Problem Set #9

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Problem 2

Is there a mistake in this question? Is the caption wrong? It says the left is $f'(x)$ and the right is for graphing $f(x)$, but the question says the left is $f(x)$ and the right is for graphing $F(x)$. The question as described in the label would be above our level as we would be finding second antiderivatives.

(a) increasing on $(0,2)\cup(5,7)$, decreasing on $(2,5)$

(b) concave up on $(0,1)\cup(4,6)$, concave down on $(1,3)\cup(6,7)$, neither on $(\infty,0]\cup\{1\}\cup[3,4]\cup\{6\}\cup[7,\infty)$

(c) relative minimum at $(0,0)$, $\left(5,\frac{\pi}{4}-\frac{3}{2}\right)$, relative maximum at $\left(2,\frac{1}{2}+\frac{\pi}{4}\right)$, $\left(7,\frac{3\pi}{4}-\frac{3}{2}\right)$

(d)

$$\begin{aligned}
F(-1) &= 1\\
F(1) &= \frac{3}{2}\\
F(2) &= \frac{3}{2}+\frac{\pi}{4}\\
F(3) &= 1+\frac{\pi}{4}\\
F(4) &= \frac{\pi}{4}\\
F(5) &= \frac{\pi}{4}-\frac{1}{2}\\
F(6) &= \frac{\pi}{2}-\frac{1}{2}\\
F(7) &= \frac{3\pi}{4}-\frac{1}{2}\\
F(8) &= \frac{3\pi}{4}-\frac{1}{2}
\end{aligned}$$

(e) Other files.

(f) Graph is shifted 1 unit down. (a) unchanged. (b) unchanged. x-coordinates for (c) are unchanged by y-coordinates shifted down by 1. Every value in (d) is subtracted by 1.

Problem 5

(b)

$$\text{Average} = \frac{1}{2-(-1)}\int_{-1}^{2} x^2\,\mathrm{d}x = \frac{1}{3}\left(\frac{2^3}{3}-\frac{(-1)^3}{3}\right) = \frac{1}{3}\cdot\frac{8+1}{3} = \frac{1}{3}\cdot 3 = 1$$

Average value occurs at $x=-1$ and $x=1$.

(c)

$$\text{Average} = \frac{1}{2-0}\int_0^2\left(1-\cos\frac{\pi t}{2}\right)\mathrm{d}t = \frac{1}{2}\left[t-\frac{2}{\pi}\sin\frac{\pi t}{2}\right]_0^2 = \frac{1}{2}\left(2-\frac{2}{\pi}\sin(\pi)-0+\frac{2}{\pi}\sin 0\right) = 2$$

Average volume of air in lungs is 2 pints between 0 and 2 seconds.

Problem 6

(i) (a)

$$\frac{1}{5-2}\int_2^5 (x-3)^2\,\mathrm{d}x = \frac{1}{3}\left[\frac{(x-3)^3}{3}\right]_2^5 = \frac{1}{3}\left(\frac{8}{3}-(-1)\frac{1}{3}\right) = \boxed{3}$$

(b)

$$\begin{aligned}
(x-3)^2 &= 3\\
x-3 &= \pm\sqrt{3}\\
&\boxed{x = 3\pm\sqrt{3}}
\end{aligned}$$

(c) Other files.

(ii) (a)

$$\begin{aligned}
\frac{1}{\pi-0}\int_0^\pi(2\sin x-\sin 2x)\,\mathrm{d}x &= \frac{1}{\pi}\left[-2\cos x+\frac{1}{2}\cos 2x\right]_0^\pi\\
&= \frac{1}{\pi}\left(-2(-1)+\frac{1}{2}(1)-\left(-2(1)+\frac{1}{2}(1)\right)\right)\\
&= \frac{1}{\pi}\left(\frac{4}{2}+\frac{1}{2}+\frac{4}{2}-\frac{1}{2}\right) = \frac{1}{\pi}\cdot 4 = \boxed{\frac{4}{\pi}}
\end{aligned}$$

(b)

$$\begin{aligned}
2\sin x-\sin 2x &= \frac{4}{\pi}\\
&\boxed{x\in\{1.238, 2.808\}}
\end{aligned}$$

(proof by calculator)

(c) Other files.

Problem 7

By the Mean Value Theorem for Integrals, it is known that there exists some $c\in[1,3]$ such that

$$f(c) = \frac{1}{3-1}\int_1^3 f(x)\,\mathrm{d}x = \frac{1}{2}\cdot 8 = 4$$

So, $f(x)=4$ for some $x\in[1,3]$. ☺

Problem 8

$$\begin{aligned}
\frac{1}{b}\int_0^b\left(2+6x-3x^2\right)\mathrm{d}x &= 3\\
\frac{1}{b}\left[2x+3x^2-x^3\right]_0^b &= 3\\
\frac{1}{b}\left(2b+3b^2-b^3\right) &= 3\\
2+3b-b^2 &= 3\\
b^2-3b+1 &= 0\\
&\boxed{b=\frac{3\pm\sqrt{5}}{2}}
\end{aligned}$$

Problem 9

$$\frac{1}{50-20}\int_{20}^{50} f(x)\,\mathrm{d}x \approx \frac{1}{30}\cdot 5\,(40+34.5+30+32+41.5+54) = \frac{1}{6}\cdot 232 = \boxed{\frac{116}{3}}$$

Problem 10

(a)

$$\frac{1}{4-1}\int_1^4 14\pi x^2\,\mathrm{d}x = \frac{1}{3}\left[\frac{14\pi x^3}{3}\right]_1^4 = \frac{1}{3}\left(\frac{896\pi}{3}-\frac{14\pi}{3}\right) = \boxed{98\pi}$$

(b)

$$\begin{aligned}
\frac{1}{k}\int_0^k k^2\sin\left(\frac{\pi x}{2k}\right)\mathrm{d}x &= 98\pi\\
k\left[-\frac{2k}{\pi}\cos\left(\frac{\pi x}{2k}\right)\right]_0^k &= 98\pi\\
k\left(-\frac{2k}{\pi}\cos\left(\frac{\pi k}{2k}\right)-\left(-\frac{2k}{\pi}\cos(0)\right)\right) &= 98\pi\\
k\left(\frac{2k}{\pi}\right) &= 98\pi\\
2k^2 &= 98\pi^2\\
&\boxed{k=7\pi}
\end{aligned}$$