

## A PROOF OF THE CANTOR THEOREM

**Theorem.** *There exists no bijection from any set to its power set.*

*Proof.* Suppose  $f : X \rightarrow \mathbf{2}^X$  is such a bijection. Consider  $s = \{x \in X : x \notin f(x)\}$ . By construction,  $s \subseteq X$  and  $s \in \mathbf{2}^X$ . Since  $f$  is bijective, there exists a unique  $r \in X$  such that  $f(r) = s$ . If  $r \in s$ , then  $r \notin (f(r) = s)$ . On the other hand, if  $r \notin s$ , then  $r \in (f(r) = s)$ . Thus,  $r \in s$  only if  $r \notin s$ , which is a contradiction.  $\square$