Preferential Attachment Coarse Projective Integration

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09/01/2013

Introduction

The edge reconnecting model, detailed in [**?**], was implemented in C++. The system naturally converges to a limiting "graphon" (see [**?**]), and in order to increase the rate of convergence, coarse projective integration (CPI) was implemented. This brief outlines the model dynamics and reviews current simulation progress.

Edge Reconnecting Model

The initial state of the edge reconnecting model is an Erdős-Rényi random graph with n vertices and $m \sim O(n^2)$ edges. The system evolves as a discrete-time Markov chain, in which the followinnng actions are performed during each step:

1. An edge e_{old} is chosen uniformly at random from the set of all possible edges, $E(G)$.
2. A vertex end of e_{old}, v_1, is chosen uniformly from the two ends.
3. e_{old} is removed from the graph, and a vertex v_2 is chosen from $V(G)$ with a probability determined based on linear preferential attachment:

$$P(v_2 = v_i) = \frac{deg(v_i) + \kappa}{2m + n\kappa}$$

   where κ is a model parameter and $deg(v_i)$ denotes the degree of vertex i in V.

4. An edge e_{new} is added between v_1 is connected to v_2.

Two distinct timescales arise from these dynamics, $T \asymp n^2$ and $T \asymp n^3$ where T is the number of steps. On the faster, $O(n^2)$ scale, the degrees of the vertices may be considered constant, while the number of parallel edges between vertices changes. On the slower $O(n^3)$ scale, the degree distribution evolves to a steady state value. While this separation of timescales has been proven on in [**?**], identifying them through numerical simulations is complicated by a couple of factors. First, the exact timescales themselves are difficult to discern. Both scales are really $O(\rho_1 n^2)$ and $O(\rho_2 n^3)$, where the ρ_i constants are evaluated at the limit of infinite-sized graphs ($n \to \infty$). This hints at the second, larger, problem: many of the results on the existence of these timescales in the first place are only valid in this large-n limit. Simulation time then becomes problematic. **??** and **??** illustrate attempts to visualize these separate scales of evolution.