Problem Set #6

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Problem 1

Completed last year.

Problem 2

Completed last year or in class.

Problem 3

(a)

$$\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x^2+1}+C=\frac{2x}{2\sqrt{x^2+1}}=\frac{x}{\sqrt{x^2+1}}$$

(b)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sin x+\cos x+C\right]=\sin x+x\cos x-\sin x=x\cos x$$

(c)

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left[\sin x-\frac{1}{3}\sin^3 x+C\right]&=\cos x-\frac{1}{3}\cdot 3\sin^2 x\cos x=\cos x-\left(1-\cos^2 x\right)\cos x\\&=\cos x-\left(\cos x-\cos^3 x\right)=\cos x-\cos x+\cos^3 x=\cos^3 x\end{aligned}$$

(d)

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{2}{3b^2}(bx-2a)\sqrt{a+bx}+C\right]&=\frac{2}{3b^2}\left(b\sqrt{a+bx}+(bx-2a)\cdot\frac{b}{2\sqrt{a+bx}}\right)\\&=\frac{2}{3b^2}\left(\frac{2b(a+bx)}{2\sqrt{a+bx}}+\frac{b(bx-2a)}{2\sqrt{a+bx}}\right)\\&=\frac{2}{3b^2}\cdot\frac{2ab+2b^2x+b^2x-2ab}{2\sqrt{a+bx}}\\&=\frac{3b^2x}{3b^2\sqrt{a+bx}}=\frac{x}{\sqrt{a+bx}}\end{aligned}$$

Problem 4

(a) Done in class.

(b)

$$\int\left(\sqrt{x^3}+\sqrt[3]{x^2}\right)\mathrm{d}x=\frac{x^{\frac{4}{3}}}{\frac{4}{3}}+\frac{x^{\frac{5}{3}}}{\frac{5}{3}}+C=\frac{3\sqrt[3]{x^4}}{4}+\frac{3\sqrt[3]{x^5}}{5}+C$$

(c)

$$\int(1-t)\left(2+t^2\right)\,\mathrm{d}t=\int\left(2+t^2-2t-t^3\right)\mathrm{d}t=2t+\frac{t^3}{3}-t^2-\frac{t^4}{4}+C$$

(d)

$$\begin{aligned}\int v\cdot\left(v^2+2\right)^2\,\mathrm{d}v&=\int v\left(v^4+4v^2+4\right)\,\mathrm{d}v=\int\left(v^5+4v^3+4v\right)\mathrm{d}v\\&=\frac{v^6}{6}+\frac{4v^4}{4}+\frac{4v^2}{2}+C=\frac{v^6}{6}+v^4+2v^2+C\end{aligned}$$

(e)

$$\int\frac{x^3-2\sqrt{x}}{x}\,\mathrm{d}x=\int\left(x^2-2x^{-\frac{1}{2}}\right)\mathrm{d}x=\frac{x^3}{3}-\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}+C=\frac{x^3}{3}-4\sqrt{x}+C$$

(f)

$$\int\left(u^2+1+\frac{1}{u^2}\right)\mathrm{d}u=\frac{u^3}{3}+u+\frac{u^{-1}}{-1}+C=\frac{u^3}{3}+u-\frac{1}{u}+C$$

(g)

$$\int(\theta-\csc\theta\cot\theta)\,\mathrm{d}\theta=\int\theta\,\mathrm{d}\theta+\int(-\csc\theta\cot\theta)\,\mathrm{d}\theta=\frac{\theta^2}{2}+\csc\theta+C$$

(h)

$$\int\sec(t)(\sec t+\tan t)\,\mathrm{d}t=\int\left(\sec^2 t+\sec(t)\tan(t)\right)\mathrm{d}t=\tan t+\sec t+C$$

(i)

$$\int\left(1+\tan^2\alpha\right)\mathrm{d}\alpha=\int\left(\frac{\cos^2\alpha}{\cos^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}\right)\mathrm{d}\alpha=\int\frac{1}{\cos^2\alpha}\,\mathrm{d}\alpha=\int\sec^2\alpha\,\mathrm{d}\alpha=\tan\alpha+C$$

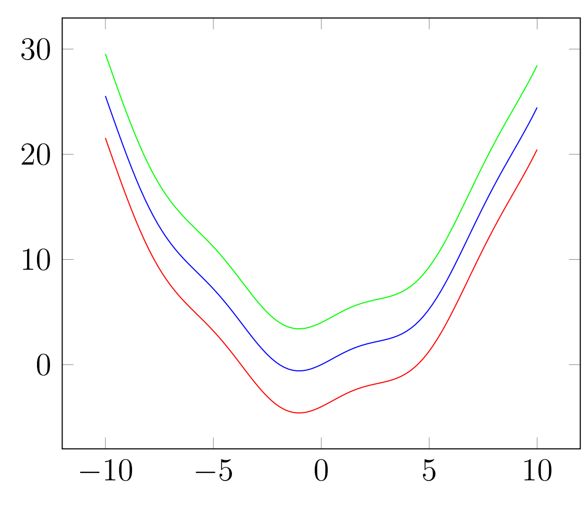
(j) Done in class.

(k) Done in class.

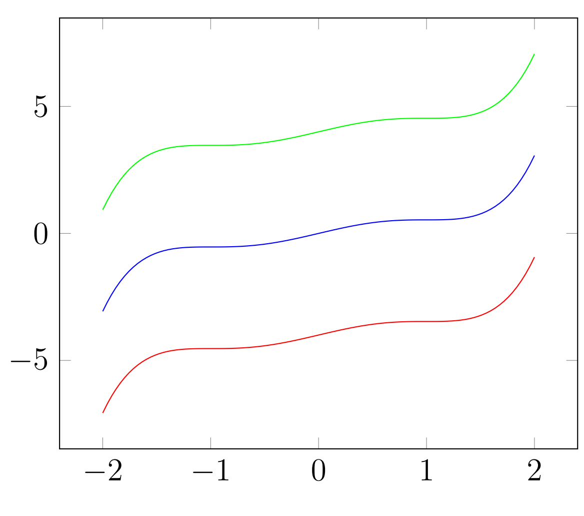
(l) Done in class.

Problem 5

(a)



(b)



Problem 6

(a)

$$\begin{gathered}f(x)=\int\left(6\sqrt{x}+5x^{3/2}\right)\mathrm{d}x=\frac{6x^{3/2}}{3/2}+\frac{5x^{5/2}}{5/2}+C=4\sqrt{x^3}+2\sqrt{x^5}+C\\f(1)=10\implies 4+2+C=10\implies C=4\\\boxed{f(x)=4\sqrt{x^3}+2\sqrt{x^5}+4}\end{gathered}$$

(b)

$$\begin{gathered}f'(\theta)=\int(\sin\theta+\cos\theta)\,\mathrm{d}\theta=-\cos\theta+\sin\theta+C\\f'(0)=1\implies -1+0+C=1\implies C=2\\f(\theta)=\int(-\cos\theta+\sin\theta+2)\,\mathrm{d}\theta=-\sin\theta-\cos\theta+2x+D\\f(0)=2\implies 0-1+0+D=2\implies D=3\\\boxed{f(\theta)=-\sin\theta-\cos\theta+2x+3}\end{gathered}$$