UNIVERSAL INQUIRY

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Contents

1. Do we have “free will”?

The fundamental laws in physics work with the use of differential equations. In the context of a physical system, such an equation allows one to calculate a final state of the system from some initial state. For example, given the properties and motions of a system of particles at some time λ, these equations make it possible—in principle—to calculate what happens to those particles at any time before or after λ. As one-on-one “maps” between states of a system, these equations are deterministic and reversible, as full information for any one state theoretically implies full information for any other state. On top of these deterministic equations, quantum mechanics uses probability to describe how random quantum jumps occur whenever measurements are made. That these quantum events are fundamentally random implies that they are not influenced by anything.

On the whole, the physics tells us that the universe's change in time is deterministic with the occasional random quantum event. The deterministic part is fixed by the past, and the random part is not influenced by anything—and therefore, anyone. From this perspective, life does not seem very analogous to walking through a “garden of forking paths”, where each path corresponds to a possible future, and it is “purely up to us” which path “becomes reality”. The laws of nature simply do not appear to work that way. For the most part, the physics suggests only one path, as quantum effects rarely manifest themselves macroscopically. What you do today would “essentially” follow from the state of the universe yesterday, which would follow from the state of the universe last Thursday, and so on, all the way back to the Big Bang. Random quantum events could, occasionally, make big differences in our lives. For instance, a driver may swerve to avoid a dead pixel on their screen caused by a random quantum fluctuation. The paths do “fork” every once in a while, but we simply have no say in this.

My conclusion is that whether free will exists depends on one's definition of free will: the physics just says that future events are fixed, with the occasional quantum event outside of anyone's control. This conclusion appears fairly uncontroversial among physicists today. Does this feel unsettling? One may think of life as a story (yet) to be told. Even if our thoughts and actions are not ultimately within our control, I am curious to see what will come out of them, and perhaps you should be too.

2. On the use of mathematics for physics

A theory in physics usually arises from a broad mixture of ideas, and mathematics is often used to "tidy up" the initially messy developmental work (e.g. to find a "neat" set of assumptions from which the whole physical theory may be derived). Apart from being, perhaps, the most economical language we know of, mathematics is highly unambiguous "by design". Compared to most other human languages, the meanings assigned to representations (e.g. symbols) are left much less room for interpretation. This keeps physics honest: a person can be mistaken, but cannot lie about the mathematics of a physical theory, so long as all of its mathematics is spelled out. But perhaps most crucially, mathematics is used in physics because it *can.* Natural phenomena may be modelled mathematically because, as far as we can tell, nature "follows patterns". Why nature appears to do so may be considered a separate question.

3. Is mathematics discovered or invented?

Mathematics can be discovered in a sense similar to how fire can be discovered, if one defines discovery as "observation". Mathematics can also be invented in a sense similar to how chairs can be invented, if one defines invention as "man-made creation". In this way, mathematics can simultaneously be discovered and invented. Indeed, mathematics can be viewed as a linguistic exercise, in which one can both pose new definitions, and make connections between existing ones.

The original question, however, seems to somehow suggest a strict dichotomy between discovery and invention. I believe most people consider mathematics at least somewhat artificial, so to me, the question seems to ask if mathematics is *entirely* artificial. If one considers "mathematics" to be the collection of things one calls "mathematical", then the original question may be rephrased to ask whether anything we call "mathematical" is (in any "part") natural. If to be "mathematical" is to be "mathematically describable", then the question may be rephrased to ask if (any "part" of) the universe is mathematically describable. As far as we can tell, this is an open question. Based on the evidence we have collected so far, the universe *seems* to work in a way that aligns with mathematical description.[1] However, in the same way that no inexhaustive collection of examples can serve as proof of a mathematical statement, no amount of physical evidence can guarantee the mathematical describability of the universe.

4. On conventions in mathematics

Mathematics, like other human languages, is written with conventions typically not made explicit to the reader. These conventions can help to communicate useful information on top of the logical content of their mathematical sentences. Consider, for instance, conventions for naming variables. In principle, any symbol may be used to denote a variable. But in practice, we choose to assign certain symbols

[1] This may remind one of the anthropic principle.

to certain variables because pre-existing connotations make such assignments feel more natural. For instance, in the context of number theory, a real number is usually denoted by symbols like x or y, whereas a natural number is usually denoted by symbols like k or n. Interchanging these notational assignments in a mathematical argument could read somewhat strangely. Similarly, in the context of analysis, the symbol ε is typically used to denote a small, positive quantity. Using ε to refer to a large or negative quantity could lead to confusion. There are also notational customs for relations. In most situations, an inequality is typically written down with the left-hand side containing "newer", "less resolved", or "less well-known" objects than those on the right-hand side. For example, writing "$x < 1$" is typically preferred to writing "$1 > x$", even though these inequalities are logically equivalent. Adhering to convention can also help make an argument easier to remember. Someone who has trouble recalling the objects in their argument may be able to guess what they are, simply by referring to their denotations.