

Mathematical Statistics. Home Assignment 1.

Point estimation and confidence intervals.

MDI, Fall 2022.

1. The scores on a verbal reasoning test are normally distributed with a population mean of $\mu = 100$ and a population standard deviation of $\sigma = 10$.
 - (a) What is the probability that a randomly chosen person scores at least 105?
 - (b) A simple random sample of size $n = 20$ is selected. What is the probability that the sample mean will be between 97 and 104? (You may use the nearest values provided in the statistical tables.)
2. Let p be equal to the proportion of New York City residents who feel that the quality of life in New York City has become worse in the past few years. How large a sample is required to be 98% confident that the maximum error of the estimate of p is 0.025?
3. To determine the average age of its customers, a large manufacturer of clothing took a random sample of 50 customers and found $\bar{x} = 36$. If you know, that population standard deviation $\sigma = 12$:
 - (a) Find a 95% confidence interval for the mean age μ of all customers,
 - (b) Suppose you need to make the 95% confidence interval narrower, say ± 2 years. How large a sample is now required?
4. A random sample of 200 students is observed. 30 of them say they are 'really enjoying' Statistics.
 - (a) Calculate the proportion of students in this sample saying they are 'really enjoying' Statistics and then provide a 95% confidence interval for the population proportion.
 - (b) You now take a further random sample, in another institution. This time there are 20 students and 8 say they are 'really enjoying' Statistics. Provide a 95% confidence interval for this value. Think about why the two confidence intervals are different.
5. A car rental company is interested in the amount of time its vehicles are out of operation for repair work. A random sample of nine cars showed that over the past year, the numbers of day, each had been inoperative were

16 10 21 22 8 17 19 14 19

- (a) Find the sample mean and the variance,

- (b) Stating any assumptions you need to make, find a 90% confidence interval for the mean number of days in a year that vehicles in the company's fleet are out of the operation.
6. There is a concern about the speed of automobiles travelling over a particular part of the highway. For a random sample of 7 automobiles, radar indicated the following speeds, in miles per hour:

79 73 68 77 86 71 69

- (a) Find the sample mean and the variance,
- (b) Assuming a normal population distribution, find a 95% confidence interval for the mean speed of all automobiles travelling over this part of the highway.
7. Suppose, that for a random sample of size n , a statistic $W(X_1, \dots, X_n)$ is unbiased estimator for the population variance σ^2 , and that W itself has a variance $\text{Var}(W) = \frac{2\sigma^4}{(n-1)}$. Find for what value of the constant c the estimator of σ^2 of the form $T = cW$ has minimum mean squared error.

Bonus

Two measurements of the side of the square were produced. Suppose the two measurements X_1 and X_2 are independent random variables with mean σ and variance σ^2 . The true length of the side of the square is a . Find mean squared error for the following estimator of the area of the square: $T = X_1X_2$.