Physics GRE:

Laboratory Methods

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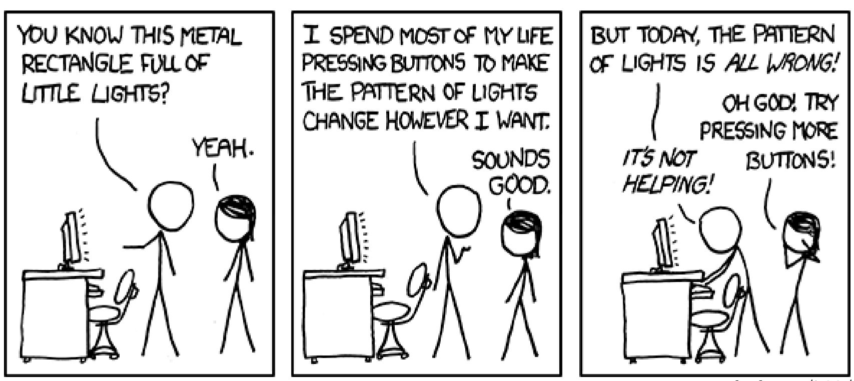

xkcd.com/722/

Contents

1 Data & Error Analysis

The arithmetic mean of a set of n numbers $\{x_i\}$ is given by

$$\mu_{\mathrm{A}} = \frac{1}{n}\sum_i x_i \tag{1}$$

The median is the "middle" element when the data are arranged in increasing order. The mode is the most common value. There are also the harmonic and geometric means, given by

$$\mu_{\mathrm{H}} = \frac{n}{\sum_i \frac{1}{x_i}} \qquad \mu_{\mathrm{G}} = \sqrt[n]{\prod_i x_i} \tag{2}$$

These three means always satisfy the following inequality:

$$\mu_{\mathrm{A}} \geq \mu_{\mathrm{G}} \geq \mu_{\mathrm{H}} \tag{3}$$

When values have varying uncertainties associated with them then one may use a weighted average to estimate the mean and its uncertainty. If each value x_i has an associate error σ_i, then

$$\overline{x} = \frac{\sum_i w_i x_i}{\sum_i w_i} \qquad \sigma_{\overline{x}}^2 = \frac{1}{\sum_i \sigma_i^{-2}} \qquad w_i = \frac{1}{\sigma_i^2} \tag{4}$$

Notice that in the case where all measurements have the same error, i.e. $\sigma_i = \sigma$, then these reduce to

$$\overline{x} = \frac{1}{n}\sum_i x_i \qquad \sigma_{\overline{x}}^2 = \frac{\sigma^2}{n} \tag{5}$$

After n measurements the uncertainty in the mean goes as $\frac{1}{\sqrt{n}}$.

2 Logic Gates

Some basic knowledge of boolean algebras and logic gates is assumed.

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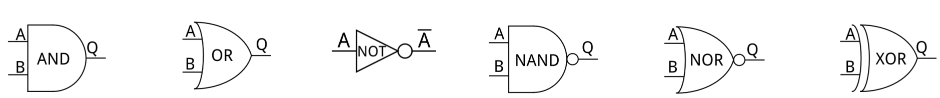

3 Lasers & Optical Interferometers

Lasers are based on the process of stimulated emission, whereby a large number of photons with the same wavelength may be created. Atoms are excited to a high-energy state and decay to a meta-stable state for which the lifetime is relatively long. When a population inversion is achieved, there are more atoms in an excited state than in the ground state and stimulated emission is more likely than absorption of a photon, leading to an amplification of light as photons travel through the material. When housed in a reflective environment, this process produces a large number of coherent photons which comprise the laser beam.

Interferometers use wave interference as the basis of observation. A Michelson interferometer uses the constructive and deconstructive interference of a split and recombined laser beam to make conclusions about the differences in path length or presence of optical materials. For example, the LIGO interferometer detects the change in optical path length as a result of passing gravitational waves. As one path is stretched and squished longer and shorter than the other the detected interference pattern shifts from constructive to destructive at a measurable frequency.

4 Dimensional Analysis

4.1 International System of Units The SI system has seven base units, as shown in the following table:

Unit	Symbol	Quantity	Dim. Symbol	Current Definition
Meter	m	Length	L	The distance travelled by light in vacuum in $\frac{1}{299\,792\,458}$ s
Kilogram	kg	Mass	M	The mass of the international prototype kilogram.
Second	s	Time	T	The duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cs-133 atom
Ampere	A	Electric Current	I	The constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $2 \cdot 10^{-7}$ N/m.
Kelvin	K	Tempereature	Θ	$\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.
Mole	mol	Amount of substance	N	The amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of C-12.
Candela	cd	Luminous intensity	J	The luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $5.4 \cdot 10^{14}$ Hz and that has a radiant intensity in that direction of $\frac{1}{683}$ W/sr.

Table 1: SI base units.

Unit	Symbol	Quantity	Expressions	
Hertz	Hz	Frequency		$1/\mathrm{s}$
Newton	N	Force		$\mathrm{kg{\cdot}m}/\mathrm{s}^2$
Joule	J	Energy; Work	N/m^2	$\mathrm{kg}/\mathrm{m{\cdot}s}^2$
Watt	W	Power	J/s	$\mathrm{kg{\cdot}m}^2/\mathrm{s}^3$
Coulomb	C	Electric charge		$\mathrm{s} \cdot \mathrm{A}$
Volt	V	Voltage	W/A	$\mathrm{kg{\cdot}m}^2/\mathrm{s}^3\mathrm{A}$
Farad	F	Capacitance	C/V	$\mathrm{s}^4\mathrm{A}^2/\mathrm{kg{\cdot}m}^2$
Ohm	Ω	Resistance	V/A	$\mathrm{kg{\cdot}m}^2/\mathrm{s}^3\mathrm{A}^2$
Weber	Wb	Magnetic flux	$\mathrm{V} \cdot \mathrm{s}$	$\mathrm{kg{\cdot}m}^2/\mathrm{s}^2\mathrm{A}$
Tesla	T	Magnetic flux density	Wb/m^2	$\mathrm{kg}/\mathrm{s}^2\mathrm{A}$
Henry	H	Inductance	Wb/A	$\mathrm{kg{\cdot}m}^2/\mathrm{s}^2\mathrm{A}^2$

Table 2: SI derived units

4.2 Planck Units

Planck units are built by combining constants of nature to create natural measurement scales. The relevant constants have the following units:

$$[c] = \mathrm{L}/\mathrm{T} \qquad [G] = \mathrm{L}^3/\mathrm{MT}^2 \qquad [\hbar] = \mathrm{L}^2\mathrm{M}/\mathrm{T} \qquad \left[\frac{1}{4\pi\epsilon_0}\right] = \mathrm{L}^3\mathrm{M}/\mathrm{T}^2\mathrm{Q}^2 \qquad [k_\mathrm{B}] = \mathrm{L}^2\mathrm{M}/\mathrm{T}^2\Theta \quad (6)$$

These may be combined to form the following Planck units:

Name	Expression	Value
Planck length	$l_\mathrm{P} = \sqrt{\frac{\hbar G}{c^3}}$	$1.616 \cdot 10^{-35}\,\mathrm{m}$
Planck mass	$m_\mathrm{P} = \sqrt{\frac{\hbar c}{G}}$	$2.177 \cdot 10^{-8}\,\mathrm{kg}$
Planck time	$t_\mathrm{P} = \sqrt{\frac{\hbar G}{c^5}}$	$5.391 \cdot 10^{-44}\,\mathrm{s}$
Planck charge	$q_\mathrm{P} = \sqrt{4\pi\epsilon_0 \hbar c}$	$1.876 \cdot 10^{-18}\,\mathrm{C}$
Planck temperature	$T_\mathrm{P} = \sqrt{\frac{\hbar c^5}{G k_\mathrm{B}^2}}$	$1.417 \cdot 10^{32}\,\mathrm{K}$

Other Planck units, such as those for volume and energy may be constructed from the above units in the obvious way.