Introduction to solar motion geometry on the basis of a simple model

2010 Phys. Educ. 45 641

(http://iopscience.iop.org/0031-9120/45/6/010)

Introduction to solar motion geometry on the basis of a simple model

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Abstract
By means of a simple mathematical model developed by the authors, the apparent movement of the Sun can be studied for arbitrary latitudes. Using this model, it is easy to gain insight into various phenomena, such as the passage of the seasons, dependences of position and time of sunrise or sunset on a specific day of year, day duration for different latitudes and seasons, and time dependence of the solar altitude reckoned from the horizontal plane. We present simulations of the Sun path alongside animated data. We show that the model adopted can also be used to explain the principle of a horizontal sundial and to determine shadow areas. We give recommendations as regards how to build energy-effective houses and how to optimize solar cell positions.

S Online supplementary data available from stacks.iop.org/physed/45/641/mmedia

Introduction

'Why do winter and summer (seasons) exist on Earth?' Following the Nobel prize winner Vitaly Ginzburg [1], one can ask a lot of people this relatively simple question and the majority of students or even highly educated people will not provide the correct answer [2]. Some of them argue that the Earth rotates around the Sun in an elliptical orbit and therefore the Earth is in the closest position to the Sun in summer, whereas in winter it is farther away. Only a small minority know that in fact the distance between the Earth and the Sun is minimal during winter in the northern hemisphere; this fact is unimportant in explaining the passage of the seasons on the Earth.

Probably, the reasons for this lack of understanding lie in the complicated explanations usually given in modern physics and astronomy textbooks which operate with three-dimensional objects and systems. The lack of clarity causes low insight into the relative movement of the simple system consisting of the Sun and the Earth. Here it is especially important to use the topocentric observation from the Earth's surface of the Sun path on the sky during different seasons rather than the three-dimensional side-view in space. Understanding the movement of the Sun and the reason of the passage of the seasons can be also helpful to activate the broad harnessing of solar energy, which sometimes can be done in a very simple manner, even in winter. The great potential of solar energy is substituting the exhausting fossil fuels and mitigating global warming [3–5]. Unfortunately, if one analyses the number of scientific publications including terms such as 'solar', 'photovoltaic', or 'climate*' it becomes evident that their recent growth is slower

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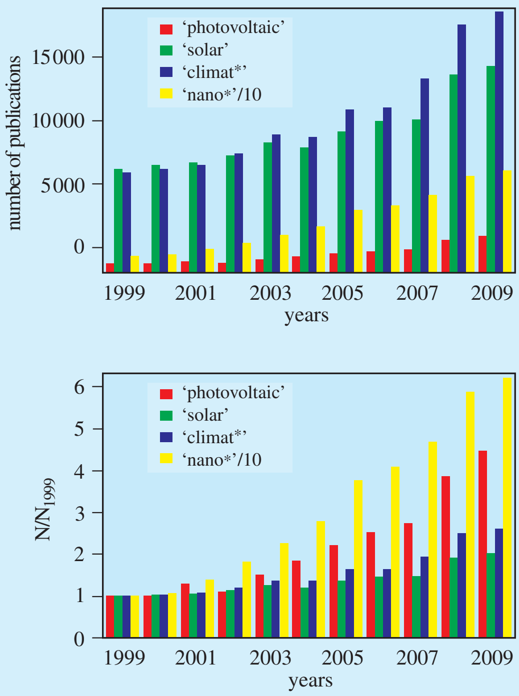

Figure 1. Histograms depicting the number of publications per year on various topics between 1999 and 2009 (data obtained from ISI Web of Knowledge, 9 February 2010). Top: absolute values; bottom: relative values with respect to the data for 1999. Note that the absolute number of publications including the key fragment 'nano*' was divided by 10 for convenience (top panel).

both in absolute and relative terms to 'hot' subject searches such as 'nano*' (figure 1). Probably, one of the reasons for such a situation is the scant public knowledge of the basic physical processes behind the apparent solar movement through the sky.

We believe that the use of renewable energy and, especially, solar energy is very important for the effective development of both small businesses and autonomous households consuming low amounts of conventional energy sources like electricity, gas, coal, etc. Our interest in this topic originates from our educational and civil activities in Ukraine which we started together a few years ago. Even after many years since graduating from secondary school we knew next to nothing about solar energy and solar movement geometry. After some studies it became clear to us that simple understandable models suitable for the broad public audience do not exist in the literature, while professional approaches are too complicated. Therefore, we developed our own simple model suitable for explaining the basic principles of the visible Sun path on the celestial sphere, the revolution of the seasons, shifts in the daily sunrise and sunset positions, etc.

Derivation of a simple parametrical model describing the visible movement of the Sun

Main assumptions

The passage of the seasons on the Earth stems from its revolution around the Sun and the tilt of the Earth's axis relative to the revolution plane. We assumed the period of such a revolution to be 365.25 days (the Julian year). The Earth revolves around the Sun in an elliptical orbit with low eccentricity ($\varepsilon = 0.0167$). The actual low eccentricity of the Earth's orbit as well as a sufficient external heating of our planet are related to the so-called anthropic principle, which describes the conditions of life abundance in the Universe [6, 7]. For the sake of simplicity we decided to neglect the ellipticity and suppose that the Earth rotates around the Sun with constant speed over an ideal circular orbit. The Earth's axial tilt (or obliquity) is the angle between its rotational axis, and a perpendicular to the orbital plane (angle α in figure 2(a)). The tilt and direction of the rotational axis with respect to the orbit change very slowly (over thousands of years) [8]. We used the value of 23.45° as the actual obliquity α.

The Earth's axial tilt is the main reason for the passage of the seasons (figure 2(a)). During the days close to the summer solstice the axis of the Earth is tilted towards the Sun and the maximal duration of a day is observed at any place in the northern hemisphere. Then the angle between the Sun's rays and the ground surface (height of the Sun, or elevation angle, or altitude) is at a maximum at midday for latitudes between the Northern tropic (the Tropic of Cancer) and the North Pole. After half a year the Earth appears at an opposite position corresponding to the winter solstice with a minimum sunlight period and a minimal altitude at midday. To find the maximum altitude, which can be observed from the Earth's surface at the extremal moments discussed above, let us consider a certain point A on the surface of an ideal spherical Earth at latitude φ. The horizontal surface for an observer at this point can be found as a tangent plane to the Earth's surface at point A. The projections of the indicated

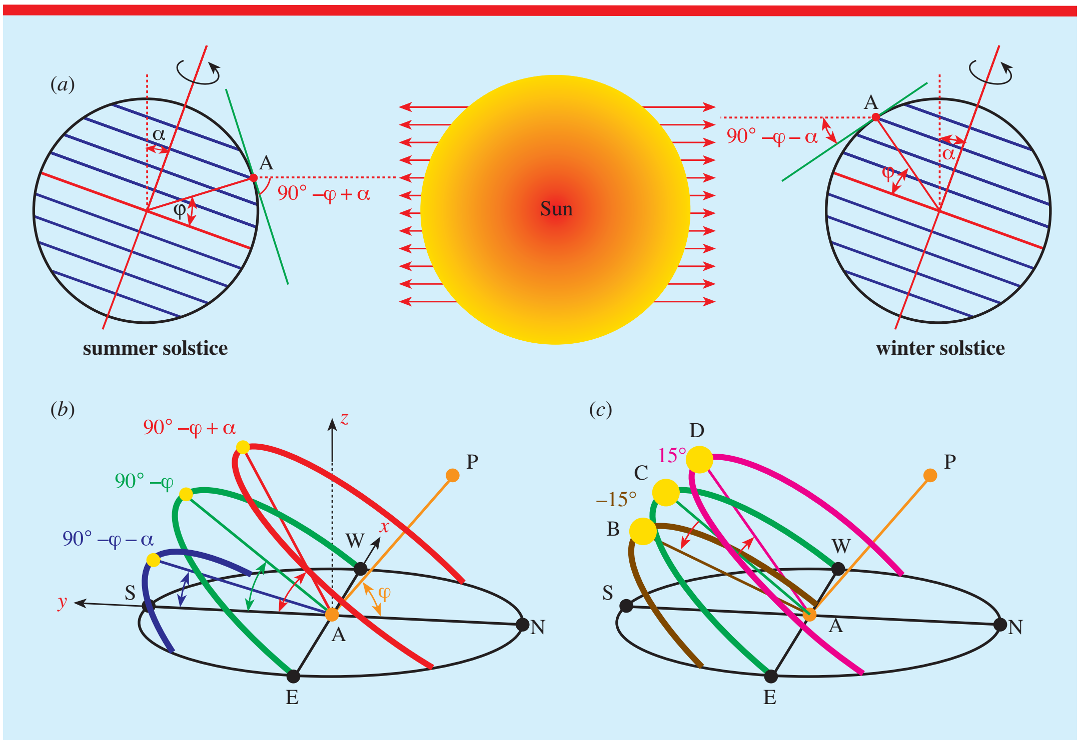

Figure 2. Scheme of the visible Sun path at solstices and equinoxes for an observer on the Earth's surface at the point A with latitude φ. (*a*) Determination of the maximal possible Sun altitudes, corresponding to summer and winter solstices. The projection of the horizontal plane at the point A is shown by the green lines; α = 23.45° is the Earth's axial tilt. (*b*) The visible movement of the Sun for a terrestrial observer and its maximal altitudes for the winter solstice (blue curve), the spring or autumn equinoxes (green curve) and the summer solstice (red curve). The polar axis AP is the rotation axis of the visible Sun's orbit at any time. The latitude φ equals the angle between this axis and the horizontal plane. (*c*) Diagram demonstrating declination angles θ of the Sun from the viewpoint of the observer at point A. The celestial equator plane is shown in green (see also (b)). Light magenta and dark yellow planes correspond to the declinations θ = 15° (∠CAD = 15°) and θ = –15° (∠CAB = 15°), respectively.

plane for summer and winter solstices are shown in figure 2(a) as green lines. It can be easily established from figure 2(a) that at the summer solstice time the maximum altitude at the point A is close to $(90° - \varphi + \alpha)$, whereas for the winter solstice this quantity is about $(90° - \varphi - \alpha)$. Small deviations from those idealized relations are due to the non-zero angular size of the Sun and the fact that the solstice occurs at a particular moment, which does not always correspond to noon for a given point. To avoid complications connected to the finite visible Sun diameter we shall consider the Sun in most cases as a point light source. Nevertheless, sometimes the finiteness of the Sun disc manifests itself conspicuously and will be discussed below. Within the same approach one can find that during the spring or autumn equinoxes the observer on the Earth surface can see the Sun at a maximum altitude of $(90° - \varphi)$. One can also easily conclude that for any latitude on the Earth the Sun rises from the East and sets in the West only during equinox times.

Figure 2(b) shows a scheme of the visible movement of the Sun on the celestial sphere observed from point A on the Earth surface in the Northern hemisphere during the winter and summer solstices, spring or autumn equinoxes. Due to the small visible diameter of the Sun, all circular arcs concerned lie in the parallel planes and the circle centres are located on the polar axis AP connecting the observer and the North celestial pole. For our purposes it is convenient to introduce a plane of the celestial equator, which is tilted relative to the horizontal plane at an angle of $(90° - \varphi)$ and intersects the latter by the straight line West–East. Note that the point A belongs to this line. Thus, for the terrestrial observer the Sun moves in the celestial equator plane during the days of the spring and autumn equinoxes. In the temporal range between the spring and autumn equinoxes the Sun is always observed above the celestial equator, otherwise it moves below the equator. To a good approximation all solar trajectories on the celestial sphere lie in planes parallel to the celestial equator.

To determine (for a certain date) the characteristic deviation between the corresponding plane and that of the celestial equator one should introduce a new parameter, declination θ, as the angle between the line AD (or AB) and the celestial equator plane (see figure 2(c)). The lines AD or AB indicate the direction between the observer and the highest position of the Sun on the sky for the date concerned. According to this definition, the summer solstice plane is characterized by the declination $\theta = \alpha$, the winter solstice plane is declined by the angle $(-\alpha)$, whereas the declination for the spring or autumn equinoxes is equal to 0. To ensure a smooth transition between the indicated points we use the following interpolation function describing the declination at any day of the year

$$\theta = \alpha \sin \frac{2\pi T}{365.25}, \qquad (1a)$$

where T is a date counted from the spring equinox: $T = 0$ for 20 March, $T = 1$ for 21 March etc. In the framework of the proposed model such a dependence shows that at any particular date the Sun rotates in a plane characterized by a constant declination during the whole day, although actually the visible movement of the Sun is more complex.

We extracted the Naval Oceanography Portal [9] data containing the annual time dependence of the solar declination in the period from 20 March 2010 till 20 March 2011 and compared those data with declination values derived according to equation (1a). The results are presented in figure 3(a). Comparison of the exact dependence with the interpolation curve shows that equation (1a) describes the observed behaviour quite well. The few-days difference between the observed solstices and the autumn equinox, on the one hand, and the model data, on the other hand, are mostly due to the Earth's orbit ellipticity.

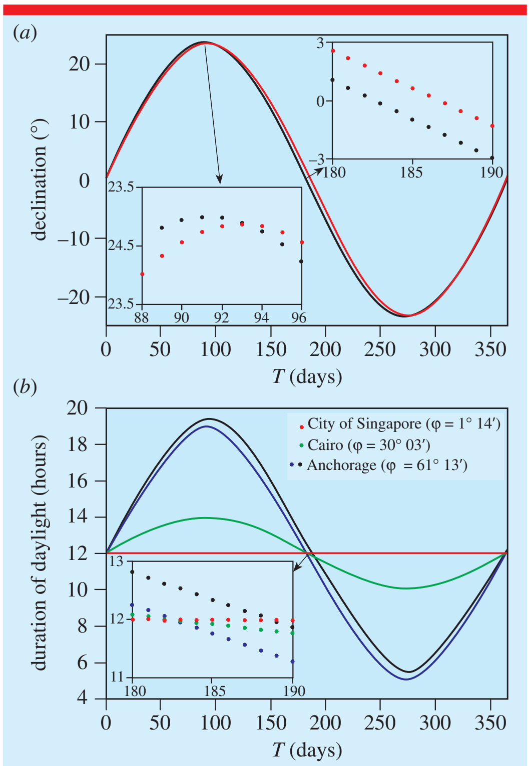

Figure 3. Model properties. (a) Comparison of the Sun declination obtained from the equation (1a) (black points) and data given by the Naval Oceanography Portal (red points) in the period between 20 March 2010 and 20 March 2011 [9]. A maximal difference of 1.726° is observed at $T = 202$ (8 October 2010); (b) Duration of daylight over the course of a year derived from the adopted model for different places on the Earth: Singapore (red curve), Cairo (green curve) and Anchorage (blue curve). The black curve shows the observed duration of daylight in Anchorage obtained from the Naval Oceanography Portal [9].

Description of the model

Hereafter, a three-dimensional Cartesian coordinate system with the origin at point A will be used (see figure 2(b)). The AX, AY, and AZ axes are pointed towards the West, South, and zenith, respectively. The Sun always revolves on the surface of the celestial sphere described in these coordinates as $x^2 + y^2 + z^2 = 1$. The declination θ for a particular day numbered by T is a constant, its value being determined by equation (1a). The assumptions presented in main assumptions are sufficient to calculate the movement of the Sun with respect to the fixed observer at the latitude φ:

$$x = -\cos\theta \sin\frac{2\pi t}{1440}, \qquad (1b)$$

$$y = -\cos\varphi \sin\theta - \sin\varphi \cos\theta \cos\frac{2\pi t}{1440}, \qquad (1c)$$

$$z = \frac{y}{\tan\varphi} + \frac{\sin\theta}{\sin\varphi}. \qquad (1d)$$

Here t is a solar time in minutes reckoned from midnight of the day numbered T, $0 \leqslant t < 1440$. It is worthwhile to note that a season-dependent difference between the local time and the solar time, accepted in our model, is non-zero for the majority of points on the Earth's surface [10]. The temporal difference can be estimated as the difference between the highest point of the Sun on the celestial sphere (the solar noon) at a particular day and the local noon of our civil time defined as 12:00 am. We emphasize that below only solar time will be used. The set of equations (1a)–(1d) is suitable to represent the visible Sun path both in the Northern ($0° \leqslant \varphi \leqslant 90°$) and Southern ($-90° \leqslant \varphi \leqslant 0°$) Earth hemispheres. In what follows the results and conclusions are applicable only for the Northern hemisphere, although they can easily be fitted for the Southern hemisphere.

Daylight duration at different latitudes

The equations system (1) can be applied to determine the daylight duration for a particular latitude φ and declination θ, the latter being dependent on T in agreement with equation (1a). It can be done by solving the equation $z = 0$, which gives the sunrise and sunset instants and, as a consequence, the duration of daylight in hours, according to the formula

$$D = 24\left(1 - \frac{\arccos(\tan\theta \tan\varphi)}{\pi}\right). \qquad (2)$$

Dependences $D(T)$ for different latitudes calculated on the basis of equation (2) are presented in figure 3(b). Analysis of the presented data shows that at the equatorial points the daylight duration is constant over the whole year. An increase in latitude causes an enhancement of the difference between daylight durations at the summer and winter solstices. If for certain θ and φ one obtains the argument of the arccosine function larger than unity, equation (2) should be augmented with a parameter-independent result $D = 24$. This phenomenon called 'polar days' ('midnight sun') happens in the summer time close to the solstice for latitudes $\varphi \geqslant 90° - \alpha$, the equality determining the ideal position of the Arctic polar circle.

A comparison between model and real data for Anchorage presented in figure 3(b) shows that equation (2) satisfactorily describes the daylight duration over the whole year, although minor differences of 10–40 min can be found. Such a discrepancy originates mostly from the non-zero size of the Sun as a light source and properties of the Earth's atmosphere. The Naval Oceanography Portal provides the following explanation: 'the Sun is not simply a geometric point. Sunrise is defined as the instant when the leading edge of the Sun's disc becomes visible on the horizon, whereas sunset is the instant when the trailing edge of the disc disappears below the horizon. These are the moments of first and last direct sunlight. At these times the centre of the disc is below the horizon. Furthermore, atmospheric refraction causes the Sun's disc to appear higher in the sky than it would if the Earth had no atmosphere. Thus, in the morning the upper edge of the disc is visible for several minutes before the geometric edge of the disc reaches the horizon. Similarly, in the evening the upper edge of the disc disappears several minutes after the geometric disc has passed below the horizon' [9]. The latter phenomenon is especially pronounced at high latitudes where the Sun's path near the horizon has a relatively moderate slope $(90° - \varphi)$. This value is equal to the angle between the celestial equator and the horizontal plane (see also figure 2(b)).

Sun path diagrams

As we have already mentioned, the model (1a)–(1d) adequately describes the Sun trajectory visible to the fixed observer located at point A for a particular latitude φ. To better understand the geometry involved it is convenient to imagine that this observer is positioned on the horizontal plane covered from the top by an imaginary topocentric celestial hemisphere centred in point A. The Sun moves on the surface of this hemisphere, the corresponding path and coordinates at the particular instant t of the day T being determined by equations (1a)–(1d) with the control parameter φ ($0° \leqslant \varphi \leqslant 90°$). To make the whole picture

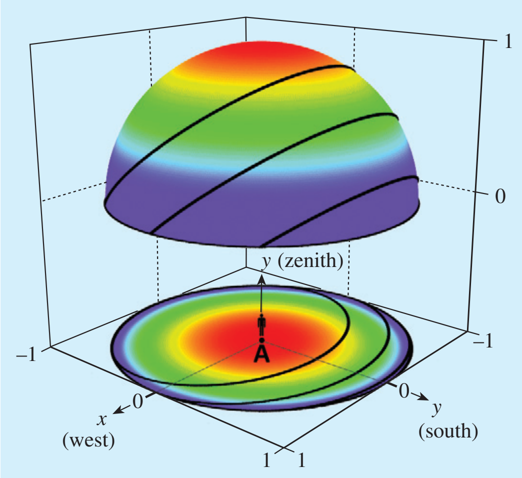

Figure 4. Three-dimensional Sun path diagram and its two-dimensional image for the data taken from figure 2*b*. Rainbow colours represent low (violet) and high (red) insolation of the horizontal plane in point A on the Earth's surface.

as simple as possible and the most descriptive let us pass from the full three-dimensional solar trajectory to its two-dimensional projection on the horizontal plane. Path diagrams obtained in this manner are limited by the skyline represented in graphs 4, 5, and 6(a) as the circular border.

A transition from small angles between the line connecting the observer with the Sun position and the horizontal plane and large angles approaching 90° (Zenith) is imitated by a transition of colours close to the violet part of the visible spectrum into those belonging to the red end of the spectrum. The graphical embodiment of the described approach using the Sun path data presented in figure 2(b) is depicted in figure 4. Circular Sun paths on the two-dimensional celestial sphere in the three-dimensional space are unambiguously imaged as elliptical arcs on the two-dimensional flat diagram. Those arcs in the framework of our model with a fixed declination during a particular day T are always symmetrical with respect to the South–North line (see figure 4). Violet regions of the Sun path diagram correspond to low insolation of the point A in the horizontal plane, i.e. to its weak heating, whereas red regions correspond to high insolation. Numerical estimations of insolation values are presented in shadow path diagrams and horizontal sundials.

Our diagrams demonstrate the different character of Sun paths at different latitudes. For this purpose, we selected a set of latitudes in the Northern hemisphere with the separation of about 30° corresponding to well-known points: Singapore, Cairo (Egypt), Anchorage (USA), and the North Pole. The comparison of their Sun path diagrams is presented in figure 5. They give a clear-cut answer to our starting question 'What is the origin of the passage of the seasons?'. Such a comparison also gives the answer to another question 'Why is it always hot at the equator and cold at the North Pole?' The observed shift with φ from the red area to the violet one establishes the sought explanation.

Shadow path diagrams and horizontal sundials

It is natural that changes of the Sun path character during the year should lead to a drastic temporal evolution of shadow patterns on the Earth's surface. In this connection, let us consider the annual evolution of the shadow of a vertical stick. This phenomenon is of practical importance, e.g., for sundials still being a decoration of city centres. The main principles of those primitive clocks were well known from antiquity [11–13] but, unfortunately, have been partially forgotten in the modern epoch. Their basic properties can be easily derived using the Sun path diagram. For example, figure 6(a) shows the solar path scheme for the latitude of Dresden (Germany) showing Sun positions at different instants of solar time for days of solstices and equinoxes. Knowledge of the Sun coordinates makes it possible to predict the temporal evolution of the shadow lengths and directions. To be specific, we analysed the simplest case of a shadow cast onto horizontal plane by a vertical stick of height L located at point A perpendicularly to the plane indicated in figure 6(b) for instants between winter and summer solstices. Hereafter, we label such dependence as a 'shadow path diagram'.

The data presented in figure 6(b) show that shadow tips trace out branches of hyperbolas in time and space. The corresponding asymptotes are determined by sunrise and sunset points on the solar map. The arms of the hyperbola branches are oriented to the North for days between autumn and spring equinoxes, otherwise they are oriented to the South. The vertex of each hyperbola

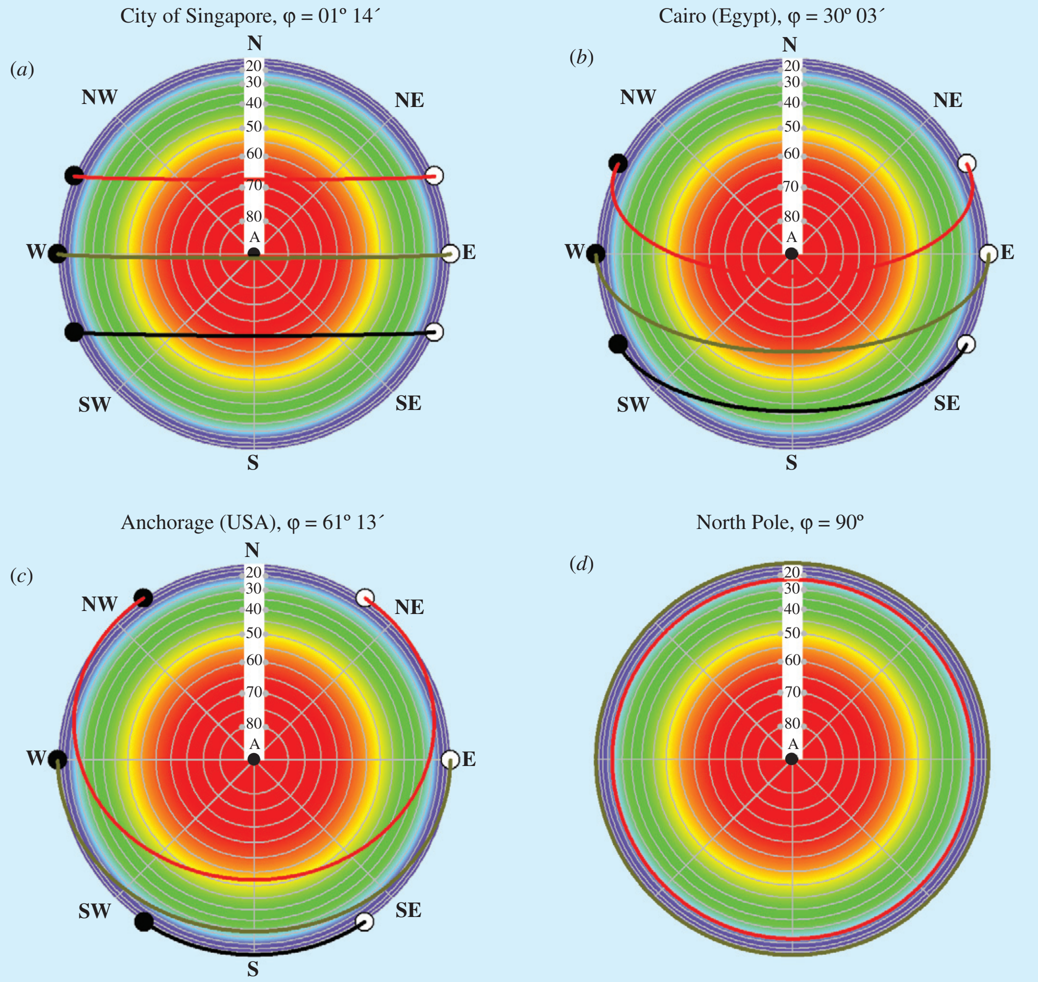

Figure 5. Sun path diagrams for the winter solstice (black curve), spring or autumn equinoxes (dark yellow curve), and the summer solstice (red curve) at different latitudes of the Earth: (*a*) φ = 01º 14′ (Singapore); (*b*) φ = 30º 03′ (Cairo); (*c*) φ = 61º 13′ (Anchorage); and (*d*) φ = 90º (the North Pole; the curve for the winter solstice is absent because the Sun is invisible during the polar night. Note also that the concept of cardinal directions becomes meaningless in the area close to the North Pole).

(observed at the solar noon) is determined by the current Sun declination as well as the latitude of the observation point, and therefore a distance between point A and a hyperbola vertex is given by the formula

$$d(T) = L \tan(\varphi - \theta(T)), \tag{3}$$

where $\theta(T)$ can be found from equation (1*a*). During equinox days the shadow path shape transforms closely to a straight line, which is oriented from the West to the East with the distance to point A equal to $L \tan \varphi$. The analysis of shadow tip positions for a particular solar time shows that all of them lie along a certain straight line. Bundle of lines corresponding to different solar times intersects at the focal point, F, located to the south of point A. The distance AF equals $L \cot \varphi$ (see figure 6(c)). Parameters of linear equations describing those lines for a particular solar time can be found as follows. First of all, we take into account the above-indicated fact that the paths of shadow tips at equinoxes are

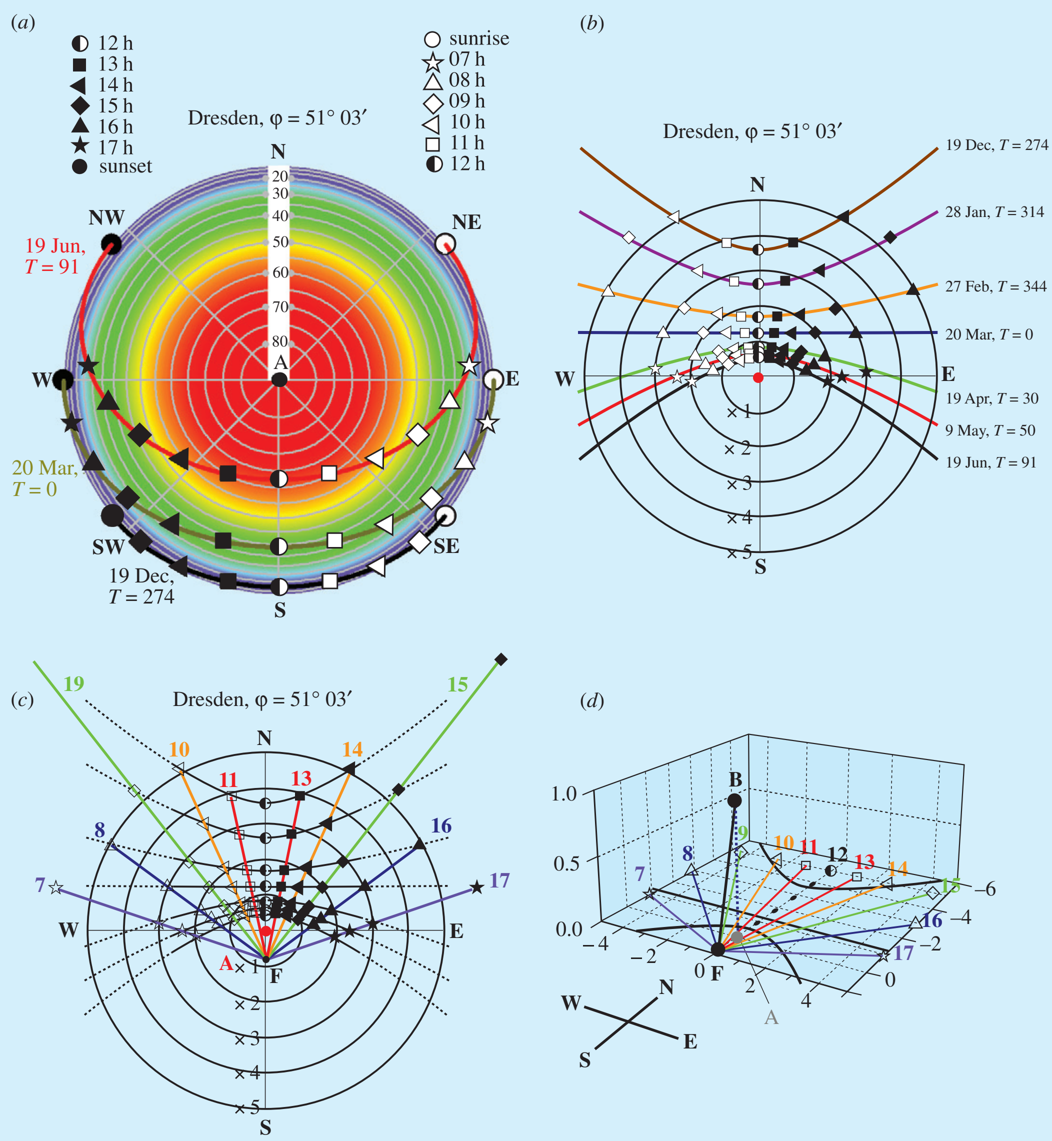

Figure 6. Explanation of the sundial idea on the basis of Sun path and shadow path diagrams. (*a*) The diagram for Dresden (latitude φ = 51° 03′) showing the Sun paths at different instants of the solar time *t* for the time *T* of the winter solstice, equinoxes, and the summer solstice. (*b*) Evolution of the shadow tip from the vertical stick located at point A of length *L* during the season transition from the winter to summer solstices. Black concentric circles show distances from point A which are equal to *L*, 2*L*, 3*L*, 4*L* and 5*L*. Marks indicating solar time are the same as in (*a*). (*c*) Demonstration of the fact that the shadow tips at any particular solar time (see (*b*)) lie on the straight lines (independent of the day *T*), which intersect at the focal point F. The distance AF equals *L*·cot(φ). The shadow paths are the same as in (*b*). (*d*) A sundial based on the data of (*c*) for practical use. A stick FB tilted by angle φ is placed at point F instead of the original vertical stick AB of length *L* located at point A.

straight lines. At the same time, only during these days is the celestial path of the Sun observed from point A uniform. Hence, we can easily find solar coordinates on the celestial sphere. For example, the Sun rises at the equinox day at 6 am exactly in the East, an hour later it changes its angular location on the celestial equator plane by $180°/12 = 15°$; at 8 a.m. the Sun can be found at an angle of 30° with respect to its position at sunrise, etc. From this data we can calculate the corresponding positions of the shadow tips at solar time instants equal to, say, N hours ($N = 7$–17). Having the latter points and bearing in mind that all lines go through the point F, one finally finds the equations for the shadow tip locations as

$$y_s(t = N, \varphi, x_s) = -L\cot(\varphi) + \frac{x_s}{\sin(\varphi)\tan(15°N)}. \tag{4}$$

Here (x_s, y_s) are coordinates of shadow tips on the horizontal plane for the stick concerned (see figures 6(b) and (c)), the axis A–x_s being oriented to the East and the axis A–y_s being oriented to the North.

The shadow path diagrams can be used for different practical purposes (see conclusions and outlook). For instance, their analysis comprises the basis of the horizontal sundial theory. Note that actually the non-zero diameter of the solar disc becomes significant. For example, a shadow calculated according to the model presented above in reality turns out to be shorter and its boundaries transform into ambiguously limited semi-shadows, especially for thin sticks and low altitudes of the Sun. To reduce the ambiguity one can use a tilted stick connecting points F and B instead of the original vertical stick AB discussed above (see figure 6(d)). The angle between the new stick FB and the horizontal plane is equal to the latitude φ, therefore the stick is parallel to the polar axis AP presented in figure 2(b). Such a trick results in the new shadow lying completely on the proper time line, which makes the time measurement much easier in practice than that for the original vertical stick where only the tip was significant. The new pattern overcomes the effect of the unavoidable distortion by the finite size of the Sun disc.

Hunt for solar energy in practical architecture as an application of the proposed model

As we have seen, solar path diagrams explain qualitatively the passage of the seasons through the annual rotation of the Earth in its orbit. Moreover, our approach allows one to quantitatively estimate the solar energy input (insolation) that can be harvested on the horizontal or any other plane. It is well known that the solar constant, i.e., the amount of radiant power falling onto a flat surface unit area oriented perpendicularly to the direction of sunlight rays outside the Earth's atmosphere, equals $\Phi_{max} \approx 1366$ W m^{-2}. After penetrating the atmosphere and partial absorption the value of the incident solar energy decreases but for the sake of simplicity we shall hereafter neglect the difference between the actual weakened Sun flux Φ_{weak} and the value Φ_{max} indicated above. In the general case where the angle between the surface concerned and the direction of Sun rays is equal to δ the insolation can be found from the following equation

$$\Phi(\delta) = \Phi_{max}\sin\delta. \tag{5}$$

Of course the angle δ permanently changes with time as a result of the Earth's rotation around its axis as well as its annual path around the Sun.

Our model gives the coordinates of the Sun on the celestial sphere at any instant. Thus, we are able to estimate the maximum energy that can be harvested on special planes oriented to the North, South, East and West or is parallel to the horizontal plane. To compare incident energies for each plane we introduce a 'solar cube' with edges equal to 1 m (figure 7(a)). Solar harvesting for different latitudes appropriate to Ukraine was chosen as an example. In figure 7(b) annual diagrams depicting daily amounts of radiant energy reaching each side of the solar cube are demonstrated in the ideal case. The calculated values were found by integration of the flux in equation (5) over the insolation time for corresponding planes. In the framework of our model eastern and western sides always absorb equal amounts of energy although in reality a little difference exists due to slow gradual variation of the declination. Analysis of the presented data for the cube top explains quantitatively the observed revolution of the seasons. Namely, it is clear that in summer the horizontal plane absorbs a few times more energy than in winter because the day is longer and, especially, because the Sun reaches higher altitudes. The energy ratio summer/winter for the horizontal plane severely depends on latitude and increases for higher φ values.

The northern side of the cube makes a small contribution only between the spring and autumn equinoxes, because in the winter time the Sun never shines on its plane. But the main practically significant conclusion can be made for the southern side. In the winter time when additional heating of our houses is required the

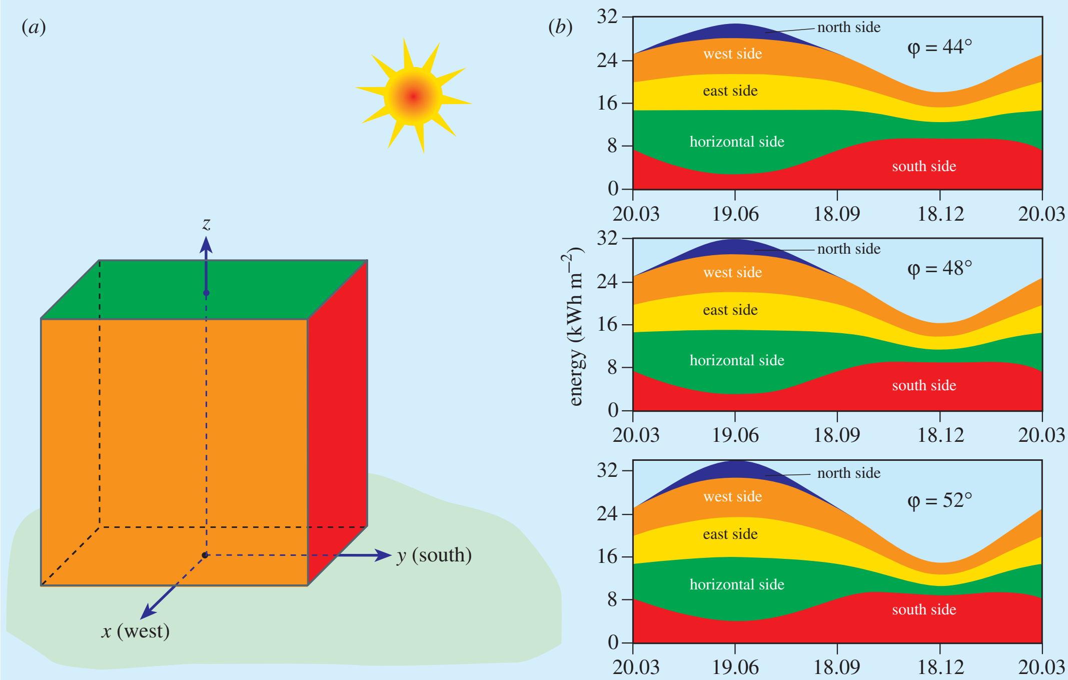

Figure 7. (*a*) 'Solar cube' and (*b*) contribution of its sides to the total daily radiant energy absorption, which can be harvested in the ideal case at different latitudes throughout the year. Example: at latitude $\varphi = 52°$ on 19 June, the south, horizontal, east, west and north sides of the solar cube absorb 4.0, 11.9, 7.4, 7.4 and 3.2 kWh m^{-2} radiant energy, respectively. The contribution from the southern side is the most significant between autumn and spring equinoxes.

southern side absorbs about half of the energy harvested by the solar cube as a whole. Thus, vertical surfaces oriented to the South provide the most effective way to absorb solar energy and additionally heat our homes either through windows or special heating devices like Trombe walls [14, 15].

The ideas presented above concerning heating of northern and southern sides of the solar cube were intuitively understood and used by peasants, e.g., in old Ukraine, to construct country cottages. Their longer walls were oriented from the East to the West, most of their windows were located on southern walls, and windows on the northern walls were absent or were rather small to decrease heat losses. An additional paramount feature of such houses was the existence of wide eaves on the southern roof edges, which protected the cottages from overheating in summer but allowed for insolation of internal rooms through the windows in winter. The width of the eave S can be easily calculated if you know the distance H between the upper edge of the window and the roof and the local latitude φ (figure 8(a)). It is apparent that a house needs sunlight access between autumn and spring equinoxes when the Sun's altitude does not exceed $(90° - \varphi)$. Therefore, the eave width S should be selected in such a way as to allow the sunlight to pass through the southern windows of the designed house during the cold season and to protect it against overheating during the day time in summer (see figure 8(a)):

$$S = H \tan \varphi. \tag{6}$$

Similar considerations can be used to propose a new kind of the greenhouse on the basis of the sector in the horizontal plane where the Sun appears in the winter time. As one sees from the solar path diagrams, in winter the Sun always rises from a direction close to Southeast and sets towards Southwest. The exact directions of sunrise and sunset are dependent on the local latitude φ and the actual declination θ defined from equation (1*a*). Obvious transformations using equations (1*a*)–(1*d*) lead to the following angle β between the sunrise (sunset) direction and the East

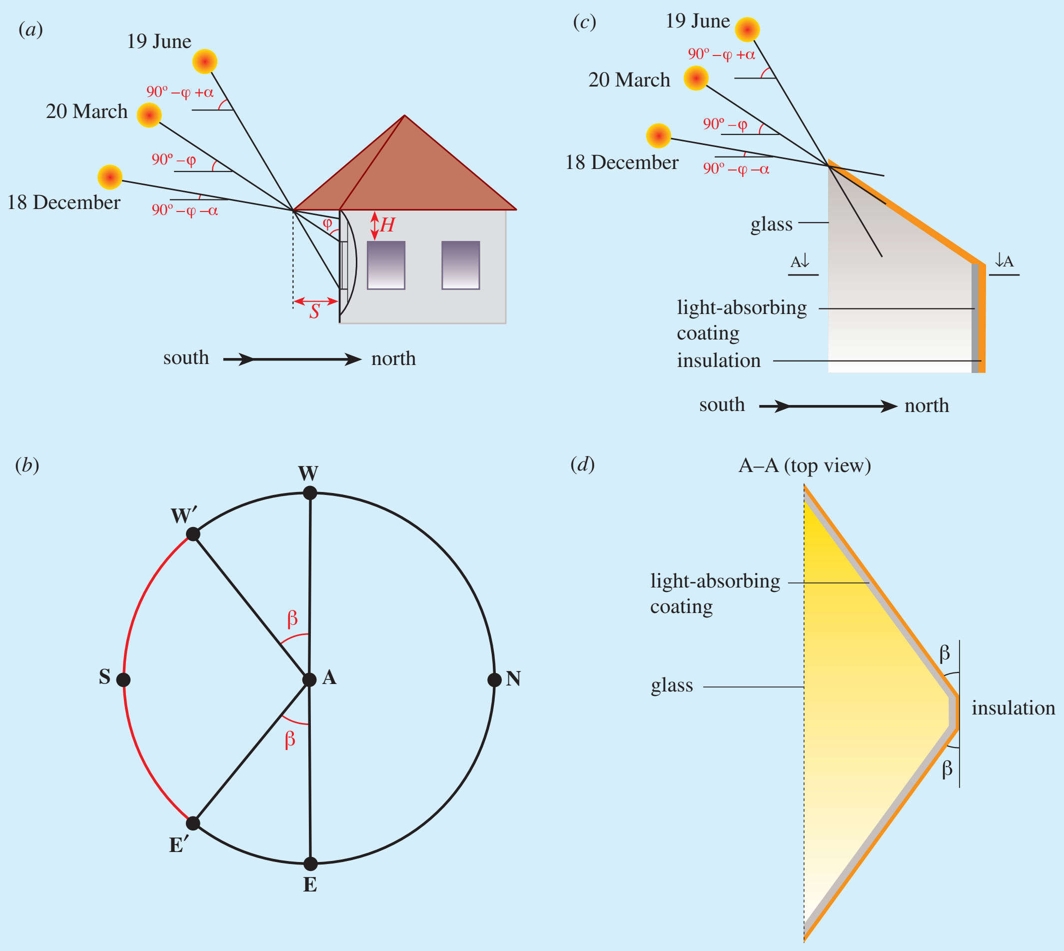

Figure 8. Solar harvest constructions based on solar path geometry. (*a*) A house with south-oriented eaves to invite sunlight through windows during winter and to shadow the southern wall during summer. The length *S* of the eaves can be determined from the maximal altitude of the Sun during the day of equinox. (*b*) The sector where the Sun can be observed between the autumn and spring equinoxes. Positions of the sunrise (E′), sunset (W′), and the angle β given by equation (7) depend on the actual solar declination. (*c*,*d*) An idea for a novel greenhouse, which takes into account parameters of the sector where the Sun can be observed. The slope of the roof and optimal angle β (see (*b*)) can be modified further according to practical need. Maximal possible values of energy harvested through glass surfaces open to the Sun can be estimated using the data presented in figure 7.

(the West)

$$\beta = \arcsin(\sin\theta / \cos\varphi). \qquad (7)$$

On the winter solstice day the absolute value of θ is equal to the Earth's axis tilt: $\theta = -\alpha = -23.45°$. In this case equation (7), e.g., for the latitude of Dresden gives $\beta \approx -39°$ (figure 8(b)). The equation concerned can be used to construct a greenhouse, which takes into account the sunshine sector allowing maximum heating by a natural energy source in the cold season (figure 8(c)).

The greenhouse's roof should be tilted at the angle $(90° - \varphi)$ with respect to the horizontal plane and two walls should be directed to the sunrise and sunset points calculated for the winter solstice. Access of the solar energy into the greenhouse should be provided by a transparent glass pane; the heat supplied by the Sun is absorbed and accumulated by the material of the walls (see side- and top-view images in figure 8(d)).

For the physical principles behind the greenhouse effect, we refer to Besson *et al*, who give a new teaching methodology of explaining thermal effects of interaction between radiation and matter [16]. In any case, to design greenhouses or solar-harvest houses, one needs to consider the Sun path and ensure the optimal absorption of solar energy by the building interior in the coldest season.

Conclusions and outlook

Our model can be applied to explain various phenomena in one way or another depending on the apparent Sun path on the celestial sphere. Furthermore, our approach can be useful to quantitatively describe the Sun path mechanically modelled, e.g., by the recently presented solar demonstrator [17]. Alternating seasons, dependences of the sunrise and sunset characteristics on the date, day durations for different latitudes, temporal dependence of the solar altitude for a fixed observer can be reproduced by rather simple computer calculations on the basis of the suggested model.

Of course, our simplified approach and corresponding conclusions have certain limitations. Some of them can be easily overcome leading to more accurate results, but to tackle other subtleties more sophisticated insights and efforts are needed. In particular, we should indicate that actually the set of equations (1a)–(1d) should be replaced by a more accurate one including a continuous time rather than integer 'days' (T) and the current time during this day (t), which would result in a continuous changes of the declination θ. Unfortunately, in that case the clarity of our model based on the everyday experience of students (24 h in one day and 365.25 days in one Julian year) will be lost. Furthermore, if exact tabulated θ values were used to perform more precise calculations on the basis of the data presented by the Naval Oceanography Portal [9] the convenient periodicity of equation (1a) would be lost as well.

There are yet other areas of application of the suggested approach. For instance, any variation of the Earth's obliquity α might significantly alter absorbed solar energy at high latitudes. Indeed, it is well known that the Earth's obliquity slowly changes with a period of about 40 000 years in the range from 22.1° to 24.5° causing glacial cycles [8]. At the same time, other naturally driven climate changes (Milankovitch cycles) require more sophisticated explanations, in particular, dealing with varying parameters of the Earth's elliptical orbit, explanations going far beyond the simplicity of the proposed model. In any case it should be noted that the Earth's orbit changes very slowly, specifically, at the scale of thousands of years and cannot be responsible for the actual sharp climate changes observed during the last 30–40 years [3–5]. Prediction of the climate on other planets with orbits close to circular may be another promising area of application of our model [7]. In this connection, the radiation flux constant (solar constant in the case of Sun) can be easily found from the power of the central star radiant energy and the orbital radius of a specific planet [5].

As for topical problems of everyday life, the presented model, being the basis of the automatic or manual tracking system, would be of help to control photovoltaic elements or solar thermal collectors to maximize their output. It should be taken into account that the solar constant Φ_{max} (5) depends on the angle between rays and the horizontal plane, since the light attenuation in the atmosphere depends on the length of the path and is stronger when the sunlight propagates at smaller angles.

The model would also be useful to construct energy-effective buildings [14, 15, 18]. We emphasized above that in order to enhance the radiant energy absorption allowance should be made for the sector where the Sun appears. However, during the cold season the shadow of any construction covers a vast area, which can be calculated using our model. Therefore, it is necessary to bear in mind the nontrivial problem of the optimal arrangement of several buildings or isolated constructions intended to avoid mutual shadowing. Doing this one should take into account the actual latitude and monitor the time-dependent shadowed areas using the appropriate software.

Similar programs might also be used to produce animated cartoons where shadow areas of animated objects are of proper size. We also want to attract attention to another application of solar path and concomitant moving shadow diagrams. Namely, Baker and Thornes used the solar positions in Monet's paintings of the Houses of Parliament in London to derive the dates and times of the depicted scenes, which allowed the authors to extract from impressionist paintings useful information about London fogs and air quality more than hundred years ago [19].

Yet another important property of the model is its ability to explain main features of horizontal sundials presented above. We are going to extend our analysis to the case of the gnomons positioned on the surfaces of vertical walls with their arbitrary orientation with respect to the East–West direction. The full sundial theory includes cumbersome mathematics hardly suitable for the high-school level [11–13, 20–22]. On the other hand, the simplicity of our mathematical model makes it appropriate to invite students to the challenging world of solar geometry.

Acknowledgments

The authors cordially thank Professors Alexander Gabovich and Vladimir Fomin for useful discussions. VK acknowledges support from the BMBF (project 03X0076C).

Received 14 May 2010, in final form 31 July 2010
doi:10.1088/0031-9120/45/6/010

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