

A PROOF OF THE CANTOR THEOREM

Theorem. *There exists no bijection from any set to its power set.*

Proof. Suppose $f : X \rightarrow \mathbf{2}^X$ is such a bijection. By the axiom schema of separation, there exists $Y = \{x \in X : x \notin f(x)\}$. $Y \subseteq X$, so $Y \in \mathbf{2}^X$. Thus, there exists a unique x' such that $Y = f(x')$. But $x' \in Y$ if and only if $x' \notin (Y = f(x'))$, which is a contradiction. \square