The Butterfly Project

Jayden Li

February 22, 2024

$$x(t) = \sin(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \frac{t}{12}\right)$$
$$y(t) = \cos(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \frac{t}{12}\right)$$

(a)
$$\begin{aligned}
\sin t &: 2\pi \\
\cos t &: 2\pi \\
e^{\cos t} &: 2\pi \\
e^{\sin t} &: 2\pi \\
\cos 4t &: \frac{\pi}{2} \\
\sin \frac{t}{12} &: 24\pi
\end{aligned}$$

LCM of everything above is 24π.

$\boxed{\text{Period} : 24\pi}$

(b) I counted the number of times it passes through the origin. The answer is $\boxed{74}$

(c)

$$\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\dfrac{\mathrm{d}}{\mathrm{d}t}\left[\cos(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \dfrac{t}{12}\right)\right]}{\dfrac{\mathrm{d}}{\mathrm{d}t}\left[\sin(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \dfrac{t}{12}\right)\right]} \\
&= \frac{-\sin(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \frac{t}{12}\right) + \cos(t)\left(e^{\cos t}(-\sin t) - 8(-\sin 4t) - 5\left(\sin \frac{t}{12}\right)^4 \cdot \frac{1}{12}\right)}{\cos(t)\left(e^{\cos t} - 2\cos 4t - \sin^5 \frac{t}{12}\right) + \sin(t)\left(e^{\cos t}(-\sin t) - 8(-\sin 4t) - 5\left(\sin \frac{t}{12}\right)^4 \cdot \frac{1}{12}\right)}
\end{aligned}$$

Slope of tangent line:

$$= \frac{-\sin(135\pi)\left(e^{\cos 135\pi} - 2\cos 4(135\pi) - \sin^5 \frac{135\pi}{12}\right) + \cos(135\pi)\left(e^{\cos 135\pi}(-\sin 135\pi) - 8(-\sin 4(135\pi)) - 5\left(\sin \frac{135\pi}{12}\right)^4 \cdot \frac{1}{12}\right)}{\cos(135\pi)\left(e^{\cos 135\pi} - 2\cos 4(135\pi) - \sin^5 \frac{135\pi}{12}\right) + \sin(135\pi)\left(e^{\cos 135\pi}(-\sin 135\pi) - 8(-\sin 4(135\pi)) - 5\left(\sin \frac{135\pi}{12}\right)^4 \cdot \frac{1}{12}\right)}$$

$$= \frac{\cancel{-\sin(\pi)\left(e^{\cos \pi} - 2\cos 540\pi - \sin^5 \frac{45\pi}{4}\right)} + \cancel{\cos(\pi)}\left(\cancel{e^{\cos \pi}(-\sin \pi)} - \cancel{8(-\sin 540\pi)} - 5\left(\sin \frac{45\pi}{4}\right)^4 \cdot \frac{1}{12}\right)}{\cancel{\cos(\pi)}\left(e^{\cos \pi} - 2\cos 540\pi - \sin^5 \frac{45\pi}{4}\right) + \cancel{\sin(\pi)\left(e^{\cos \pi}(-\sin \pi) - 8(-\sin 540\pi) - 5\left(\sin \frac{45\pi}{4}\right)^4 \cdot \frac{1}{12}\right)}}$$

$$= -\frac{5\left(\sin \frac{5\pi}{4}\right)^4}{12} \Big/ \left(e^{-1} - 2 - \left(\sin \frac{5\pi}{4}\right)^5\right)$$

$$= -\frac{5\left(-\frac{\sqrt{2}}{2}\right)^4}{12} / \left(e^{-1} - 2 - \left(-\frac{\sqrt{2}}{2}\right)^5\right)$$

$$= -\frac{5\left(\frac{1}{4}\right)}{12} / \left(e^{-1} - 2 - \left(-\frac{4\sqrt{2}}{32}\right)\right)$$

$$= -\frac{5}{48} \cdot \frac{1}{e^{-1} - 2 + \frac{\sqrt{2}}{8}}$$

$$= -\frac{5}{48e^{-1} - 96 + 6\sqrt{2}}$$

$$x(135\pi) = \sin(135\pi)\left(e^{\cos 135\pi} - 2\cos 4(135\pi) - \sin^5 \frac{135\pi}{12}\right)$$

$$= \sin(\pi)\left(e^{\cos\pi} - 2\cos 540\pi - \left(\sin\frac{45\pi}{4}\right)^5\right)$$

$$= 0$$

$$y(135\pi) = \cos(135\pi)\left(e^{\cos 135\pi} - 2\cos 4(135\pi) - \sin^5 \frac{135\pi}{12}\right)$$

$$= -\left(e^{\cos\pi} - 2\cos 540\pi - \left(\sin\frac{5\pi}{4}\right)^5\right)$$

$$= -\left(e^{-1} - 2 + \frac{\sqrt{2}}{8}\right)$$

$$\boxed{y + \frac{1}{e} - 2 + \frac{\sqrt{2}}{8} = -\frac{5x}{\frac{48}{e} - 96 + 6\sqrt{2}}}$$

copy to desmos: `y+\frac{1}{e}-2+\frac{\sqrt{2}}{8}=-\frac{5x}{\frac{48}{e}-96+6\sqrt{2}}`

(d) – The changing the power of $\cos 4t$ changes the size of the curve. Increasing it will enlarge the "head" and "abdomen" of the butterfly.

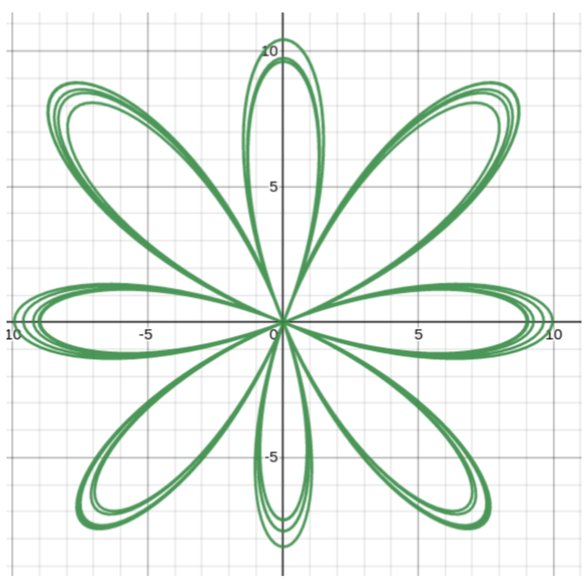

Figure 1: Curve created by $x(t) = \sin(t)\left(e^{\cos t} - 10\cos 4t - \left(\sin\frac{t}{12}\right)^5\right)$, $y(t) = \cos(t)\left(e^{\cos t} - 10\cos 4t - \left(\sin\frac{t}{12}\right)^5\right)$

– Changing the coefficient of $\sin\frac{t}{12}$ changes the curve significantly, as the inner lines "expand" outside and become larger.

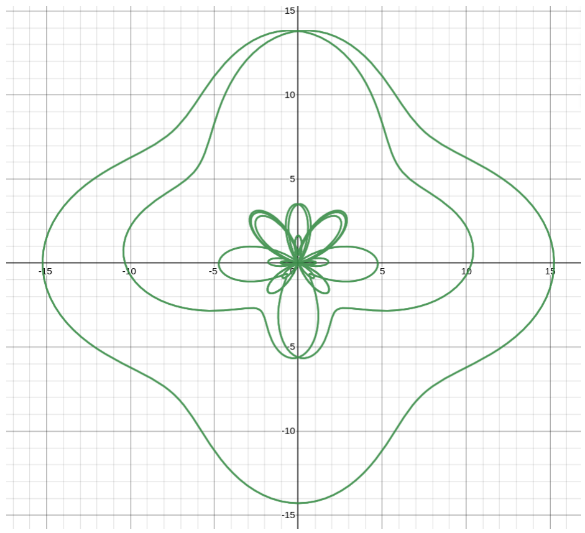

Figure 2: Curve created by $x(t) = \sin(t)\left(e^{\cos t} - 2\cos 4t - 15\left(\sin\frac{t}{12}\right)^5\right)$, $y(t) = \cos(t)\left(e^{\cos t} - 2\cos 4t - 15\left(\sin\frac{t}{12}\right)^5\right)$

– Changing the exponent of $\sin\frac{t}{12}$ does not change the curve much. Increasing it makes the "inner lines" of the wings and other body parts closer to its edge.

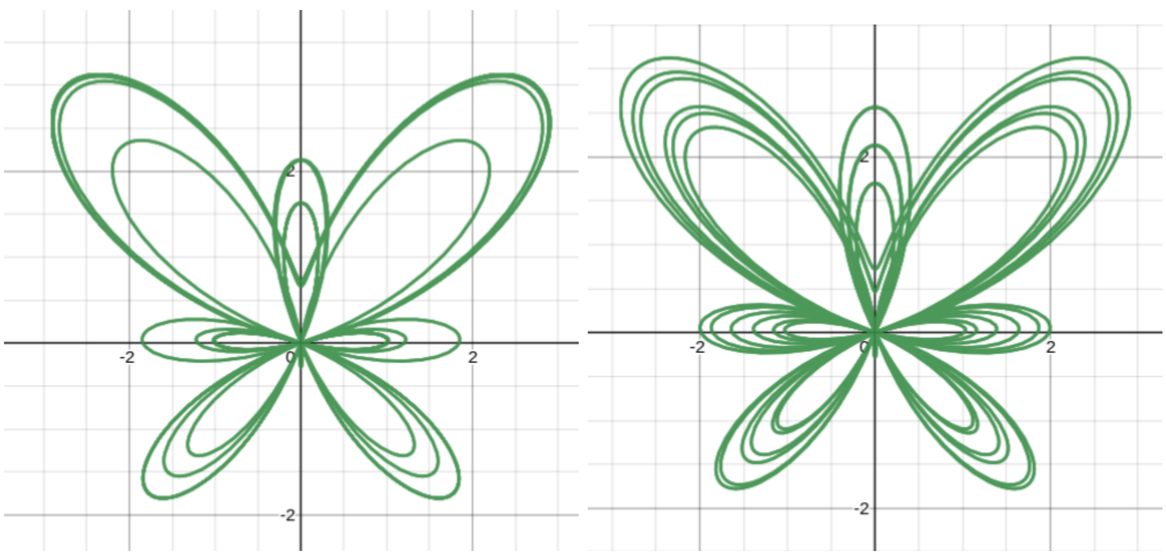

Figure 3: Curves created by $x(t) = \sin(t)\left(e^{\cos t} - 2\cos 4t - \left(\sin\frac{t}{12}\right)^a\right)$, $y(t) = \cos(t)\left(e^{\cos t} - 2\cos 4t - \left(\sin\frac{t}{12}\right)^a\right)$. Left image shows the curve with $a = 20$, right image has $a = 2$.

– When any of these are negative, weird stuff happens.

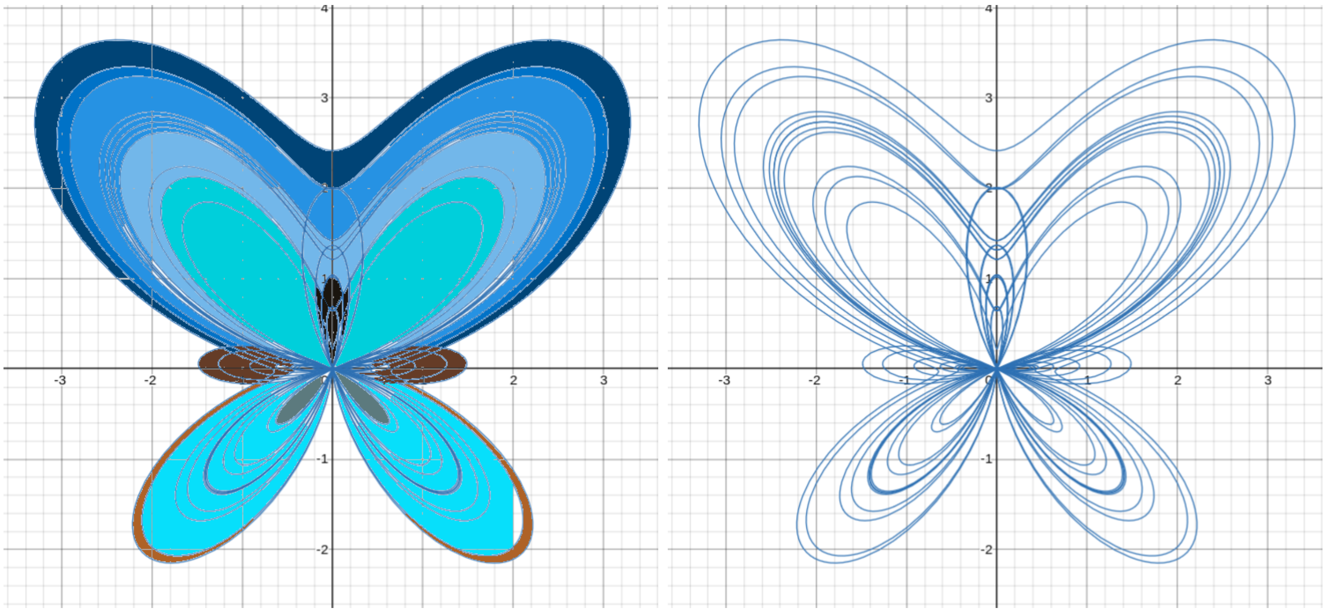

Figure 4: Art.

$$x(t) = \sin(t)\left(e^{\cos t} - 1.4\cos 4t - 1.1\left(\sin\left(\frac{t}{12}\right)\right)^{3.4}\right)$$
$$y(t) = \cos(t)\left(e^{\cos t} - 1.4\cos 4t - 1.1\left(\sin\left(\frac{t}{12}\right)\right)^{3.4}\right)$$