

AP Calculus BC Notes

Jayden Li

Fall Term, 2024-25

1 The Fundamental Theorem of Calculus

1.1 Part 1: Definite integral as a function

Let $x \in [a, b]$ and f is an integrable function. Then we define the antiderivative F :

$$F(x) = \int_a^x f(t) \, dt$$

F is a function as the bounds of integration is a variable. The fundamental theorem of calculus states that:

$$F'(x) = \frac{d}{dx} \int_a^x f(x) \, dx = f(x)$$

If the bounds of integration is the function g , we use the chain rule:

$$\frac{d}{dx} \int_a^{g(x)} f(t) \, dt = \frac{d}{dx} F(g(x)) = F'(g(x))g'(x) = f(g(x))g'(x)$$

1.2 Part 2: Definite integral as a number

Suppose F is the antiderivative of f (then $F'(x) = f(x)$). Then:

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

In this statement, $f(x)$ is called the *integrand*. To evaluate the statement, we first find the antiderivative F .

1.3 Net Change Theorem

We can rewrite the second part of the FTC as follows:

$$\int_a^b F'(x) \, dx = F(b) - F(a)$$

This is the *Net Change Theorem*.