A PROOF OF THE CANTOR THEOREM

Remark. Assume Zermelo–Frænkel set theory.

Theorem. *For any set, there exists no bijection from the set to its power set.*

Proof. Suppose $f : X \to \mathbf{2}^X$ is such a bijection. By the axiom schema of separation, there exists $Y = \{x \in X : x \notin f(x)\}$. $Y \subseteq X$, so $Y \in \mathbf{2}^X$. Thus, there exists a unique x' such that $Y = f(x')$. But $x' \in Y$ if and only if $x' \notin (Y = f(x'))$, which is a contradiction. $\square$