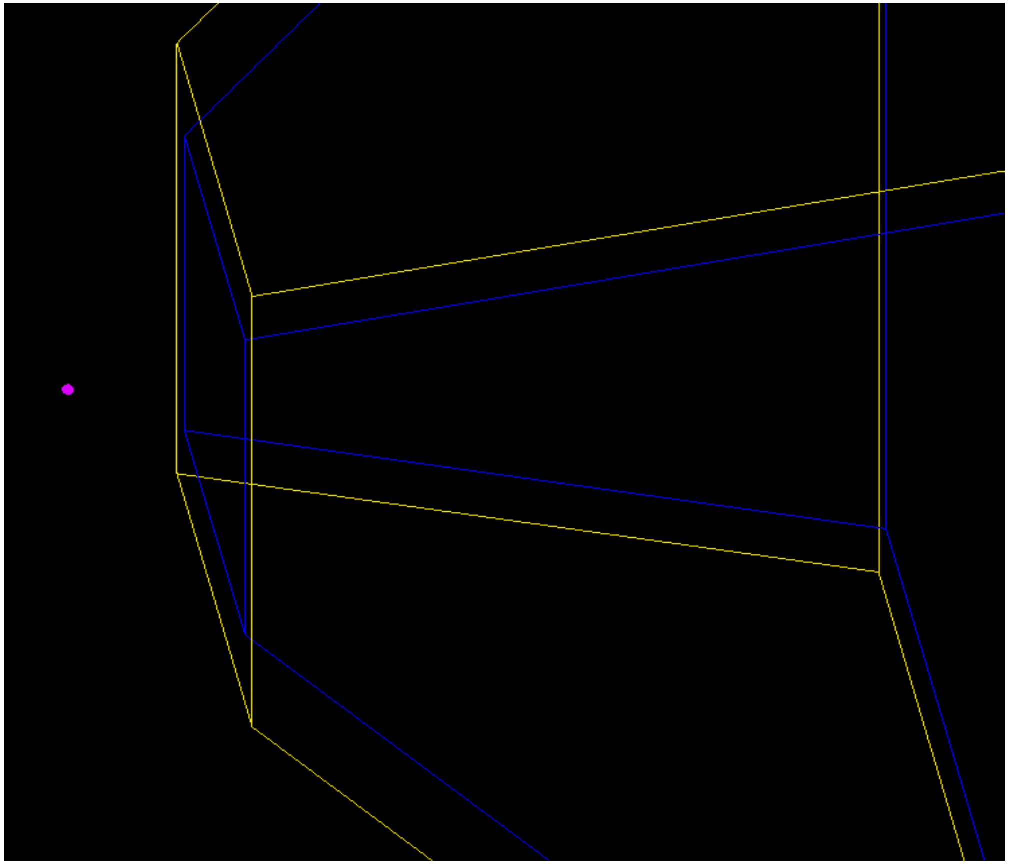
Capture Section

- Beryllium walls reflect neutrons back into system
  - Probability density function for walls solved
  - $dN_{scatter} = N_0 \Sigma e^{-\Sigma x} dx$
- Reflecting events lead to diffusion events
- Angle of scatter calculated by Fresnel's law
  - $(\theta', \phi') = \arccos(\frac{\cos(\theta, \phi)}{n})$
  - $n = 1 - \frac{\lambda^2 \rho}{2\pi}$



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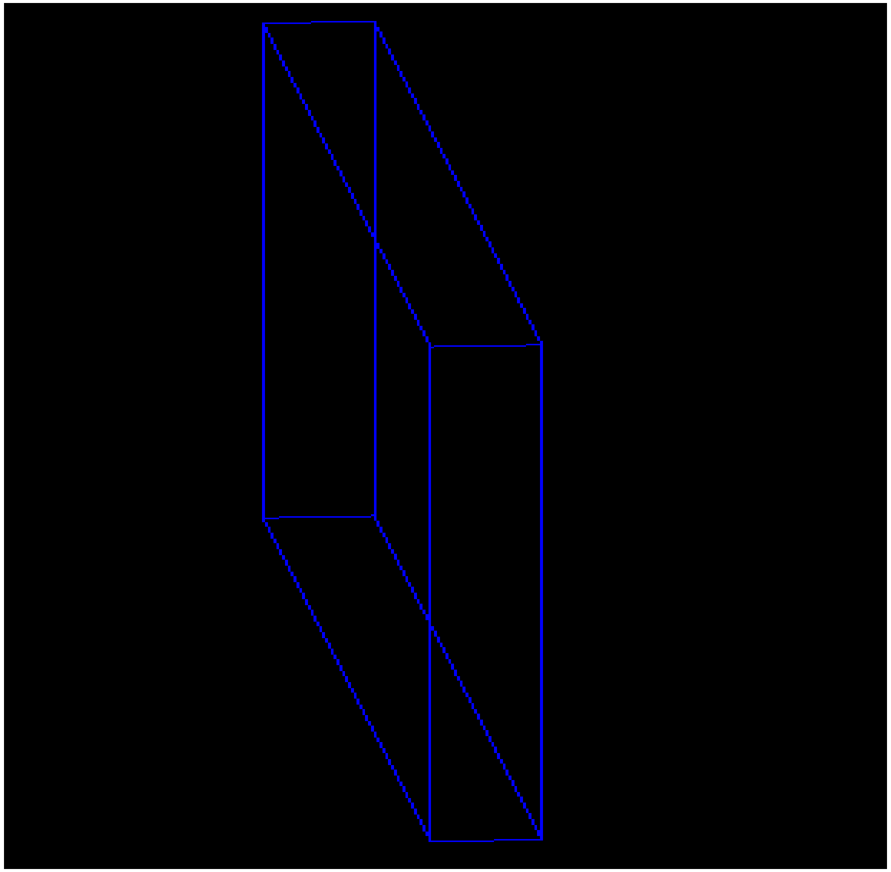
Calculating dx Values

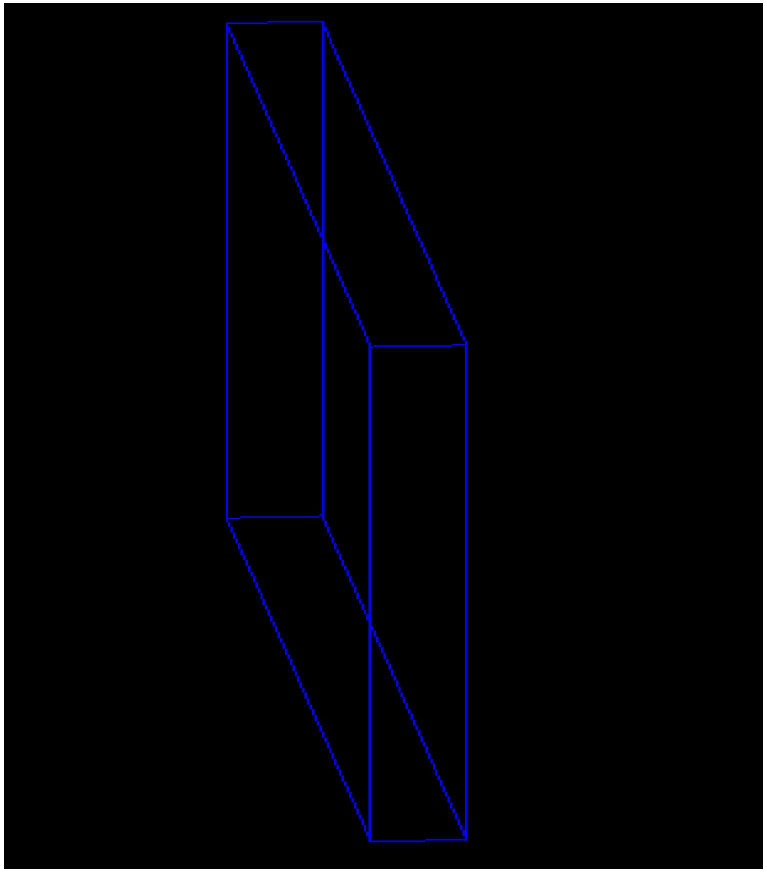
- dx represents the path length of our particles through one voxel
- Each voxel has dimensions 2.5 cm x 2.5 cm x 2.5 cm
- Using the trajectory of the particles, we can create a line and determine where that line intersects with the first and second boundaries
  - Before the first intersection, our particles are assumed to have no interaction due to the physical geometry of the system
  - Between the first and second boundaries, our particles are scattered with some probability
  - The dx values are calculated as the intersection of the trajectory line with the meshgrid voxels
  - After the particles cross the second boundary they have left the system and are no longer of interest
- This is done for all 36 angles in the radial and azimuthal planes
- The x value in the exponent is then the sum of all dx values up to the voxel of interest.

Multiplying Target



- Made from Beryllium
- (n,2n) reactions
  - Increase fluence in shaping region
- Thickness is 30 cm.

Multiplying Target



- Entering third region:
  - $N_{(n,2n)} = 2 * N_0(1 - e^{-\sigma_{(n,2n)}\rho_{atomic}x})$
  - $x = \frac{w}{\cos(\theta',\phi')}$
- Energy of these particles
  - $E_{f,(n,2n)} = \frac{1}{2}\left(\frac{1}{2} + \left(\frac{A-1}{A+1}\right)^2\right) * E_0 = 0.43 * E_0$

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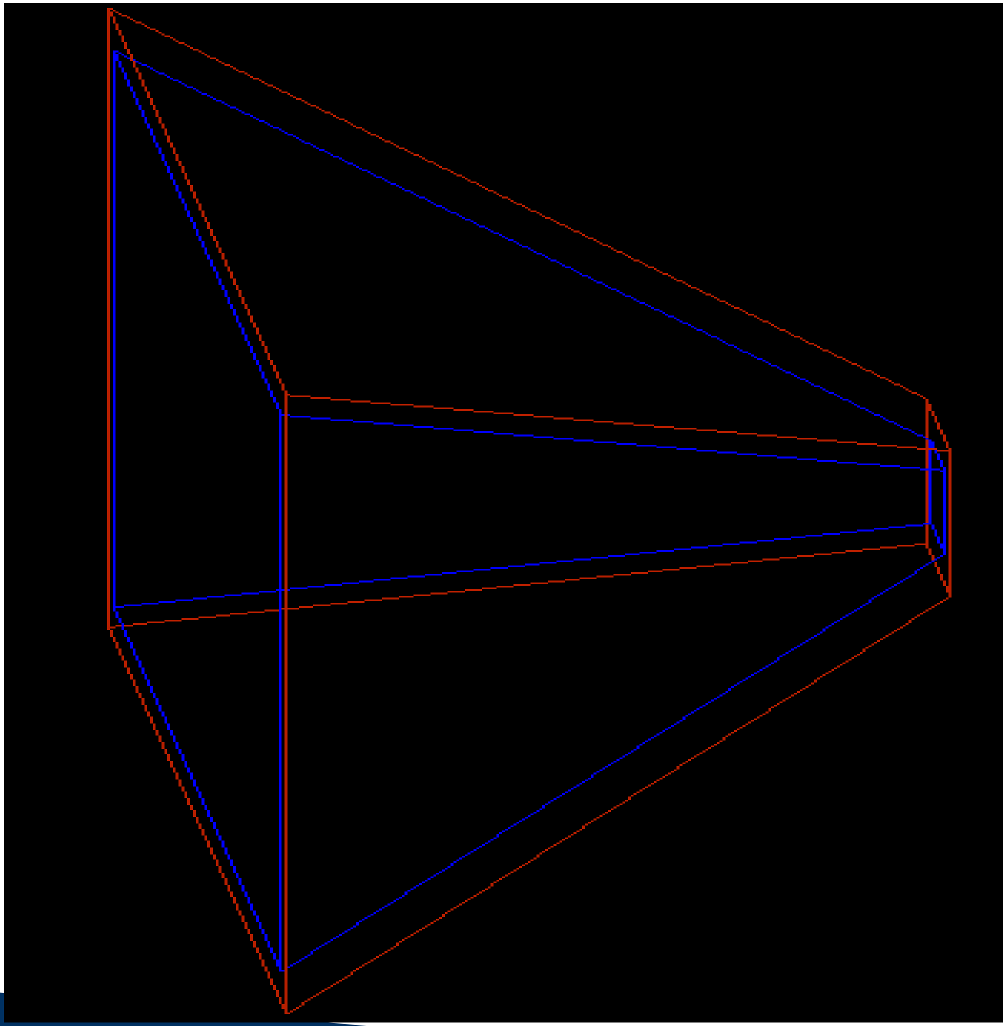
Shaping Region

- Bismuth
- ZrH_2 is inside the Beryllium walls
- Walls diverge at 49.49^o radially by 58.66^o azimuthally
- Scattering events are calculated the same way as in the capture region but with new Σ and dx values



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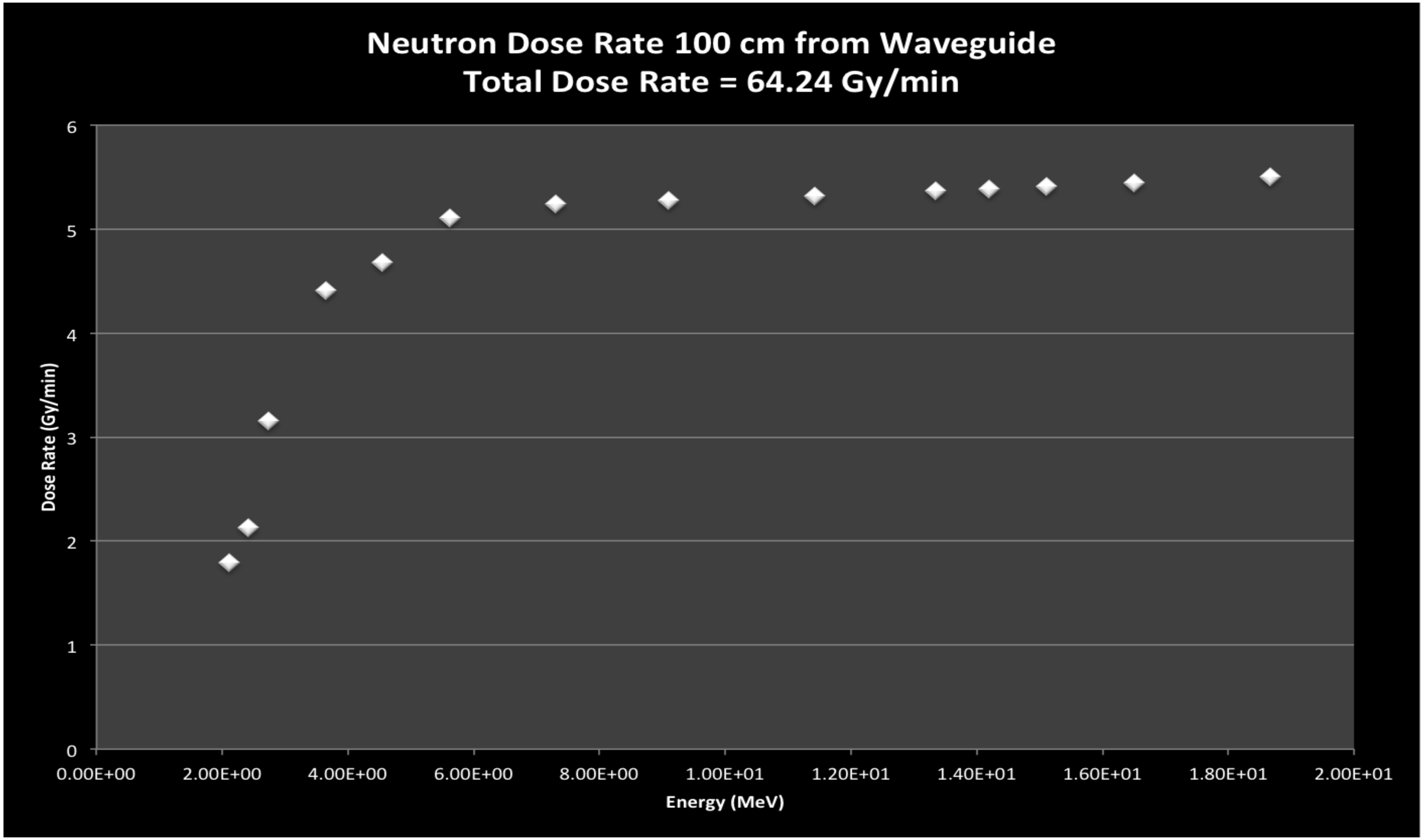
Results



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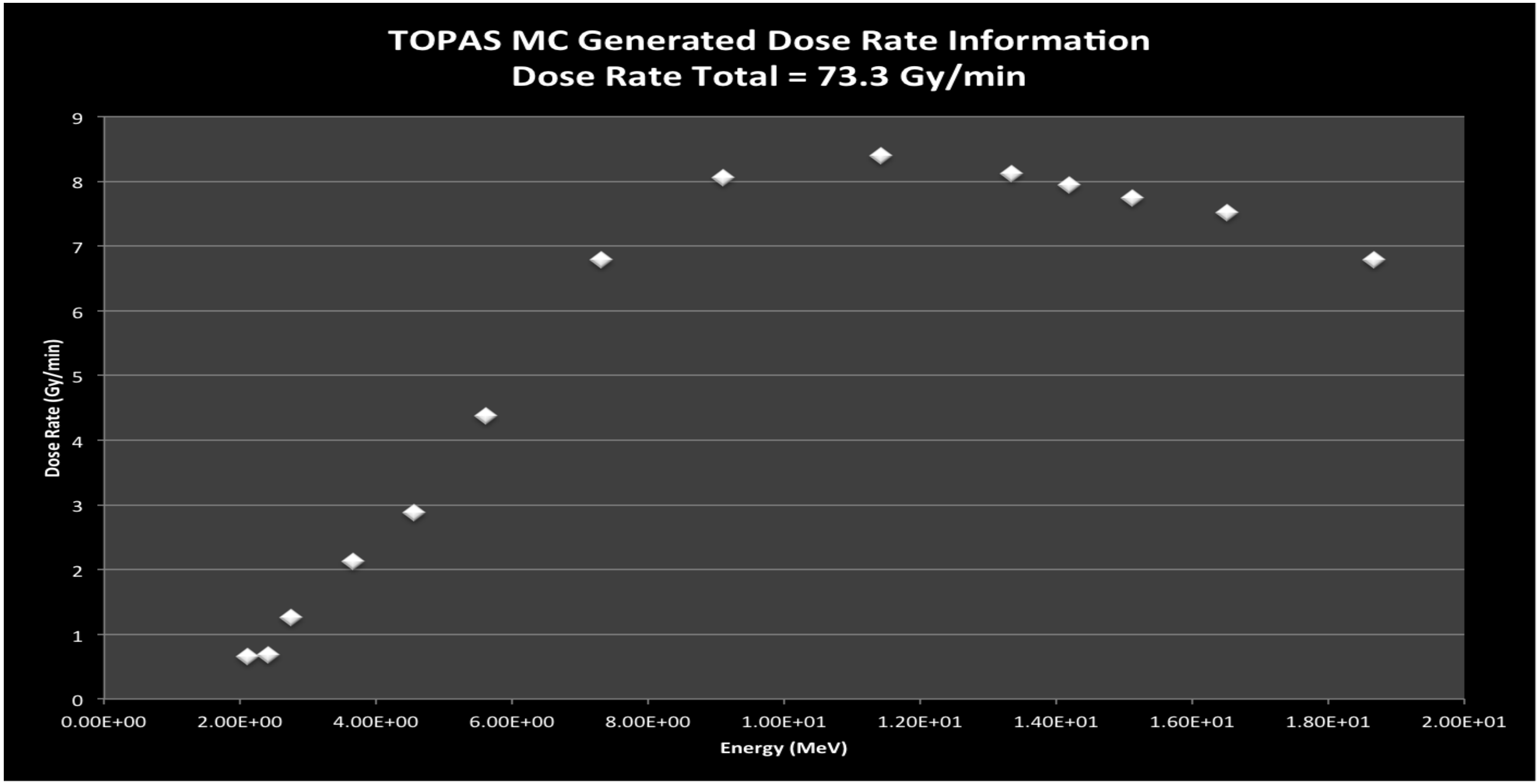
Results for similar geometry in TOPAS MC



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Validation

- Run this geometry Monte Carlo
    - Results close to already existing geometry.
- Particle Tracking
    - In = Out
- Switch from isotropic to planar source
- Extreme Cases

Summary

- We created a "hybrid" MC-deterministic code to improve the speed it takes to simulate many particles in a specific geometry
- Dose rate calculated was 64.24 Gy/min
- Time taken for our code = 3.5 hours on one thread
  - Time for TOPAS MC of similar geometry but same source = 72 hours on 6 threads
- Accuracy can be increased by sweeping across many more angles and using a smaller meshgrid

Reference

- Harling, O; Riley, K., “Fission reactor neutron sources for neutron capture therapy - a critical review”, Journal of Neuro-Oncology 62: 7-17, 2003.
- National Nuclear Data Center, *Evaluated Nuclear Data File B-VII.1 Library*, http://www.nndc.bnl.gov/sigma/
- Perl J, Shin J, Schumann J, Faddegon B, Paganetti H. TOPAS: an innovative proton Monte Carlo platform for research and clinical applications. Med Phys. 2012 Nov; 39(11):6818-37.
- Slaybaugh, Rachel. *NE 255 Section 08 Notes, Monte Carlo*, https://github.com/rachelslaybaugh/NE255/tree/gh-pages/08-monte-carlo