Problem Set #40

Jayden Li

January 10, 2025

Problem 5

Let a_n be the amount of money received by the nth person. $a_n = Dc^{n-1}$.

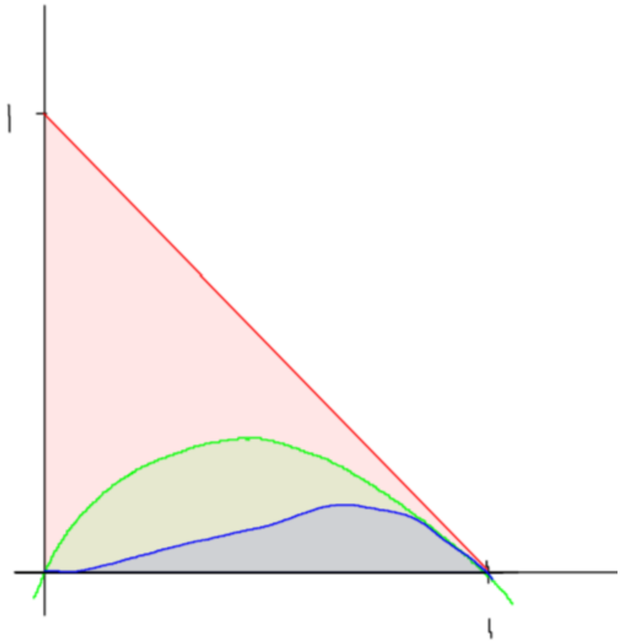
(a) $S_n = \sum_{k=1}^{n} S_k = \sum_{k=1}^{n} Dc^{k-1}$

(b) $\lim_{n\to\infty} S_n = D\sum_{i=1}^{\infty} c^{i-1} = \frac{D}{1-c} = \frac{D}{s} = \underbrace{\frac{1}{s}}_{k} D$

Marginal propensity to consume $= c = 0.8$ then multiplier $= k = 1/s = 1/(1-c) = 1/0.2 = 5$.

Problem 8

(a)



(b)

$$\int_0^1 (1-x)\,\mathrm{d}x = \left[x - \frac{x^2}{2}\right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$$
$$\int_0^1 \left(x - x^2\right)\mathrm{d}x = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$
$$\int_0^1 \left(x^2 - x^3\right)\mathrm{d}x = \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

(c) $$a_n = \int_0^1 \left(x^{n-1} - x^n\right)\mathrm{d}x = \left[\frac{x^n}{n} - \frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \boxed{\frac{1}{n^2+n}}$$

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \int_0^1 \left(x^{n-1} - x^n\right)\mathrm{d}x = \int_0^1 \left(x^0 - \cancel{x^1} + \cancel{x^1} - \cancel{x^2} + \ldots + \cancel{x^{n-1}} - x^n\right)\mathrm{d}x = \int_0^1 (1 - x^n)\,\mathrm{d}x$$
$$= \left[x - \frac{x^{n+1}}{n+1}\right]_0^1 = 1 - \frac{1}{n+1} = \frac{n+1}{n+1} - \frac{1}{n+1} = \boxed{\frac{n}{n+1}}$$