other consequence of this result is that we can now define the area under a continuous curve. Here we do need the function to be non-negative.

DEFINITION 1. Let f be a non-negative, continuous function on the closed interval $[a, b]$. The area bounded above by the graph of f, below by the $x-axis$, on the left by the line $x = a$, and on the right by $x = b$ is given by

$$Area = \lim_{n \to \infty} \sum_{k=1}^{n} f(c_k)\Delta x$$

,

where $\Delta x = \dfrac{b-a}{n}$ and c_k is any point in the k^{th} subinterval of the regular partition of $[a, b]$ into n subintervals.

1. Use Definition 1 to determine the area under the curve $y = f(x) = -x^2 + 4x - 3$ on the interval $[1, 3]$.

   Notice that because the $f(x)$ is both increasing and decreasing on the interval, if we computed $Upper(n)$, the max values of f would sometimes occur at the right-hand endpoints and sometimes at the left. This makes it hard to compute $Upper(n)$ (or $Lower(n)$). However, by Theorem 2, we can use any convenient set of evaluation points for our Riemann sum. As noted earlier, right-hand endpoints x_k are convenient because the general formula is fairly simple.

   The general form of the **right Riemann sum** is:

$$Right\ (n) = \sum_{k=1}^{n} f(x_k)\Delta x$$

2. (a) Notation Practice: Fill in the following table for the Riemann sums using regular partitions and right endpoints. Do not try to evaluate the sums.

$f(x)$	$[a,b]$	Δx	$x_k = a + k\Delta x$	$f(x_k)$	Right$(n) = \sum_{k=1}^{n} f(x_k)\Delta x$
$x^2 - 1$	$[0,2]$	$\frac{2}{n}$	$\frac{2k}{n}$	$\frac{4k^2}{n^2} - 1$	$\frac{2}{n}\sum_{k=1}^{n}\left(\frac{4k^2}{n^2} - 1\right)$
$2(x-1)^2$	$[1,4]$	$\frac{3}{n}$	$1 + \frac{3k}{n}$	$\frac{18k^2}{n^2}$	$\frac{3}{n}\sum_{k=1}^{n}\frac{18k^2}{n^2}$
$\sin(x)$	$[0,\pi]$	$\frac{\pi}{n}$	$\frac{\pi k}{n}$	$\sin\left(\frac{\pi k}{n}\right)$	$\frac{\pi}{n}\sum_{k=1}^{n}\sin\left(\frac{\pi k}{n}\right)$

   (b) For $f(x) = 2(x-1)^2$ on $[1, 4]$, use algebra to simplify $Right\ (n) = \sum_{k=1}^{n} f(c_k)\Delta x$. Then calculate the area under $f(x) = 2(x-1)^2$ on $[1, 4]$.

   Left Riemann Sum. We have been working with right-hand Riemann sums. But we could use left endpoint sums instead. The k^{th} subinterval is (x_{k-1}, x_k), so its left-hand endpoint is $x_{k-1} = a + (k-1)\Delta x$. The form of a general left Riemann sum is

$$Left\ (n) = \sum_{k=1}^{n} f(x_{k-1})\Delta x$$