

Problem Set #16

Jayden Li

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Problem 6

Let $g(a) = \int_{-\pi}^{\pi} (f(x) - a \cos(nx))^2 dx$.

$$g'(a) = \frac{d}{da} \int_{-\pi}^{\pi} (f(x) - a \cos(nx))^2 dx = \int_{-\pi}^{\pi} \frac{\partial}{\partial a} [(f(x) - a \cos(nx))^2] dx$$

$$0 = \int_{-\pi}^{\pi} 2(f(x) - a \cos(nx))(-\cos(nx)) dx$$

$$0 = - \int_{-\pi}^{\pi} f(x) \cos(nx) dx + \int_{-\pi}^{\pi} a \cos^2(nx) dx$$

$$\begin{aligned} \int_{-\pi}^{\pi} a \cos^2(nx) dx &= a \int_{-\pi}^{\pi} \frac{1 + \cos(2nx)}{2} dx = \frac{1}{2} a \int_{-\pi}^{\pi} (1 + \cos(2nx)) dx = \frac{1}{2} a \left[x + \frac{\sin(2nx)}{2n} \right]_{-\pi}^{\pi} \\ &= \frac{1}{2} a (\pi + 0 + \pi - 0) = \frac{1}{2} a \cdot 2\pi = a\pi \end{aligned}$$

$$0 = - \int_{-\pi}^{\pi} f(x) \cos(nx) dx + a\pi$$

$$a = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$$

