

## A PROOF OF THE CANTOR THEOREM

*Remark.* Assume Zermelo–Fränkel set theory.

**Theorem.** *There exists no bijection from any set to its power set.*

*Proof.* Suppose  $f : X \rightarrow \mathbf{2}^X$  is such a bijection. By the axiom schema of separation, there exists  $Y = \{x \in X : x \notin f(x)\}$ .  $Y \subseteq X$ , so  $Y \in \mathbf{2}^X$ . Thus, there exists a unique  $x'$  such that  $Y = f(x')$ . But  $x' \in Y$  if and only if  $x' \notin (Y = f(x'))$ , which is a contradiction.  $\square$