Topic 3. Counting Principles: combinations. Classical Probability. Conditional Probability.

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1 Counting Principles: Combinations

1. An bag contains 15 marbles of which 10 are red and 5 are white. 4 marbles are selected from the bag.

   (a) How many (different) samples (of size 4) are possible?
   (b) How many samples (of size 4) consist entirely of red marbles?
   (c) How many samples have 2 red and 2 white marbles?
   (d) How many samples (of size 4) have exactly 3 red marbles?
   (e) How many samples (of size 4) have at least 3 red?
   (f) How many samples (of size 4) contain at least one red marble?

2. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if:

   (a) there are no other restrictions?
   (b) 2 of the men refuse to serve together?
   (c) 2 of the women refuse to serve together?
   (d) 1 man and 1 woman refuse to serve together?

2 Conditional Probability

1. What is the probability that a family of two children has

   (a) two boys given that it has at least one boy?
   (b) two boys given that the first child is a boy?

2. We toss a fair coin three successive times. Find the conditional probability $P(A|B)$ when A and B are the events: $A = \{$more heads than tails come up$\}$, $B = \{$1st toss is a head$\}$.

3. A conservative design team, call it C, and an innovative design team, call it N, are asked to separately design a new product within a month. From past experience we know that:

   - The probability that team C is successful is 2/3
   - The probability that team N is successful is 1/2
   - The probability that at least one team is successful is 3/4

   Assuming that exactly one successful design is produced, what is the probability that it was designed by N?

4. Radar Detection. If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present. the radar generates a (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.05. What is the probability of no aircraft presence and a false alarm? What is the probability of aircraft presence and no detection?

5. A fair six-sided die is thrown twice. Let B be the event that the first number thrown is no larger than 3, and let C be the event that the sum of the two numbers thrown equals 6. Find the probabilities of B and C, and the conditional probabilities of C given B, and of B given C.