Problem Set #12

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Problem 3

Let a be the length of the edges of the cube. Then $a = 30, \mathrm{d}a = 0.1$.

(a)

$$V = a^3$$
$$\text{Maximum error} = \mathrm{d}V = 3a^2 \cdot \mathrm{d}a = 3(30)^2 \cdot 0.1 = \boxed{270 \text{ cm}^3}$$
$$\text{Relative error} = \frac{270}{V} = \frac{270}{27000} = \boxed{0.01}$$
$$\text{Percentage error} = 0.01 \cdot 100 = \boxed{1\%}$$

(b)

$$A = 6a^2$$
$$\text{Maximum error} = \mathrm{d}A = 12a \cdot \mathrm{d}a = 12(30) \cdot 0.1 = \boxed{36 \text{ cm}^2}$$
$$\text{Relative error} = \frac{36}{A} = \frac{36}{5400} = \boxed{0.00667}$$
$$\text{Percentage error} = 0.00667 \cdot 100 = \boxed{0.667\%}$$

Problem 6

Lemma. Let $\tan_d(x)$ be tangent in terms of degrees. We calculate the derivative of $\tan_d$.

$$\frac{\mathrm{d}}{\mathrm{d}x} \tan_d(x) = \frac{\mathrm{d}}{\mathrm{d}x} \tan\left(\frac{\pi x}{180}\right) = \frac{\pi}{180} \sec^2(x)$$

We calculate the linearization of tangent around 45°.

$$L_{\tan}(x) = \tan_d'(45^\circ)(x - 45^\circ) + \tan(45^\circ) = \frac{\pi}{180} \sec^2\left(\frac{\pi}{4}\right)(x - 45^\circ) + 1$$
$$= \frac{\pi}{180} \left(\frac{1}{\frac{\sqrt{2}}{2}}\right)^2 (x - 45^\circ) + 1 = \frac{\pi}{90}(x - 45^\circ) + 1$$
$$L_{\tan}(44^\circ) = \frac{\pi}{90}(44^\circ - 45^\circ) + 1 = 1 - \frac{\pi}{90} = \boxed{0.965}$$

Problem 8

$$L_f(x) = f'(1)(x - 1) + f(1) = 2(x - 1) + 5 = 2x + 3$$

(a)

$$f(0.9) \approx L_f(0.9) = 2(0.9) + 3 = \boxed{4.8}$$
$$f(1.1) \approx L_f(1.1) = 2(1.1) + 3 = \boxed{5.2}$$

(b) f is concave down because f' is decreasing. So estimates in part (a) are $\boxed{\text{overestimates}}$.