Problem Set #10

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Problem 1

(a)

$$L_3 = 5.000$$

(b)

$$\begin{aligned} R_3 &= 14.000 \\ M_3 &= 8.750 \end{aligned}$$

(c)

$$\int_0^3 x^2 \,\mathrm{d}x = \frac{x^2}{2}\bigg|_0^3 = \frac{9}{2}$$

(d)

$$\begin{aligned} \text{Error } L_3 &= -4.000 \\ \text{Error } R_3 &= 5.000 \\ \text{Error } M_3 &= -0.250 \end{aligned}$$

(e)

$$\text{Area} = \frac{(b_1 + b_2)\,h}{2}$$

(f)

$$\int_0^3 x^2 \,\mathrm{d}x \approx \frac{(0+1)1}{2} + \frac{(1+4)1}{2} + \frac{(4+9)1}{2} = \frac{1}{2} + \frac{5}{2} + \frac{13}{2} = \frac{19}{2}$$

Problem 2

(a)

$$\int_1^2 \frac{1}{x^2} \,\mathrm{d}x = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$

(b)

$$\begin{aligned} T_4 &= 0.509 \\ M_4 &= 0.496 \\ T_8 &= 0.502 \\ M_8 &= 0.499 \end{aligned}$$

(c)

$$\begin{aligned} E_{T_4} &= 0.509 - \frac{1}{2} = 0.009 \\ E_{M_4} &= 0.496 - \frac{1}{2} = -0.004 \\ E_{T_8} &= 0.502 - \frac{1}{2} = 0.002 \\ E_{M_8} &= 0.499 - \frac{1}{2} = -0.001 \end{aligned}$$

(d) Trapezoidal rule overestimates, midpoint rule underestimates

(e) Concave up on $[1, 2]$

Problem 3

(a)

$$\int_0^1 \left(1 - x^2\right) \mathrm{d}x = \left[x - \frac{x^3}{3}\right]_0^1 = \frac{3}{3} - \frac{1}{3} - 0 = \frac{2}{3}$$

(b)

$$\begin{aligned} T_4 &= 0.656 \\ M_4 &= 0.672 \\ T_8 &= 0.664 \\ M_8 &= 0.668 \end{aligned}$$

(c)

$$\begin{aligned} E_{T_4} &= 0.656 - 0.666 = -0.010 \\ E_{M_4} &= 0.672 - 0.666 = 0.006 \\ E_{T_8} &= 0.664 - 0.666 = -0.002 \\ E_{M_8} &= 0.668 - 0.666 = 0.002 \end{aligned}$$

(d) Midpoint rule overestimates, trapezoidal rule underestimates

(e) Concave down on $[0, 1]$

(f) Error from midpoint and trapezoid has opposite sign, and absolute error from midpoint is approximately half of absolute error from trapezoidal

Problem 4

(a) Uploaded in other files

(b)

$$\text{Distance} = \int_0^{1.8} |v(t)| \,\mathrm{d}t = \int_0^{1.8} v(t) \,\mathrm{d}t$$

(c)

$$\begin{aligned} L_3 &= 0.6(100 + 96 + 80) = 165.6 \\ R_3 &= 0.6(96 + 80 + 0) = 105.6 \\ T_3 &= (L_3 + R_3)/2 = 135.6 \end{aligned}$$

(d)

$$M_3 = 0.6(99 + 90 + 50) = 143.4$$

(e)

$$S_6 = \frac{2M_3 + T_3}{3} = \frac{2 \cdot 71.7 + 67.8}{3} = 140.8$$

(f) 140.8 feet is the best estimate because it is known that Simpson's Rule has the lowest error

Problem 5

(a) Graphs in other files

$$\begin{aligned} L_{1_f} = L_{1_g} = L1_h = 2 \\ R_{1_f} = R_{1_g} = R1_h = 1 \end{aligned}$$

L_1 and R_1 are the same for each function

(b)

$$\begin{aligned} \int_0^1 f(x) \,\mathrm{d}x &\approx M_{1_f} = \frac{7}{4} \\ \int_0^1 g(x) \,\mathrm{d}x &\approx M_{1_g} = \frac{15}{8} \\ \int_0^1 h(x) \,\mathrm{d}x &\approx M_{1_h} = \frac{31}{16} \end{aligned}$$

(c)

$$\begin{aligned} T_{1_f} &= \frac{3}{2} & S_{2_f} &= \frac{2 \cdot \frac{7}{4} + \frac{3}{2}}{3} = \frac{5}{3} \\ T_{1_g} &= \frac{3}{2} & S_{2_g} &= \frac{2 \cdot \frac{15}{8} + \frac{3}{2}}{3} = \frac{7}{4} \\ T_{1_h} &= \frac{3}{2} & S_{2_h} &= \frac{2 \cdot \frac{31}{16} + \frac{3}{2}}{3} = \frac{43}{24} \end{aligned}$$

(d)

$$\begin{aligned} \int_0^1 f(x) \,\mathrm{d}x &= \left[2x - \frac{x^3}{3}\right]_0^1 = \frac{5}{3} \\ \int_0^1 g(x) \,\mathrm{d}x &= \left[2x - \frac{x^4}{4}\right]_0^1 = \frac{7}{4} \\ \int_0^1 h(x) \,\mathrm{d}x &= \left[2x - \frac{x^5}{5}\right]_0^1 = \frac{9}{5} \end{aligned}$$

(e) Simpson's Rule is very accurate even with just a single subdivision

Problem 6

(a)

$$\text{Total Water} = \int_0^{60} r(t) \,\mathrm{d}t$$

(b)

$$\int_0^{60} r(t) \,\mathrm{d}t \approx M_3 = 20\,(2100 + 3000 + 5100) = 204000$$

The total volume of water to flow through the dam in 60 seconds is approximately 204000 cubic feet. M_3 underestimates the value of the integral because r is concave up.

(c)

$$\begin{aligned} T_3 &= 20\left(\frac{2000 + 2400}{2} + \frac{2400 + 3900}{2} + \frac{3900 + 6500}{2}\right) = 211000 \\ S_6 &= \frac{2M_3 + T_3}{3} = \frac{2 \cdot 204000 + 211000}{3} = \boxed{\frac{619000}{3}} \end{aligned}$$

(d)

$$\begin{aligned} \frac{1}{60} S_6 &= \frac{619000}{3 \cdot 60} = \frac{30950}{9} \\ \frac{2000 + 2100 + 2400 + 3000 + 3900 + 5100 + 6500}{7} &= \frac{25000}{7} \end{aligned}$$

Both values esimate the average rate at which water flows through the dam in one minute. The first value (with Simpson's Rule) is more accurate because the second only takes the values of the function at the endpoints of intervals into account.