# Control Systems

## Assignment 1

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# Summary

# Question

Given the rotational system shown in Figure P2.24, find the transfer function:

$$G(s) = \frac{\theta_6(s)}{\theta_1(s)}$$

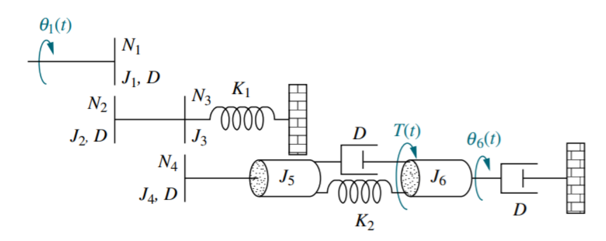


FIGURE P2.24

Figure: Diagram of the Problem Statement

# Understanding the Question

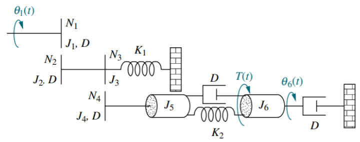


FIGURE P2.24

The above system involves gears, springs, viscous dampers and inertia.

| | | |
|---|---|---|
| $N_1$, $N_2$, $N_3$ and $N_4$ | $\implies$ | Number of teeth on the gears |
| $J_1$, $J_2$, $J_3$, $J_4$, $J_5$ and $J_5$ | $\implies$ | Inertia |
| $D$ | $\implies$ | Viscous damper |
| $K_1$, and $K_2$ | $\implies$ | Spring constants |

Angular velocity $\theta_1(t)$ is applied on the first gear, torque $T(t)$ is observed at $J_6$ and angular velocity $\theta_6(t)$ is observed in the end.

# How to Solve

Following steps must be followed to solve this problem:

1. Derive a compact circuit by generating equivalent impedance from the left to $J_5$.
2. Determine the equations of motion for the circuit obtained from Step 1.
3. Substitute the equivalent impedences in the equations obtained from Step 2.
4. Compare the equations and generate the transfer function $\frac{\theta_6(s)}{\theta_1(s)}$.

# Compact Circuit

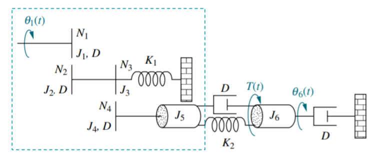


Figure: We will derive a compact circuit for the dotted part of the system.

Rotational mechanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio

$$\frac{\textit{Number of teeth of gear on destination shaft}}{\textit{Number of teeth of gear on source shaft}} \tag{1}$$

# Compact Circuit

Therefore,

$$J_{eq} = J_1\left(\frac{N_1N_3}{N_2N_4}\right)^2 + (J_2 + J_3)\left(\frac{N_4}{N_3}\right)^2 + (J_4 + J_5) \tag{2}$$

$$K_{eq} = K_1\left(\frac{N_4}{N_3}\right)^2 \tag{3}$$

$$D_{eq} = D\left[\left(\frac{N_1N_3}{N_2N_4}\right)^2 + \left(\frac{N_4}{N_3}\right)^2 + 1\right] \tag{4}$$

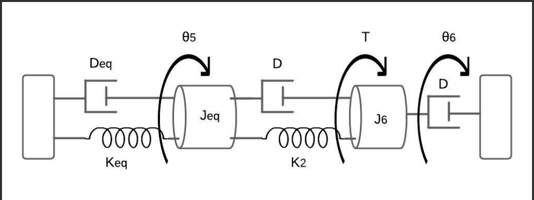


Figure: This is the compact circuit for the system.

# Equations of Motion

The Laplace transform corresponding to each impedance is as given in the table.

| Component | Impedence |
|---|---|
| Spring | $K$ |
| Viscous Damper | $Ds$ |
| Inertia | $Js^2$ |

Equations of motion are given by:

$$(\Sigma_5 \times \theta_5) - (\Sigma_{56} \times \theta_6) = T_5 \tag{5}$$

$$-(\Sigma_{56} \times \theta_5) + (\Sigma_6 \times \theta_6) = T_6 \tag{6}$$

Where, $\Sigma_5 =$ Sum of impedances connected to the motion at $\theta_5$
$\Sigma_{56} =$ Sum of impedances connected between $\theta_5$ and $\theta_6$
$\Sigma_6 =$ Sum of impedances connected to the motion at $\theta_6$
$T_5 =$ Sum of applied torques at $\theta_5$
$T_6 =$ Sum of applied torques at $\theta_6$

# Equations of Motion

Terms involved in equations of motion:

$$\Sigma_5 = J_{eq}s^2 + (D_{eq} + D)s + (K_2 + K_{eq}) \tag{7}$$

$$\Sigma_{56} = Ds + K_2 \tag{8}$$

$$\Sigma_6 = J_6 s^2 + 2Ds + K_2 \tag{9}$$

$$T_5 = 0 \tag{10}$$

$$T_6 = T(s) \tag{11}$$

Substituting above equations in (5) and (6), we get:

$$[(J_{eq}s^2 + (D_{eq} + D)s + (K_2 + K_{eq})) \times \theta_5] - [(Ds + K_2) \times \theta_6] = 0 \tag{12}$$

$$-[(Ds + K_2) \times \theta_5] + [(J_6 s^2 + 2Ds + K_2) \times \theta_6] = T(s) \tag{13}$$

# Solution on Substituting

From previous equations,

$$\frac{\theta_6(s)}{\theta_5(s)} = \frac{J_{eq}s^2 + (D_{eq} + D)s + (K_2 + K_{eq})}{Ds + K_2} \tag{14}$$

$$\frac{\theta_5(s)}{\theta_1(s)} = \frac{N_1 N_3}{N_2 N_4} \tag{15}$$

Therefore,

$$\boxed{\frac{\theta_6(s)}{\theta_1(s)} = \frac{N_1 N_3}{N_2 N_4}\left(\frac{J_{eq}s^2 + (D_{eq} + D)s + (K_2 + K_{eq})}{Ds + K_2}\right)} \tag{16}$$

Where $J_{eq}$, $D_{eq}$ and $K_{eq}$ are obtained from (2), (3) and (4) respectively.

# The End