Homework One

Analytic Geometry and Calculus

UC Berkeley Math N16B, Summer 2021

Upload your responses to the prompts marked (Submit) to Gradescope before 8pm Friday; you will receive feedback on these.

gradescope.com/courses/275664

The rest of the exercises you should complete at your discretion. Note that *Calculus with Applications, 11th Edition* has some select solutions, usually to odd-numbered exercises, in the back.

Goals this Week

Here are some goals you should have in mind while exercising:

1. Be able to identify when an integral will need to be integrated "by parts", and effectively do it.

2. Be able to visualize (and draw) the three-dimensional solids described in this homework, and write down an integral that represents their volume. Understand that the integrand in these integrals represents a cross-sectional area, and integrating that area along an axis calculates how much volume you're accumulating.

3. Be able to set up integrals that calculate total capital (money) accumulated given only information about the money flow and any interest rate imposed on that money.

Exercises

1. From Chapter 8.1 of *Calculus with Applications, 11th Edition* (page 473) work through the exercises:

   1–12 odd (SUBMIT) 13–22* 33 37 (SUBMIT) 41

   *just make sure you know *how you'd start* each of those integrals.

2. From Chapter 8.2 of *Calculus with Applications, 11th Edition* (page 480) work through the exercises:

   1–18 every-other-odd 24–37 odd 35 40

3. There are two methods you could use to find the volume of a solid of revolution: the "disk"/"washer" method and the "shell" method; for one you integrate with respect to x, and the other with respect to y. Find the volume of the solid formed by rotating the region bound by the graphs of $y = \ln(x)$, $y = 2$, the x-axis, and the y-axis about the x-axis using both of these methods. Then find the volume of the solid formed by rotating this same region about the y-axis using both of these methods. See

   `math.ucr.edu/~mpierce/7b/docs/solid-of-revolution.pdf`

   for my write-up of these calculations.

4. (SUBMIT) Calculate the following general geometric volume formulas. Drawing a picture of the solid being described will help you set up the correct integral.

   (a) (A SPHERE) Find the volume of a sphere of radius 1 by taking the half-circular region of radius 1 that lives below the curve $y = \sqrt{1 - x^2}$ and rotating it about the x-axis. Then alter this calculation to find the general formula for the volume of a sphere of radius r.

(b) (A CONE) Determine a formula for the volume of a right-circular cone (a cone with a circular base such that that the "point" of the cone is directly over the center of the base) with height h and circular base of radius r. How would the volume change in the case that the "point" is not directly over the center of the base?

(c) (A SQUARE-BASED PYRAMID) Determine a formula for the volume of a square-based pyramid with height h and with a base of side-length ℓ.

5. (SPIVAK) Imagine a solid that has circular base with diameter $\overline{AB}$ with length ℓ such that each plane that is perpendicular to $\overline{AB}$ intersects the solid in a square. Express the volume of this solid as an integral and then evaluate the integral.

6. (CHALLENGE) You have a bowl full of water, the shape of which is exactly half of a sphere of radius r. You tilt the bowl thirty degrees, spilling out some of the water. What is the volume of the remaining water?

7. From Chapter 8.3 of *Calculus with Applications, 11th Edition* (page 489) work through the exercises:

   15 17 19

8. (RECREATIONAL) Fifty natural numbers are written in such a way so that sum of any four consecutive numbers is 53. The first number is 3, the 19th number is eight times the 13th number, and

the 28th number is five times the 37th number. What is the 44th number?