WE CONSIDER THE EDGE RECONNECTING MODEL WITH $\boxed{\varkappa = 1}$, STARTED FROM THE COMPLETE GRAPH K_n ON n VERTICES. THUS THE EDGE DENSITY $\boxed{\varrho = 1}$

DENOTE BY $X_n(T)$ THE ADJACENCY MATRIX OF THE GRAPH AFTER T STEPS.

DENOTE BY $\boxed{v^n(T) = \frac{1}{\binom{n}{2}} \sum_{i=1}^{n} \sum_{j=1}^{i-1} \mathbb{1}\left[X_n(T) = 0 \right]}$

IN WORDS: $v^n(T)$ IS THE THE DENSITY OF UNORDERED PAIRS OF VERTICES WITH NO EDGE BETWEEN THEM.

WE HAVE $v^n(0) = 0$, BECAUSE IN THE COMPLETE GRAPH THERE IS NO PAIR OF VERTICES WITH NO EDGE BETWEEN THEM.

<u>CLAIM</u>: IF n IS LARGE AND $\boxed{T = \lfloor t \cdot \frac{n^2}{2} \rfloor}$

$$v^n\left(\left\lfloor t \cdot \frac{n^2}{2} \right\rfloor\right) \approx (1 - e^{-t}) \cdot e^{-(1-e^{-t})}$$

$$v^n(\lfloor 5 \cdot n^2 \rfloor) \approx (1 - e^{-2.5}) \cdot e^{-(1-e^{-2.5})}$$

THUS FOR $\boxed{s=3}$ WE ALREADY HAVE

$$v^m(3 \cdot m^2) \approx (1-e^{-6}) \cdot e^{-(1-e^{-6})} \approx \frac{1}{e}$$

SIMILARLY FOR $\boxed{s=4}$, $\boxed{s=5}$ ...

PROOF OF CLAIM: BY THEOREM 1,

$$v^m\left(\left\lfloor t \cdot \frac{m^2}{2} \right\rfloor\right) = v^m\left(\left\lfloor t \cdot \frac{\varsigma(W_0) \cdot m^2}{2} \right\rfloor\right) \approx$$

$$\approx \int_0^1 \int_0^1 W_t(x,y,0)\, dx\, dy \overset{(32)}{=}$$

$$= \int_0^1 \int_0^1 \sum_{h=0}^{\infty} W_0(x,y,h) \cdot q\left(t, h, 0, \frac{D(W_0,x) \cdot D(W_0,y)}{\varsigma(W_0)}\right) = ☆$$

$\boxed{W_0(x,y,h) = \mathbb{1}[h=1]}$ ← MULTIGRAPH LIMIT OF COMPLETE GRAPHS

THUS $\varsigma(W_0) = 1$, $D(W_0,x) \overset{(12)}{=} 1$

$$☆ = \int_0^1 \int_0^1 q(t,1,0,1)\, dx\, dy = q(t,1,0,1) \overset{(24)}{=}$$

$$\sum_{\ell=0}^{0} b(\ell, 1, e^{-t}) \cdot p(0-\ell, (1-e^{-t})) = b(0,1,e^{-t}) \cdot p(0, 1-e^{-t})$$

$$\overset{(18),(19)}{=} \binom{1}{0} \cdot 1 \cdot (1-e^{-t})^1 \cdot e^{-(1-e^{-t})} = (1-e^{-t}) \cdot \exp(-(1-e^{-t}))$$

BUT $\frac{1}{e}$ IS NOT THE STATIONARY VALUE OF $\nu^m(T)$, $T \to \infty$!

If $\varsigma = 1$ AND $\varkappa = 1$, THEN $\hat{w}_\infty(x,y,\varkappa)$ IS DEFINED BY (4.1), THEREFORE

$$\nu^m(\infty) \approx \int_0^1 \int_0^1 p\left(0, F^{-1}(x) \cdot F^{-1}(y)\right) dx\, dy \text{ , WHERE}$$

$$F(x) = \int_0^x g(y,1,1)\, dy = 1 - e^{-x} \text{. THAT IS}$$

$\nu^m(\infty) = \mathbb{E}\left(\exp(-\mathcal{X} \cdot \mathcal{Y})\right)$, WHERE $\mathcal{X}$ AND $\mathcal{Y}$ ARE INDEPENDENT RANDOM VARIABLES WITH $EXP(1)$ DISTRIBUTION.

$$\nu^m(\infty) = \int_0^\infty \int_0^\infty \exp(-x \cdot y) \cdot e^{-x} \cdot e^{-y}\, dx\, dy =$$

$$= \int_0^\infty \frac{1}{1+x} \cdot e^{-x}\, dx \approx 0.5963\ldots$$

NOTE THAT $0.5963 \neq 1/e = 0.3678\ldots$

$$\boxed{\text{THUS } 1/e \text{ APPEARS TO BE THE STATIONARY VALUE OF } \nu^m(T) \text{ ON THE } m^2 \text{ TIMESCALE, BUT IT IS NOT!}}$$

ABOUT YOUR FIGURE 3:
EVOLUTION OF DEGREES ON THE m^2 TIMESCALE.

WE FOLLOW THE NOTATION OF (38), (39):

$$\mathcal{D}_m(T) = \frac{1}{m} \cdot d(\mathcal{X}_m(T), i)$$

NOW $\boxed{\mathcal{D}_m(0) = 1}$ $\boxed{m = \binom{n}{2}}$ $\boxed{\varsigma = 1}$ $\boxed{\mathcal{K} = 1}$

THUS

$$\mathbb{E}\left(\mathcal{D}_m(T+1) - \mathcal{D}_m(T) \,\middle|\, \mathcal{X}_m(T)\right) \approx \frac{1}{m^3} \cdot \left(1 - \mathcal{D}_m(T)\right) \quad (38)$$

$$\mathrm{Var}\left(\mathcal{D}_m(T+1) - \mathcal{D}_m(T) \,\middle|\, \mathcal{X}_m(T)\right) \approx \frac{1}{m^3} \cdot 2 \cdot \mathcal{D}_m(T) \quad (39)$$

HEURISTICS: IF $T \asymp m^2$, THEN $\mathcal{D}_m(T)$ IS NOT TOO DIFFERENT FROM $\mathcal{D}_m(0) = 1$, SO LET'S SAY

$$\mathbb{E}\left(\mathcal{D}_m(T+1) - \mathcal{D}_m(T)\right) \approx 0$$

$$\mathrm{Var}\left(\mathcal{D}_m(T+1) - \mathcal{D}_m(T)\right) \approx \frac{2}{m^3}$$

THUS IF $\boxed{T = s \cdot m^2}$

THEN

$$\mathbb{E}(\mathcal{D}_m(T)) \approx 1$$

$$\mathrm{Var}(\mathcal{D}_m(T)) \approx T \cdot \frac{2}{m^3} = \frac{2 \cdot s}{m}$$

"INDEPENDENT" INCREMENTS $\Rightarrow$

$\Rightarrow$ IF $\boxed{T = s \cdot m^2}$, THEN

$\mathcal{D}_m(s \cdot m^2)$ IS NORMALLY DISTRIBUTED: $\mathcal{N}\left(\mu = 1, \sigma = \sqrt{\frac{2s}{m}}\right)$

THUS THE EMPIRICAL DEGREE DISTRIBUTION AT TIME $\boxed{T = s \cdot m^2}$ IS NORMAL WITH

$\boxed{\text{MEAN} \approx m}$ $\boxed{\text{STANDARD DEVIATION} \approx \sqrt{2 \cdot s \cdot m}}$