LOOKING INTO THE EARTH

An Introduction to Geological Geophysics

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SUBPART **I.1**

Data Acquisition and Processing

The first stage of most of the geophysical methods described in this book is making measurements in the field. Then usually follow further stages of working with the data, before they are ready for geological deductions to be drawn. As these stages are common to many geophysical methods they are described in this subpart.

Chapter 2 – **Data acquisition and reduction** *– describes the necessary basic stages, from taking measurements to converting the data into a relevant form. It also includes graphical ways of displaying the results more clearly. Chapter 3 –* **Data processing** *– describes special mathematical ways for separating wanted from unwanted parts of the results.*

Chapter 2

Data Acquisition and Reduction: Carrying Out a Geophysical Survey

This chapter describes the general principles of taking measurements – data acquisition – and the subsequent stages of correction and calculation – data reduction – often needed to make deductions about the subsurface.

A number of important geophysical terms are introduced, some of which are used in everyday language but have specialised meanings in geophysics.

2.1 Data acquisition: Taking measurements

Most geophysical measurements are made at the Earth's surface, either to save the expense and time of drilling, or because it is not feasible to go deep enough. Therefore the first actual step, after planning, to carry out a geophysical survey, is **data acquisition,** a set of measurements made with a geophysical instrument. Often the instrumental readings are taken along a line or **traverse** (Fig. 2.1a). Usually, readings are not taken continuously along the traverse but are taken at intervals – usually regular – and each place where a reading is taken is called a **station.** When the readings are plotted – often after calculations – they form a **profile** (Fig. 2.1b).

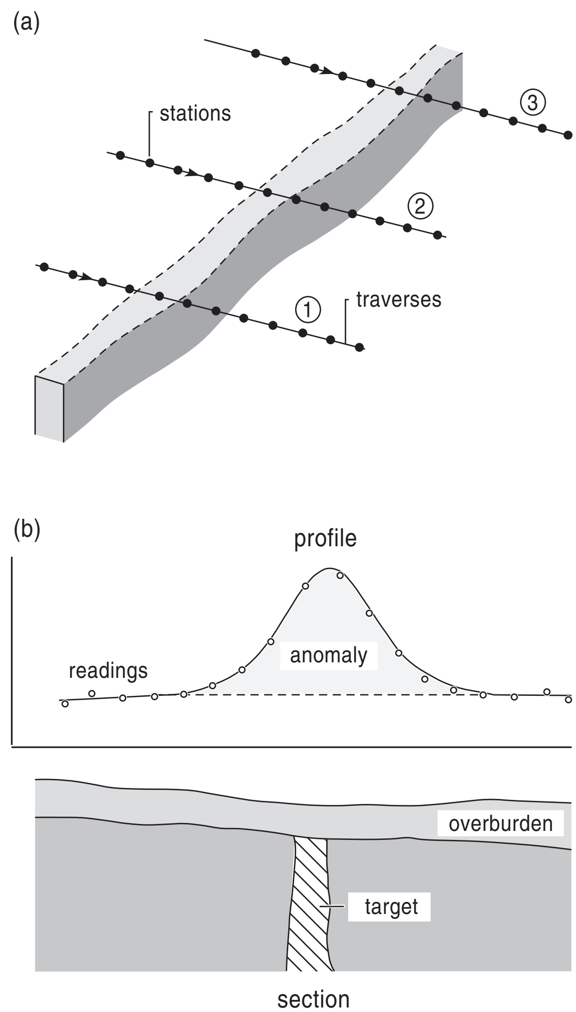

Figure 2.1 Traverses, stations, and profiles.

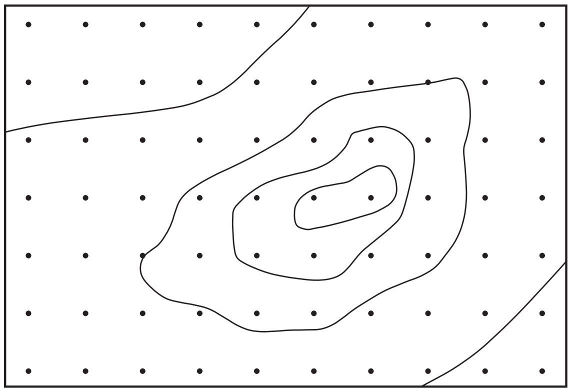

Figure 2.2 Grid of stations and contoured results.

If the **causative body** or **target** – the subsurface feature to be detected by the survey – is elongated, such as a mineral vein or a fault, the profile is best taken across it, perpendicular to the strike, so far as this can be estimated. Often, several parallel profiles are taken to see how far the body continues, or whether it changes its dimensions; if the target is not elongated, and especially if it is irregular, traverses may be close to one another, the stations then forming an array or **grid,** and the results are often contoured (Fig. 2.2).

2.2 Data reduction

Often the 'raw readings' provided by the instrument are not directly useful. For instance, in a gravity survey to detect the presence of a dense ore body by the extra pull of gravity above it, allowances, or corrections, have to be made for any undulations of the surface, for the pull of gravity also varies with height above sea level (Fig. 2.3); in a magnetic survey allowance is made for change of the Earth's magnetic field during the survey. Converting the readings into a more useful form is called **data reduction.**

The presence of a target is often revealed by an **anomaly.** In everyday language an 'anomaly' is something out of the ordinary, but in geophysics it is very common, for an anomaly is simply that part of a profile, or contour map, that is above or below the surrounding average (e.g., Figs. 2.1b and 2.3b).

Not all types of geophysical targets reveal themselves as spatial anomalies. In a seismic refraction survey (Chapter 6), the measured travel-times are plotted on a graph and from this the depths to interfaces are calculated. This too is a form of data reduction.

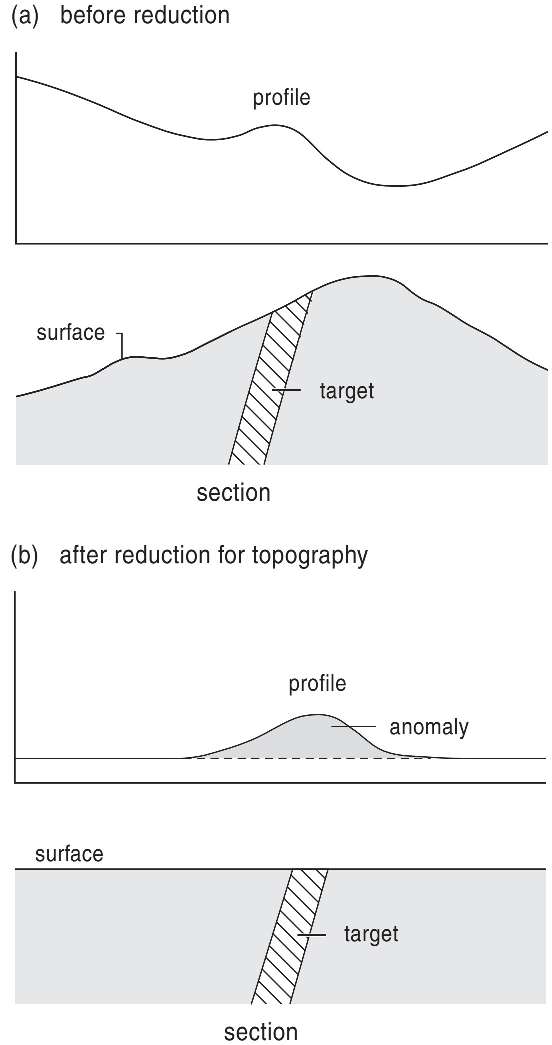

Figure 2.3 Gravity survey before and after reduction.

2.3 Signal and noise

Even after the data has been reduced, the profile may not reveal the presence of the target as clearly as one would like, because of noise. **Noise** is not sound but simply unwanted variations or fluctuations in the quantities being measured; it contrasts with the wanted part, the **signal.** In seismology, where small ground movements have to be detected, noise can be vibrations due to passing traffic or anything else that shakes the ground (Fig. 2.4a). This is not unlike the background noise at a party through which you have to try to hear your partner's conversation. Noise can also be spatial: In a magnetic survey noise may be

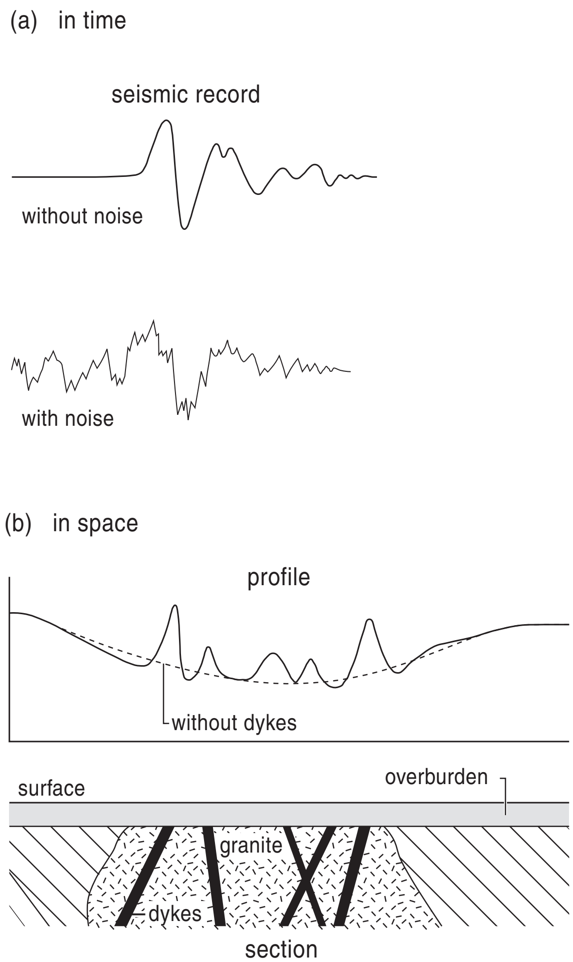

Figure 2.4 Noise (a) in time and (b) in space.

due to wire fences, or buried bits of defunct cars, which obscure the signal (i.e., the anomaly due to the buried body that has to be detected). What is noise – and what signal – may depend on the purpose of the survey; when searching for a weak magnetic anomaly of a granite intrusion the strong anomalies of basaltic dykes would be noise, whereas to a person looking for dykes they are the signal (Fig. 2.4b). Here, 'noise' is like the definition of a weed, as a plant unwanted in its place.

One common method to improve the **signal-to-noise ratio** is to repeat readings and take their average: The signal parts of each reading add, whereas the noise, usually being random, tends to cancel. This is called **stacking,** and it can also be done with profiles, which is particularly important in reflection seismology (Section 7.3). A more general method to make the wanted signal clearer is to use **signal processing.** However, as this often uses fairly sophisticated mathematics, it has been described separately in the following chapter. Improving the sound of old sound recordings, to remove hisses and scratches, is done by signal processing.

2.4 Modelling

Because a geophysical anomaly is two steps remote from a geological description (being measured at the surface, and of a physical rather than a geological quantity), there are usually two further stages. The first is to model the reduced data in physical terms. In common language, a **model** is often a small version of the real thing, such as a model boat, but in geophysical usage it is a body or structure (described by such physical properties as depth, size, density, etc.) that could account for the data measured. Figure 2.5 shows a section that could account for the observed negative (gravity) anomaly: Values calculated from the model are compared with the actual measured values to test how well it accounts for the observations.

The model is almost always simpler than the reality. This is for several reasons. Firstly, the signal observed is often 'blurred' compared with the causative body; for example, the anomaly of Figure 2.1b is wider than the target and tails off without sharp margins, which makes it hard to decide the

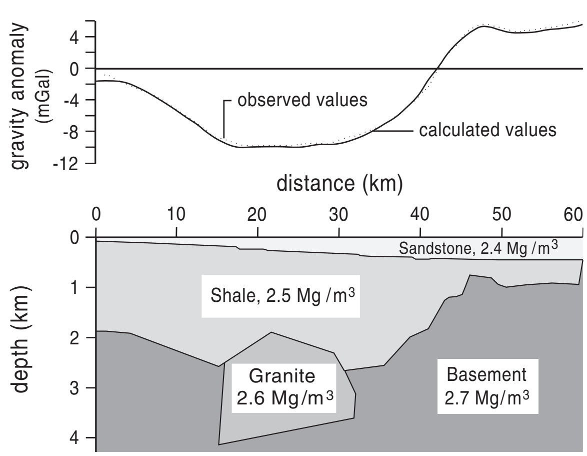

Figure 2.5 A simple density model to account for the variation in the pull of gravity.

exact position of the target. Trying to deduce the form of the causative body from the anomaly is the **inversion problem;** this is often difficult in practice and may be impossible even in theory, for more than one body could produce the results (see, e.g., Section 8.8.1 and Fig. 8.17).

Secondly, because of noise and errors of measurement, it may be difficult to discern the exact shape and size of the anomaly, as Figure 2.4b illustrates.

Thirdly, the stations may not be close enough to reveal all the details of the signal, and hence of the target (Fig. 2.6); then we say the **resolution** is not sufficient. An example of resolution is that a mosaic picture can show no detail smaller than the pieces used. Sometimes, there are theoretical as well as practical limits on resolution (e.g., distinguishing a thin layer using seismic reflection, Section 7.8.2). These factors should be taken into account when planning a survey, and decisions have to be made as to the effort worth taking to try to reduce them, supposing this is possible.

Sometimes it is possible to deduce the causative body directly from the results: this is **inverse modelling.** More often, it is easier – using a computer – to 'guess' a model, calculate the values it would produce, compare them with the observations, and then modify the model until it matches the results sufficiently well. This iterative, or trial-and-error, process is **forward modelling.** Even with modern computers the amount of computation can take a long time, so models often have to be simple, perhaps with constant cross-section – that is, two-dimensional, or 2D, models – and consisting of simple geometrical shapes, as in Figure 2.5. A more advanced type of model, though still assuming the body is elongated, allows its length to be limited; this is termed 2½D. If the model is not simple or clearly not elongated, a full 3D model may be used. Because of the various limitations, there is little point in refining the details of a model once the calculated values are indistinguishable from the measured ones.

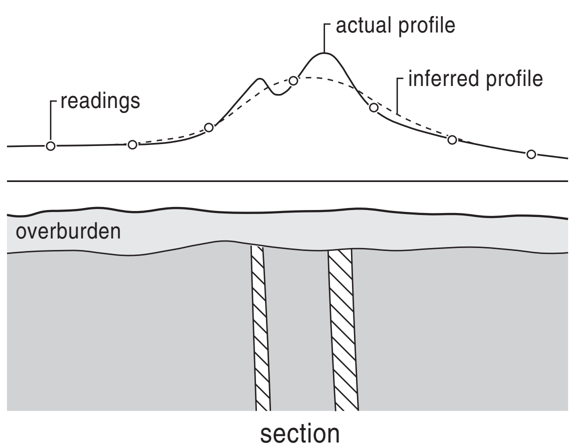

Figure 2.6 Apparent anomalies with different station spacings.

A model usually has a simple shape, abrupt boundaries, and uniform physical properties, whereas the causative body may have an irregular shape, have gradational boundaries, and the properties of both it and its surroundings may vary spatially. Such simplification is not always a drawback, for omission of detail may emphasise the essential features.

2.5 Geological interpretation

Finally, the physical model has to be translated into geological terms; a large, roughly cylindrical mass with density less than its surroundings may be interpreted as a granite pluton; an elongated anomaly of low electrical resistance as a galena vein, and so on. This stage in the interpretation needs to take account of all available information, from the general geological context within which the survey was carried out, to information from outcrops, boreholes, and any other geophysical surveys. To neglect the other available information, either geophysical or geological, is like relying only on your hearing when you can see as well. Such information may allow you to choose between different physical models that match the geophysical observations.

2.6 Displaying the results

At the end of a survey, the results have to be presented in some form, and this offers an opportunity to emphasise features, particularly for 2D gridded data. The human eye and brain together have very sophisticated abilities to pick out lines, circles, and so on, but results need to be presented in a suitable form.

One way is **contouring**, with which you are familiar at least for topographic heights; it is far easier to grasp the relief from contours than from the values of the height printed at many positions. Since

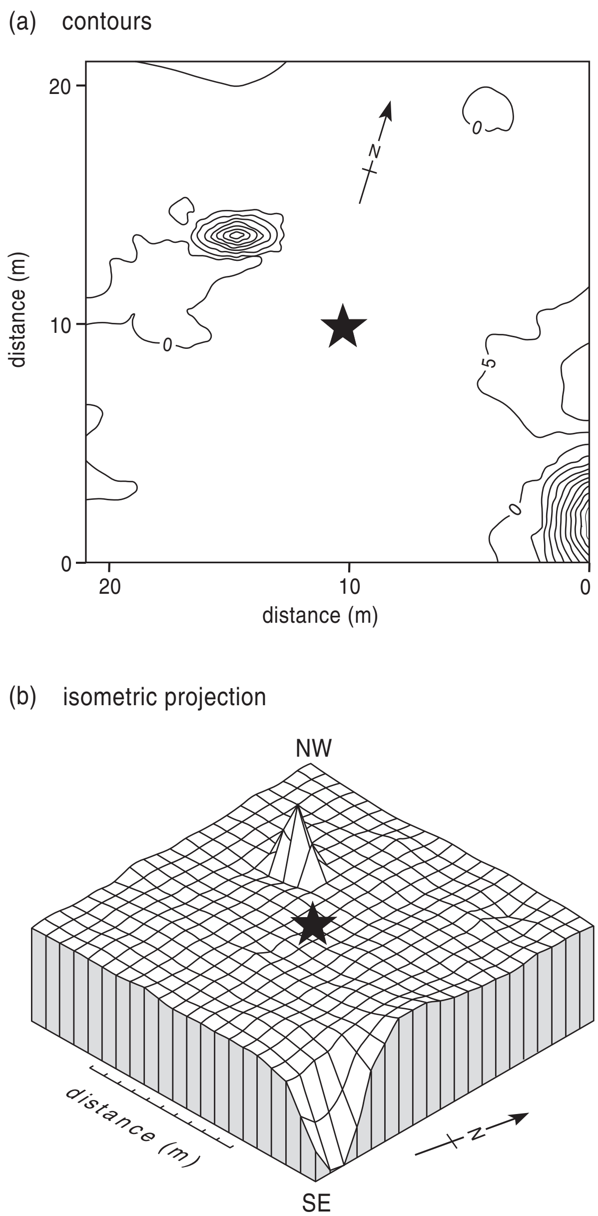

Figure 2.7 Contours and isometric projection of the results of a magnetic gradient survey.

many geophysical surveys result in numbers distributed over an area, they can be displayed in the same way, as already shown in Fig. 2.2, and we treat them like topographic contours by referring to peaks and troughs. Figure 2.7a, for example, shows contours of magnetic field gradient (their significance and that of the star are explained in Section 27.2.3).

Relief can also be made clearer by showing a perspective view, sometimes combined with contouring, as in Figure 27.5. Real topographic relief can be enhanced by oblique sunlight; this is exploited in archaeology to reveal shallow ditches and mounds, by taking an aerial photograph when the sun is low. Computer programs are available that give **false illumination** to gridded data; the direction chosen for the 'sun' is important, for it will not show up 'valleys' and 'ridges' aligned in the direction of illumination, just as furrows in a field are evenly illuminated if they are aligned towards the sun. Figure 11.5 shows an example.

A common alternative to contouring is **isometric projection** (Fig. 2.7b). Imagine first sculpting the top surface of a block to match the gridded values, and then cutting the block into regularly spaced slices, columns, or cubes. The projection shows where the surface intersects the columns, as viewed from above at some angle.

The use of colour can strongly emphasise features (e.g., Plate 1), but it needs to be used with care, for the eye is drawn to the boundary between contrasting colours and will tend to assume that their boundary is particularly significant, whether it is or not.

Some of these methods of display can be combined effectively, such as contours and shading, or colour with false illumination in perspective as seen in Plates 4 and 5.

Yet another alternative is a dot density plot and its derivative the grey-scale plot, but as these are mainly used in surveys of archaeological sites they are described later, in Section 28.3.5.

When measurements are made along closely spaced traverses, the resulting profiles are sometimes displayed one above the other, as **stacked profiles;** Figure 11.18 is an example. If traverses are widely and perhaps irregularly spaced the resulting profiles or sections may be shown in their relative positions in a **fence projection,** as in Figure 21.11.

Summary

1. Geophysical survey data are acquired by taking measurements, usually along a line or grid.
2. These data often need to be reduced in order to remove unwanted effects or to compute the data into a more useful form.
3. The data will contain the wanted signal and

unwanted noise, though which is which will partly depend upon what geological information is being sought.

4. Modelling is finding a physical body or structure that would approximately account for the observed data, and models usually are simpler than reality. Modelling may be forward or inverse.
5. Geological interpretation translates physical models into geologically feasible bodies or structures, taking account of all available information.
6. Gridded data may be displayed in several ways: Contouring, often with shading or colour, can be done; features can be further enhanced by using false illumination; or isometric projection may be used. Fence projection is useful when profiles are widely spaced.
7. Important terms: survey, station, traverse, profile, grid; data acquisition, data reduction; target, causative body, anomaly; signal, noise, signal-to-noise ratio, stacking; resolution; models, inversion problem, forward and inverse modelling, interpretation; contouring, isometric projection, false illumination, stacked profiles, fence projection.

Further reading

Milsom (1996) covers some of the preceding points in his first chapter; otherwise they are distributed throughout standard textbooks such as Kearey and Brooks (1991), Robinson and Coruh (1988), and Telford et al. (1990).

Problems

1. What is the difference between a positive and a negative anomaly?
2. A profile across a known subsurface body fails to show an anomaly. This might be because of which of the following?
   (i) Small signal-to-noise ratio.
   (ii) Data has not been reduced.
   (iii) Station spacing was too large.
   (iv) Instrument was not sensitive enough.
   (v) The body does not differ from its surrounding in the physical property being measured.
3. The purpose of stacking is to:
   (i) Display the result more clearly.
   (ii) Improve the signal-to-noise ratio.
   (iii) Help reduce the data.
4. To deduce the possible shape of the body producing a very elongated anomaly, the observed anomaly found by surveying along a single traverse is compared with one calculated for a body assumed to be uniform and horizontal, and to extend a long way to either side of the traverse. This is an example of:
   (i) Forward modelling in 2D.
   (ii) Inverse modelling in 2D.
   (iii) Forward modelling in 2½D.
   (iv) Inverse modelling in 2½D.
   (v) Forward modelling in 3D.
   (vi) Inverse modelling in 3D.
5. Ore veins in an area strike roughly E–W. The area is surveyed on a grid and, after reduction, any anomalies are to be made more obvious by use of false illumination. This should be from which of the following directions?
   (i) N. (ii) NE. (iii) E. (iv) SE. (v) S. (vi) SW. (vii) W. (viii) NW.
6. In a survey negative but not positive anomalies could be significant. How could you display the results to pick up only the negative anomalies?
7. An area has been surveyed along a few traverses, some of which intersect. The most appropriate way to display the results would be by:
   (i) Contouring.
   (ii) Stacked profiles.
   (iii) Fence projection.
   (iv) Isometric projection.
8. An anomaly has been found for a survey over a grid. You wish to know not just where the anomaly is but also if it has any particularly large values and where these are. You could achieve this by which of the following?
   (i) Contouring.
   (ii) Contouring plus colour.
   (iii) Use of false illumination.
   (iv) Isometric projection.

Chapter 3

Data Processing: Getting More Information from the Data

Even after the results of a survey have been reduced and displayed, as described in the previous chapter, the features of interest may not be obvious. If so, there may be further stages of processing that will enhance the features.

The methods described, which are mathematical, are applicable to the results of most geophysical techniques (and widely outside geophysics). This presentation emphasises the underlying concepts, which are referred to in later parts of the book.

3.1 Fourier analysis

3.1.1 Wavelength

To fix our ideas, we invent a simple example. Suppose a granite was intruded below some area in the past and later was exposed by erosion. Today, the uneven surface of the granite and surrounding country rock is buried beneath overburden, which – to keep it simple – has a level surface (Fig. 3.1). To detect the presence and extent of the granite, we exploit the granite having a lower density than the country rock, by measuring the variation in gravity (g) along a traverse (the method will be explained in Chapter 8, but detailed understanding is not needed here).

The granite alone causes the value of gravity to be lower in the middle of the profile, a negative anomaly (Fig. 3.1b); however, the varying thickness of overburden produces a varying value of gravity, least where it is thickest because its density is lower than that of the rocks beneath (Fig. 3.1c). Together (Fig. 3.1d), they produce a profile in which the anomaly of the granite is not obvious; however, it is responsible for the dip in the middle part of the profile because it is much wider than the anomalies due to the variations in overburden thickness, which contribute fairly narrow peaks and troughs. The essence of **Fourier analysis** is to sort features by their widths, from which we can then select the ones we want.

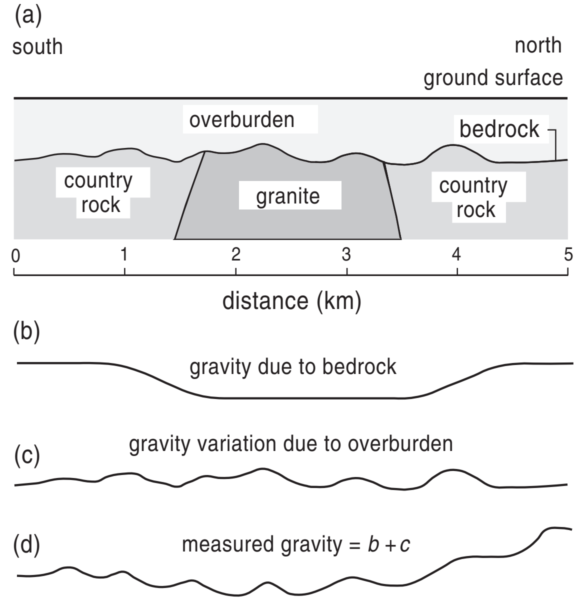

Figure 3.1 Components of a gravity anomaly.

These widths are measured as wavelengths. Waves are common in nature: One obvious example is water waves; the undulations that move along a rope shaken at one end are another. In geophysics, we shall meet both seismic and electromagnetic waves. For the present, we ignore that they move; what is important here is that they have the same general shape, shown in Figure 3.2. The repeat distance – conveniently measured between successive crests – is the **wavelength,** λ (pronounced lamda), and the maximum deviation from the undisturbed position is the **amplitude,** a (the trough-to-crest height is therefore twice the amplitude). This curve is called a **sinusoid,** because it is described by the mathematical sine function (Box 3.1).

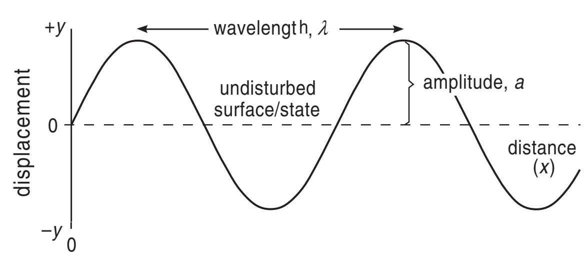

Figure 3.2 Wavelength and amplitude.

BOX 3.1 Fourier harmonic analysis equations

A sinusoid has the mathematical form described by a sine curve:

$$y = a \sin\left(\frac{2\pi x}{\lambda}\right) \qquad \text{Eq. 1}$$

where x is the distance along a profile. We can use the equation to calculate the value of the displacement, y, for any distance, x.

When x is zero, the displacement y is zero. As x increases, y increases, up to a maximum of a at $x = \lambda/4$; then it decreases, becoming zero at $x = \lambda/2$, and continues to decrease to a minimum of $-a$ at $x = 3\lambda/4$, after which it increases again, reaching zero at $x = \lambda$. This cycle repeats, as shown in Figure 3.2. Whenever the quantity in brackets equals π, 2π, 3π, . . . , y passes through zero. (A cosine curve has the same shape but is displaced a quarter wavelength along the x axis, compared to a sine curve.)

A harmonic series consists of several sinusoids, each such that an exact number of half-wavelengths equals the length, L, of the signal (Fig. 3.3), that is, $\lambda/2 = L$, $\lambda = L$, $3\lambda/2 = L$, $2\lambda = L$, ...

$$\begin{aligned} y &= a_0 + a_1 \sin\left(2\pi x \frac{1}{2L}\right) \\ &\quad + a_2 \sin\left(2\pi x \frac{2}{2L}\right) \\ &\quad + a_3 \sin\left(2\pi x \frac{3}{2L}\right) + \ldots \end{aligned} \qquad \text{Eq. 2}$$

Therefore their wavelengths are $2L$, L, $\tfrac{2}{3}L$, $L/2$, . . . In a Fourier series, these add together to match the signal, which is done by giving the amplitude of each harmonic a particular value. Therefore the series is found by replacing λ of Eq. 1 by $2L$, L, . . . in turn and giving different values to a: a_1, a_2, . . . ; a_0 is an extra term used to adjust the whole curve up or down; y is the addition of the series and gives the value of the signal at any value of x along the signal.

The same ideas apply to time-varying signals as well as spatial ones. The equations can be converted by replacing each spatial quantity by the corresponding temporal one:

Spatial quantities	Temporal quantities
L, length of signal	T, duration of signal
x, distance along signal	t, time since signal began
λ, wavelength	τ, period of harmonic $= 1/\text{frequency}, f$

τ (tau, to rhyme with cow) is the time it takes the oscillation at a place to complete one cycle (e.g., the time for a cork on the sea to bob from crest to trough to the next crest). However, it is usually more convenient to use the frequency, f, so that Eq. 1 becomes, for time-varying signals,

$$y = a \sin(2\pi f t) \qquad \text{Eq. 3}$$

The values of the amplitudes are found from the signal using these expressions:

$$a_0 = \frac{1}{L}\int_0^L y\,dx \quad \text{or} \quad \frac{1}{T}\int_0^T y\,dt$$
$$= \text{average value of signal} \qquad \text{Eq. 4a}$$

$$a_n = \frac{2}{L}\int_0^L y \sin\left(\frac{n\pi x}{L}\right)dx$$
$$\text{or} \quad \frac{2}{T}\int_0^T y \sin\left(\frac{n\pi t}{T}\right)dt \qquad \text{Eq. 4b}$$

3.1.2 Harmonic analysis

Few profiles look like Figure 3.2, so why bother with waves? The essence of Fourier analysis is that a wiggly line, such as the gravity profile of Figure 3.1, can be reproduced by adding together a series of waves. Only certain wavelengths are used, such that 1, 2, 3, . . . half-wavelengths exactly fit the length of the profile; these are called **harmonics.** Their amplitudes are adjusted so that, added together, they match the required shape. Figure 3.3a shows that an approximate sawtooth shape can be built up using the first 10 harmonics, because at different distances the harmonics have different values, which may add or partially cancel. The approximation improves as more harmonics are added. Figure 3.3b shows another profile or waveform produced using different proportions of the same harmonics. Remarkably, any wiggly line can be matched, though the finer details require higher harmonics, which have shorter wavelengths.

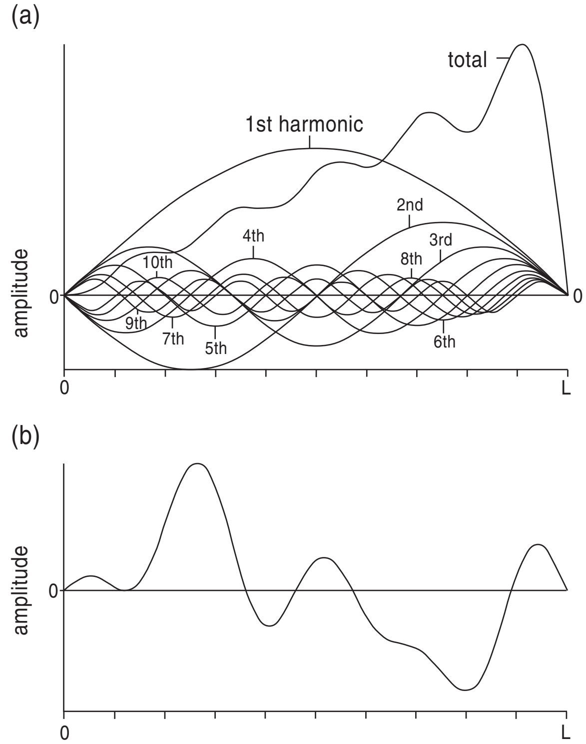

Figure 3.3 Addition of waves.

How the values of the amplitudes of the harmonics depend on the curve to be reproduced is explained in Box 3.1.

Signals in time. Some signals vary with *time* rather than *space,* such as the variation with time of the height of the sea surface at some place (Fig. 3.4), rather than the shape of the surface at some instant: The height fluctuates due to the passage of waves and also rises and falls over a period of about 13 hr due to the tides. This can be analysed in exactly the same way as the spatial gravity profile except that time replaces distance (Box 3.1). However, instead of describing features in terms of wavelength we now use **frequency,** ***f*,** which is measured in Hertz, Hz, the number of times a complete cycle repeats in 1 sec.

In signal processing, time-varying signals are often referred to as if they were in space, or vice versa, so a low-frequency signal (one that varies slowly with time) may be described as having a long wavelength, or a wide granite as producing a low-frequency anomaly.

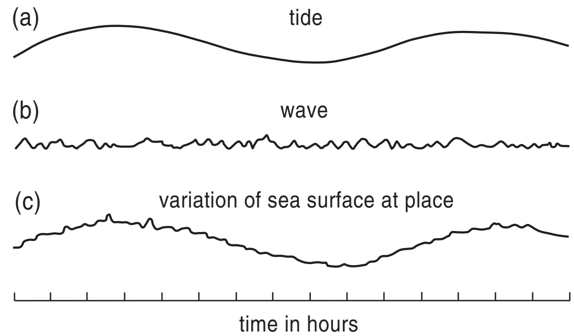

Figure 3.4 Variation in sea level at a point, due to waves and tides.

3.1.3 Fourier analysis of a profile

We now apply this idea of wavelength analysis to the gravity profiles of Figure 3.1. The profile due to the granite alone consists mostly of long wavelengths (Fig. 3.5a) – the main one being about 6 km long – while the overburden profile clearly has mostly shorter wavelengths. Therefore we can make the anomaly of the granite clearer by analysing the measured profile, Figure 3.1d, into harmonics, then rejecting wavelengths shorter than the fifth harmonic and finally recombining the remaining harmonics into a single wiggly curve (Fig. 3.5a). How well the recombined harmonics match the observed profile can be judged by showing their differences as a residual anomaly (Fig. 3.5b); this also indicates whether the unmatched part consists mainly of unwanted short wavelengths.

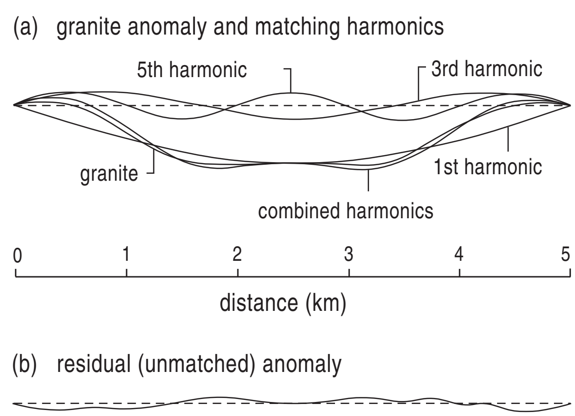

Figure 3.5 Analysis of anomaly.

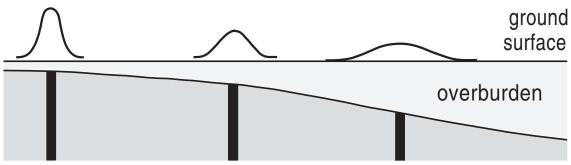

Figure 3.6 An anomaly broadens as the body is buried more deeply.

Though this enhances the wanted anomaly compared to unwanted ones, there are some points to note. Each component curve due to a different geological body usually contains a range of wavelengths, so separation of wanted from unwanted anomalies, or signals, is only partial; for instance, the overburden and granite have some wavelengths in common. Secondly, the wavelengths of the harmonics deduced depend upon the length of profile being analysed, for they have to fit its length exactly, and as the length of the profile is rather arbitrary so are the harmonics. These two points emphasise that Fourier harmonic analysis (to give it its full title) is a mathematical procedure and does not analyse the measured signal into its geophysical or geological components as such.

Thirdly, rejecting the shorter wavelengths is often regarded as removing the anomalies of bodies near the surface to leave deeper ones, but this is not necessarily so. It is true that for many geophysical techniques a body near the surface gives a narrower (as well as stronger) anomaly than when buried deeper (Fig. 3.6; also Figs. 8.9 and 8.10). But whereas a narrow anomaly cannot be due to a deep body, the converse is not true, for a broad anomaly can be due to either a narrow body at depth or to a broad body near the surface (Fig. 3.7; also Fig. 8.17a).

The unwanted anomalies of the overburden of Figure 3.1 are an example of noise (Section 2.3). As noise has a shorter wavelength than the signal, removing shorter wavelengths often improves the signal-to-noise ratio, so making the anomaly more obvious.

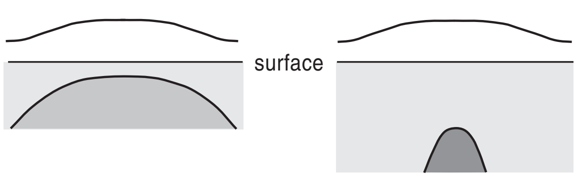

Figure 3.7 Similar anomalies from shallow and deep bodies.

3.1.4 Fourier analysis in 2D: Gridded data

The examples so far have been in one dimension, such as a profile, but readings may be taken on a grid and contoured, as described in Section 2.6, and we may wish to remove the short-wavelength anomalies in much the same way.

Such surveys are analysed by sets of waves at right angles to one another, parallel to the sides of the rectangle. Waves with a single wavelength, λ, extending over an area with their crests and troughs parallel, look rather like a ploughed field or piece of corrugated iron (Fig. 3.8a and b); two such sets at right angles, when added, resemble an egg tray (Fig. 3.8c). The readings of an areal survey are analysed into harmonic series in each of the two directions, and as with analysis of profiles, various ranges of wavelengths can be eliminated and the remaining wavelengths recombined. (An example using the related process of filtering is shown in Figure 3.13.) However, there are other possibilities, such as eliminating *all* wavelengths in one of the two directions: This will emphasise features elongated in the other direction.

Fourier analysis is a large subject, just touched upon here. It can, for instance, be extended to readings taken over the whole surface of the Earth (surface spherical harmonic analysis, mentioned in Section 11.1.4), and even throughout the Earth (spherical harmonic analysis), but these will not be considered here.

3.1.5 Why a harmonic series?

What is special about harmonics? Why not use sinusoidal waves that don't fit an exact number of half-wavelengths into the length of the profile or some shape of curve other than a sinusoid? There are two reasons. The first is that some natural signals are made up of such a series. For instance, oscillations of musical instruments, such as a vibrating string of a guitar, can be analysed into harmonics that correspond to an exact number of half-wavelengths fitting into the length of the string. It is the different proportions of the harmonics that make a guitar sound different from another type of instrument playing the same note, and a trained musician can hear the individual harmonics. In fact, it is from music that the term 'harmonic' derives.

The second and more important reason is a mathematical one, that the amount of each har-

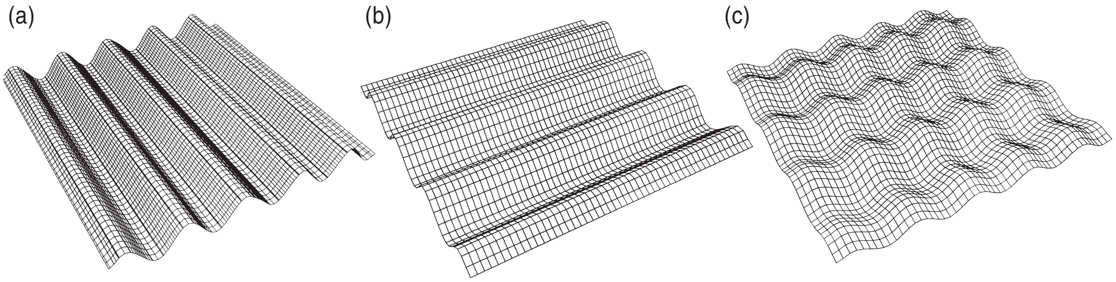

Figure 3.8 Combination of waves at right angles.

monic can be calculated independently. Thus, for example, to calculate the amount of the third harmonic, it is not necessary to know the amounts of the first, second, and fourth harmonics, and so on. So it is necessary only to calculate the amplitudes of relevant harmonics and not every one down to the extremely short wavelengths that may be present.

3.2. Digital filtering

3.2.1 Simple filters

An alternative way of rejecting unwanted wavelengths or frequencies is by **filtering.** In everyday language, a filter can be used to separate large from small particles; for example, if a mixture of rice and beans were put into a filter (or sieve) with suitable mesh size, the beans would be retained while the rice grains would be let through. In electronics, filters are used to remove unwanted frequencies, such as sharp 'spikes' and pulses from the electricity supply, or to change the proportions of bass and treble when recording music. Though neither of these filters works in the same way as the digital filters we shall be considering, they embody the same idea of separating wanted from unwanted things by size. Digital filters do this mathematically and have much in common with Fourier analysis.

Digital filters are usually applied to values taken at regularly spaced **sampling intervals,** either along a line or on a grid. (If readings are continuous, values at regular intervals are just read off; if irregularly spaced, regular ones are found by interpolation.)

One of the simplest digital filters takes the average of three successive readings along a profile and records the result at their midpoint (Fig. 3.9; 3-pt filter). This is repeated at each sampling position in turn, so that each reading is 'used' three times. This simple 3-point average is sometimes called a running average; or we talk of a 'moving window': Imagine a card with a hole just wide enough to see three readings, and average only the points that you can 'see through the window' as the card is moved point by point along the profile.

The average value of y, $\bar{y}$, is:

$$\bar{y}_n = \frac{1}{3}\left(y_{n-1} + y_n + y_{n+1}\right) \qquad \text{Eq. 3.1}$$

The subscript n denotes the point where we shall record the result (i.e., the midpoint of the three readings), $n-1$ is the previous point, and $n+1$ the following point. Then the process is repeated with the window centred on the next point (i.e., with n increased by 1).

The short wavelength 'jaggedness' on the left part of the unfiltered line is 'smoothed' much more than the hump in the middle, while the straight line at the right is not affected at all. In terms of Fourier analysis, we would say that shorter-wavelength amplitudes (or higher frequencies, for a time-varying signal) are the most reduced. A slightly more complex filter of this type could have a window of five points. Figure 3.9 shows that it smooths short wavelengths more effectively than the 3-point filter.

Other filters could have yet more points, but another way to change their effectiveness is to use a weighted average rather than a simple average (i.e., the values of the points are multiplied by different amounts, or coefficients). For example, a weighted 7-point filter could be

$$\begin{aligned}\bar{y}_n = (&-0.115y_{n-3} + 0y_{n-2} + 0.344y_{n-1} + 0.541y_n \\ &+ 0.344y_{n+1} + 0y_{n+2} - 0.115y_{n+3}\end{aligned} \qquad \text{Eq. 3.2}$$

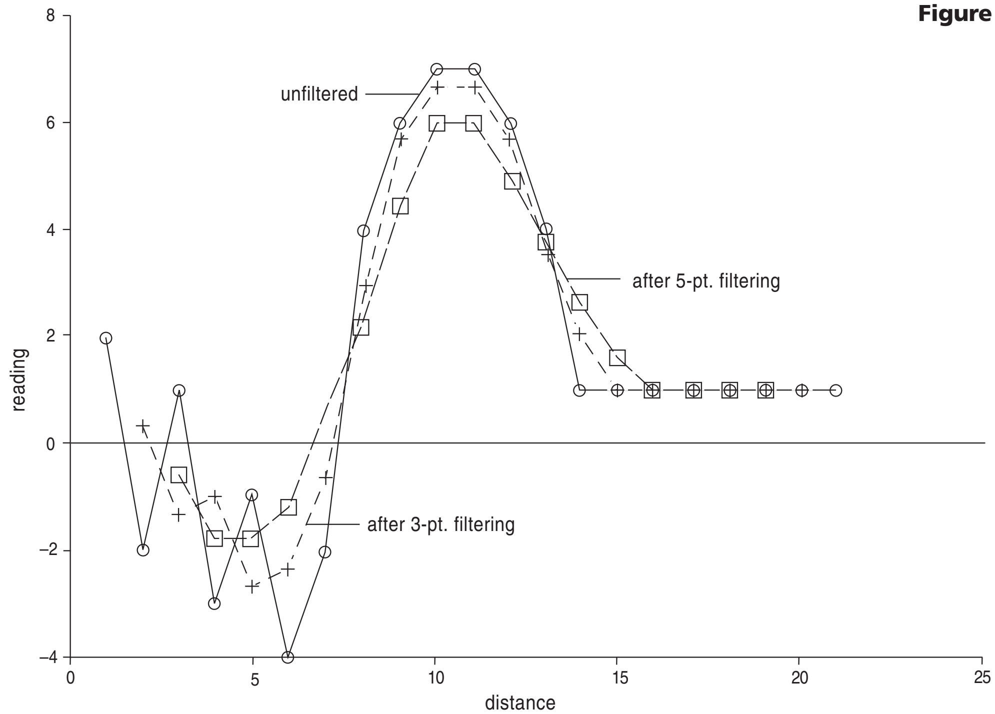

Figure 3.9 Simple filters.

To find out which wavelengths are being reduced and by how much, we can apply the filters to a range of pure sinusoidal waves and see how much they are reduced (Fig. 3.10). Wavelengths several times the sampling interval are hardly affected (Fig. 3.10a), while those comparable to the sampling interval are greatly reduced (Fig. 3.10d); the 5- and 7-point filters generally reduce them more than the 3-point one.

These filters are called **low-pass** filters because they let 'pass through' with little or no reduction all wavelengths longer than some value (or frequencies below some value), but greatly reduce those shorter. In Figure 3.10 this cutoff wavelength is roughly that of Figure 3.10c. These are also called smoothing filters, because the jaggedness is reduced, as Figure 3.9 showed.

The converse to a low-pass filter is a **high-pass** one, which lets through only wavelengths shorter than some value, while a **band-pass** filter is one that lets through only a range of intermediate wavelengths (or frequencies).

Filters have various limitations. One is that they differ in how completely they pass or reject the different wavelengths. A second is that the differing window lengths of the 3-, 5-, and 7-point filters can sometimes result in the 7-point filter being less effective than the 3-point one (compare the filtered curves of Fig. 3.10c). A third is that the sampled values shown in the left-hand column of Figure 3.10c to e, when connected together, do not define a regular wave, even though they are at regular intervals on a regular sinusoid. This can result in shorter wavelengths sometimes not being reduced as much as longer ones (compare Fig. 3.10d and e); this effect will be considered further in the next section. Although filters can be very useful, they have to be applied with understanding.

3.2.2 Aliasing

A potential drawback of sampling the signal at intervals, rather than continuously, is that wavelengths that do not exist can appear to be present.

Consider a single sinusoid sampled at different intervals (Fig. 3.11). When the sample interval is short compared to the wavelength, the undulations of the wave are followed faithfully (Fig. 3.11a). Even when there are only two samplings in each wavelength (Fig. 3.11b) the wavelength is still obvious, though the amplitude is not correct. However, as the interval is increased further, curious things happen. When the interval exactly equals the wavelength (Fig. 3.11d) all the readings are the same, so there seems to be no variation. The value depends

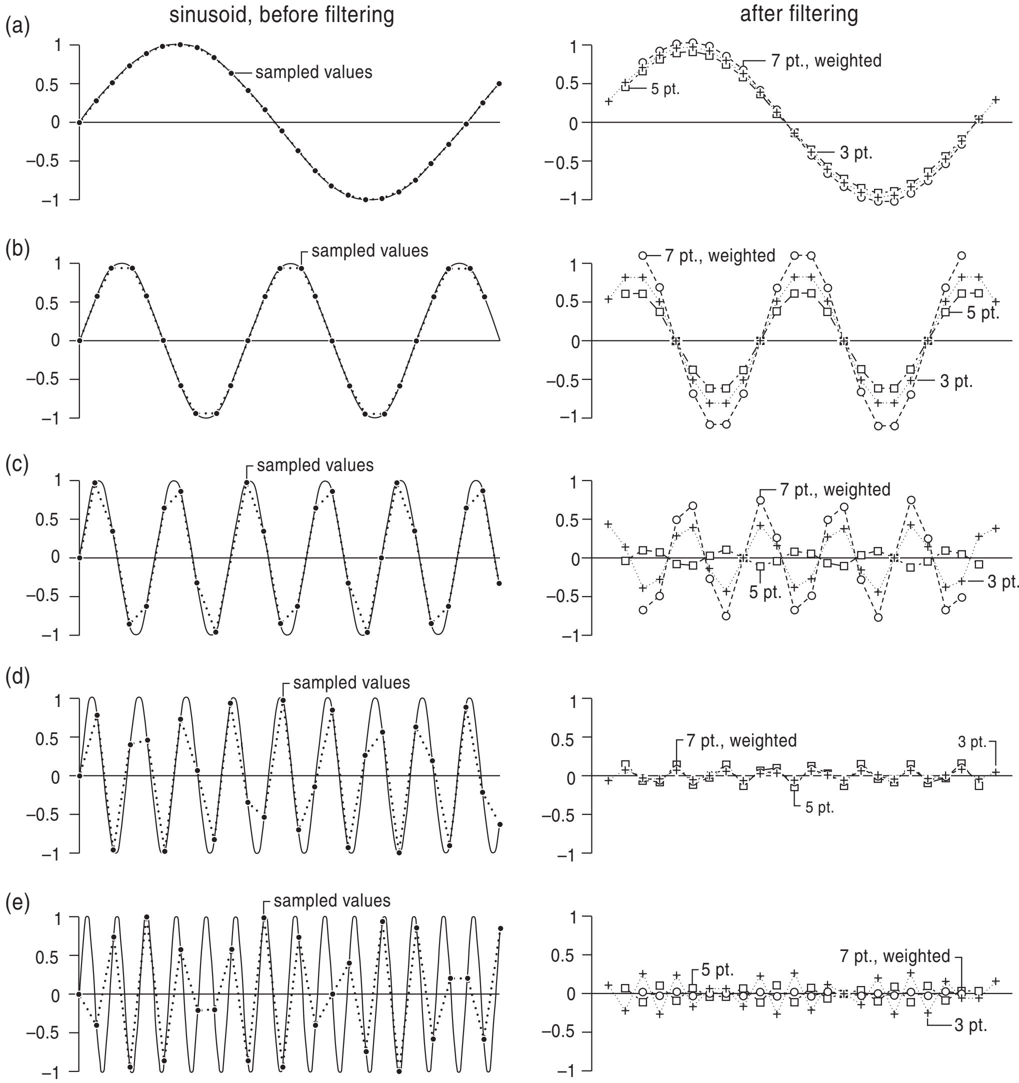

Figure 3.10 Effect of filters on different wavelengths.

on where in the wave cycle the samplings happen: Compare Figure 3.11d(i) and (ii). When the sampling interval is only a little different from the wavelength (Fig. 3.11c and e), the signal appears to have a very long wavelength, which is spurious. This production of spurious wavelengths is called **aliasing,** and if a real signal contains a harmonic with a wavelength close to the sampling interval, a spuriously long wavelength will be found. This effect recurs whenever the interval is close to a multiple of the wavelength; in fact, once the sampling interval *exceeds half the wavelength* the apparent wavelength is always longer than the true wavelength. This critical wavelength – twice the sampling interval – is called the **Nyquist wavelength.**

Aliasing can occur in time as well as space. A common example is when the wheels of a wagon in a western film appear to be near-stationary or even going backwards, despite evidently being driven furiously. This occurs because between frames of the film each spoke of a wheel rotates to approximately the former position of another spoke, but the eye cannot tell that one spoke has been replaced by another. To avoid the effect, the time between frames needs to be no longer than half the time it takes one spoke to replace the next. In general,

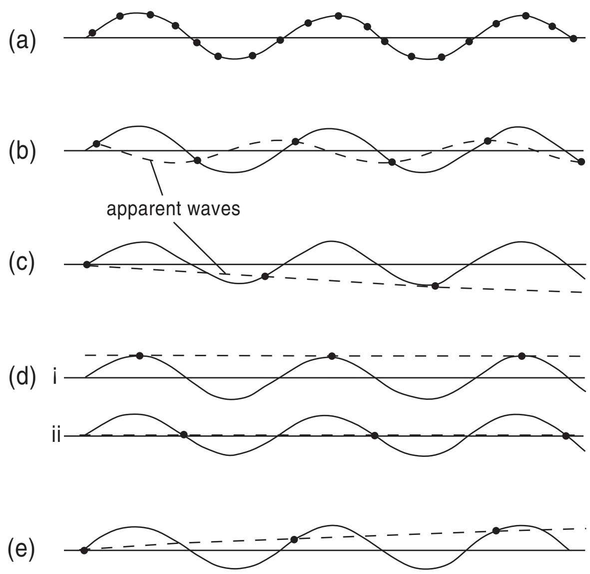

Figure 3.11 Effect of sampling intervals.

aliasing produces spurious periods when a harmonic has a period less than twice the sampling interval. Since period is the reciprocal of the frequency, $1/f$, this is equivalent to saying that spurious frequencies are produced when the number of samplings each second is less than twice the frequency of a harmonic. The critical frequency, half the sampling frequency, is called the **Nyquist frequency.** To avoid the spurious frequencies, any higher frequencies have to be removed before sampling, usually by using a nondigital electronic filter.

In geophysics, aliasing is more commonly a potential problem with time-varying signals – such as seismic recording of ground motions – than with spatial anomalies.

3.2.3 Designing a simple filter

Filters can be designed to remove, to a considerable extent, whichever range of wavelengths we wish, by our choice of the sampling interval, the number of points in the window, and the values of the coefficients of the points. As we saw earlier, the sampling interval is important because wavelengths much less than the sampling interval are automatically largely rejected (though aliasing can occur). The sampling interval is therefore chosen to be somewhat shorter than the shortest wavelength we wish to retain; a quarter of this length is a suitable sampling distance. Provided the correct sampling interval has been chosen, there is little advantage in having a filter with more than seven points. Then the coefficients are chosen (by mathematical calculations beyond the scope of this book) to give the maximum discrimination between wanted and unwanted wavelengths, or frequencies, though this cannot be done as well as by Fourier analysis.

3.2.4 Filtering in 2D: Gridded data

In Section 3.1.4 we explained that Fourier analysis could be applied to gridded or 2D data; similarly, filters can be used in 2D, with points sampled all round the point in question. The larger dots of Figure 3.12 show the sampling positions for one particular filter, while the numbers are the weightings, which are the same for all points on a circle. Figure 3.13 shows the effect of applying a low-pass filter that progressively reduces wavelengths in the range 16 to 10 km and entirely removes those shorter than 10 km. It reveals a large, near-circular anomaly. (The anomaly is actually centred somewhat to the north, because there is also a steady increase from north to south – a 'regional anomaly' in the terminology of gravity and magnetic surveying; see Section 8.6.2 – which also needs to be removed.)

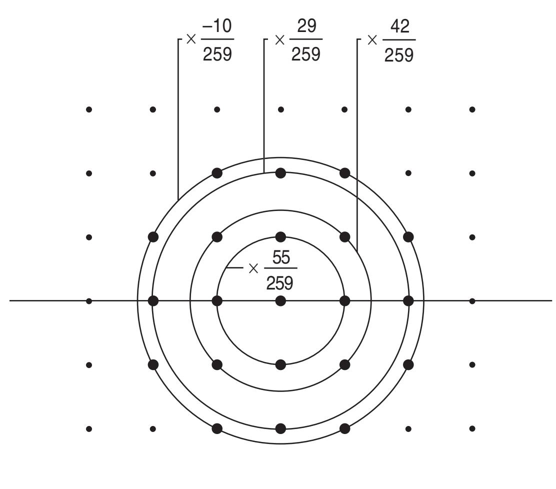

Figure 3.12 Filter window and coefficients for gridded data.

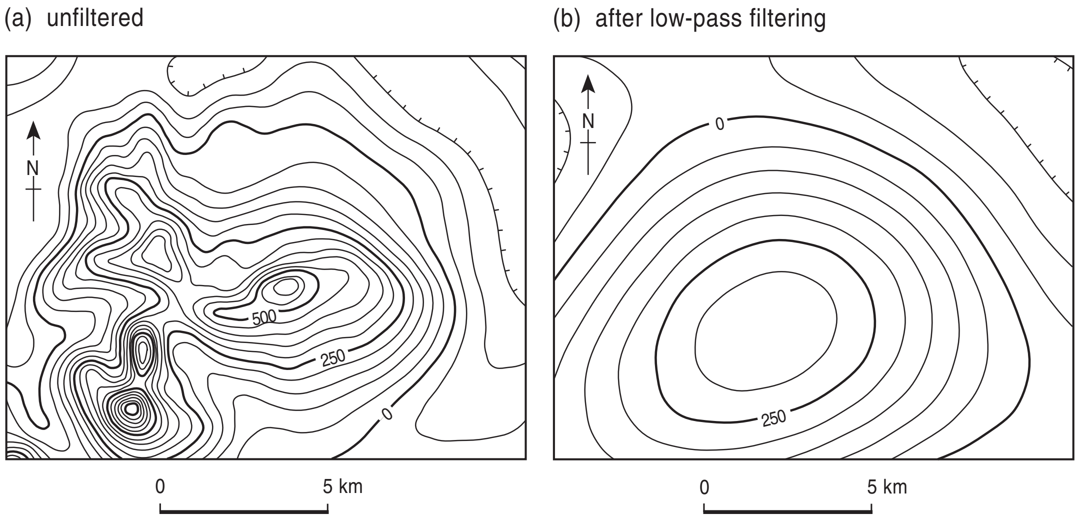

Figure 3.13 Gridded data before and after low-pass filtering.

3.2.5 Using filters to enhance various types of features

The filters described so far can enhance the signal simply by reducing unwanted wavelengths, such as the short wavelengths due to noise, as is done with Fourier analysis. However, with 2D data there are other possibilities, as with Fourier analysis. One is to use directional filters to separate elongated features by their direction. For instance, this technique could be used to emphasise anomalies due to ore veins in a region where their likely direction is known. An example of directional filtering is given in Section 27.4.2.

Another type of filter can be used to emphasise edges of an anomaly, and so help outline the positions of the causative body, by selecting where short wavelengths are concentrated. This is in contrast to enhancing large anomalies by filtering out the short wavelengths, as described in Sections 3.1.3 and 3.2.4, which tends to deemphasise the edges.

An alternative way to pick out edges is by finding where values are changing most rapidly. In Figure 3.1 the value of the anomaly changes most rapidly near the edges of the granite, so picking out where this occurs may outline a body; similarly, the steepest slopes or gradients occur around the edge of a broad hill. An example using gradient to pick out edges is shown in Figure 25.5a, in Section 25.3.2.

3.3 Summing up: Fourier analysis and filtering

A filter achieves much the same result as a Fourier analysis, so which should be used? Fourier analysis requires that the whole signal or profile be analysed, whereas a filter needs only a few successive readings at a time, those in the 'window'. Therefore, a Fourier analysis cannot be made before the signal is complete, and it requires more computing as it uses more data at a time. However, Fourier analysis gives the more complete separation of frequencies or wavelengths, and the performance of a filter is specified in the terms of Fourier analysis, rather than vice versa, so Fourier analysis underpins filters. Fourier analysis can analyse a continuous curve, whereas a filter operates on discrete data, but as much geophysical data is discontinuous this is of little consequence, provided the sampling interval is chosen suitably and precautions are taken against aliasing. In practice, filters are more often used than Fourier analysis.

Fourier harmonic analysis and the various forms of filtering are used to emphasise wanted features in 1D or 2D, but none of the methods are guaranteed to enhance the desired feature, because the causative body often generates a signal with a range of wavelengths that may overlap those due to 'noise'. Signal processing is no substitute for careful design of a survey to optimise the quality of data that can be acquired for the available effort and cost, but it can be useful to extract more information out of the data collected, and it is widely used in geophysical surveying.

Some other types of data processing are described in this book, but as these are limited to a few methods they will be explained in the relevant chapters.

Summary

1. Data processing is used to emphasise, or enhance, wanted anomalies or signals at the expense of unwanted ones.
2. Data processing uses mathematical methods, the two main ones being Fourier harmonic analysis and filtering.
3. Fourier analysis matches the signal (in time or space) by a series of sinusoids (sine or cosine waves). Each member of the series fits an exact number of half-wavelengths into the length of the signal; together they form a harmonic series. The analysis finds the amplitudes of these harmonics.
4. Once a Fourier analysis has been made, unwanted wavelengths, or frequencies, are deleted, and the remainder recombined, to enhance wanted anomalies or other features.
5. Fourier analysis can be carried out in 1D for a profile or time-varying signal, or 2D for gridded data.
6. Digital filtering is carried out by multiplying the values of several successive samplings within a 'window' by various amounts (weighting coefficients), repeating this progressively by moving the window along the signal. The range of unwanted wavelengths or frequencies to be rejected is controlled by choice of the sampling interval, the number of points in the window, and their weighting coefficients.
7. Aliasing produces spurious longer wavelengths when the sampling interval is longer than the Nyquist wavelength (i.e., half that of any harmonic present); or spurious frequencies if the number of samplings each second is less than the frequency of any harmonic present in the signal. The sampling interval should be chosen to be about a quarter of the shortest required wavelength or period, and shorter wavelengths/higher frequencies need to be absent or removed.
8. Filters can be low-, high-, or band-pass, and can operate in 1D or 2D. Filters can be designed to emphasise linear features with a particular direction or to enhance edges.
9. Filters need only a limited sequence of the data at a time, whereas Fourier analysis requires the whole signal or profile, so filters are the more used. However, Fourier analysis is the more fundamental and underpins filters.
10. Important terms: Fourier analysis, filtering; harmonic, sinusoid, wavelength, frequency, amplitude; high-, low-, and band-pass filters; sampling interval, aliasing, Nyquist frequency, and Nyquist wavelength.

Further reading

There are many books on signal processing, but they involve mathematics much more advanced than used in this book. Kearey and Brooks (1991) has a chapter at an intermediate level.

Problems

1. The brightness of daylight at a place is recorded continuously for about a week and then Fourier analysed. What is the lowest period found? What is this as a frequency?
2. (a) Explain with sketches why a wagon wheel shown in films can appear to be stationary. What is this effect called?
   (b) How will the wheel appear to be rotating if it then revolves just a little slower? Or just a little faster?
   (c) A wheel has 8 spokes. Through what different angles must the wheel rotate between film frames to appear stationary?
3. If you were sampling with an interval of 0.02 sec, what Nyquist frequency should you use to prevent aliasing? Would the filter used be a high-, low-, or band-pass one?
4. If you were told that two spheres with different masses were buried in different places at different depths below a plain, how would you recognise the deeper one from its gravity anomaly?
5. Which of the following are true?
   (i) Wide anomalies are always due to deep bodies.

(ii) Narrow anomalies are always due to shallow bodies.
(iii) The deeper a body, the wider its anomaly.

6. A sinusoid has a wavelength of 10 m. What apparent wavelengths would result from sampling intervals of 1, 2, 9, 10, and 11 m?
7. The following values were measured at 1-metre intervals. Plot and apply simple 3-point and 5-point filters. What are the maximum and minimum values of the unfiltered and two filtered data sets? What are the most obvious wavelengths in each case?
–3, 2, 1, 4, 0, 4, –1, 3, 0, 1, –4, 0, –4, –1, –4, 1, 0, 2, 4.