RPIS

Zadanie 1. egzaminacyjne

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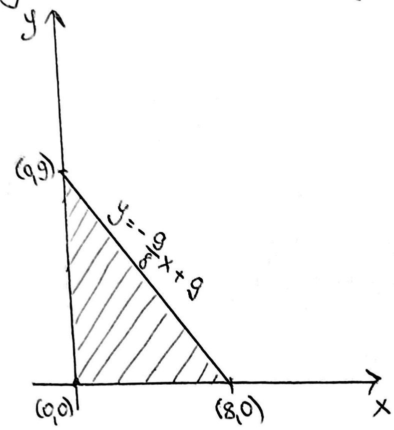
$9 \to m$, $8 \to n$

1. Wyznaczenie wartości stałej C



$$y = ax + b$$
$$9 = a \cdot 0 + b$$
$$b = 9$$

$$0 = a \cdot 8 + b$$
$$0 = a \cdot 8 + 9$$
$$a = -\frac{9}{8}$$

$$y = -\frac{9}{8}x + 9$$

$$1 = \iint_{\mathbb{R}\,\mathbb{R}} f(x,y)\,dx\,dy = \int_0^8 \underbrace{\int_0^{-\frac{9}{8}y+9} c \cdot dx}\,dy =$$

$$\int_0^{-\frac{9}{8}y+9} c \cdot dx = c \cdot x \Big|_0^{-\frac{9}{8}y+9} = c \cdot \left(-\frac{9}{8}y + 9\right)$$

$$= \int_0^8 c \cdot \left(-\frac{9}{8}y + 9\right) dy = c \cdot \left(-\frac{9}{8} \int_0^8 y\,dy + 9 \int_0^8 1\,dy \right) = c \cdot \left(-\frac{9}{8} \cdot \frac{y^2}{2} \Big|_0^8 + 9y \Big|_0^8 \right) =$$

$$= c \cdot \left(-\frac{9}{\cancel{8}} \cdot \frac{\cancel{64}^{\,4}}{\cancel{2}} + 72 \right) = c \cdot 36$$

$$c \cdot 36 = 1$$
$$c = \frac{1}{36}$$

2. Przejście do nowej zmiennej (S,T), odwrócenie tego przejścia, moduł Jakobianu

$$\begin{cases} S = 2x + 3y \\ T = y \end{cases} \Leftrightarrow \begin{cases} x = \frac{S - 3T}{2} \\ y = T \end{cases}$$

$$|J| = \begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 \end{vmatrix} = \frac{1}{2}$$

3. Całka nieoznaczona z gęstości $g(s,t)$

$$g(s,t) = f(x(s,t), y(s,t)) \cdot |J| = \frac{1}{2} \cdot \frac{1}{36}$$

$$\int g(s,t) \cdot |J|\,dt = \int \frac{1}{72}\,dt = \frac{1}{72} \cdot t + c$$