Problem Set #2

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Problem 1

(a) Done in class.

(b)

$$
\begin{aligned}
L_4 &= \frac{3}{4}\left[v(2+0) + v\left(2+\frac{3}{4}\right) + v\left(2+\frac{3}{2}\right) + v\left(2+\frac{9}{4}\right)\right] \approx 6.479 \\
M_4 &= \frac{3}{4}\left[v\left(2+\frac{3}{8}\right) + v\left(2+\frac{9}{8}\right) + v\left(2+\frac{15}{8}\right) + v\left(2+\frac{21}{8}\right)\right] \approx 6.635 \\
R_4 &= \frac{3}{4}\left[v\left(2+\frac{3}{4}\right) + v\left(2+\frac{3}{2}\right) + v\left(2+\frac{9}{4}\right) + v\left(2+3\right)\right] \approx 6.979
\end{aligned}
$$

(c) Average does not equal midpoint sum; average should be the Riemann sum if the height of the rectangle is the average of the function evaluated at the left and right ($\Delta x \cdot f(x_i + x_{i+1})/2$).

(d) L_n underestimates on increasing functions and overestimates on decreasing functions. R_n underestimates on decreasing functions and overestimates on increasing functions.

Problem 3

(a) $\Delta t = 4/5, t_0 = 1, t_1 = 9/5, t_2 = 13/5, t_3 = 17/5, t_4 = 21/5, t_5 = 5.$

$$
\begin{aligned}
M_5 &= v\left(\frac{t_0+t_1}{2}\right)\Delta t + v\left(\frac{t_1+t_2}{2}\right)\Delta t + v\left(\frac{t_2+t_3}{2}\right)\Delta t + v\left(\frac{t_3+t_4}{2}\right)\Delta t + v\left(\frac{t_4+t_5}{2}\right)\Delta t \\
&= \frac{4}{5}\left[v\left(\frac{7}{5}\right) + v\left(\frac{11}{5}\right) + v\left(\frac{15}{5}\right) + v\left(\frac{19}{5}\right) + v\left(\frac{23}{5}\right)\right] \\
&= -1.44
\end{aligned}
$$

(b) The object moves -1.44 feet to the left.

(c)

$$
\begin{aligned}
\text{Distance} &= \frac{4}{5}\left[\left|v\left(\frac{7}{5}\right)\right| + \left|v\left(\frac{11}{5}\right)\right| + \left|v\left(\frac{15}{5}\right)\right| + \left|v\left(\frac{19}{5}\right)\right| + \left|v\left(\frac{23}{5}\right)\right|\right] \\
&= 2.336
\end{aligned}
$$

The object travels 2.336 feet in total.

(d) The value of $\lim_{n\to\infty} M_n = \int_1^5 v(t)\,\mathrm{d}t$, or the actual net signed area under/above v on $[1, 5]$. This is the change in position of the object.

Problem 4

(a)

$$\sum_{i=1}^{5} 2i = 2 + 4 + 6 + 8 + 10$$

(b)

$$\sum_{i=3}^{7} \left(i^2 + 1\right) = 10 + 17 + 26 + 37 + 50$$

(c)

$$\sum_{j=1}^{4} \frac{1}{j} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

(d)

$$\sum_{k=1}^{n} \frac{1}{n}(2k+3) = \frac{1}{n}\left(5 + 7 + 9 + \ldots + 2n + 3\right)$$

Problem 7

(a)

$$
\begin{aligned}
\sum_{i=1}^{n} \frac{i^2-10}{n^3} &= \frac{1}{n^3}\sum_{i=1}^{n}\left[i^2 - 10\right] = \frac{1}{n^3}\left[\sum_{i=1}^{n} i^2 - 10n\right] = \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6} - 10n\right] \\
&= \frac{\left(n^2+n\right)(2n+1) - 60n}{6n^3} = \frac{2n^2+3n-59}{6n^2}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^2\left(\frac{1}{n}\right) &= \frac{1}{n}\sum_{i=1}^{n}\left(1 + \frac{2i}{n} + \frac{i^2}{n^2}\right) = \frac{1}{n}\left[n + \frac{2}{n}\sum_{i=1}^{n} i + \frac{1}{n^2}\sum_{i=1}^{n} i^2\right] \\
&= \frac{1}{n}\left[n + \frac{2n(n+1)}{2n} + \frac{n(n+1)(2n+1)}{6n^2}\right] \\
&= 1 + \frac{n+1}{n} + \frac{2n^2+3n+1}{6n^2}
\end{aligned}
$$

Problem 8

(a)

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2-10}{n^3} = \lim_{n\to\infty}\frac{2n^2+3n-59}{6n^2} = \lim_{n\to\infty}\left[\frac{1}{3} + \cancelto{0}{\frac{1}{2n}} - \cancelto{0}{\frac{59}{6n^2}}\right] = \frac{1}{3}$$

(b)

$$
\begin{aligned}
\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{i}{n}\right)^2\left(\frac{1}{n}\right) &= \lim_{n\to\infty}\left[1 + \frac{n+1}{n} + \frac{2n^2+3n+1}{6n^2}\right] \\
&= \lim_{n\to\infty}\left[1 + 1 + \frac{1}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right] = 2 + \frac{1}{3} = \frac{7}{3}
\end{aligned}
$$