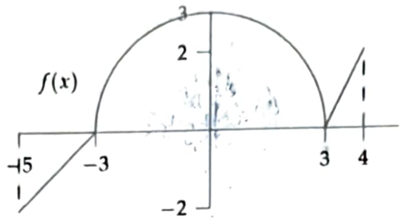

Figure 1. Find the area between f and the $x-axis$.

7. Prove that $\int_a^b x\,dx = \frac{b^2-a^2}{2}$, by considering partitions into n equal subintervals.

8. Find the constants a and b that maximize the value of $\int_a^b (1-x^2)\,dx$. Explain your reasoning.

9. Use the properties of integrals to verify the inequality without evaluating the integrals

$$\int_0^1 \sqrt{1+x^2}\,dx \leq \int_0^1 \sqrt{1+x}\,dx$$

10. Evaluate without doing any computations:

(a) $\int_{-1}^{1} x^3\sqrt{1-x^2}\,dx.$ (b) $\int_{-1}^{1} x^5+3\sqrt{1-x^2}\,dx.$

11. This problem uses the online applet https://integral-domain.org/lwilliams/Applets/calculus/reimann.php

(a) What is the area of a semi-circle of radius 1? Give your answer in terms of π and also as a decimal rounded correctly to four places. $\frac{\pi}{2} \approx 1.571$

(b) The equation of the semicircle of radius 1 centered at the origin is $f(x)=\sqrt{1-x^2}$ on the interval $[-1,1]$. You should be able to find the area of this region using calculus. According to our theory, since $f(x)=\sqrt{1-x^2}$ is continuous, it is integrable so

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \lim_{n\to\infty} Right\ (n) = \lim_{n\to\infty} Left\ (n)$$

So you should be able to approximate the answer using left and right Riemann sums with increasingly large values of n. Use the link above to find: $Left\ (5)$, $Right\ (5)$, and $Midpoint\ (5)$. Then $Left\ (52)$, $Right\ (52)$, and $Midpoint\ (52)$. Finally $Left\ (512)$, $Right\ (512)$, and $Midpoint\ (512)$. Correctly round to four decimal places.

Left(5) ≈ 1.424
Right(5) ≈ 1.424
Midpoint(5) = 1.613

Left(5) = 1.566
Right(5) = NaN
Midpoint(5) = 1.572

Left(512) = 1.571
Right(512) = 1.571
Midpoint(512) = 1.571

n	$Left\ (n)$	$Right\ (n)$	$Midpoint\ (n)$
5			
52			
512			

(c) Are these estimates getting closer to your answer in part (a) as n gets larger?

Yes.