Problem Set #63

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Problem 1

(a) f is a polynomial so it is differentiable on $(1,3)$ and continuous on $[1,3]$.

$$f(1) = 5 - 12(1) + 3(1)^2 = 5 - 12 + 3 = -4$$
$$f(3) = 5 - 12(3) + 3(3)^2 = 5 - 36 + 27 = -4$$

$$\begin{aligned} f'(x) &= 0 \\ -12 + 6x &= 0 \\ x &= 2 \end{aligned}$$

(b) $\sqrt{x}$ is continuous on $[0,9]$ and differentiable on $(0,9)$, and $x/3$ is continuous and differentiable on $\mathbb{R}$. So f is continuous on $[0,9]$ and differentiable on $(0,9)$.

$$f(0) = \sqrt{0} - 0/3 = 0$$
$$f(9) = \sqrt{9} - 9/3 = 0$$

$$\begin{aligned} f'(x) &= 0 \\ \frac{1}{2\sqrt{x}} - \frac{1}{3} &= 0 \\ 2\sqrt{x} &= 3 \\ \sqrt{x} &= \frac{3}{2} \\ x &= \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \end{aligned}$$

Discard negative case since $x \in (0,9)$.

$$x = \frac{\sqrt{6}}{2}$$

Problem 2

$$\begin{aligned} f(x) &= 1 - \left(\sqrt[3]{x}\right)^2 \\ f(-1) &= 1 - \left(\sqrt[3]{-1}\right)^2 = 0 \\ f(1) &= 1 - \left(\sqrt[3]{1}\right)^2 = 0 \end{aligned}$$

$$
\begin{aligned}
f'(c) &= 0 \\
-\frac{2}{3\sqrt[3]{c}} &= 0 \\
-2 &= 0 \cdot 3\sqrt[3]{c}
\end{aligned}
$$

Zero multiplied by any real number is 0, so the above statement is false.

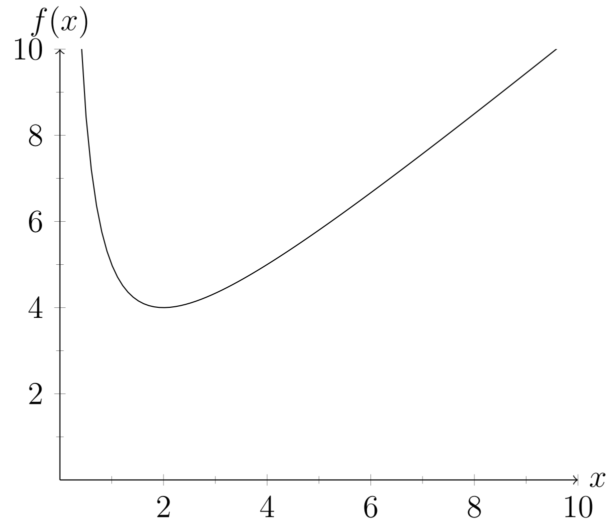
This does not contradict Rolle's Theorem because f is not differentiable at $x = 0$, so f is not differentiable on $(-1, 1)$, which is one of the conditions of Rolle's Theorem.

Problem 3

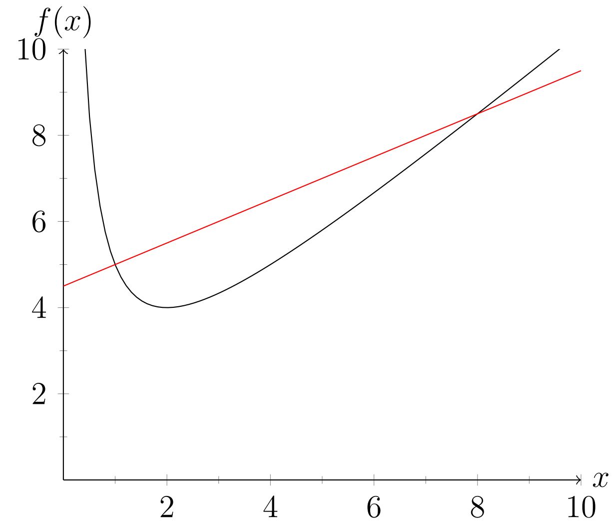
$x \in \{0.9, 3.2, 4.4, 6.1\}$

Problem 4

(a)



(b)

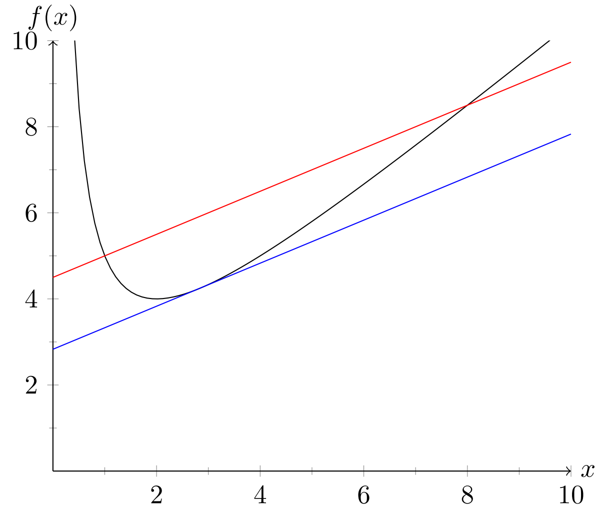


(c)

$$
\begin{aligned}
f'(c) &= \frac{f(8) - f(1)}{8 - 1} \\
1 + 4\left(-1 \cdot x^{-2}\right) &= \frac{8.5 - 5}{7} \\
1 - \frac{4}{x^2} &= \frac{3.5}{7} \\
\frac{4}{x^2} &= \frac{1}{2} \\
x^2 &= 8
\end{aligned}
$$

Ignore negative case since $-\sqrt{8} \notin [1, 8]$.

$$x = 2\sqrt{2}$$



Problem 5

(a) f is differentiable on $(-1,1)$ and continuous on $[-1,1]$ because it is a polynomial.

$$\begin{aligned}
f'(x) &= \frac{f(1)-f(-1)}{1-(-1)} \\
6x+2 &= \frac{10-6}{2} \\
6x &= 2-2 \\
x &= 0
\end{aligned}$$

(b) $\sqrt[3]{x}$ is differentiable at all real numbers except 0, so it is differentiable on $(0,1)$ and continuous on $[0,1]$.

$$\begin{aligned}
f'(x) &= \frac{\sqrt[3]{1}-\sqrt[3]{0}}{1-0} \\
\frac{1}{3}x^{-2/3} &= 1 \\
\frac{1}{(\sqrt[3]{x})^2} &= 3 \\
\left(\sqrt[3]{x}\right)^2 &= \frac{1}{3}
\end{aligned}
\qquad
\begin{aligned}
\sqrt[3]{x} &= \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \\
x &= \frac{\left(\sqrt{3}\right)^3}{3^3} \\
x &= \frac{3\sqrt{3}}{27} \\
x &= \frac{\sqrt{3}}{9}
\end{aligned}$$

Problem 6

$$\begin{aligned}
f(4)-f(1) &= f'(c)(4-1) \\
(4-3)^{-2}-(1-2)^{-2} &= -2(c-3)^{-3}\cdot 1\cdot 3 \\
\frac{1}{1^2}-\frac{1}{(-1)^2} &= -\frac{6}{(c-3)^3} \\
0\cdot(c-3)^3 &= -6
\end{aligned}$$

Which is impossible since any number multiplied by 0 is 0.

This does not contradict the MVT because f is undefined at $x = 3$, so it is not continuous on $[1, 4]$.

Problem 7

Problem 8

Problem 9

Problem 10

Problem 11

Problem 12

Problem 13

Problem 14

Problem 15