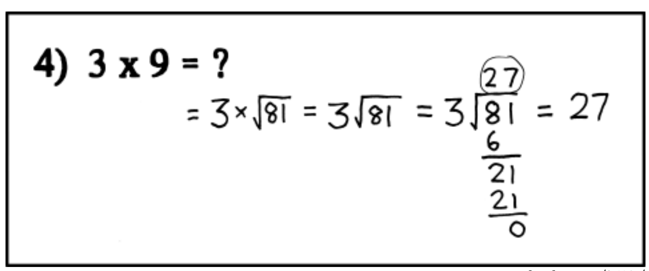
# Physics GRE:

# Summary


*G. J. Loges*[1]

University of Rochester
Dept. of Physics & Astronomy


xkcd.com/759/

# Contents

# 1 Test-Taking

**1.1 Strategies** The Physics GRE subject test consist of 100 multiple-choice questions in 170 minutes. Each question has five choices; a correct response earns you one point and an incorrect response deducts a quarter of a point. Your raw score is converted to a scale in 10 point jumps out of 990, with a median score in the mid-six-hundreds. The test covers most aspects of an undergraduate physics curriculum. While it may not be the best indicator of your ability to do well in a physics graduate program, it does show that you have your foundations covered. Doing well will only help your chances of standing out.

- Move quickly! You have fewer than two minutes per question. Some questions you will know very quickly and others will take a good five minutes to finish the calculations. Mark involved questions and come back to them later. Set a timer for the practice tests and hold yourself to it!
- Have a bunch of common constants on quick recall such as $m_e = 0.5\,\mathrm{MeV}/c^2$ and $hc = 1240\,\mathrm{eV\,nm}$. Also, estimate like crazy when you can. If the five answers span many orders of magnitude chop everything down to one significant figure and go from there.
- Use dimensional analysis and limiting cases to narrow down the choices.
- Answer questions, even if you are not sure. Taking a random guess has an expected value of zero, and if you can eliminate one or more options then guessing is likely to be worthwhile.

**1.2 Useful Quantities** While there is a list of constants at the beginning of the test, it is useful to have a collection of common quantities available for quick use.

| Name | Symbol | Approximate Value |
|---|---|---|
| Speed of light | $c$ | $3.0 \cdot 10^8\,\mathrm{m/s}$ |
| Electric charge | $e$ | $1.6 \cdot 10^{-19}\,\mathrm{C}$ |
| Electron mass | $m_e$ | $0.5\,\mathrm{MeV}/c^2$ |
| Proton mass | $m_p$ | $940\,\mathrm{MeV}/c^2$ |
| Proton/Electron mass ratio | $m_p/m_e$ | 1800 |
| Hydrogen ground state energy | $E_1$ | $-13.6\,\mathrm{eV}$ |
| – | $hc$ | $1240\,\mathrm{eV\,nm}$ |
| Fine structure constant | $\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$ | $\frac{1}{137}$ |
| Bohr radius | $a_0 = \frac{\hbar}{m_e c\alpha}$ | $0.53\,\mathrm{Å}$ |
| Electron Compton wavelength | $\lambda_C = \frac{h}{m_e c}$ | $0.0024\,\mathrm{nm}$ |
| Visible spectrum wavelengths | – | $400 - 700\,\mathrm{nm}$ |
| Room temperature thermal energy | $kT_{\mathrm{room}}$ | $\frac{1}{40}\,\mathrm{eV}$ |

**1.3 Binomial Expansion** For integral powers we may expand binomials as

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \tag{1}$$

| $\beta$ | $0$ | $\frac{1}{2}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{\sqrt{3}}{2}$ | $\frac{9}{10}$ | $\frac{12}{13}$ | $\frac{95}{100}$ | $\frac{98}{100}$ | $\frac{99}{100}$ | $\frac{995}{1000}$ | $\frac{999}{1000}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 0.5 | 0.6 | 0.8 | 0.87 | 0.9 | 0.92 | 0.95 | 0.98 | 0.99 | 0.995 | 0.999 |
| $\gamma$ | $1$ | $\frac{2}{\sqrt{3}}$ | $\frac{5}{4}$ | $\frac{5}{3}$ | $2$ | $\frac{10}{\sqrt{19}}$ | $\frac{13}{5}$ | $\frac{20}{\sqrt{39}}$ | $\frac{50}{3\sqrt{11}}$ | $\frac{100}{\sqrt{199}}$ | $\frac{200}{\sqrt{399}}$ | $\frac{1000}{\sqrt{1999}}$ |
| | 1 | 1.15 | 1.25 | 1.67 | 2 | 2.29 | 2.6 | 3.2 | 5.0 | 7.1 | 10.0 | 22.4 |

Table 1: Values of the Lorentz factor for several values of $\beta = \frac{v}{c}$.

This generalizes to nonintegral powers through the use of the gamma function for the binomial coefficients:

$$(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k y^{r-k} = y^r + rxy^{r-1} + \frac{r(r-1)}{2}x^2y^{r-2} + \frac{r(r-1)(r-2)}{6}x^3y^{r-3} + \cdots \tag{2}$$

However, due to the infinities the sum may not converge:

$$(1+x)^{-1} = 1 - x + x^2 - \cdots \qquad |x| < 1 \tag{3}$$

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots \qquad |x| < 1 \tag{4}$$

$$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \cdots \qquad |x| < 1 \tag{5}$$

These provide a quick way to estimate quantities such as those arising in doppler shift and special relativity topics.

# 2 Classical Mechanics

### Kinematics

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \boldsymbol{v}_0 t + \frac{1}{2}\boldsymbol{a}t^2 \tag{Constant Acceleration}$$

$$\boldsymbol{v}(t) = \boldsymbol{v}_0 + \boldsymbol{a}t$$

$$y_{\max} = y_0 + \frac{v_{0y}^2}{2g} \tag{Max Height}$$

$$R = \frac{v^2}{g}\sin 2\theta \tag{Range Equation}$$

### Newton's Laws

$$\sum \boldsymbol{F} = 0 \iff \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = 0 \tag{1st Law}$$

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\boldsymbol{v})}{\mathrm{d}t} \tag{2d Law}$$

$$\boldsymbol{F}_{\mathrm{AB}} = -\boldsymbol{F}_{\mathrm{BA}} \tag{3d Law}$$

### Work & Energy

$$W = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} \tag{Work}$$

$$\sum W = \Delta E \tag{Work-Energy Theorem}$$
$$\boldsymbol{F} = -\boldsymbol{\nabla} U \tag{Conservative Force}$$
$$T = T_t + T_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{p^2}{2m} + \frac{L^2}{2I} \tag{Kinetic Energy}$$
$$f_\mathrm{s} \le \mu_\mathrm{s} N \tag{Static Friction}$$
$$f_\mathrm{k} = \mu_\mathrm{k} N \tag{Kinetic Friction}$$
$$\boldsymbol{J} = \Delta \boldsymbol{p} = \int \boldsymbol{F}(t)\,\mathrm{d}t \tag{Impulse}$$

**Rotational Motion**

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{Constant Angular Acceleration}$$
$$\omega(t) = \omega_0 + \alpha t$$
$$T = \frac{1}{f} = \frac{2\pi}{\omega} \tag{Period $\leftrightarrow$ Frequency}$$
$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} \tag{Torque}$$
$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = I\boldsymbol{\omega} \tag{Angular Momentum}$$
$$I = \sum_i m_i r_i^2 = \int r^2\,\mathrm{d}m = \int r^2 \rho(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} \tag{Momentum of Inertia}$$
$$\boldsymbol{\tau} = \frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t} = \frac{\mathrm{d}(\boldsymbol{r} \times \boldsymbol{p})}{\mathrm{d}t} = \frac{\mathrm{d}(I\boldsymbol{\omega})}{\mathrm{d}t} \tag{Angular 2$^\mathrm{d}$ Law}$$

**Noninertial Reference Frames**

$$\boldsymbol{F}_\mathrm{cf} = -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \tag{Centrifugal Force}$$
$$\boldsymbol{F}_\mathrm{Co} = -2m\boldsymbol{\omega} \times \boldsymbol{v} \tag{Coriolis Force}$$

**Dynamics of Systems of Particles**

$$\boldsymbol{r}_\mathrm{cm} = \frac{1}{M}\sum_i m_i \boldsymbol{r}_i = \frac{1}{M}\int \boldsymbol{r}\,\mathrm{d}m \tag{Center of Mass}$$

**Central Forces & Celestial Mechanics**

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \text{const.} \tag{Kepler's 2$^\mathrm{d}$ Law}$$
$$\frac{T^2}{a^3} = \frac{4\pi^2}{G(M+m)} \approx \frac{4\pi^2}{GM} \tag{Kepler's 3$^\mathrm{d}$ Law}$$
$$g = \frac{GM_\oplus}{R_\oplus^2} \approx 9.81\,{}^\mathrm{N}/_\mathrm{kg} \tag{Earth Surface Gravity}$$

**Lagrangian Mechanics**

$$S = \int L(\boldsymbol{q}, \dot{\boldsymbol{q}}; t)\,\mathrm{d}t \tag{Action}$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\boldsymbol{q}}} - \frac{\partial L}{\partial \boldsymbol{q}} \qquad \text{(Euler-Lagrange Equations)}$$

**Hamiltonian Mechanics**

$$\boldsymbol{p} = \frac{\partial L}{\partial \dot{\boldsymbol{q}}} \qquad \text{(Conjugate Momentum)}$$

$$H(\boldsymbol{q}, \boldsymbol{p}; t) = \dot{\boldsymbol{q}} \cdot \boldsymbol{p} - L(\boldsymbol{q}, \dot{\boldsymbol{q}}; t) \qquad \text{(Hamiltonian)}$$

$$\dot{\boldsymbol{q}} = \frac{\partial H}{\partial \boldsymbol{p}} = \{\boldsymbol{q}, H\} \qquad \text{(Hamilton's Equations)}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{q}} = \{\boldsymbol{p}, H\}$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

**Fluid Dynamics**

$$P = \frac{\mathrm{d}F}{\mathrm{d}A} \qquad \text{(Pressure)}$$

$$\text{const.} = P + \frac{1}{2}\rho v^2 + \rho g h \qquad \text{(Bernoulli Equation)}$$

$$0 = \frac{\mathrm{d}\rho}{\mathrm{d}t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{u}) \qquad \text{(Continuity Equation)}$$

$$Av = \text{const.} \qquad \text{(Incompressible Tube Flow)}$$

$$F_{\text{buoy}} = \rho_{\text{fl}} V g \qquad \text{(Archimedes' Principle)}$$

# 3 Electromagnetism

**Electrostatics**

$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\boldsymbol{r}} \qquad \text{(Coulomb's Law)}$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\boldsymbol{F}}{q} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\boldsymbol{r} - \boldsymbol{r}_i|^2}\frac{\boldsymbol{r} - \boldsymbol{r}_i}{|\boldsymbol{r} - \boldsymbol{r}_i|} \qquad \text{(Electric Field)}$$

$$\boldsymbol{E} = -\boldsymbol{\nabla} V \qquad \text{(Electric Potential)}$$

$$V(\boldsymbol{r}) = -\int_\infty^{\boldsymbol{r}} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$$

**Magnetostatics**

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B} = I\boldsymbol{l} \times \boldsymbol{B} \qquad \text{(Magnetic Force)}$$

$$\omega = \frac{qB}{m} \qquad \text{(Cyclotron Frequency)}$$

$$\mathrm{d}\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\boldsymbol{l} \times \hat{\boldsymbol{r}}}{r^2} \qquad \text{(Biot-Savart Law)}$$

### Maxwell's Equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{Gauss' Law}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{Gauss' Law for Magnetism}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{Faraday's Law}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \tag{Ampère's Law}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} = \epsilon \boldsymbol{E} = \epsilon_0 (1 + \chi_e) \boldsymbol{E} \tag{Auxillary Fields}$$

$$\boldsymbol{H} = \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M} = \frac{1}{\mu} \boldsymbol{B} = \frac{1}{\mu_0 (1 + \chi_m)} \boldsymbol{B}$$

### Electromagnetic Induction

$$\Phi_{\mathrm{E}} = \int_A \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{A} \tag{Electric Flux}$$

$$\Phi_{\mathrm{B}} = \int_A \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A} \tag{Magnetic Flux}$$

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{d}t} \tag{Induced EMF}$$

$$L = \mu_0 n^2 A l \tag{Solenoid Self-Inductance}$$

### Electromagnetic Waves

$$\frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \nabla^2 \boldsymbol{E} \tag{Vacuum Wave Equations}$$

$$\frac{1}{c^2} \frac{\partial^2 \boldsymbol{B}}{\partial t^2} = \nabla^2 \boldsymbol{B}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \tag{Speed of Light}$$

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{n} \tag{Speed of Light in Medium}$$

### Circuits

$$u = \frac{I}{nAq} \tag{Drift Velocity}$$

$$V = IR \tag{Ohm's Law}$$

$$P = IV = I^2 R = \frac{V^2}{R} \tag{Disspated Power}$$

$$Q = CV \tag{Capacitance}$$

$$U = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{Q^2}{2C} \tag{Energy of Capacitor}$$

$$V = L \frac{\mathrm{d}I}{\mathrm{d}t} \tag{Inductor Potential}$$

$$U = \frac{1}{2} L I^2 \tag{Energy of Inductor}$$

$$R_{\mathrm{ser}} = \sum_i R_i \qquad \text{(Effective Resistance)}$$

$$\frac{1}{R_{\mathrm{par}}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{C_{\mathrm{ser}}} = \sum_i \frac{1}{C_i} \qquad \text{(Effective Capacitance)}$$

$$C_{\mathrm{par}} = \sum_i C_i$$

$$\oint_C V \,\mathrm{d}l = 0 \qquad \text{(Kirchhoff Loop Rule)}$$

$$\sum_i I_i = 0 \qquad \text{(Kirchhoff Current Rule)}$$

$$\tau = RC \qquad \text{(RC Time Constant)}$$

$$\tau = \frac{R}{L} \qquad \text{(RL Time Constant)}$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad \text{(LC Frequency)}$$

$$Z_{\mathrm{C}} = \frac{1}{i\omega C} \qquad \text{(Capacitor Impedance)}$$

$$Z_{\mathrm{L}} = i\omega L \qquad \text{(Inductor Impedance)}$$

# 4 Quantum Mechanics

$$p = \frac{h}{\lambda} \qquad \text{(de Broglie Wavelength)}$$

$$K_{\max} = h\nu - W = eV_{\mathrm{stop}} \qquad \text{(Photoelectric Effect)}$$

**Operators**

$$\langle A \rangle = \langle A | \Psi \rangle = \int \Psi^* A \psi \,\mathrm{d}\boldsymbol{r} \qquad \text{(Expected Value)}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle A \rangle = \frac{1}{i\hbar} \langle [A, H] \rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle \qquad \text{(Ehrenfest Theorem)}$$

$$p_i = -i\hbar \frac{\partial}{\partial x^i} \qquad \text{(Momemtum Operator)}$$

$$T = \frac{\boldsymbol{p} \cdot \boldsymbol{p}}{2m} = -\frac{\hbar^2}{2m} \nabla^2 \qquad \text{(Kinetic Energy)}$$

$$L_z = -i\hbar \frac{\partial}{\partial \phi} \qquad \text{(Angular Momentum)}$$

$$H = T + V(x) = -\frac{\hbar^2}{2m} \nabla^2 + V(x) \qquad \text{(Nonrelativistic Hamiltonian)}$$

$$a = \sqrt{\frac{m\omega}{2\hbar}} \left( x + \frac{ip}{m\omega} \right) = \sqrt{\frac{m\omega}{2\hbar}} \left( x + \frac{\hbar}{m\omega} \frac{\partial}{\partial x} \right) \qquad \text{(Harmonic Oscillator)}$$

$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left( x - \frac{ip}{m\omega} \right) = \sqrt{\frac{m\omega}{2\hbar}} \left( x - \frac{\hbar}{m\omega} \frac{\partial}{\partial x} \right)$$

$$H = \hbar\omega\left(N + \frac{1}{2}\right) = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$[x_i, p_j] = i\hbar\delta_{ij} \qquad \text{(Cannonical coordinates)}$$

$$[f(x), p_x] = i\hbar f'(x) \qquad \text{(Function of position)}$$

$$[x, g(p_x)] = i\hbar g'(p_x) \qquad \text{(Function of momentum)}$$

$$[L_i, L_j] = i\hbar\epsilon_{ijk}L^k \qquad \text{(Angular momentum)}$$

$$[L^2, L_i] = 0$$

$$[a, a^\dagger] = 1 \qquad \text{(Ladder Operators)}$$

$$[N, a] = -a$$

$$[N, a^\dagger] = a^\dagger$$

**One-Dimensional Potentials**

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad E_n = \frac{\hbar^2\pi^2n^2}{2mL^2} \qquad n \in \{1, 2, 3, \dots\} \qquad \text{(Infinite square well)}$$

$$R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 \qquad T = \frac{4k_1k_2}{(k_1 + k_2)^2} \qquad \text{(Step: } E > V_0\text{)}$$

$$T = \left(1 + \frac{V_0^2\sin^2(k_2L)}{4E(E - V_0)}\right)^{-1} \qquad \text{(Finite Barrier: } E > V_0\text{)}$$

$$T = \left(1 + \frac{V_0^2\sinh^2(k_2L)}{4E(V_0 - E)}\right)^{-1} \qquad \text{(Finite Barrier: } E < V_0\text{)}$$

$$\psi(x) = \sqrt{\frac{m\beta}{\hbar^2}}\exp\left(-\frac{m\beta|x|}{\hbar^2}\right) \qquad V(x) = -\beta\delta(x) \qquad \text{(Delta Well Bound State)}$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\sqrt[4]{\frac{m\omega}{\pi\hbar}}H_n(\xi)e^{-\xi^2/2} \qquad \xi = \sqrt{\frac{m\omega}{\hbar}}x \qquad \text{(Harmonic Oscillator)}$$

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right) \qquad n \in \{0, 1, 2, \dots\}$$

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right) \qquad a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{ip}{m\omega}\right)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}\left(a^\dagger + a\right) \qquad p = i\sqrt{\frac{\hbar m\omega}{2}}\left(a^\dagger - a\right)$$

**Angular Momentum & Spin**

$$L^2|l, m\rangle = \hbar^2 l(l+1)|l, m\rangle \qquad \text{(Eigenvalue Expressions)}$$

$$L_z|l, m\rangle = \hbar m|l, m\rangle$$

$$L_\pm|l, m\rangle = \hbar\sqrt{l(l+1) - m(m \pm 1)}|l, m \pm 1\rangle \qquad \text{(Ladder Operators)}$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \text{(Pauli Matrices)}$$

$$\sigma_i^2 = \mathbb{I}_2 \qquad \det\sigma_i = -1 \qquad \operatorname{tr}\sigma_i = 0$$

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij}\mathbb{I}_2 \qquad [\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma^k$$

$$\boldsymbol{A}\cdot\boldsymbol{B} = \frac{1}{2}\left(C^2 - A^2 - B^2\right) \qquad \boldsymbol{C} \equiv \boldsymbol{A}+\boldsymbol{B} \qquad \text{(The "Trick")}$$

**Time-Independent Non-Degenerate Perturbation Theory**

$$E_n^{(1)} = \langle n^{(0)}|H'|n^{(0)}\rangle \qquad \text{(1}^{\text{st}}\text{-order energy correction)}$$

$$|n^{(1)}\rangle = \sum_{k\neq n} \frac{\langle k^{(0)}|H'|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}}\,|k^{(0)}\rangle \qquad \text{(1}^{\text{st}}\text{-order eigenstate correction)}$$

$$E_n^{(2)} = \sum_{k\neq n} \frac{|\langle k^{(0)}|H'|n^{(0)}\rangle|^2}{E_n^{(0)} - E_k^{(0)}} \qquad \text{(2}^{\text{d}}\text{-order energy correction)}$$

# 5 Thermodynamics & Statistical Mechanics

**Laws of Thermodynamics**

$$T_1 = T_2 \;\wedge\; T_2 = T_3 \qquad \Longrightarrow \qquad T_1 = T_3 \qquad \text{(1}^{\text{st}}\text{ Law)}$$

$$đQ = \mathrm{d}E + đW \qquad \text{(2}^{\text{d}}\text{ Law)}$$

$$\Delta S \geq 0 \qquad \text{(3}^{\text{d}}\text{ Law)}$$

$$T \to 0 \qquad \Longrightarrow \qquad S \to S_0 \qquad \text{(4}^{\text{th}}\text{ Law)}$$

**Thermodynamic Processes**

$$S = k\log\Omega \qquad \text{(Entropy)}$$

$$\mathrm{d}S = \frac{đQ}{T}$$

$$\beta = \frac{1}{kT} = \frac{\partial \log\Omega}{\partial E} = \frac{1}{k}\frac{\partial S}{\partial E} \qquad \text{(Temperature)}$$

$$S + S' = \text{maximal} \;\wedge\; T = T' \qquad \text{(Thermal Equilibrium)}$$

$$đQ = 0 \qquad \text{(Adiabatic Processes)}$$

$$PV^\gamma = \text{const.}$$

$$TV^{\gamma-1} = \text{const.}$$

$$P^{1-\gamma}T^\gamma = \text{const.}$$

$$\mathrm{d}T = 0 \qquad \text{(Isothermal Processes)}$$

$$W = nRT\log\left(\frac{V_2}{V_1}\right)$$

$$\mathrm{d}P = 0 \qquad \text{(Isobaric Processes)}$$

$$W = P\Delta V$$

**Ideal Gases**

$$PV = nRT = NkT \qquad \text{(Ideal Gas Law)}$$

$$\mathrm{d}E = \left(\frac{\partial E}{\partial T}\right)_V \mathrm{d}T \qquad \text{(Ideal Gas Energy)}$$

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT = NkT \qquad \text{(Van der Waals Equation)}$$

**Equipartition Theorem**

$$E(q_1, q_2, \ldots, q_n) = E_1(q_1) + E'(q_2, \ldots, q_n) = E_1 + E' \qquad \text{(Assumptions)}$$
$$E_1(q_1) = Aq_1^r$$
$$\langle E_1 \rangle = \frac{1}{r} kT \qquad \text{(Conclusion)}$$

**Maxwell Relations**

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V \qquad \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \qquad \text{(Maxwell Relations)}$$
$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \qquad \left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$
$$H = E + PV \qquad \text{(Enthalpy)}$$
$$G = H - TS \qquad \text{(Gibb's Free Energy)}$$
$$F = E - TS \qquad \text{(Helmholtz Free Energy)}$$

| | | |
|---|---|---|
| $-S$ | $U$ | $V$ |
| $H$ | | $F$ |
| $-P$ | $G$ | $T$ |

(Mnemonic)

**Statistical Mechanics**

$$Z = \sum_i e^{-\beta E_i} = \sum_E \Omega(E) e^{-\beta E} \qquad \text{(Partition Function)}$$
$$Z = \prod_i \zeta_i \qquad \text{(Weakly Interacting Subsystems)}$$
$$Z = \frac{\zeta^N}{N!} \qquad \text{(Indistinguishable Subsystems)}$$
$$\langle E \rangle = -\frac{\partial \log Z}{\partial \beta} \qquad \text{(Average Energy)}$$
$$S = k(\log Z + \beta \langle E \rangle) \qquad \text{(Entropy)}$$
$$C_{\mathrm{V}} = \frac{1}{kT^2} \frac{\partial^2 \log Z}{\partial T^2} \qquad \text{(Specific Heat)}$$

**Particle Statistics**

$$\langle n_i \rangle = N \cdot \frac{e^{-\beta E_i}}{Z} \qquad \text{(Maxwell-Boltzmann Statistics)}$$
$$\langle n_i \rangle = \frac{1}{e^{(E_i - \mu)/kT} - 1} \qquad \text{(Bose-Einstein Statistics)}$$
$$\langle n_i \rangle = \frac{1}{e^{(E_i - \mu)/kT} + 1} \qquad \text{(Fermi-Dirac Statistics)}$$
$$\langle n_i \rangle = \frac{1}{e^{E_i/kT} - 1} \qquad \text{(Photon Statistics)}$$

**Theories of Specific Heat**

$$c_{\mathrm{V}} = 3N_{\mathrm{A}}k\left(\frac{h\nu}{kT}\right)^2 \frac{e^{h\nu/kT}}{\left(e^{h\nu/kT}-1\right)^2} \qquad \text{(Einstein Theory)}$$
$$c_{\mathrm{V}} \to \alpha T^3 \qquad \text{(Debye Low Temperature)}$$
$$c_{\mathrm{V}} \to 3N_{\mathrm{A}}k = 3R \qquad \text{(Einstein \& Debye High Temperature)}$$
(6)

# 6 Atomic Physics

**Bohr Model**

$$L = n\hbar \qquad n \in \{1, 2, 3, \ldots\} \qquad \text{(Quantization Assumption)}$$
$$r_n = \frac{\hbar^2 n^2}{Zke^2\mu} = \frac{a_0 n^2}{Z} \qquad \text{(Radii)}$$
$$a_0 = \frac{\hbar^2}{kme^2} \approx 0.529\,\text{Å} \qquad \text{(Bohr Radius)}$$
$$E_n = -\frac{Z^2k^2e^4\mu}{2\hbar^2 n^2} = -13.6\,\mathrm{eV}\frac{Z^2}{n^2} \qquad \text{(Hydrogen Energy Levels)}$$
$$n \in \{1, 2, 3, \ldots\} \qquad \text{(Principle Quantum Nº)}$$
$$l \in \{0, 1, \ldots, n-1\} \qquad \text{(Azimuthal Quantum Nº)}$$
$$m_l \in \{-l, -l+1, \ldots, l\} \qquad \text{(Magnetic Quantum Nº)}$$
$$m_s \in \{-s, -s+1, \ldots, s\} \qquad \text{(Spin Projection Quantum Nº)}$$

**Selection Rules**

$$\Delta l = \pm 1 \qquad \text{(Electric Dipole Selection Rules)}$$
$$\Delta m = 0, \pm 1$$
$$\Delta s = 0$$
$$j_i = 0 \not\to j_f = 0$$

**Blackbody Radiation**

$$I_{\mathrm{P}}(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \qquad \text{(Planck's Law)}$$
$$\lambda_{\mathrm{peak}}T = 2.9 \cdot 10^6\,\mathrm{nm\,K} \qquad \text{(Wien's Displacement Law)}$$
$$I_{\mathrm{RJ}}(\lambda, T) = \frac{2ckT}{\lambda^4} \qquad \text{(Raleigh-Jeans Law)}$$
$$I_{\mathrm{W}} = \frac{2hc^2}{\lambda^5}e^{-\frac{hc}{\lambda kT}} \qquad \text{(Wien's Law)}$$

**Compton Scattering**

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta) \qquad \text{(Compton Shift)}$$

$$\lambda_{\mathrm{C}} = \frac{h}{mc} \qquad \text{(Compton Wavelength)}$$

$$\lambda_{\mathrm{C}} = \frac{h}{m_e c} \approx 2.43\,\mathrm{pm} \qquad \text{(Electron Compton Wavelength)}$$

**Moseley's Law**

$$E = 13.6\,\mathrm{eV}\left(\frac{3}{4}\right)(Z-1)^2 \qquad (K\alpha \text{ Photons: } n = 2 \to 1)$$

$$E = 13.6\,\mathrm{eV}\left(\frac{5}{36}\right)(Z-7.4)^2 \qquad (L\alpha \text{ Photons: } n = 3 \to 2)$$

# 7 Optics & Wave Phenomena

**Wave Properties**

$$\frac{1}{v_{\mathrm{p}}^2}\frac{\partial^2\phi}{\partial t^2} = \nabla^2\phi \qquad \text{(Wave Equation)}$$

$$v_{\mathrm{p}} = \frac{\omega}{k} = \frac{\lambda}{T} \qquad \text{(Phase Velocity)}$$

$$v_{\mathrm{g}} = \frac{\mathrm{d}\omega(k)}{\mathrm{d}k} \qquad \text{(Group Velocity)}$$

$$2\pi f_{\mathrm{B}} = \omega_{\mathrm{B}} = |\omega_1 - \omega_2| \qquad \text{(Beat Frequency)}$$

**Interference & Diffraction**

$$n\lambda = d\sin\theta \approx d\cdot\frac{y}{D} \qquad \text{(Double-Slit Maxima)}$$

$$\left(n+\tfrac{1}{2}\right)\lambda = d\sin\theta \approx d\cdot\frac{y}{D} \qquad \text{(Double-Slit Minima)}$$

$$\left(n+\tfrac{1}{2}\right)\lambda = a\sin\theta \approx a\cdot\frac{y}{D} \qquad \text{(Single-Slit Maxima)}$$

$$n\lambda = a\sin\theta \approx a\cdot\frac{y}{D} \qquad \text{(Single-Slit Minima)}$$

$$I = I_0\cos^2\left(\frac{\pi d\,y}{\lambda D}\right)\mathrm{sinc}^2\left(\frac{\pi a\,y}{\lambda D}\right) \qquad \text{(Double-Slit Intensity)}$$

$$f_n = \frac{c}{\lambda_n} = \frac{(2n+1)c}{4L} = (2n+1)f_0 \qquad \text{(Half-Open Frequencies)}$$

$$f_n = \frac{c}{\lambda_n} = \frac{nc}{2L} = nf_1 \qquad \text{(Both Open Freqencies)}$$

**Geometrical Optics**

$$n_1\sin\theta_1 = n_2\sin\theta_2 \qquad \text{(Snell's Law)}$$

$$\theta_{\mathrm{crit}} = \arcsin\left(\frac{n_2}{n_1}\right) \qquad \text{(Critical Angle)}$$

$$\theta_{\mathrm{Brew}} = \arctan\left(\frac{n_2}{n_1}\right) \qquad \text{(Brewster's Angle)}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{(Lensmaker's Equation)}$$

$$\frac{1}{o} + \frac{1}{f} = \frac{1}{i} \qquad \text{(Thin Lens Equation)}$$

$$M = \frac{i}{o} \qquad \text{(Magnification)}$$

**Doppler Effect**

$$f = \left(\frac{c + v_{\mathrm{o}}}{c + v_{\mathrm{s}}}\right) f_0 \qquad \text{(Doppler Shift)}$$

$$f = \left(1 + \frac{\Delta v}{c}\right) f_0 \qquad \text{(Low Velocity Approximation)}$$

# 8 Relativity

**Four-Vectors & Lorentz Transformations**

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z) \qquad \text{(Four-Position)}$$

$$\eta_{\mu\nu} = \eta^{\mu\nu} = \operatorname{diag}(1, -1, -1, -1) \qquad \text{(Minkowski Metric)}$$

$$M_\mu = \eta_{\mu\nu} M^\nu \qquad M^\mu = \eta^{\mu\nu} M_\nu \qquad \text{(Raising \& Lowering of Indices)}$$

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu = c^2\mathrm{d}t^2 - \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) = c^2\mathrm{d}t^2 - \mathrm{d}\boldsymbol{x}^2 \qquad \text{(Spacetime Interval)}$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \beta^2}} \qquad \text{(Lorentz Factor)}$$

$$\begin{bmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} \qquad \text{(Boost Along } x\text{-Axis)}$$

$$v' = \frac{v + u}{1 + \frac{vu}{c^2}} \qquad \text{(Colinear Velocity Addition)}$$

$$\boldsymbol{p} = \gamma m \boldsymbol{v} \qquad \text{(Momentum)}$$

$$E = \gamma m c^2 \qquad \text{(Energy)}$$

$$E^2 = (mc^2)^2 + |\boldsymbol{p}|^2 c^2$$

$$p^\mu = (p^0, p^1, p^2, p^3) = (\tfrac{E}{c}, p_x, p_y, p_z) \qquad \text{(Four-Momentum)}$$

$$p_\mu p^\mu = m^2 c^2$$

**Time Dilation & Length Contraction**

$$\Delta t = \gamma \Delta\tau \qquad \text{(Time Dilation)}$$

$$L = \frac{L_0}{\gamma} \qquad \text{(Length Contraction)}$$

### Electromagnetism

$$A^\mu = (A^0, A^1, A^2, A^3) = (\tfrac{V}{c}, A_x, A_y, A_z) \qquad \text{(Four-Potential)}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \text{(Electromagnetic Field Tensor)}$$

$$\begin{aligned} E'_x &= E_x & B'_x &= B_x \\ E'_y &= \gamma(E_y - vB_z) & B'_y &= \gamma(B_y + vE_z/c^2) \\ E'_z &= \gamma(E_z + vB_y) & B'_z &= \gamma(B_z - vE_y/c^2) \end{aligned} \qquad \text{(Boost Along } x\text{-Axis)}$$

### Doppler Effect

$$\lambda = \frac{c}{f} = \lambda_0 \sqrt{\frac{1 \pm \beta}{1 \mp \beta}} = \frac{c}{f_0} \sqrt{\frac{1 \pm \beta}{1 \mp \beta}} \qquad \text{(Doppler Effect)}$$

### General Relativity

$$r_{\mathrm{s}} = \frac{2GM}{c^2} \qquad \text{(Schwarzchild Radius)}$$

## 9 Specialized Topics

### Radioactive Decay

$$\begin{aligned} \frac{\mathrm{d}N}{\mathrm{d}t} &= -\gamma N \qquad \text{(Exponential Decay)} \\ N(t) &= N_0 e^{-\gamma t} = N_0 e^{-t/\tau} = N_0 2^{-t/t_{1/2}} \\ \tau &= \frac{1}{\gamma} = \frac{t_{1/2}}{\log 2} \end{aligned}$$

### X-Ray Diffraction

$$2d \sin\theta = n\lambda \qquad n \in \{1, 2, 3, \ldots\} \qquad \text{(Bragg Diffraction)}$$

### Vector Differential Operators

$$\boldsymbol{\nabla} f = \frac{\partial f}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{k}} \qquad \text{(Gradient)}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \qquad \text{(Divergence)}$$

$$\boldsymbol{\nabla} \times \boldsymbol{F} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\boldsymbol{\imath}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\boldsymbol{\jmath}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\boldsymbol{k}} \qquad \text{(Curl)}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \qquad \text{(Laplacian)}$$

**Fourier Series**

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \qquad \text{(Fourier Series)}$$

$$a_m = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos mx \, \mathrm{d}x \qquad m \in \{0, 1, 2, \ldots\}$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin kx \, \mathrm{d}x \qquad k \in \{1, 2, 3, \ldots\}$$

**Matrix Algebra**

$$\operatorname{tr} A = \sum_i a_{ii} = \sum_i \lambda_i \qquad \text{(Trace)}$$

$$\det A = \prod_i \lambda_i \qquad \text{(Determinant)}$$

$$\det A_{2\times 2} = a_{11}a_{22} - a_{12}a_{21} \qquad (2 \times 2 \text{ Determinant})$$

$$A_{2\times 2}^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \qquad (2 \times 2 \text{ Inverse})$$

$$\lambda_{\pm} = \frac{\operatorname{tr} A}{2} \pm \sqrt{\left(\frac{\operatorname{tr} A}{2}\right)^2 - \det A} \qquad (2 \times 2 \text{ Eigenvalues})$$