$X_1 \ldots X_n$

IID $\quad X \sim F(\mu, \sigma^2)$

$X \sim F(x, \theta)$

$N(\mu, \sigma^2)$

$Bin(n, p)$

$Y = T(X_1 \ldots X_n)$ - Statistic

$T^{(1)} = \sum_{i=1}^{5} X_i^{(1)} \quad X_1^{(1)} \ldots X_5^{(1)}$

$T^{(2)} = \sum X_i^{(2)} \quad X_1^{(2)} \ldots X_5^{(2)}$

$T, F_T, E[T]$

$Var[T]$

F_T - Sampling Distribution

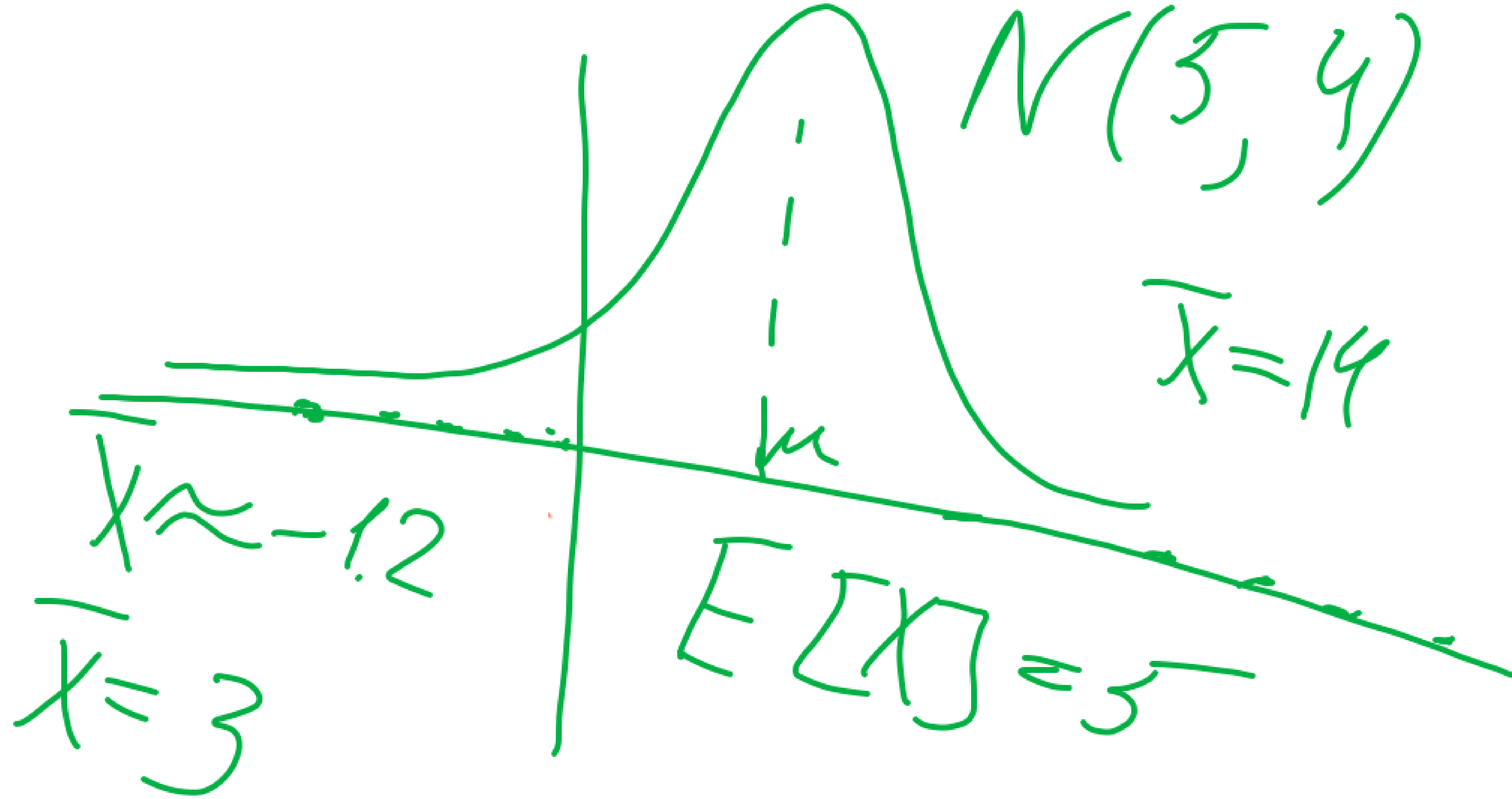
$$\bar{X} = \frac{\sum X_i}{n}$$

$$E[\bar{X}] = \mu$$

$$Var[\bar{X}] = \frac{\sigma^2}{n}$$

$$S = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$E[S] = \sigma^2$$

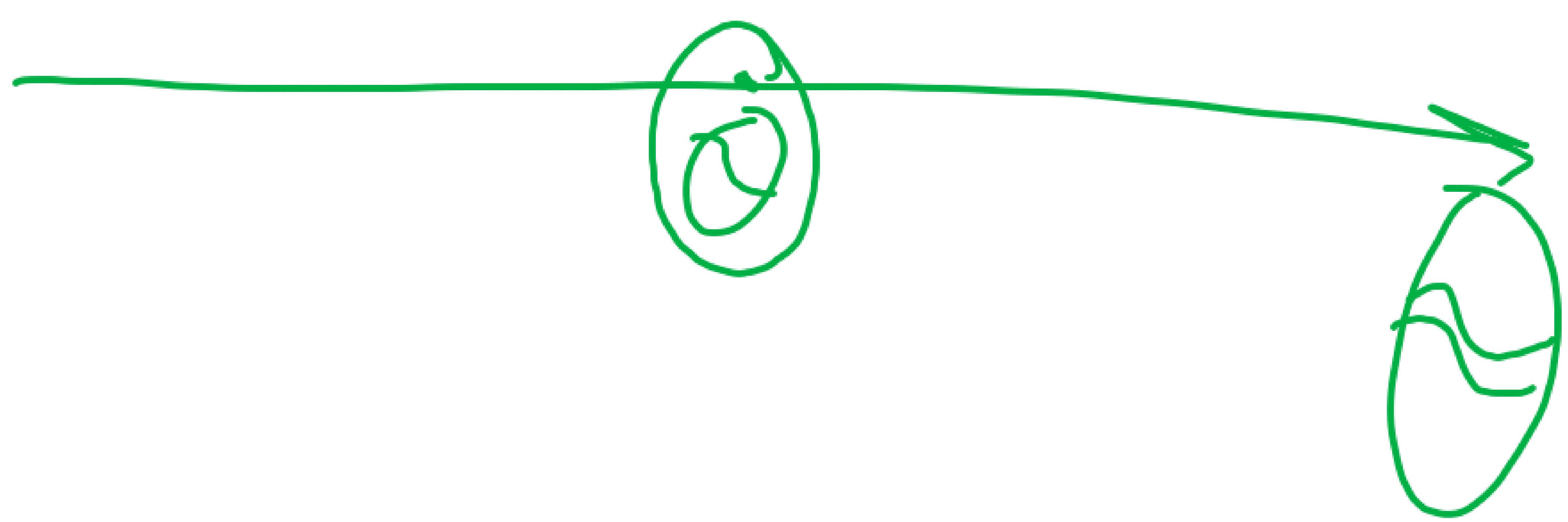


$X_1 \_\_\_ X_n$ $\quad X \sim f(x; \theta)$

I.I.D.

$\hat{\theta} = T(X_1 \ldots X_n)$; $\hat{\theta} \to \theta$

$\hat{\theta}$ _ point estimator



$$\hat{\Theta} = T(X_1 \ldots X_n)$$

$E[\hat{\Theta}]$

$Var[\hat{\Theta}]$

$$Bias_{\hat{\Theta}} = E[\hat{\Theta}] - \Theta$$

$Bias = 0 \rightarrow \hat{\Theta}$ - unbiased.

$$MSE(\hat{\theta}) = E\left((\hat{\theta} - \theta)^2\right) = Var(\hat{\theta}) + (Bias_{\hat{\theta}})^2$$

$X=0$	$X=1$
$1/2$	$1/2$

$E[X]=\mu=0\cdot 1/2+1\cdot 1/2=1/2$

$Var(X)=1\cdot 1/2-1/4=1/4$

$X_1 \perp \dots \perp X_9$ I.I.D

(1) $\mu_1=0.45$; $E[\hat{\mu}_1]=\hat{\mu}_1\cdot P(\mu=\hat{\mu}_1)=\hat{\mu}_1$; $Bias=\hat{\mu}_1-\mu=-0.05$

(2) $\hat{\mu}_2=T(X_1\dots X_9)=X_1$, $E[\hat{\mu}_2]=E[X_1]=\mu=0.5 \to Bias=0$

$E[X_1]=\dots=E[X_9]=\mu$

$Var(X_1)\dots=Var(X_9)=\sigma^2$

$$\hat{\mu}_3 = \overline{X} \qquad E[c \cdot X] = c \cdot E[X]$$

$$E[\hat{\mu}_3] = E[\overline{X}] = E\left[\frac{X_1 + \dots + X_9}{9}\right] = \frac{1}{9} \cdot \sum_{i=1}^{9} E[X_i] =$$

$$= \frac{1}{9} \cdot 9 \cdot \mu = \mu = 0.5$$

$$\text{Bias} = 0$$

$$E\left[\frac{\sum_{1}^{n} X_i}{n}\right] = \frac{1}{n} \cdot n \cdot \mu = \mu$$

$$\hat{\mu}_4 = X_1 + \frac{1}{3} X_2$$

$$E[\hat{\mu}_4] = \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} = \frac{4}{6}, \quad \text{Bias} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

$$\hat{\mu}_5 = \frac{2}{3}X_1 + \frac{2}{3}X_2 - \frac{1}{3}X_3$$

$$E[\hat{\mu}_5] = \frac{2}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{1}{2} - \frac{1}{3}\cdot\frac{1}{2} = \frac{3}{6}; \quad Bias = 0.$$

$$Var[\hat{\mu}_5] = \frac{4}{9}var(X_1) + \frac{4}{9}var(X_2) - \frac{1}{9}var(X_3) = \frac{7}{9}var(X) = \frac{7}{9\cdot 4}$$

$$Var(c\cdot X) = c^2 Var(X) \qquad MSE = \frac{7}{36}$$

$$Var(\bar{X}) = Var\left(\frac{\sum X_i}{n}\right) = \frac{1}{n^2}\cdot\sum_{i=1}^{n} Var(X_i) = \frac{1}{n^2}\cdot n\cdot\sigma^2 = \frac{\sigma^2}{n}$$

$$Var(\bar{X}) = \frac{1}{4 \cdot 9} = \frac{1}{36} \rightarrow MSE(\bar{X}) = \frac{1}{36}$$

σ^2 n

$$MSE(\bar{X}) < MSE(\hat{\mu}_5)$$

$$1/36 < \frac{7}{36}$$