Problem Set #7

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September 16, 2024

Problem 2

(j) $\displaystyle\int_{-1}^{2} \frac{s^4+1}{s^2}\,\mathrm{d}s$ is undefined because $\frac{s^4+1}{s^2}$ is undefined at $s=0$ and is not continuous on $[-1,9]$.

(k)

$$\int_0^{\pi/4} \sec^2 t\,\mathrm{d}t = \tan t\big|_0^{\pi/4} = 1$$

(l)

$$\int_0^{\pi/4} \sec\theta\tan\theta\,\mathrm{d}\theta = \sec(\theta)\big|_0^{\pi/4} = \frac{2}{\sqrt{2}} - \frac{1}{1} = \sqrt{2}-1$$

Problem 3

(b)

$$g'(y) = \frac{\mathrm{d}}{\mathrm{d}y}\int_0^y t^2\sin t\,\mathrm{d}t = y^2\sin y$$

(c)

$$h'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\int_x^{\pi}\sqrt{1+\sec t}\,\mathrm{d}t = -\frac{\mathrm{d}}{\mathrm{d}x}\int_{\pi}^{x}\sqrt{1+\sec t}\,\mathrm{d}t = -\sqrt{1+\sec t},\ \text{where } x \neq \frac{\pi}{2}+\pi n$$

(d)

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_1^{x^4}\sec t\,\mathrm{d}t = 4x^3\sec\left(x^4\right),\ \text{where } x \neq \sqrt[4]{\frac{\pi}{2}+\pi n}$$

Problem 4

(c)

$$\begin{aligned} g'(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^{x^3}\sqrt{w}\sin w\,\mathrm{d}w - \int_0^{\sqrt{x}}\sqrt{w}\sin w\,\mathrm{d}w\right] = 3x^2\sqrt{x^3}\sin\left(x^3\right) - \sqrt{\sqrt{x}}\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}} \\ &= 3x^3\sqrt{x}\sin\left(x^3\right) - \frac{\sqrt{\sqrt{x}}\sin\left(\sqrt{x}\right)}{2\sqrt{x}} \end{aligned}$$

(d)

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^{5x}\cos\left(u^2\right)\,\mathrm{d}u - \int_0^{\cos x}\cos\left(u^2\right)\,\mathrm{d}u\right] = 5\cos\left(25x^2\right) - \cos\left(\cos^2 x\right)(-\sin x) \\ &= 5\cos\left(25x^2\right) + \cos\left(\cos^2 x\right)\sin x \end{aligned}$$

Problem 9

$$\int_1^2 h''(u)\,\mathrm{d}u = h'(2) - h'(1) = 5 - 2 = 3$$