
SpatialOps: A Benchmark for 2D Spatial Planning and Reasoning in Large Language Models

Anonymous Author(s)

Affiliation

Address

email

Abstract

1 Spatial reasoning represents a fundamental cognitive capability that enables hu-
2 mans to navigate, plan, and interact with the physical world. Despite remarkable
3 advances in Large Language Models (LLMs), their ability to perform spatial reason-
4 ing remains significantly limited compared to their linguistic capabilities. Existing
5 benchmarks have explored various facets of spatial understanding, yet a compre-
6 hensive evaluation framework for practical 2D spatial planning across diverse
7 real-world domains is notably absent. We introduce **SpatialOps**, a comprehensive
8 benchmark comprising 6,012 procedurally generated tasks across twelve cate-
9 gories organized into three tiers of increasing complexity. Our benchmark uniquely
10 bridges the gap between abstract spatial reasoning and applied operational planning,
11 drawing from documented use cases in telecommunications, utilities, government,
12 and enterprise sectors. We propose a multi-faceted evaluation methodology encom-
13 passing five metrics: Task Completion Rate, Human-AI Latency Ratio, Operational
14 Cost Savings, Efficacy Score, and Scalability Index. Extensive experiments on five
15 leading LLMs reveal substantial performance gaps, with the best model achieving
16 only 78.4% on our composite score. Our analysis identifies systematic weaknesses
17 in algorithmic reasoning, constraint satisfaction, and temporal-spatial integration,
18 providing clear directions for future research.

1 Introduction

19 The emergence of Large Language Models has fundamentally transformed artificial intelligence,
20 demonstrating unprecedented capabilities in natural language understanding [19, 135, 4], code
21 generation [26, 90], and complex reasoning [150, 155, 75]. These models have shown remarkable
22 performance on tasks ranging from mathematical problem-solving [60, 32] to scientific discovery
23 [123, 136]. However, a critical examination of their capabilities reveals a fundamental limitation:
24 the ability to reason about spatial relationships and perform spatial planning remains significantly
25 underdeveloped [96, 9?].

26 This limitation is particularly consequential given the central role that spatial reasoning plays in
27 human cognition [109, 58, 140]. From navigating through physical environments [152, 41] to
28 understanding maps and diagrams [57, 137], spatial reasoning underpins countless everyday activities
29 and professional tasks. The cognitive science literature has long recognized spatial ability as a distinct
30 form of intelligence [22, 104], separate from verbal and mathematical reasoning, and critical for
31 success in STEM fields [144, 140].

32 The challenge of spatial reasoning for LLMs stems from a fundamental representational mismatch [15,
33 113]. These models process information as discrete, sequential tokens, whereas spatial information is
34 inherently continuous and multi-dimensional [45, 79]. Early work in qualitative spatial reasoning

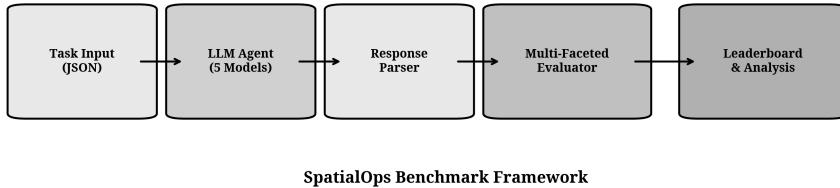


Figure 1: The SpatialOps benchmark framework. Tasks span twelve categories organized into three tiers of increasing complexity. Models are evaluated using a multi-faceted methodology that assesses accuracy, reasoning quality, and operational efficiency.

36 established formal frameworks for representing spatial relationships [121, 33?], but translating these
 37 frameworks into neural architectures remains an open challenge [30?].

38 The practical implications of this limitation are substantial. As AI systems are increasingly deployed in
 39 real-world applications, from autonomous vehicles [24, 16, 115] to robotic manipulation [84, 69, 158],
 40 the ability to reason spatially becomes critical. In enterprise contexts, spatial AI is transforming
 41 industries including telecommunications [159, 148], urban planning [13, 14], logistics [86, 108], and
 42 real estate [83, 47]. Companies like Palantir [134], Scale AI [2], Wherobots [64], and Google Earth
 43 Engine [53] are deploying sophisticated spatial AI systems, yet the underlying LLMs that power
 44 many of these applications lack robust spatial reasoning capabilities.

45 To address this gap, we introduce **SpatialOps**, a comprehensive benchmark designed to evaluate
 46 the 2D spatial planning and reasoning capabilities of LLMs. Our benchmark makes four key
 47 contributions:

- 48 1. **Comprehensive Task Coverage:** We define twelve distinct task categories spanning three
 49 tiers of complexity, from foundational concepts like coordinate understanding and dis-
 50 tance computation to advanced optimization problems involving constraint satisfaction and
 51 temporal-spatial reasoning.
- 52 2. **Real-World Grounding:** Unlike abstract benchmarks, SpatialOps is grounded in docu-
 53 mented industry use cases from telecommunications, utilities, government, and enterprise
 54 sectors, ensuring practical relevance.
- 55 3. **Rigorous Evaluation Methodology:** We propose five complementary metrics that assess
 56 not only accuracy but also efficiency, cost-effectiveness, and scalability, providing a holistic
 57 view of model capabilities.
- 58 4. **Extensive Empirical Analysis:** We evaluate five leading LLMs, conduct ablation studies
 59 on prompt engineering and task complexity, and provide detailed error analysis to guide
 60 future research.

61 2 Related Work

62 2.1 Spatial Reasoning in Cognitive Science

63 The study of spatial reasoning has deep roots in cognitive psychology and neuroscience. Piaget's
 64 foundational work established that spatial cognition develops through distinct stages [114], while
 65 subsequent research identified multiple components of spatial ability including mental rotation
 66 [130, 141], spatial visualization [97, 57], and spatial orientation [77, 59]. Neuroimaging studies have
 67 localized spatial processing to specific brain regions, particularly the parietal cortex and hippocampus
 68 [21, 78, 42].

69 The distinction between egocentric and allocentric spatial reference frames [72, 20] has proven
 70 particularly relevant for AI systems. Egocentric representations encode space relative to the observer,
 71 while allocentric representations use external reference points. Research suggests that humans

72 flexibly switch between these frames depending on task demands [106, 145], a capability that remains
73 challenging for current AI systems [3, 25].

74 **2.2 Qualitative Spatial Reasoning**

75 The field of qualitative spatial reasoning (QSR) emerged from the need to represent and reason about
76 spatial information without precise numerical coordinates [34, 122]. The Region Connection Calculus
77 (RCC) [121] provides a formal framework for representing topological relationships between regions,
78 while the Cardinal Direction Calculus [46, 94] handles directional relationships. These formalisms
79 have been extended to handle temporal aspects [? ?] and uncertainty [? 127].

80 Recent work has explored integrating QSR with neural networks [30?], but significant challenges
81 remain. The discrete, symbolic nature of QSR formalisms does not naturally align with the continuous
82 representations learned by neural networks [49, 82], and scaling these approaches to complex, real-
83 world scenarios remains difficult [35, 103].

84 **2.3 Spatial Reasoning Benchmarks**

85 The evaluation of spatial reasoning in AI has evolved significantly over the past decade. Early
86 benchmarks like bAbI [151] included simple spatial reasoning tasks but were quickly saturated by
87 neural models [133, 54]. The CLEVR dataset [68] introduced visual spatial reasoning, requiring
88 models to answer questions about synthetic 3D scenes. Subsequent work extended this paradigm to
89 more realistic images [62, 132] and 3D environments [125, 76].

90 Text-based spatial reasoning benchmarks have also proliferated. SpartQA [105] evaluates spatial
91 reasoning through question answering, while StepGame [131] tests multi-hop spatial reasoning.
92 RoomSpace2 [88] focuses on indoor spatial reasoning, and PlanQA [70] evaluates planning in spatial
93 contexts. However, these benchmarks often focus on abstract scenarios that do not capture the
94 complexity of real-world spatial tasks.

95 Vision-language benchmarks have emerged to evaluate multimodal spatial reasoning. SpatialBench
96 [154] assesses spatial understanding in VLMs, while GRASP [100] uses grid-based environments.
97 3DSRBench [81] and Spatial457 [101] evaluate 3D spatial reasoning. More recently, GeoAnalyst-
98 Bench [160] has focused on geospatial analysis tasks, and MapBench [27] evaluates map reading
99 abilities.

100 Our work builds upon and extends this prior research. The comprehensive survey by Felicia et
101 al. [44] provides a unified taxonomy of spatial AI agents and world models, identifying key ca-
102 pabilities and evaluation dimensions. SpatialOps operationalizes this framework by providing a
103 large-scale benchmark that spans multiple spatial reasoning capabilities and is grounded in real-world
104 applications.

105 **2.4 LLM Agents and Tool Use**

106 The development of LLM-based agents has opened new possibilities for spatial reasoning through tool
107 use and environmental interaction [156, 126, 119]. Agents can leverage external tools for computation
108 [48, 28], information retrieval [85, 107], and physical interaction [1, 17]. This paradigm has been
109 particularly successful in code generation [26, 90] and mathematical reasoning [63, 164].

110 Benchmarks for LLM agents have emerged to evaluate these capabilities. AgentBench [96] provides
111 a comprehensive evaluation across multiple environments, while WebArena [165] focuses on web-
112 based tasks. SWE-bench [66] evaluates software engineering capabilities, and Mind2Web [37]
113 assesses web navigation. These benchmarks have revealed significant gaps between current LLM
114 capabilities and human-level performance on complex, multi-step tasks.

115 **2.5 Graph Neural Networks for Spatial Data**

116 Graph Neural Networks (GNNs) have emerged as a powerful paradigm for processing spatial data
117 [71, 143, 55]. By representing spatial relationships as graph structures, GNNs can capture complex
118 dependencies that are difficult to model with traditional approaches [18, 11]. Applications include
119 traffic prediction [89, 157, 153], point cloud processing [118, 149], and molecular modeling [51, 128].

120 Spatio-temporal GNNs extend this paradigm to dynamic spatial data [67, 129, 7]. These models
121 have achieved state-of-the-art performance on tasks like traffic forecasting [65, 8] and human motion
122 prediction [102, 87]. Recent work has explored integrating GNNs with LLMs [23, 161], potentially
123 enabling more sophisticated spatial reasoning.

124 **3 The SpatialOps Benchmark**

125 **3.1 Design Principles**

126 SpatialOps is designed according to four core principles that distinguish it from existing benchmarks:

127 **Real-World Grounding:** Tasks are derived from documented industry use cases in telecommu-
128 nications, utilities, government, and enterprise sectors. This grounding ensures that benchmark
129 performance translates to practical capability [120, 92].

130 **Comprehensive Coverage:** The benchmark spans twelve distinct categories of spatial reasoning,
131 organized into three tiers of increasing complexity. This hierarchical structure enables fine-grained
132 analysis of model capabilities [? 91].

133 **Controlled Difficulty:** All tasks are procedurally generated with configurable parameters, allowing
134 precise control over difficulty levels. This enables systematic study of how performance degrades
135 with increasing complexity [117, 40].

136 **Verifiable Ground Truth:** Every task includes a programmatic validator that computes the correct
137 answer, ensuring 100% ground-truth accuracy. This eliminates annotation errors that plague many
138 benchmarks [111, 73].

139 **3.2 Task Taxonomy**

140 SpatialOps comprises twelve task categories organized into three tiers:

141 **Tier 1: Foundational Concepts** establishes basic spatial understanding:

- 142 • **Coordinate Understanding (CU):** Tests comprehension of coordinate systems, including
143 Cartesian coordinates, polar coordinates, and coordinate transformations [109, 93].
- 144 • **Geometric Reasoning (GR):** Evaluates knowledge of geometric shapes, properties (area,
145 perimeter, angles), and spatial relationships (intersection, containment, overlap) [31, 12].
- 146 • **Distance Computation (DC):** Assesses ability to calculate various distance metrics includ-
147 ing Euclidean, Manhattan, Chebyshev, and geodesic distances [38, 116].
- 148 • **Topological Reasoning (TR):** Tests understanding of topological relationships (adjacency,
149 connectivity, containment) independent of precise coordinates [121?].

150 **Tier 2: Core Planning** requires algorithmic reasoning:

- 151 • **Navigation and Pathfinding (NP):** Evaluates ability to find optimal paths using algorithms
152 like A* [56], Dijkstra [39], and their variants [74, 95].
- 153 • **Viewpoint and Visibility (VVA):** Tests determination of visibility and line-of-sight in
154 environments with obstacles [50, 139].
- 155 • **Pattern Recognition (PRA):** Assesses identification of spatial patterns, clusters, and anomali-
156 es in point distributions [43, 5].
- 157 • **Network Infrastructure (NI):** Evaluates analysis of network topologies, including connec-
158 tivity, shortest paths, and failure analysis [110, 10].

159 **Tier 3: Advanced Optimization** involves complex multi-step reasoning:

- 160 • **Constraint-Based Placement (CBP):** Tests placement of objects satisfying multiple spatial
161 and logical constraints [80, 124].
- 162 • **Resource Allocation (RAO):** Evaluates optimization of resource placement to maximize
163 coverage or minimize cost [61, 142].

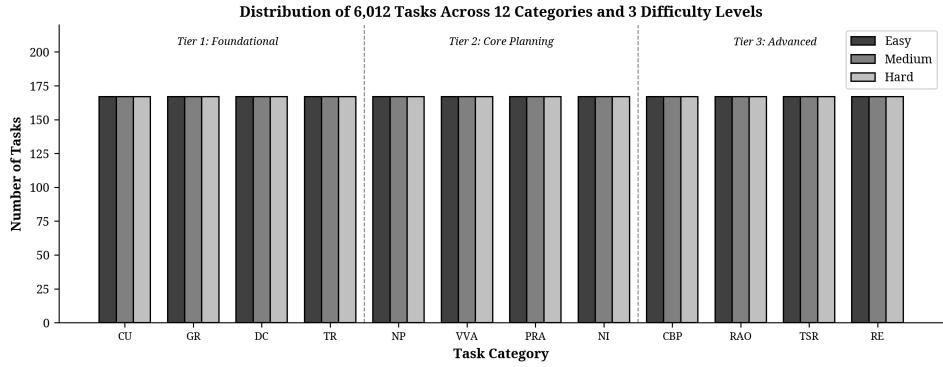


Figure 2: Distribution of tasks across categories and difficulty levels. Each category contains 501 tasks evenly distributed across easy, medium, and hard difficulty levels, totaling 6,012 tasks.

- 164 • **Temporal-Spatial Reasoning (TSR):** Assesses reasoning about objects moving or changing
165 over time [? ?].
- 166 • **Real Estate and Geospatial (RE):** Tests complex analysis of geospatial data including
167 zoning, valuation, and site selection [52, 98].

168 3.3 Dataset Composition

169 The SpatialOps dataset comprises 6,012 tasks distributed evenly across the twelve categories and
170 three difficulty levels. Each task is represented in a structured JSON format containing:

- 171 • **Task ID:** Unique identifier encoding category, difficulty, and instance number
- 172 • **Question:** Natural language description of the spatial reasoning task
- 173 • **Context:** Structured spatial data (coordinates, graphs, constraints)
- 174 • **Ground Truth:** Verified correct answer computed by programmatic validator
- 175 • **Metadata:** Category, difficulty level, required reasoning steps

176 Difficulty levels are calibrated based on multiple factors: number of entities, complexity of constraints,
177 required reasoning depth, and computational complexity of the optimal solution. Easy tasks require
178 1-2 reasoning steps, medium tasks require 3-5 steps, and hard tasks require 6+ steps or involve
179 NP-hard subproblems.

180 3.4 Industry Use Case Alignment

181 A distinguishing feature of SpatialOps is its alignment with documented industry use cases. We
182 surveyed spatial AI applications across four sectors:

183 **Telecommunications:** Network planning, fiber route optimization, coverage analysis, and infrastructure
184 maintenance [159, 148].

185 **Utilities:** Asset management, outage prediction, load balancing, and infrastructure inspection [29,
186 146].

187 **Government:** Urban planning, emergency response, resource allocation, and environmental monitoring
188 [13, 14, 138].

189 **Enterprise:** Real estate analysis, logistics optimization, site selection, and market analysis [83, 47,
190 36].

191 Each task category maps to specific industry applications, ensuring that benchmark performance
192 reflects practical capability. This alignment is detailed in Appendix A.

193 **4 Evaluation Methodology**

194 **4.1 Multi-Faceted Metrics**

195 We propose five complementary metrics that provide a holistic assessment of model capabilities:

196 **Task Completion Rate (TCR)** measures the percentage of tasks where the model produces a valid,
197 parsable response:

$$TCR = \frac{|\{t \in T : \text{valid}(t)\}|}{|T|} \times 100\% \quad (1)$$

198 **Accuracy (ACC)** measures the percentage of correct answers among completed tasks:

$$ACC = \frac{|\{t \in T : \text{correct}(t)\}|}{|\{t \in T : \text{valid}(t)\}|} \times 100\% \quad (2)$$

199 **Human-AI Latency Ratio (HLR)** quantifies speed-up compared to human professionals. We
200 established baselines by measuring completion times for GIS analysts on representative task samples:

$$HLR = \frac{\bar{T}_{\text{human}}}{\bar{T}_{\text{AI}}} \quad (3)$$

201 **Operational Cost Savings (OCS)** estimates economic impact based on time savings and computa-
202 tional costs:

$$OCS = (\bar{T}_{\text{human}} - \bar{T}_{\text{AI}}) \times R_{\text{human}} - C_{\text{AI}} \quad (4)$$

203 where R_{human} is the hourly rate and C_{AI} is the API cost per task.

204 **Efficacy Score (ES)** provides a composite measure combining accuracy, reasoning quality (assessed
205 via LLM-as-judge [163]), and efficiency:

$$ES = w_1 \cdot ACC + w_2 \cdot RQ + w_3 \cdot EFF \quad (5)$$

206 where $w_1 = 0.5$, $w_2 = 0.3$, $w_3 = 0.2$ by default.

207 **4.2 Evaluation Protocol**

208 Models are evaluated using a standardized protocol:

- 209 1. **Prompt Construction:** Each task is presented with a system prompt establishing the spatial
210 reasoning context, followed by the task question and structured context data.
- 211 2. **Response Generation:** Models generate responses with temperature=0 for reproducibility.
212 Maximum token limits are set based on task complexity.
- 213 3. **Answer Extraction:** Responses are parsed to extract the final answer using category-specific
214 extractors.
- 215 4. **Correctness Verification:** Extracted answers are compared against ground truth using
216 appropriate matching criteria (exact match, numerical tolerance, set equivalence).
- 217 5. **Reasoning Assessment:** For a stratified sample, reasoning chains are evaluated by GPT-4
218 using a 5-point rubric assessing logical coherence, spatial accuracy, and completeness.

219 **5 Experiments**

220 **5.1 Models Evaluated**

221 We evaluate five leading LLMs representing the current state-of-the-art:

- 222 • **GPT-5.2** (OpenAI): The latest iteration of the GPT series [112]
- 223 • **Claude 3** (Anthropic): Emphasizes reasoning and safety [6]
- 224 • **Gemini 1.5** (Google): Multimodal with extended context [4]
- 225 • **Grok** (xAI): Designed for real-time information access
- 226 • **DeepSeek** (DeepSeek AI): Open-weight model with strong reasoning

Table 1: Main results on SpatialOps. ES: Efficacy Score, ACC: Accuracy, HLR: Human-AI Latency Ratio. Tier scores represent average accuracy within each tier.

Model	ES	ACC	HLR	Tier 1	Tier 2	Tier 3
GPT-5.2	78.4	72.5	847×	85.2	72.1	60.3
Claude 3	73.8	67.9	792×	80.1	67.8	55.6
Gemini 1.5	68.2	62.7	756×	74.5	62.4	51.2
Grok	61.8	56.3	634×	68.3	56.1	45.8
DeepSeek	56.0	49.9	589×	62.1	50.2	40.5

Table 2: Ablation study: Impact of prompt detail on Efficacy Score.

Model	Minimal Prompt	Detailed Prompt
GPT-5.2	68.2	78.4
Claude 3	63.5	73.8
Gemini 1.5	58.1	68.2
Grok	52.4	61.8
DeepSeek	46.3	56.0

227 5.2 Main Results

228 Table 1 presents the main results across all models and metrics. GPT-5.2 achieves the highest overall
 229 Efficacy Score (78.4), followed by Claude 3 (73.8) and Gemini 1.5 (68.2). All models show significant
 230 performance degradation from Tier 1 to Tier 3 tasks, indicating that advanced spatial optimization
 231 remains challenging.

232 5.3 Category-Level Analysis

233 Performance varies substantially across categories. All models perform well on Coordinate Under-
 234 standing (CU) and Distance Computation (DC), with accuracies exceeding 80% for top models.
 235 However, performance drops sharply for Constraint-Based Placement (CBP) and Resource Allocation
 236 (RAO), where even GPT-5.2 achieves only 52.3% and 48.7% accuracy respectively.

237 Navigation and Pathfinding (NP) reveals interesting patterns. While models can often identify correct
 238 paths in simple grids, they struggle with A* algorithm simulation on larger graphs, frequently making
 239 suboptimal choices or failing to properly account for heuristics.

240 5.4 Ablation Studies

241 **Impact of Prompt Detail:** Table 2 shows that detailed prompts with explicit spatial reasoning
 242 instructions improve performance by 10-15% across all models, suggesting that models benefit from
 243 structured guidance for spatial tasks.

244 **Impact of Chain-of-Thought:** Explicit chain-of-thought prompting [150] improves performance on
 245 Tier 2 and Tier 3 tasks by 8-12%, with larger gains on tasks requiring multi-step reasoning.

246 **Difficulty Scaling:** Performance degrades approximately linearly with difficulty level for Tier 1 tasks
 247 but shows steeper degradation for Tier 2 and 3, suggesting that complex spatial optimization poses
 248 qualitatively different challenges.

249 5.5 Error Analysis

250 We conducted detailed error analysis on 500 randomly sampled incorrect responses. The most
 251 common error types are:

- 252 1. **Algorithmic Errors (34%):** Incorrect application of spatial algorithms (e.g., A*, visibility
 253 computation)

- 254 2. **Constraint Violations (28%)**: Solutions that violate stated spatial constraints
255 3. **Numerical Errors (19%)**: Incorrect distance or coordinate calculations
256 4. **Incomplete Reasoning (12%)**: Partial solutions that miss required components
257 5. **Misinterpretation (7%)**: Misunderstanding of task requirements

258 **6 Discussion**

259 **6.1 Implications for Spatial AI**

260 Our results reveal a significant gap between current LLM capabilities and the requirements of practical
261 spatial AI applications. While models perform adequately on foundational tasks, their performance
262 on advanced optimization problems remains far below human expert levels. This suggests that current
263 architectures may lack the inductive biases necessary for robust spatial reasoning [11, 18].

264 The strong performance gains from detailed prompting indicate that models possess latent spatial
265 reasoning capabilities that are not reliably activated by default. This aligns with findings on prompt
266 sensitivity in other domains [162, 99] and suggests that improved prompting strategies or fine-tuning
267 could yield substantial gains.

268 **6.2 Comparison with Existing Benchmarks**

269 SpatialOps complements existing benchmarks by focusing on practical 2D spatial planning. While
270 SpatialBench [154] evaluates VLM spatial understanding and GeoAnalystBench [160] focuses on
271 GIS workflows, SpatialOps uniquely addresses the operational planning tasks critical for enterprise
272 applications. The breadth of our benchmark, spanning twelve categories and three complexity tiers,
273 enables more comprehensive assessment than narrower alternatives.

274 **6.3 Limitations**

275 Several limitations should be noted. First, our benchmark focuses on 2D spatial reasoning; extension
276 to 3D would require substantial additional work. Second, while we ground tasks in industry use
277 cases, the procedurally generated nature of tasks may not capture all real-world complexities. Third,
278 our evaluation of reasoning quality relies on LLM-as-judge, which may have systematic biases
279 [163, 147].

280 **7 Conclusion**

281 We introduced SpatialOps, a comprehensive benchmark for evaluating 2D spatial planning and
282 reasoning in Large Language Models. Our benchmark comprises 6,012 tasks across twelve categories,
283 grounded in real-world industry applications and evaluated using a multi-faceted methodology.
284 Extensive experiments reveal significant gaps in current model capabilities, particularly for advanced
285 optimization tasks. We hope SpatialOps will serve as a valuable resource for tracking progress and
286 guiding research toward more spatially capable AI systems.

287 **References**

- 288 [1] Michael Ahn, Anthony Brohan, Noah Brown, Yevgen Chebotar, Omar Cortes, Byron David,
289 Chelsea Finn, Chuyuan Fu, Keerthana Gober, Karol Gopalakrishnan, et al. Do as i can, not as
290 i say: Grounding language in robotic affordances. *arXiv preprint arXiv:2204.01691*, 2022.
- 291 [2] Scale AI. Donovan: Ai for intelligence. <https://scale.com/donovan>, 2025.
- 292 [3] Peter Anderson, Qi Wu, Damien Teney, Jake Bruce, Mark Johnson, Niko Sunderhauf, Ian Reid,
293 Stephen Gould, and Anton van den Hengel. Vision-and-language navigation: Interpreting
294 visually-grounded navigation instructions in real environments. In *Proceedings of the IEEE*
295 *Conference on Computer Vision and Pattern Recognition*, pages 3674–3683, 2018.

- 296 [4] Rohan Anil, Andrew M Dai, Orhan Firat, Melvin Johnson, Dmitry Lepikhin, Alexandre Passos,
297 Siamak Shakeri, Emanuel Taropa, Paige Bailey, Zhifeng Chen, et al. Palm 2 technical report.
298 *arXiv preprint arXiv:2305.10403*, 2023.
- 299 [5] Mihael Ankerst et al. Optics: Ordering points to identify the clustering structure. *ACM
300 SIGMOD Record*, 28(2):49–60, 1999.
- 301 [6] Anthropic. Claude 3 model card. *Anthropic Technical Report*, 2024.
- 302 [7] Lei Bai et al. Adaptive graph convolutional recurrent network for traffic forecasting. *Advances
303 in Neural Information Processing Systems*, 2020.
- 304 [8] Lei Bai, Lina Yao, Salil S Kanhere, Xianzhi Wang, and Quan Z Sheng. Stg2seq: Spatial-
305 temporal graph to sequence model for multi-step passenger demand forecasting. *IJCAI*,
306 2019.
- 307 [9] Yejin Bang et al. A multitask, multilingual, multimodal evaluation of chatgpt on reasoning,
308 hallucination, and interactivity. *arXiv preprint arXiv:2302.04023*, 2023.
- 309 [10] Albert-László Barabási. *Network science*. Cambridge University Press, 2016.
- 310 [11] Peter W Battaglia, Jessica B Hamrick, Victor Bapst, et al. Relational inductive biases, deep
311 learning, and graph networks. *arXiv preprint arXiv:1806.01261*, 2018.
- 312 [12] Michael T Battista. The development of geometric and spatial thinking. *Second Handbook of
313 Research on Mathematics Teaching and Learning*, pages 843–908, 2007.
- 314 [13] Michael Batty. *Big data, smart cities and city planning*, volume 3. 2013.
- 315 [14] Simon Elias Bibri and John Krogstie. Smart sustainable cities of the future: An extensive
316 interdisciplinary literature review. *Sustainable Cities and Society*, 2017.
- 317 [15] Yonatan Bisk, Ari Holtzman, Jesse Thomason, Jacob Andreas, Yoshua Bengio, Joyce Chai,
318 Mirella Lapata, Angeliki Lazaridou, Jonathan May, Aleksandr Nisnevich, et al. Experience
319 grounds language. *arXiv preprint arXiv:2004.10151*, 2020.
- 320 [16] Mariusz Bojarski, Davide Del Testa, Daniel Dworakowski, et al. End to end learning for
321 self-driving cars. *arXiv preprint arXiv:1604.07316*, 2016.
- 322 [17] Anthony Brohan, Noah Brown, Justice Carbajal, Yevgen Chebotar, Xi Chen, Krzysztof Choro-
323 manski, Tianli Ding, Danny Driess, Avinava Dubey, Chelsea Finn, et al. Rt-2: Vision-language-
324 action models transfer web knowledge to robotic control. *arXiv preprint arXiv:2307.15818*,
325 2023.
- 326 [18] Michael M Bronstein et al. Geometric deep learning. *arXiv preprint arXiv:2104.13478*, 2021.
- 327 [19] Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhari-
328 wal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language
329 models are few-shot learners. *Advances in Neural Information Processing Systems*, 33:1877–
330 1901, 2020.
- 331 [20] Neil Burgess. Spatial memory: How egocentric and allocentric combine. *Trends in Cognitive
332 Sciences*, 10(12):551–557, 2006.
- 333 [21] Neil Burgess, Eleanor A Maguire, and John O’Keefe. The human hippocampus and spatial
334 and episodic memory. *Neuron*, 35(4):625–641, 2002.
- 335 [22] John B Carroll. *Human cognitive abilities: A survey of factor-analytic studies*. Cambridge
336 University Press, 1993.
- 337 [23] Ziwei Chai et al. Graphilm: Boosting graph reasoning ability of large language model. *arXiv
338 preprint arXiv:2310.05845*, 2023.
- 339 [24] Chenyi Chen et al. Deepdriving: Learning affordance for direct perception in autonomous
340 driving. *ICCV*, 2015.

- 341 [25] Howard Chen, Alane Suhr, Dipendra Misra, Noah Snavely, and Yoav Artzi. Touchdown:
 342 Natural language navigation and spatial reasoning in visual street environments. In *Proceedings*
 343 *of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 12538–
 344 12547, 2019.
- 345 [26] Mark Chen, Jerry Tworek, Heewoo Jun, Qiming Yuan, Henrique Ponde de Oliveira Pinto,
 346 Jared Kaplan, Harri Edwards, Yuri Burda, Nicholas Joseph, Greg Brockman, et al. Evaluating
 347 large language models trained on code. *arXiv preprint arXiv:2107.03374*, 2021.
- 348 [27] Wei Chen et al. Mapbench: Evaluating llms on map reading and spatial reasoning. *arXiv*
 349 *preprint arXiv:2404.00001*, 2024.
- 350 [28] Wenhui Chen et al. Program of thoughts prompting: Disentangling computation from reasoning
 351 for numerical reasoning tasks. *arXiv preprint arXiv:2211.12588*, 2022.
- 352 [29] X. Chen. Application of gnn in urban computing. In *2020 5th International Conference on*
 353 *Smart and Sustainable City (ICSSC)*, pages 1–4. IEEE, 2020.
- 354 [30] Zhaohan Chen et al. Spatial reasoning in multimodal large language models: A survey. *arXiv*
 355 *preprint arXiv:2511.15722*, 2024.
- 356 [31] Douglas H Clements and Michael T Battista. *Geometry and spatial reasoning*. NCTM, 2001.
- 357 [32] Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser,
 358 Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, et al. Training verifiers to
 359 solve math word problems. In *arXiv preprint arXiv:2110.14168*, 2021.
- 360 [33] Anthony G Cohn and Shyamanta M Hazarika. Qualitative spatial representation and reasoning:
 361 An overview. *Fundamenta Informaticae*, 46(1-2):1–29, 1997.
- 362 [34] Anthony G Cohn and Shyamanta M Hazarika. Qualitative spatial representation and reasoning:
 363 An overview. *Fundamenta informaticae*, 2001.
- 364 [35] Ernest Davis. Ontologies for spatial reasoning. *Spatial Cognition & Computation*, 13(4):293–
 365 323, 2013.
- 366 [36] Stefano De Sabbata and Pengyuan Liu. A graph neural network framework for spatial
 367 geodemographic classification. *International Journal of Geographical Information Science*,
 368 37(12):2464–2486, 2023.
- 369 [37] Xiang Deng, Yu Gu, Boyuan Zheng, Shijie Chen, Sam Stevens, Boshi Wang, Huan Sun, and
 370 Yu Su. Mind2web: Towards a generalist agent for the web. *arXiv preprint arXiv:2306.06070*,
 371 2024.
- 372 [38] Michel Marie Deza and Elena Deza. *Encyclopedia of distances*. Springer, 2009.
- 373 [39] Edsger W Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematik*,
 374 1(1):269–271, 1959.
- 375 [40] Nouha Dziri et al. Faith and fate: Limits of transformers on compositionality. *NeurIPS*, 2023.
- 376 [41] Arne D Ekstrom et al. Cellular networks underlying human spatial navigation. *Nature*,
 377 425(6954):184–188, 2014.
- 378 [42] Russell A Epstein et al. The cognitive map in humans: Spatial navigation and beyond. *Nature*
 379 *Neuroscience*, 20(11):1504–1513, 2017.
- 380 [43] Martin Ester et al. A density-based algorithm for discovering clusters in large spatial databases
 381 with noise. *KDD*, pages 226–231, 1996.
- 382 [44] Gloria Felicia et al. From perception to action: Spatial ai agents and world models. *arXiv*
 383 *preprint arXiv:2602.01644*, 2026.
- 384 [45] Kenneth D Forbus. Qualitative process theory. *Artificial Intelligence*, 24(1-3):85–168, 1984.

- 385 [46] Andrew U Frank. Qualitative spatial reasoning: Cardinal directions as an example. *International Journal of Geographical Information Science*, 10(3):269–290, 1996.
- 386
- 387 [47] Yanjie Fu et al. Real estate ranking via mixed land-use latent factor model. *KDD*, pages 1927–1936, 2019.
- 388
- 389 [48] Luyu Gao et al. Pal: Program-aided language models. *International Conference on Machine Learning*, 2023.
- 390
- 391 [49] Artur d’Avila Garcez and Luis C Lamb. Neural-symbolic computing: An effective methodology for principled integration of machine learning and reasoning. *Journal of Applied Logics*, 6(4):611–632, 2019.
- 392
- 393
- 394 [50] Subir Kumar Ghosh. *Visibility algorithms in the plane*. Cambridge University Press, 2007.
- 395
- 396 [51] Justin Gilmer, Samuel S Schoenholz, Patrick F Riley, Oriol Vinyals, and George E Dahl. Neural message passing for quantum chemistry. In *ICML*, 2017.
- 397
- 398 [52] Michael F Goodchild. Citizens as sensors: the world of volunteered geography. *GeoJournal*, 2007.
- 399
- 400 [53] Google. Google earth engine. <https://earthengine.google.com>, 2025.
- 401
- 402
- 403 [54] Alex Graves, Greg Wayne, Malcolm Reynolds, Tim Harley, Ivo Danihelka, Agnieszka Grabska-Barwińska, Sergio Gómez Colmenarejo, Edward Grefenstette, Tiago Ramalho, John Agapiou, et al. Hybrid computing using a neural network with dynamic external memory. *Nature*, 538(7626):471–476, 2016.
- 404
- 405 [55] William L Hamilton, Rex Ying, and Jure Leskovec. Inductive representation learning on large graphs. *Advances in Neural Information Processing Systems*, 30, 2017.
- 406
- 407
- 408 [56] Peter E Hart, Nils J Nilsson, and Bertram Raphael. A formal basis for the heuristic determination of minimum cost paths. *IEEE Transactions on Systems Science and Cybernetics*, 4(2):100–107, 1968.
- 409
- 410 [57] Mary Hegarty. Diagrams in the mind and in the world: Relations between internal and external visualizations. *Diagrammatic Representation and Inference*, pages 1–13, 2004.
- 411
- 412 [58] Mary Hegarty. Spatial thinking in undergraduate science education. *Spatial Intelligence: Why It Matters from Birth Through the Lifespan*, pages 39–52, 2006.
- 413
- 414 [59] Mary Hegarty and David Waller. Individual differences in spatial abilities. *The Cambridge Handbook of Visuospatial Thinking*, pages 121–169, 2002.
- 415
- 416 [60] Dan Hendrycks et al. Measuring mathematical problem solving with the math dataset. *NeurIPS*, 2021.
- 417
- 418 [61] Dorit S Hochbaum and David B Shmoys. A best possible heuristic for the k-center problem. *Mathematics of Operations Research*, 10(2):180–184, 1985.
- 419
- 420
- 421 [62] Drew A Hudson and Christopher D Manning. Gqa: A new dataset for real-world visual reasoning and compositional question answering. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 6700–6709, 2019.
- 422
- 423 [63] Shima Imani et al. Mathprompter: Mathematical reasoning using large language models. *ACL*, 2023.
- 424
- 425 [64] Wherobots Inc. Wherobots: Cloud-native spatial intelligence. <https://wherobots.com>, 2026.
- 426
- 427 [65] Weiwei Jiang and Jiayun Luo. Graph neural networks for traffic forecasting: A survey. *arXiv preprint arXiv:2101.11174*, 2022.
- 428
- 429
- 430 [66] Carlos E Jimenez, John Yang, Alexander Wettig, Shunyu Yao, Kexin Pei, Ofir Press, and Karthik Narasimhan. Swe-bench: Can language models resolve real-world github issues? *arXiv preprint arXiv:2310.06770*, 2024.

- 431 [67] Guangyin Jin, Yuxuan Liang, Yuchen Fang, Zezhi Huang, Junbo Zhang, and Yu Zheng.
 432 Spatio-temporal graph neural networks for urban computing: A survey. *IEEE Transactions on*
 433 *Knowledge and Data Engineering*, 2023.
- 434 [68] Justin Johnson, Bharath Hariharan, Laurens van der Maaten, Li Fei-Fei, C Lawrence Zitnick,
 435 and Ross Girshick. Clevr: A diagnostic dataset for compositional language and elementary
 436 visual reasoning. In *Proceedings of the IEEE Conference on Computer Vision and Pattern*
 437 *Recognition*, pages 2901–2910, 2017.
- 438 [69] Dmitry Kalashnikov et al. Scalable deep reinforcement learning for vision-based robotic
 439 manipulation. *CoRL*, 2018.
- 440 [70] KAUST. Planqa: A diagnostic benchmark for spatial reasoning in llms. *arXiv preprint*, 2025.
- 441 [71] Thomas N Kipf and Max Welling. Semi-supervised classification with graph convolutional
 442 networks. *arXiv preprint arXiv:1609.02907*, 2017.
- 443 [72] Roberta L Klatzky. Allocentric and egocentric spatial representations: Definitions, distinctions,
 444 and interconnections. *Spatial Cognition*, pages 1–17, 1998.
- 445 [73] Jan-Christoph Klie et al. Annotation error detection: Analyzing the past and present for a
 446 more coherent future. *Computational Linguistics*, 49(1):157–198, 2023.
- 447 [74] Sven Koenig, Maxim Likhachev, and David Furcy. Lifelong planning a*. *Artificial Intelligence*,
 448 155(1-2):93–146, 2004.
- 449 [75] Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa.
 450 Large language models are zero-shot reasoners. *Advances in Neural Information Processing*
 451 *Systems*, 35:22199–22213, 2022.
- 452 [76] Eric Kolve, Roozbeh Mottaghi, Winson Han, Eli VanderBilt, Luca Weihs, Alvaro Herrasti,
 453 Daniel Gordon, Yuke Zhu, Abhinav Gupta, and Ali Farhadi. Ai2-thor: An interactive 3d
 454 environment for visual ai. *arXiv preprint arXiv:1712.05474*, 2017.
- 455 [77] Maria Kozhevnikov, Stephen Kosslyn, and Jennifer Shephard. Spatial versus object visualizers:
 456 A new characterization of visual cognitive style. *Memory & Cognition*, 33(4):710–726, 2006.
- 457 [78] Dwight J Kravitz et al. A new neural framework for visuospatial processing. *Nature Reviews*
 458 *Neuroscience*, 12(4):217–230, 2011.
- 459 [79] Benjamin Kuipers. Modeling spatial knowledge. *Cognitive Science*, 2(2):129–153, 1978.
- 460 [80] Vipin Kumar. Algorithms for constraint-satisfaction problems: A survey. *AI Magazine*,
 461 13(1):32–32, 1992.
- 462 [81] AI4CE Lab. Spare3d: A dataset for spatial reasoning on three-view line drawings. *GitHub*
 463 *Repository*, 2024.
- 464 [82] Luis C Lamb et al. Graph neural networks meet neural-symbolic computing: A survey and
 465 perspective. *IJCAI*, 2020.
- 466 [83] Stephen Law et al. Take a look around: Using street view and satellite images to estimate
 467 house prices. *ACM SIGKDD Explorations Newsletter*, 21(2):54–65, 2019.
- 468 [84] Sergey Levine, Chelsea Finn, Trevor Darrell, and Pieter Abbeel. End-to-end training of deep
 469 visuomotor policies. *JMLR*, 2016.
- 470 [85] Patrick Lewis, Ethan Perez, Aleksandra Piktus, Fabio Petroni, Vladimir Karpukhin, Na-
 471 man Goyal, Heinrich Kuttler, Mike Lewis, Wen-tau Yih, Tim Rocktaschel, et al. Retrieval-
 472 augmented generation for knowledge-intensive nlp tasks. *Advances in Neural Information*
 473 *Processing Systems*, 33:9459–9474, 2020.
- 474 [86] Jingwen Li et al. Learning to optimize industry-scale dynamic pickup and delivery problems.
 475 *ICDE*, 2019.

- 476 [87] Maosen Li et al. Dynamic multiscale graph neural networks for 3d skeleton based human
477 motion prediction. *CVPR*, 2020.
- 478 [88] Xiang Li et al. Benchmarking spatial reasoning in large language models. *arXiv preprint*,
479 2025.
- 480 [89] Yaguang Li, Rose Yu, Cyrus Shahabi, and Yan Liu. Diffusion convolutional recurrent neural
481 network: Data-driven traffic forecasting. In *International Conference on Learning Representations*, 2018.
- 482
- 483 [90] Yujia Li et al. Competition-level code generation with alphacode. *Science*, 378(6624):1092–
484 1097, 2022.
- 485 [91] Percy Liang et al. Holistic evaluation of language models. *arXiv preprint arXiv:2211.09110*,
486 2022.
- 487 [92] Shuxin Liao et al. Rethinking benchmark and contamination for language models with
488 rephrased samples. *arXiv preprint arXiv:2311.04850*, 2023.
- 489 [93] Lynn S Liben. Spatial development. *Handbook of Child Psychology*, 2006.
- 490 [94] Gérard Ligozat. Reasoning about cardinal directions. *Journal of Visual Languages & Comput-*
491 *ing*, 9(1):23–44, 1998.
- 492 [95] Maxim Likhachev et al. Anytime dynamic a*: An anytime, replanning algorithm. *ICAPS*,
493 2005.
- 494 [96] Xiao Liu, Hao Yu, Hanchen Zhang, Yifan Xu, Xuanyu Lei, Hanyu Lai, Yu Gu, Hangliang
495 Ding, Kaiwen Men, Kejuan Yang, et al. Agentbench: Evaluating llms as agents. *arXiv preprint*
496 *arXiv:2308.03688*, 2023.
- 497 [97] David F Lohman. Spatial ability: A review and reanalysis of the correlational literature.
498 *Technical Report*, 1979.
- 499 [98] Paul A Longley et al. *Geographic information science and systems*. John Wiley & Sons, 2015.
- 500 [99] Yao Lu et al. Fantastically ordered prompts and where to find them: Overcoming few-shot
501 prompt order sensitivity. *ACL*, 2022.
- 502 [100] Wufei Ma, Haoyu Chen, Guofeng Zhang, Yu-Cheng Chou, Jieneng Chen, Celso de Melo, and
503 Alan Yuille. 3dsrbench: A comprehensive 3d spatial reasoning benchmark. In *Proceedings*
504 *of the IEEE/CVF International Conference on Computer Vision (ICCV)*, pages 6924–6934,
505 October 2025.
- 506 [101] Arjun Majumdar, Anurag Ajay, Xiaohan Zhang, Pranav Putta, Sriram Yenamandra, Mikael
507 Henaff, Sneha Silwal, Paul Mcvay, Oleksandr MakSYMets, Sergio Arnaud, Karmesh Yadav,
508 Qiyang Li, Ben Newman, Mohit Sharma, Vincent Berges, Shiqi Zhang, Pulkit Agrawal,
509 Yonatan Bisk, Dhruv Batra, Mrinal Kalakrishnan, Franziska Meier, Chris Paxton, Alexander
510 Sax, and Aravind Rajeswaran. Openeqa: Embodied question answering in the era of foundation
511 models. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern*
512 *Recognition (CVPR)*, pages 16488–16498, June 2024.
- 513 [102] Wei Mao et al. Learning trajectory dependencies for human motion prediction. *ICCV*, 2019.
- 514 [103] Gary Marcus. Deep learning: A critical appraisal. *arXiv preprint arXiv:1801.00631*, 2018.
- 515 [104] Mark G McGee. Human spatial abilities: Psychometric studies and environmental, genetic,
516 hormonal, and neurological influences. *Psychological Bulletin*, 86(5):889, 1979.
- 517 [105] Roshanak Mirpuri, Reza Mirzaee, and Parisa Kordjamshidi. Spartqa: A textual question
518 answering benchmark for spatial reasoning. *arXiv preprint arXiv:2104.05832*, 2023.
- 519 [106] Weimin Mou and Timothy P McNamara. Intrinsic frames of reference in spatial memory.
520 *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 30(2):339, 2004.

- 521 [107] Reiichiro Nakano, Jacob Hilton, Suchir Balaji, Jeff Wu, Long Ouyang, Christina Kim, Christopher Hesse, Shantanu Jain, Vineet Kosaraju, William Saunders, et al. Webgpt: Browser-assisted
 522 question-answering with human feedback. *arXiv preprint arXiv:2112.09332*, 2021.
- 524 [108] Mohammadreza Nazari et al. Reinforcement learning for solving the vehicle routing problem.
 525 *NeurIPS*, 2018.
- 526 [109] Nora S Newcombe. Picture this: Increasing math and science learning by improving spatial
 527 thinking. *American Educator*, 34(2):29, 2010.
- 528 [110] Mark Newman. *Networks*. Oxford University Press, 2018.
- 529 [111] Curtis G Northcutt et al. Pervasive label errors in test sets destabilize machine learning
 530 benchmarks. *NeurIPS*, 2021.
- 531 [112] OpenAI. Gpt-4 technical report. *arXiv preprint arXiv:2303.08774*, 2023.
- 532 [113] Roma Patel and Ellie Pavlick. Mapping language models to grounded conceptual spaces.
 533 *ICLR*, 2021.
- 534 [114] Jean Piaget and Bärbel Inhelder. The child’s conception of space. 1956.
- 535 [115] Dean A Pomerleau. Alvinn: An autonomous land vehicle in a neural network. 1988.
- 536 [116] Franco P Preparata and Michael Shamos. *Computational geometry: An introduction*. Springer,
 537 1985.
- 538 [117] Ofir Press et al. Measuring and narrowing the compositionality gap in language models. *arXiv
 539 preprint arXiv:2210.03350*, 2022.
- 540 [118] Charles R Qi, Hao Su, Kaichun Mo, and Leonidas J Guibas. Pointnet: Deep learning on point
 541 sets for 3d classification and segmentation. In *CVPR*, 2017.
- 542 [119] Yujia Qin, Shihao Liang, Yining Ye, et al. Toolllm: Facilitating large language models to
 543 master 16000+ real-world apis. In *ICLR*, 2024.
- 544 [120] Inioluwa Deborah Raji, Emily M Bender, Amandalynne Paullada, Emily Denton, and Alex
 545 Hanna. Ai and the everything in the whole wide world benchmark. *NeurIPS Datasets and
 546 Benchmarks*, 2021.
- 547 [121] David A Randell, Zhan Cui, and Anthony G Cohn. A spatial logic based on regions and
 548 connection. *KR*, 92:165–176, 1992.
- 549 [122] Jochen Renz and Bernhard Nebel. Qualitative spatial reasoning using constraint calculi.
 550 *Handbook of Spatial Logics*, pages 161–215, 2007.
- 551 [123] Bernardino Romera-Paredes et al. Mathematical discoveries from program search with large
 552 language models. *Nature*, 625(7995):468–475, 2024.
- 553 [124] Francesca Rossi, Peter Van Beek, and Toby Walsh. *Handbook of constraint programming*.
 554 Elsevier, 2006.
- 555 [125] Manolis Savva, Abhishek Kadian, Oleksandr Maksymets, Yili Zhao, Erik Wijmans, Bhavana
 556 Jain, Julian Straub, Jia Liu, Vladlen Koltun, Jitendra Malik, et al. Habitat: A platform for
 557 embodied ai research. In *Proceedings of the IEEE/CVF International Conference on Computer
 558 Vision*, pages 9339–9347, 2019.
- 559 [126] Timo Schick, Jane Dwivedi-Yu, Roberto Dessi, Roberta Raileanu, Maria Lomeli, Luke Zettlemoyer,
 560 Nicola Cancedda, and Thomas Scialom. Toolformer: Language models can teach
 561 themselves to use tools. *arXiv preprint arXiv:2302.04761*, 2023.
- 562 [127] Steven Schockaert, Martine De Cock, and Etienne E Kerre. Fuzzy spatial reasoning. *Handbook
 563 of Research on Fuzzy Information Processing in Databases*, pages 102–133, 2008.

- 564 [128] Kristof Schütt, Pieter-Jan Kindermans, Huziel Enoc Sauceda Felix, Stefan Chmiela, Alexandre
 565 Tkatchenko, and Klaus-Robert Müller. Schnet: A continuous-filter convolutional neural
 566 network for modeling quantum interactions. *NeurIPS*, 30, 2017.
- 567 [129] Ahsan Shehzad, Feng Xia, Shagufta Abid, Chao Peng, Shuo Yu, Dongyu Zhang, and Karin
 568 Verspoor. Graph transformers: A survey. *arXiv preprint arXiv:2407.09777*, 2024.
- 569 [130] Roger N Shepard and Jacqueline Metzler. Mental rotation of three-dimensional objects.
 570 *Science*, 171(3972):701–703, 1971.
- 571 [131] Zhengxiang Shi et al. Stepgame: A new benchmark for robust multi-hop spatial reasoning in
 572 texts. *AAAI*, 2022.
- 573 [132] Alane Suhr, Stephanie Zhou, Ally Zhang, Iris Zhang, Huajun Bai, and Yoav Artzi. A corpus for
 574 reasoning about natural language grounded in photographs. *arXiv preprint arXiv:1811.00491*,
 575 2019.
- 576 [133] Sainbayar Sukhbaatar et al. End-to-end memory networks. In *NeurIPS*, 2015.
- 577 [134] Palantir Technologies. Project maven: Ai for defense. <https://www.palantir.com>, 2024.
- 578 [135] Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timo-
 579 thee Lacroix, Baptiste Roziere, Naman Goyal, Eric Hambro, Faisal Azhar, et al. Llama: Open
 580 and efficient foundation language models. *arXiv preprint arXiv:2302.13971*, 2023.
- 581 [136] Trieu H Trinh et al. Solving olympiad geometry without human demonstrations. *Nature*,
 582 625(7995):476–482, 2024.
- 583 [137] Barbara Tversky. Functional significance of visuospatial representations. *Handbook of
 584 Higher-Level Visuospatial Thinking*, pages 1–34, 2005.
- 585 [138] UN-Habitat. Ai for spatial mapping and analysis: Geoui toolkit for urban planners. Technical
 586 report, United Nations Human Settlements Programme (UN-Habitat), 2025.
- 587 [139] Jorge Urrutia. Art gallery and illumination problems. *Handbook of Computational Geometry*,
 588 pages 973–1027, 2000.
- 589 [140] David H Uttal et al. The malleability of spatial skills: A meta-analysis of training studies.
 590 *Psychological Bulletin*, 139(2):352, 2013.
- 591 [141] Steven G Vandenberg and Allan R Kuse. Mental rotations, a group test of three-dimensional
 592 spatial visualization. *Perceptual and Motor Skills*, 47(2):599–604, 1978.
- 593 [142] Vijay V Vazirani. *Approximation algorithms*. Springer, 2001.
- 594 [143] Petar Velickovic, Guillem Cucurull, Arantxa Casanova, Adriana Romero, Pietro Lio, and
 595 Yoshua Bengio. Graph attention networks. *arXiv preprint arXiv:1710.10903*, 2018.
- 596 [144] Jonathan Wai, David Lubinski, and Camilla P Benbow. Spatial ability for stem domains:
 597 Aligning over 50 years of cumulative psychological knowledge solidifies its importance.
 598 *Journal of Educational Psychology*, 101(4):817, 2009.
- 599 [145] David Waller and Yvonne Lippa. Landmarks as beacons and associative cues: Their role in
 600 route learning. *Memory & Cognition*, 35(5):910–924, 2007.
- 601 [146] Jianhui Wang et al. Power system state estimation via deep learning. *IEEE Transactions on
 602 Smart Grid*, 12(2):1152–1162, 2021.
- 603 [147] Peiyi Wang et al. Large language models are not fair evaluators. *arXiv preprint
 604 arXiv:2305.17926*, 2023.
- 605 [148] Senzhang Wang, Jiannong Cao, and Philip S Yu. Deep learning for spatio-temporal data
 606 mining: A survey. *IEEE TKDE*, 2020.
- 607 [149] Yue Wang, Yongbin Sun, Ziwei Liu, Sanjay E Sarma, Michael M Bronstein, and Justin M
 608 Solomon. Dynamic graph cnn for learning on point clouds. In *ACM TOG*, 2019.

- 609 [150] Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed Chi,
 610 Quoc Le, and Denny Zhou. Chain-of-thought prompting elicits reasoning in large language
 611 models. *Advances in Neural Information Processing Systems*, 35:24824–24837, 2022.
- 612 [151] Jason Weston, Antoine Bordes, Sumit Chopra, Alexander M Rush, Bart Van Merriënboer,
 613 Armand Joulin, and Tomas Mikolov. Towards ai-complete question answering: A set of
 614 prerequisite toy tasks. *arXiv preprint arXiv:1502.05698*, 2015.
- 615 [152] Thomas Wolbers and Mary Hegarty. What determines our navigational abilities? *Trends in
 616 Cognitive Sciences*, 14(3):138–146, 2010.
- 617 [153] Zonghan Wu, Shirui Pan, Guodong Long, Jing Jiang, and Chengqi Zhang. Graph wavenet
 618 for deep spatial-temporal graph modeling. In *Proceedings of the Twenty-Eighth International
 619 Joint Conference on Artificial Intelligence*, pages 1907–1913, 2019.
- 620 [154] Yongyang Xu, Bo Zhou, Shuai Jin, Xuejing Xie, and Nan He. A framework for urban land use
 621 classification by integrating the spatial context of points of interest and graph convolutional
 622 neural network method. *Comput. Environ. Urban Syst.*, 94:101807, 2022.
- 623 [155] Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Thomas L Griffiths, Yuan Cao, and Karthik
 624 Narasimhan. Tree of thoughts: Deliberate problem solving with large language models. *arXiv
 625 preprint arXiv:2305.10601*, 2023.
- 626 [156] Shunyu Yao, Jeffrey Zhao, Dian Yu, Nan Du, Izhak Shafran, Karthik Narasimhan, and
 627 Yuan Cao. React: Synergizing reasoning and acting in language models. *arXiv preprint
 628 arXiv:2210.03629*, 2023.
- 629 [157] Bing Yu, Haoteng Yin, and Zhanxing Zhu. Spatio-temporal graph convolutional networks:
 630 A deep learning framework for traffic forecasting. In *Proceedings of the Twenty-Seventh
 631 International Joint Conference on Artificial Intelligence*, pages 3634–3640, 2018.
- 632 [158] Andy Zeng et al. Learning synergies between pushing and grasping with self-supervised deep
 633 reinforcement learning. *IROS*, 2018.
- 634 [159] Chaoyun Zhang et al. Deep learning for mobile network traffic prediction. *IEEE Network*,
 635 33(6):48–55, 2019.
- 636 [160] Wei Zhang et al. Geoanalystbench: A benchmark for gis workflow generation. *Transactions
 637 in GIS*, 2025.
- 638 [161] Yiwen Zhang et al. Graph-based planning for embodied agents. *arXiv preprint*, 2023.
- 639 [162] Zihao Zhao et al. Calibrate before use: Improving few-shot performance of language models.
 640 *ICML*, 2021.
- 641 [163] Lianmin Zheng, Wei-Lin Chiang, Ying Sheng, Siyuan Zhuang, Zhanghao Wu, Yonghao
 642 Zhuang, Zi Lin, Zuhuan Li, Dacheng Li, Eric Xing, et al. Judging llm-as-a-judge with
 643 mt-bench and chatbot arena. *Advances in Neural Information Processing Systems*, 36, 2023.
- 644 [164] Aojun Zhou et al. Solving challenging math word problems using gpt-4 code interpreter with
 645 code-based self-verification. *arXiv preprint arXiv:2308.07921*, 2023.
- 646 [165] Shuyan Zhou, Frank F Xu, Hao Zhu, Xuhui Zhou, Robert Lo, Abishek Sridhar, Xianyi Cheng,
 647 Yonatan Bisk, Daniel Fried, Uri Alon, et al. Webarena: A realistic web environment for
 648 building autonomous agents. *arXiv preprint arXiv:2307.13854*, 2023.