d) $f(x) = e^{-x^2}\cos x = \left(\sum_{n=0}^{\infty}\frac{(-x^2)^n}{n!}\right)\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

$$= \left(1 - \frac{x^2}{1} + \frac{x^4}{2} + \dots\right)\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots\right)$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^2}{1} + \frac{x^4}{2} - \cancel{\frac{x^6}{24}} + \frac{x^4}{2} - \cancel{\frac{x^6}{4}} + \cancel{\frac{x^8}{48}} + \dots$$

$$= 1 - \frac{3x^2}{2} + \frac{x^4}{24} + \frac{24x^4}{24x^4} + \dots$$

$$\boxed{= 1 - \frac{3x^2}{2} + \frac{25x^4}{24} + \dots}$$

e) $f(x) = y = \frac{x}{\sin x} = \frac{x}{\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}}$

$$= \frac{x}{x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots} = \frac{x}{x - \frac{x^3}{6} + \frac{x^5}{120} + \dots}$$

$$1 + \frac{x^2}{6} + \frac{7x^4}{360}$$

$$x - \frac{x^3}{6} + \frac{x^5}{120} \;\Big)\; x$$

$$-\left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)$$

$$0 + \frac{x^3}{6} - \frac{x^5}{120}$$

$$-\left(\frac{x^3}{6} - \frac{x^5}{36} + \frac{x^7}{720}\right)$$

$$0 + \frac{7x^5}{360} + \dots$$

$$-\frac{x^5}{120} + \frac{x^5}{36} = \frac{-3x^5}{360} + \frac{10x^5}{360} = \frac{7x^5}{360}$$

$$\boxed{= 1 + \frac{x^2}{6} + \frac{7x^4}{360} + \dots}$$

f) $f(x) = e^x \ln(1+x)$

$$= \left(\sum_{n=0}^{\infty} \frac{x^n}{n!}\right) \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

$$= \left(1 + x + \frac{x^2}{2} + \dots\right)\left(x - \frac{x^2}{2} + \frac{x^3}{3} + \dots\right)$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} + x^2 - \cancel{\frac{x^3}{2}} + \cancel{\frac{x^4}{3}} + \cancel{\frac{x^3}{2}} - \cancel{\frac{x^4}{4}} + \cancel{\frac{x^3}{6}} + \dots$$

$$\boxed{= x + \frac{x^2}{2} + \frac{x^3}{3} + \dots}$$