Topic 1. Intro to Probability Theory. Random events. Probability Space.

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1 In-class tasks

1. Find unions and intersections of the following events. In which case one event is a subset of the other?

   (a) $A = \{1, 2, 5, 6\}$, $B = \{1, 5\}$

   (b) $A = \{$Ann, Mary, Mike$\}$, $B = \{$Tom, Mike, John$\}$

   (c) $A = \{$Moscow, London, Paris$\}$, $B = \{$Paris, Berlin, Tokyo$\}$, $C = \{$Tokyo, Rome$\}$.

2. A survey of the houses in an old residential area found 30% with holes in the roof, 40% with broken windows, and 25% with the both problems.

   - What is the proportion of houses with one or the other (or both) problems?
   - What is the proportion of houses with exactly one of these problems?
   - What is the proportion of houses with none of these problems?

3. A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.

4. A fair six-sided dice has 3 faces that are painted blue (B), 2 faces that are red (R) and 1 face that is green (G). We toss the dice twice. List the complete sample space of all possible outcomes.

   (a) if we are interested in the color facing upward on each of the two tosses.

   (b) if the outcome of interest is the number of red we observe on the two tosses.

5. Consider the experiment of rolling a pair of dice. Suppose that we are interested in the sum of the face values on the dice.

   - How many sample points are there in this random experiment?
   - Describe the following random events using sample points described before:
     A="We obtained even value",
     B="We obtained a value that is less than 7",
     C="We obtained a value of 7",
   - Which of the events are mutually exclusive?

6. Consider the experiment of rolling a pair of 4-sided dices. Describe the complete sample space for two scenarios:

   (a) if we believe that dices are completely identical same, and we do not take any order into account,

   (b) if dices are visually different, or we somehow distinguish between them.

7. Basketball player throw a ball 3 times. Suppose we observe if the player succeeds. Let us use the designation $x_i = $ 'He succeeded when did a throw number i, and do the following tasks:

   - Describe sample space for this experiment and say how many sample points are there.
   - Think of four random events that could happen in this experiment and describe them using sample points.