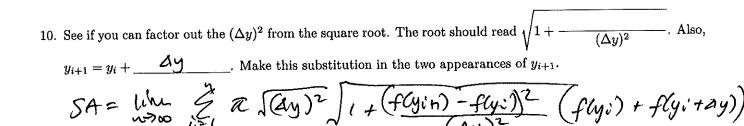$$f(y) = \cos(y-1) + 2$$

(a) 1. First, we are revolving $f(y)$ around the y-axis. Since the function f is in terms of y, we integrate along the

   __y__ axis (with respect to __y__), which is parallel to the axis of rotation. Consider the cross section of

   the vase along the axis of integration. This means we can use (disc / washer / cylindrical shells).

2. The base of the vase is at $y =$ __0__, and since the vase's height is 6 meters, the top of the vase is at

   $y =$ __6__. The limits of integration is [__0__ , __6__].

3. The radius of each disc is __$f(y) = \cos(y-1) + 2$__ .

4. Recall the disc method formula: $V = \pi \int_a^b (\text{Radius})^2 \, dy$. As an integral, the volume of the solid is:

$$V = \pi \int_0^6 (f(y))^2 \, dy = \pi \int_0^6 (\cos(y-1) + 2)^2 \, dy$$

5. Evaluate the integral below. **The vase should store more than 80 cubic meters of water.**

$$u = y - 1 \qquad du = dy$$

$$V = \pi \int_{-1}^5 (\cos u + 2)^2 \, du = \pi \int_{-1}^5 (\cos^2 u + 4\cos u + 4) \, du$$

$$= \pi \int_{-1}^5 \frac{1 + \cos 2u}{2} \, du + \pi \int_{-1}^5 (4\cos u + 4) \, du$$

$$= \frac{\pi}{2} \left[u + \frac{\sin 2u}{2} \right]_{-1}^5 + \pi \left[4\sin u + 4u \right]_{-1}^5$$

$$= \frac{\pi}{2} \left(5 + \frac{\sin 10}{2} - \left(-1 + \frac{\sin(-2)}{2} \right) \right) + \pi \left(4\sin 5 + 20 - (4\sin(-1) - 4) \right)$$

$$= \frac{\pi}{2} \left(6 + \frac{\sin 10}{2} - \frac{\sin(-2)}{2} \right) + \pi \left(24 + 4\sin 5 - 4\sin(-1) \right)$$

$$= \quad \ldots \quad [\text{more simplification}]$$

$$\approx 83.634 \, m^3 \geq 80 \, m^3$$

10. See if you can factor out the $(\Delta y)^2$ from the square root. The root should read $\sqrt{1 + \dfrac{}{(\Delta y)^2}}$. Also,

$y_{i+1} = y_i + \underline{\quad \Delta y \quad}$. Make this substitution in the two appearances of y_{i+1}.

$$SA = \lim_{n \to \infty} \sum_{i=1}^{n} \pi \sqrt{(\Delta y)^2} \sqrt{1 + \frac{(f(y_i + n) - f(y_i))^2}{(\Delta y)^2}} \left(f(y_i) + f(y_i + \Delta y) \right)$$

11. As $n \to \infty$, $\Delta y \to \underline{\quad 0 \quad}$. So $y_{i+1} = y_i + \Delta y \to \underline{\quad y_i \quad}$. In which of the two appearances

of $f(y_i + \Delta y)$ in the above equation can we actually make this substitution? (hint: notice $(\Delta y)^2$ in the

denominator, and remember the limit definition of the derivative) _not in root sic_ _dy in denominator_

$$SA = \lim_{n \to \infty} \sum_{i=1}^{n} \pi \Delta y \sqrt{1 + \left(\frac{f(y_i + n) - f(y_i)}{dy} \right)^2} \left(f(y_i) + f(y_i) \right)$$

> You should have $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} \pi \Delta y \sqrt{1 + \frac{(f(y_i + \Delta y) - f(y_i))^2}{(\Delta y)^2}} \, (f(y_i) + f(y_i))$, or something similar.

12. By the limit definition, $f'(y_i) = \displaystyle\lim_{\Delta y \to 0} \dfrac{f(y_i + \Delta y) - f(y_i)}{\Delta y}$

Do you see this in your answer to 11? If yes, rewrite it in terms of $f'(y_i)$. Make any trivial simplifications.

$$SA = \lim_{n \to \infty} \sum_{i=1}^{n} \pi \Delta y (2 f(y_i)) \sqrt{1 + (f'(y_i))^2}$$

> You should have $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} 2\pi f(y_i) \Delta y \sqrt{1 + (f'(y))^2}$, or something similar.

recall bounds of integ. [0,6].

13. Is this in the form of a Riemann sum? Write the above sum as an integral, and evaluate it by calculator.

$$SA = \int_{a}^{b} 2\pi f(y) \sqrt{1 + (f'(y))^2} \, dy = \int_{0}^{6} 2\pi f(y) \sqrt{1 + (f'(y))^2} \, dy$$

$$\approx 90.216 \; m^2$$