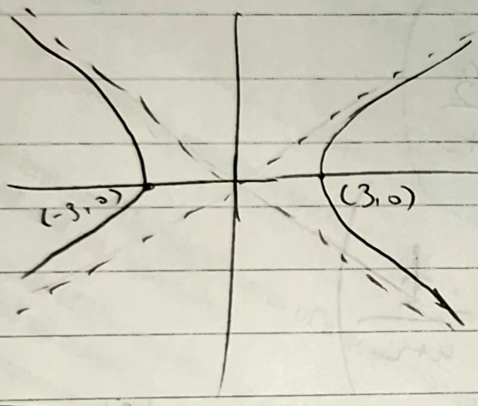
21. $4x^2 - 9y^2 = 36$

$\frac{x^2}{9} - \frac{y^2}{4} = 1$] ⇒ horizontal.

$a = 3,\ b = 2$

Asymptote : $y = \pm \frac{b}{a} x$

$y = \pm \frac{2}{3} x$



25. $9x^2 - y^2 - 36x - 6y + 18 = 0$

$9(x^2 - 4x) - (y^2 + 6y) = -18$

$9((x-2)^2 - 4) - ((y+3)^2 - 9) = -18$

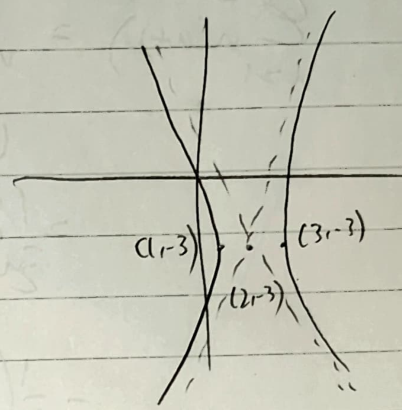
$9(x-2)^2 - 36 - (y+3)^2 + 9 = -18$

$9(x-2)^2 - (y+3)^2 = 9$

horizontal $\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$

$a = 1,\ b = 3,\ h = 2,\ k = -3$

$(y+3) = \pm 3(x-2)$



31. vertices : $(0, \pm 2)$

⇒ $a = 2,\ h = 0,\ k = 0$, vertical.

Foci : $(0, \pm 4)$

⇒ $c = 4,\ h = 0,\ k = 0$

$a^2 + b^2 = c^2$

$4 + b^2 = 16$

$b = \sqrt{12}$

$$\boxed{\frac{y^2}{4} - \frac{x^2}{12} = 1}$$

35. Foci : $(0, \pm 8)$

⇒ $c = 8,\ h = 0,\ k = 0$, vertical.

Asymptotes : $y = \pm 4x$

$\begin{cases} \frac{a}{b} = 4 \\ a^2 + b^2 = c^2 \end{cases} \Rightarrow \begin{cases} a^2 = 16b^2 \\ a^2 + b^2 = 64 \end{cases}$

$b^2 = 64 - 16b^2 \qquad a^2 = 16\left(\frac{64}{17}\right)$

$b^2 = \frac{64}{17} \qquad a^2 = \frac{1024}{17}$

$$\boxed{\frac{y^2}{\frac{1024}{17}} - \frac{x^2}{\frac{64}{17}} = 1}$$

39. Vertices: $(4,1),(4,9)$

$\Rightarrow$ vertical, $h=4$, $k=5$, $a=4$

Foci: $(4,0),(4,10)$

$\Rightarrow$ vertical, $h=4$, $k=5$, $c=5$

$a^2+b^2=c^2$

$16+b^2=25$

$b^2=9$

$$\boxed{\frac{(y-5)^2}{16}-\frac{(x-4)^2}{9}=1}$$

45. vertices: $(1,2),(3,2)$

$\Rightarrow$ horizontal, $h=2$, $k=2$, $a=1$

asymptotes: $y=x$, $y=4-x$

$\pm\frac{b}{a}=\pm1$

$\frac{b}{1}=1$

$b=1$

$$\frac{(x-2)^2}{1}-\frac{(y-2)^2}{1}=1$$

$$\boxed{(x-2)^2-(y-2)^2=1}$$

41. Vertices: $(2,3),(2,-3)$

pass through $(0,5)$

$\Rightarrow$ vertical, $h=2$, $k=0$, $a=3$

$$\frac{y^2}{9}-\frac{(x-2)^2}{b^2}=1$$

$$\frac{5^2}{9}-\frac{(0-2)^2}{b^2}=1$$

$$\frac{25}{9}-\frac{4}{b^2}=1$$

$$\frac{4}{b^2}=\frac{16}{9}$$

$16b^2=36$

$b^2=\frac{9}{4}$

$$\boxed{\frac{y^2}{9}-\frac{(x-2)^2}{\frac{9}{4}}=1}$$