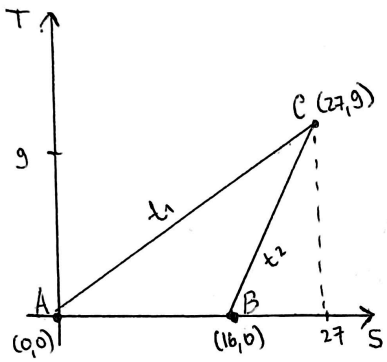
4. Jak dla ustalonej wartości s zmienia się t

(x,y)	$(0,0)$	$(0,9)$	$(8,0)$
$\downarrow$			
(s,t)	$(0,0)$	$(27,9)$	$(16,0)$

- $(0,9)$
  $S = 2x + 3y = 2\cdot 0 + 3\cdot 9 = 27$
  $T = 9$
- $(8,0)$
  $S = 2x + 3y = 16 + 0 = 16$
  $T = 0$



$t_1 = AC$ $\quad A=(0,0)$, $C=(27,9)$

$t_1 = as+b$

$0 = a\cdot 0 + b$ $\quad b=0$

$9 = 27a + 0$ $\quad a = \frac{1}{3}$

$t_1 = \frac{1}{3}s + 0 = \frac{1}{3}s$

$t_2 = CB$ $\quad C=(27,9)$, $B=(16,0)$

$t_2 = as+b$

$9 = 27a+b$ $\quad 0 = 16a+b$

$\begin{cases} 16a+b=0 \\ 27a+b=9 \end{cases}$

$11a = 9$ $\quad a = \frac{9}{11}$

$b = -\frac{16\cdot 9}{11} = -\frac{144}{11}$

$t_2 = \frac{9}{11}s - \frac{144}{11}$

Zatem:

$t \in [0, \frac{1}{3}s]$ dla $s \in [0,16]$

$t \in [\frac{1}{3}s, \frac{9}{11}s - \frac{144}{11}]$ dla $s \in [16,27]$

5. Wzory końcowe

$$f_S(s) = \begin{cases} \int_0^{\frac{1}{3}s} \frac{1}{72}\,dt = \frac{t}{72}\Big|_0^{\frac{1}{3}s} = \frac{1\cdot s}{72\cdot 3} & \text{dla } s\in[0,16] \\ \int_{\frac{9}{11}s-\frac{144}{11}}^{\frac{1}{3}s} \frac{1}{72}\,dt = \frac{t}{72}\Big|_{\frac{9}{11}s-\frac{144}{11}}^{\frac{1}{3}s} = \frac{1}{72}\cdot\left(\frac{1}{3}s - \frac{9}{11}s + \frac{144}{11}\right) = \frac{1}{72}\cdot\left(\frac{11s}{33} - \frac{27s}{33} + \frac{144\cdot 3}{33}\right) = \\ = \frac{144\cdot 8}{11\cdot 33\cdot 72\cdot 9} - \frac{16s}{33\cdot 72\cdot 9} = \frac{18}{11\cdot 9} - \frac{2s}{33\cdot 9} & \text{dla } s\in[16,27] \end{cases}$$

Sprawdzenie poprawności:

1) $\int_0^{16} \frac{1}{72\cdot 3}\cdot s\,ds = \frac{1}{72\cdot 3}\cdot\frac{s^2}{2}\Big|_0^{16} = \frac{1}{72\cdot 3}\cdot\frac{16\cdot 16}{2} = \frac{16}{27}$

2) $\int_{16}^{27} \frac{18}{11\cdot 9}\,ds - \int_{16}^{27}\frac{2\cdot s}{33\cdot 9}\,ds = \frac{18}{11\cdot 9}\cdot s\Big|_{16}^{27} - \frac{2}{33\cdot 9}\cdot\frac{s^2}{2}\Big|_{16}^{27} = \frac{18}{11\cdot 9}\cdot 11 - \frac{1}{33\cdot 9}\cdot 473 = \frac{18}{9} - \frac{43}{27} = \frac{11}{27}$

$\frac{16}{27} + \frac{11}{27} = \frac{27}{27}$ ☺