Larson Chapter 6.5

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133.

$$
\begin{aligned}
z^2 &= 2 - 2i \\
z^2 &= 2\sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right) \\
z &= \sqrt{2\sqrt{2}}\operatorname{cis}\left(\frac{-\frac{\pi}{4} + 2\pi k}{2}\right) \\
k = 0 : z &= \sqrt{2\sqrt{2}}\operatorname{cis}\left(-\frac{\pi}{8}\right) \\
z &= \sqrt{2\sqrt{2}}\left(\sqrt{\frac{\frac{\sqrt{2}}{2} + 1}{2}} - i\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}\right) \\
z &= \frac{\sqrt{2\sqrt{2}}\sqrt{\sqrt{2} + 2}}{2} - i\frac{\sqrt{2\sqrt{2}}\sqrt{2 - \sqrt{2}}}{2} \\
z &= \sqrt{\frac{4 + 4\sqrt{2}}{4}} - i\sqrt{\frac{4\sqrt{2} - 4}{4}} \\
&\boxed{z = \sqrt{1 + \sqrt{2}} - i\sqrt{\sqrt{2} - 1}} \\
k = 1 : z &= \sqrt{2\sqrt{2}}\operatorname{cis}\left(\frac{-\frac{\pi}{4} + \frac{8\pi}{4}}{2}\right) \\
z &= \sqrt{2\sqrt{2}}\operatorname{cis}\left(-\frac{\pi}{8} + \pi\right) \\
z &= \sqrt{2\sqrt{2}}\left(\cos\left(-\frac{\pi}{8} + \pi\right) + i\sin\left(-\frac{\pi}{8} + \pi\right)\right) \\
z &= \sqrt{2\sqrt{2}}\left(-\frac{\sqrt{\sqrt{2} + 2}}{2} + i\left(-\left(-\frac{\sqrt{2 - \sqrt{2}}}{2}\right)\right)\right) \\
z &= -\frac{\sqrt{2\sqrt{2}}\sqrt{\sqrt{2} + 2}}{2} + i\frac{\sqrt{2\sqrt{2}}\sqrt{2 - \sqrt{2}}}{2} \\
z &= -\sqrt{\frac{4 + 4\sqrt{2}}{4}} + i\sqrt{\frac{4\sqrt{2} - 4}{4}} \\
&\boxed{z = -\sqrt{1 + \sqrt{2}} + i\sqrt{\sqrt{2} - 1}}
\end{aligned}
$$

135.

$$z^2 = 1 + \sqrt{3}i$$

$$z^2 = 2\,\mathrm{cis}\,\frac{\pi}{3}$$

$$z = \sqrt{2}\,\mathrm{cis}\left(\frac{\frac{\pi}{3}+2\pi k}{2}\right)$$

$$k=0: z = \sqrt{2}\,\mathrm{cis}\,\frac{\pi}{6}$$

$$z = \sqrt{2}\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$$

$$\boxed{z = \frac{\sqrt{6}}{2}+i\frac{\sqrt{2}}{2}}$$

$$k=1: z = \sqrt{2}\,\mathrm{cis}\left(\frac{\pi}{6}+\pi\right)$$

$$z = \sqrt{2}\left(-\frac{\sqrt{3}}{2}-\frac{i}{2}\right)$$

$$\boxed{z = -\frac{\sqrt{6}}{2}-i\frac{\sqrt{2}}{2}}$$

139.

$$z^4 = 8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$z = \sqrt[4]{8}\left(\cos\left(\frac{\frac{2\pi}{3}+\frac{6\pi k}{3}}{4}\right)+i\sin\left(\frac{\frac{2\pi}{3}+\frac{6\pi k}{3}}{4}\right)\right)$$

$$z = \sqrt[4]{8}\left(\cos\left(\frac{\pi}{6}+\frac{3\pi k}{6}\right)+i\sin\left(\frac{\pi}{6}+\frac{3\pi k}{6}\right)\right)$$

$$k=0: z = \sqrt[4]{8}\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$$

$$\boxed{z = \frac{\sqrt[4]{8}\sqrt{3}}{2}+i\frac{\sqrt[4]{8}}{2}}$$

$$k=1: z = \sqrt[4]{8}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$\boxed{z = -\frac{\sqrt[4]{8}}{2}+i\frac{\sqrt[4]{8}\sqrt{3}}{2}}$$

$$k=2: z = \sqrt[4]{8}\left(-\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$$

$$\boxed{z = -\frac{\sqrt[4]{8}\sqrt{3}}{2}-i\frac{\sqrt[4]{8}}{2}}$$

$$k=1: z = \sqrt[4]{8}\left(-\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)$$

$$\boxed{z = \frac{\sqrt[4]{8}}{2}-i\frac{\sqrt[4]{8}\sqrt{3}}{2}}$$

141.

$$
\begin{aligned}
z^3 &= -25i \\
z^3 &= 25\operatorname{cis}\frac{3\pi}{2} \\
z &= \sqrt[3]{25}\operatorname{cis}\left(\frac{\frac{3\pi}{2}+\frac{4\pi k}{2}}{3}\right) \\
k=0: z &= \sqrt[3]{25}\operatorname{cis}\frac{\pi}{2} \\
&\boxed{z = i\sqrt[3]{25}} \\
k=1: z &= \sqrt[3]{25}\operatorname{cis}\frac{7\pi}{6} \\
z &= \sqrt[3]{25}\left(-\frac{\sqrt{3}}{2}-\frac{i}{2}\right) \\
&\boxed{z = -\frac{\sqrt[3]{25}\sqrt{3}}{2} - i\frac{\sqrt[3]{25}}{2}} \\
k=2: z &= \sqrt[3]{25}\operatorname{cis}\frac{11\pi}{6} \\
z &= \sqrt[3]{25}\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right) \\
&\boxed{z = \frac{\sqrt[3]{25}\sqrt{3}}{2} - i\frac{\sqrt[3]{25}}{2}}
\end{aligned}
$$

151.

$$
\begin{aligned}
z^5 &= 128(-1+i) \\
z^5 &= 128\left(\sqrt{2}\operatorname{cis}\frac{3\pi}{4}\right) \\
z &= \sqrt[5]{128\sqrt{2}}\operatorname{cis}\left(\frac{\frac{3\pi}{4}+\frac{8\pi k}{4}}{5}\right) \\
k=0: z &= \sqrt[5]{128\sqrt{2}}\left(\cos\frac{3\pi}{20}+i\sin\frac{3\pi}{20}\right) \\
&\boxed{z = \sqrt[5]{128\sqrt{2}}\cos\frac{3\pi}{20} + i\sqrt[5]{128\sqrt{2}}\sin\frac{3\pi}{20}} \\
k=1: &\boxed{z = \sqrt[5]{128\sqrt{2}}\cos\frac{11\pi}{20} + i\sqrt[5]{128\sqrt{2}}\sin\frac{11\pi}{20}} \\
k=2: &\boxed{z = \sqrt[5]{128\sqrt{2}}\cos\frac{19\pi}{20} + i\sqrt[5]{128\sqrt{2}}\sin\frac{19\pi}{20}} \\
k=3: &\boxed{z = \sqrt[5]{128\sqrt{2}}\cos\frac{27\pi}{20} + i\sqrt[5]{128\sqrt{2}}\sin\frac{27\pi}{20}} \\
k=4: z &= \sqrt[5]{128\sqrt{2}}\cos\frac{35\pi}{20} + i\sqrt[5]{128\sqrt{2}}\sin\frac{35\pi}{20}
\end{aligned}
$$

$$z = \sqrt[5]{128\sqrt{2}}\cos\frac{7\pi}{4} + i\sqrt[5]{128\sqrt{2}}\sin\frac{7\pi}{4}$$
$$z = \sqrt[5]{128\sqrt{2}}\sqrt[5]{\left(\frac{\sqrt{2}}{2}\right)^5} + i\sqrt[5]{128\sqrt{2}}\sqrt[5]{\left(-\frac{\sqrt{2}}{2}\right)^5}$$
$$z = \sqrt[5]{\frac{128\sqrt{2}\cdot\sqrt{2^5}}{32}} + i\sqrt[5]{\frac{128\sqrt{2}\cdot\sqrt{2^5}}{-32}}$$
$$z = \sqrt[5]{4\sqrt{64}} + i\sqrt[5]{-4\sqrt{64}}$$
$$z = \sqrt[5]{32} + i\sqrt[5]{-32}$$
$$\boxed{z = 2 - 2i}$$

153.

$$x^4 - i = 0$$
$$x^4 = \operatorname{cis}\frac{\pi}{2}$$
$$x = \operatorname{cis}\left(\frac{\frac{\pi}{2} + \frac{4\pi k}{2}}{4}\right)$$
$$x = \operatorname{cis}\frac{\pi + 4\pi k}{8}$$
$$k = 0 : x = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$$
$$\boxed{x = \frac{\sqrt{\sqrt{2}+2}}{2} + i\frac{\sqrt{2-\sqrt{2}}}{2}}$$
$$k = 1 : x = \cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}$$
$$\boxed{x = -\frac{\sqrt{2-\sqrt{2}}}{2} + i\frac{\sqrt{\sqrt{2}+2}}{2}}$$
$$k = 2 : x = \cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}$$
$$\boxed{x = -\frac{\sqrt{\sqrt{2}+2}}{2} - i\frac{\sqrt{2-\sqrt{2}}}{2}}$$
$$k = 3 : x = \cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}$$
$$\boxed{x = \frac{\sqrt{2-\sqrt{2}}}{2} - i\frac{\sqrt{\sqrt{2}+2}}{2}}$$

159.

$$x^3 - (1 - i) = 0$$
$$x^3 = \sqrt{2}\operatorname{cis}\left(-\frac{\pi}{4}\right)$$
$$x = \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}}\operatorname{cis}\left(\frac{-\frac{\pi}{4} + \frac{8\pi k}{4}}{3}\right)$$

$$x = \sqrt[6]{2}\operatorname{cis}\left(\frac{-\pi + 8\pi k}{12}\right)$$

$$k = 0 : x = \sqrt[6]{2}\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$$

$$x = \sqrt[6]{2}\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} - i\sqrt[6]{2}\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$\boxed{x = \frac{\sqrt[6]{2}\sqrt{2+\sqrt{3}}}{2} - i\frac{\sqrt[6]{2}\sqrt{2-\sqrt{3}}}{2}}$$

$$k = 1 : x = \sqrt[6]{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$$

$$x = \sqrt[6]{2}\left(-\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}\right) + i\sqrt[6]{2}\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$\boxed{x = -\frac{\sqrt[6]{2}\sqrt{2-\sqrt{3}}}{2} + i\frac{\sqrt[6]{2}\sqrt{2+\sqrt{3}}}{2}}$$

$$k = 2 : x = \sqrt[6]{2}\left(\cos\frac{15\pi}{12} + i\sin\frac{15\pi}{12}\right)$$

$$x = \sqrt[6]{2}\cos\frac{5\pi}{4} + i\sqrt[6]{2}\sin\frac{5\pi}{4}$$

$$\boxed{x = -\frac{\sqrt[6]{2}\sqrt{2}}{2} - i\frac{\sqrt[6]{2}\sqrt{2}}{2}}$$