Problem Set #15

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Problem 1

(a) $$\int \cos\sqrt{x}\,\mathrm{d}x = \int 2\sqrt{x}\cos\left(\sqrt{x}\right)\frac{1}{2\sqrt{x}}\,\mathrm{d}x = \begin{bmatrix} t = \sqrt{x} \\ \mathrm{d}t = \dfrac{\mathrm{d}x}{2\sqrt{x}} \end{bmatrix} 2\int t\cos t\,\mathrm{d}t$$

$$= \begin{bmatrix} u = t & \mathrm{d}u = \mathrm{d}t \\ \mathrm{d}v = \cos t\,\mathrm{d}t & v = \sin t \end{bmatrix} 2\left(t\sin t - \int \sin t\,\mathrm{d}t\right)$$

$$= 2t\sin t - 2(-\cos t) = \boxed{2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\sqrt{x} + C}$$

Problem 2

(a) $$\int (2x+3)e^x\,\mathrm{d}x = \begin{bmatrix} u = 2x+3 & \mathrm{d}u = 2\,\mathrm{d}x \\ \mathrm{d}v = e^x\,\mathrm{d}x & v = e^x \end{bmatrix} (2x+3)e^x - 2\int e^x\,\mathrm{d}x = \boxed{2xe^x + e^x + C}$$

Problem 4

(a) $$\int \sin^2 x\,\mathrm{d}x = -\frac{1}{2}\cos(x)\sin^{2-1}(x) + \frac{2-1}{2}\int \sin^{2-2}(x)\,\mathrm{d}x = -\frac{1}{4}\cdot 2\cos(x)\sin(x) + \frac{1}{2}\int 1\,\mathrm{d}x$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(b) $$\int \sin^4 x\,\mathrm{d}x = -\frac{1}{4}\cos(x)\sin^{4-1}(x) + \frac{4-1}{4}\int \sin^{4-2}(x)\,\mathrm{d}x = -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \sin^2(x)\,\mathrm{d}x$$

$$= -\frac{\sin 2x}{8}\cdot\sin^2 x + \frac{3}{4}\left(\frac{x}{2} - \frac{\sin 2x}{4}\right) = -\frac{2\sin 2x}{16}\cdot\sin^2 x + \frac{3x}{8} - \frac{3\sin 2x}{16}$$

$$= \boxed{\frac{\sin 2x}{16}\left(-2\sin^2 x - 3\right) + \frac{3x}{8} + C}$$

Problem 6

(b) $$\int \left(\tan^n x + \tan^{n-2} x\right)\,\mathrm{d}x = \begin{bmatrix} u = \tan^{n-2} x & \mathrm{d}u = (n-2)\tan^{n-3}(x)\sec^2(x)\,\mathrm{d}x \\ \mathrm{d}v = \left(\tan^2(x) + 1\right)\,\mathrm{d}x & v = \tan x \end{bmatrix}$$

$$\tan^{n-2}(x)\tan(x) - (n-2)\int \tan^{n-3}(x)\tan(x)\sec^2(x)\,\mathrm{d}x$$

$$\int \tan^n x\,\mathrm{d}x + \int \tan^{n-2} x\,\mathrm{d}x = \tan^{n-1} x - (n-2)\int \tan^{n-2}(x)\sec^2(x)\,\mathrm{d}x$$

$$= \begin{bmatrix} t = \tan x \\ \mathrm{d}t = \sec^2(x)\,\mathrm{d}x \end{bmatrix} \tan^{n-1} x - (n-2)\int t^{n-2}\,\mathrm{d}t$$

$$\int \tan^n x\,\mathrm{d}x = \tan^{n-1} x - \frac{(n-2)t^{n-1}}{n-1} - \int \tan^{n-2} x\,\mathrm{d}x$$

$$= \frac{(n-1)\tan^{n-1} x - (n-2)\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,\mathrm{d}x$$

$$= \frac{\cancel{n\tan^{n-1} x} - \tan^{n-1} x - \cancel{n\tan^{n-1} x} + 2\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,\mathrm{d}x$$

$$\boxed{\int \tan^n x\,\mathrm{d}x = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2}\,\mathrm{d}x}$$

Problem 7

$$\text{Distance traveled} = \int_0^t x^2 e^{-x}\mathrm{d}x = \begin{bmatrix} u_1 = x^2 & \mathrm{d}u_1 = 2x\,\mathrm{d}x \\ \mathrm{d}v_1 = e^{-x}\,\mathrm{d}x & v_1 = -e^{-x} \end{bmatrix} \left[-x^2 e^{-x}\right]_0^t + 2\int_0^t xe^{-x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u_2 = x & \mathrm{d}u_2 = \mathrm{d}x \\ \mathrm{d}v_2 = e^{-x}\,\mathrm{d}x & v_2 = -e^{-x} \end{bmatrix} - t^2e^{-t} + 2\left[-xe^{-x}\right]_0^t + 2\int_0^t e^{-x}\,\mathrm{d}x$$

$$= -t^2e^{-t} - 2te^{-t} + 2\left[-e^{-x}\right]_0^t = -t^2e^{-t} - 2te^{-t} + 2\left(-e^{-t} - (-1)\right)$$

$$= \boxed{-t^2e^{-t} - 2te^{-t} - 2e^{-t} + 2}$$

Problem 8

$$\int_1^4 xf''(x)\,\mathrm{d}x = \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = f''(x)\,\mathrm{d}x & v = f'(x) \end{bmatrix} \left[xf'(x)\right]_1^4 - \int_1^4 f'(x)\,\mathrm{d}x = 4f'(4) - f'(1) - \left[f(x)\right]_1^4$$

$$= 4\cdot 3 - 5 - f(4) + f(1) = 12 - 5 - 7 + 2 = \boxed{2}$$