AP Physics C – Class Notes

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1 Introduction

1.1 Jumping Monsters

See Figure 1.1 in Notebook.

We investigate the relationship between the mass of the toy m and the change in height Δh. Equipment:

- Meter stick (not ruler, since ruler is only 30cm long)
- Phone (to record video)
- Balance (to measure mass in grams and kilograms, a scale measures weight in Newtons)
- Washers, paper clips and tape (to increase mass of toy)

We collect many data points. We will collect 5 data points, which is 5 conditions, which is 5 different masses to test. We want to repeat every mass a few times too; we will test every mass 3 times ("3 trials"). In total, the toy will jump $5 \cdot 3 = 15$ times. Trial means that conditions/masses are the same.

Results/data are in Table 1.2 in Notebook.

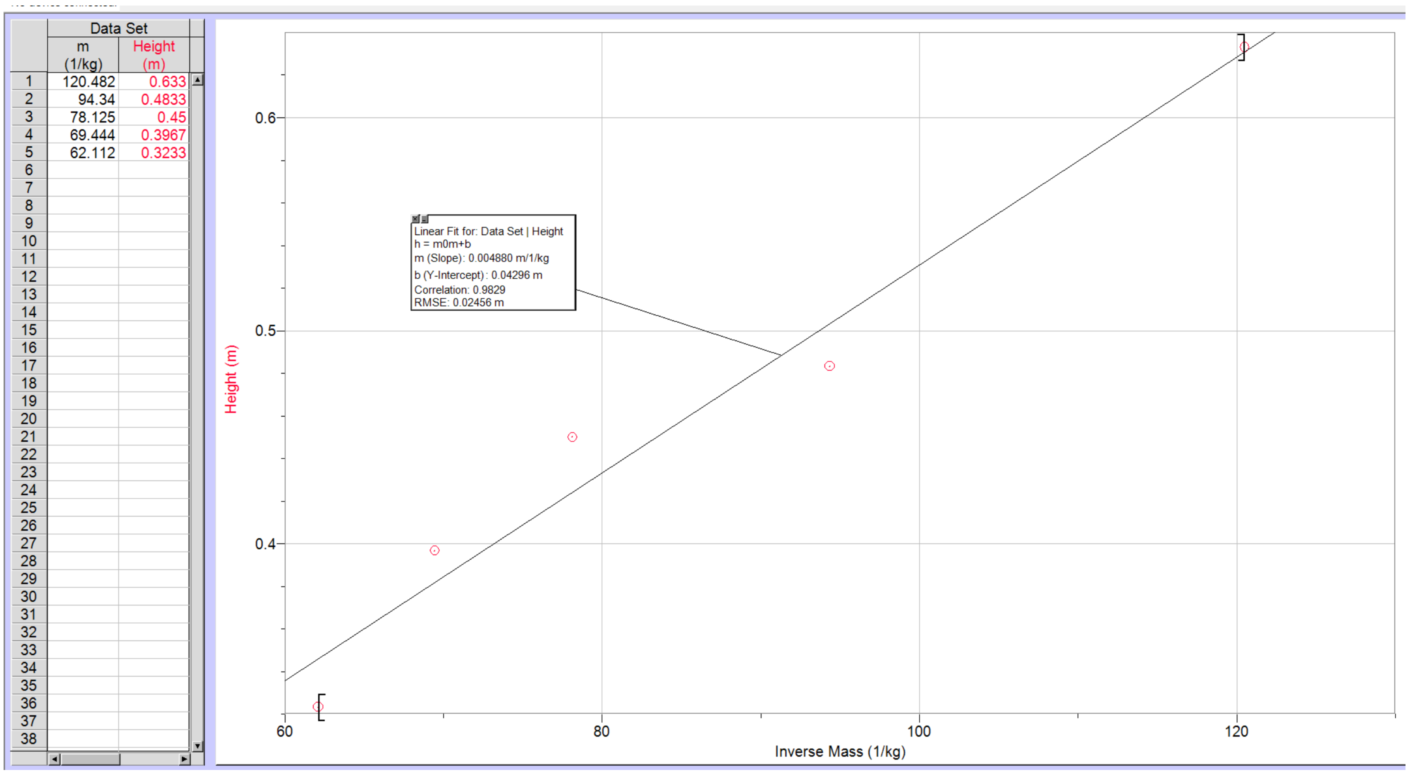
Based on conservation of energy:

$$\mathrm{PE_s} = \mathrm{PE_g} \implies \frac{1}{2}kx^2 = mgh \implies h = \frac{kx^2}{2mg} = \frac{kx^2}{2g} \cdot \frac{1}{m}$$

If we graph mass m against height Δh, this is an inverse relationship, as $kx^2/2g$ is a constant (the spring distance x does not change for one toy, k is spring constant, and g is acceleration due to gravity).

Because we want a linear relationship, we can graph inverse mass $1/m$ against height Δh. This becomes a line with slope $kx^2/2g$.

Logger pro graph, plotting inverse mass in 1/kg against height in meters:



$$\text{Slope} = m = \frac{kx^2}{2g} = 0.004880 \implies k = m \cdot \frac{2g}{x^2} = 0.004880 \cdot \frac{2(9.81)}{(1.5/100)^2} = \boxed{425.536\,\text{N/m}}$$

Linearization

Linearization is a powerful technique. For example, with Kepler's third law of planetary motion:

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

is annoying to plot when we plot T against r. We rearrange and plot T^2 against r^3 for a linear relationship:

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$4\pi^2/GM$ is a constant/the slope. We take $y = T^2$ and $x = r^3$, and we have the fitted slope $m = 4\pi^2/GM$.

1.2 Projectile Motion

Two-dimensional motion, x and y-directions. The type of motion in the two directions are different.

x-direction has constant velocity because there is no net force and no acceleration acting in the x-direction.

$$v_x = \frac{\Delta x}{\Delta t} \iff \Delta x = v_x(\Delta t)$$

y-direction has constant acceleration due to gravity $-g$.

$$v_y = v_{0y} + at$$
$$\Delta y = v_{0y}t + \frac{1}{2}at^2$$
$$v_y^2 = v + 0y^2 + 2a\Delta y$$

1.2.1 Special cases

In horizontal projectile motion, $v_{0y} = 0\,\mathrm{m/s}$.

When the projectile reaches its highest point in general 2-dimensional motion, then at the highest point, there is no y-velocity: $v_y = 0\,\mathrm{m/s}$.

If the projectile is launched at a certain velocity v_0 at an angle θ, we have initial x and initial y-velocities:

$$v_x = v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$

1.3 Momentum Lab

Equipment:

- Motion sensor/detector
- Force sensor: measures force
- Cart connected to plunger; plunger is a metal rod with a spring inside

Push cart on a track, measure force and velocity to obtain the mass of the cart. To improve the accuracy and lower the percent error:

- Check if the track is level, if the track is inclined then the cart is not moving at a constant velocity (disregarding friction) before and after the collision.
- Reduce friction
- Increase the number of **experimental conditions** (cannot increase the number of trials, because trial has the same conditions like initial speed, which is not possible to make sure) by pushing it more times
- Line up force sensor and cart/track

1.4 AP Formula Sheet

G universal gravitational constant is important. We should take acceleration due to gravity on Earth $g = 10\,\mathrm{m/s^2}$. Magnitude of gravitational field of earth is also $g = 10\,\mathrm{N/kg}$. These units are equivalent.

Kinematic equations for **constant acceleration** and therefore constant force:

$$\Delta x = v_0 t + \frac{1}{2}at^2 \qquad v = v_0 + at \qquad v^2 = v_0^2 + 2a\Delta x$$

Newton's second law does not have to be complicated; $\Sigma F = ma$ is fine.

Maximum friction $\|\vec{F}_f\| = \|\mu \vec{F}_N\|$ tells us, where $\vec{F}_N$ is normal force:

- kinetic friction $\vec{F}_k = \mu_k \vec{F}_N$
- Maximum static friction $\vec{F}_{\text{s,max}} = \mu_{\text{s,max}} \vec{F}_N$
- Static friction $\|\vec{F}_{\text{s}}\| < \vec{F}_{\text{s,max}}$.

2 Calculus

Kinematics

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad \Delta x = \int v \,\mathrm{d}t$$
$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \qquad \Delta v = \int a \,\mathrm{d}t$$

where x is position, v is velocity and a is acceleration. These will apply to all situations (for constant velocity, constant acceleration or nonconstant acceleration).

Integrate with respect to independent variable; integrand is dependent variable. See Figure 2.1.

Connecting dynamics/forces and kinematics

Newton's second law: $\Sigma F = ma$. Acceleration a connects forces and dynamics to kinematics.

Suppose we are given position x. Then we can calculate the acceleration by differentiation:

$$a = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \implies F = ma = m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$$

Connecting energy and kinematics by kinetic energy $K = mv^2/2$.

Work and force-position graphs

Consider a force-position graph (Figure 2.2). The area under the curve is the work done, or change in energy: $W = \Delta E$. The relationship is as follows, where ΣF is the net force:

$$\Delta E = \int \Sigma F \,\mathrm{d}x \qquad \Sigma F = \frac{\mathrm{d}E}{\mathrm{d}x}$$

Power and time

Power is the rate of change of energy with respect to time:

$$\Delta E = \int P \,\mathrm{d}t \qquad P = \frac{\mathrm{d}E}{\mathrm{d}t}$$

The change in energy is the area under the curve of a power-time graph (Figure 3.3).

Force and time

Recall that for constant net force ΣF, impulse $J = \Delta p = \Sigma F \cdot \Delta t$. For a force-time graph in Figure 2.4:

$$J = \Delta p = \int F \, \mathrm{d}t \qquad F = \frac{\mathrm{d}p}{\mathrm{d}t} = m \cdot \frac{\mathrm{d}v}{\mathrm{d}t} = ma$$

(Impulse $\Delta p = I = J = \Delta(mv) = m \cdot \Delta v$, since mass usually does not change. $\mathrm{d}p = F\mathrm{d}t \implies \mathrm{d}(mv) = m\mathrm{d}v = F\mathrm{d}t \implies F = m(\mathrm{d}v/\mathrm{d}t) = ma$)

Rotational kinematics

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} \qquad \Delta\theta = \int \omega \, \mathrm{d}t \qquad \Delta\omega = \int \alpha \, \mathrm{d}t$$

2.1 Potential Energy

Suppose we drop a ball from a certain height. We know that the work done by the force of gravity is positive work, but since energy is conserved, the potential energy in the ball must decrease. Therefore, we have:

$$-W = \Delta U$$

But, we know that work is force times distance:

$$\Delta U = -F_y \cdot \Delta y \implies F_y = -\frac{\Delta U}{\Delta y}$$

where Δy is the distance the ball is dropped, or the distance over which the force acts. By black magic, we can change this into a derivative:

$$F_y = -\frac{\mathrm{d}U}{\mathrm{d}y}$$

Must have negative sign. So, we can also have potential energy in terms of force:

$$F_y = -\frac{\mathrm{d}U}{\mathrm{d}y} \implies -F_y \, \mathrm{d}y = \mathrm{d}U \implies U = -\int F_y \, \mathrm{d}y$$

Potential energy

The relationship between potential energy and force is, where x is position:

$$\Delta U_x = -\int F_x \, \mathrm{d}x \qquad F_x = -\frac{\mathrm{d}U}{\mathrm{d}x}$$

The work done by a **conservative force** only depends on the initial and final position of the object, not on the path taken.

A lot of forces are path-dependent. For example, friction depends on the length of the path, so it is not conservative.

Forces associated with change in potential energy are conservative.

Example: Gravitional potential

Change in gravitational potential energy is $\Delta U_g = mgy$. We can calculate the force:

$$F_g = -\frac{\mathrm{d}U_g}{\mathrm{d}y} = -\frac{\mathrm{d}}{\mathrm{d}y}(mgy) = -mg$$

is consistent with the result we get from $F = ma$.

Example: Spring potential

Change in spring potential is $U_s = kx^2/2$:

$$F_s = -\frac{\mathrm{d}U_s}{\mathrm{d}x} = -\frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{2}kx^2) = -kx$$

2.2 Perfect Ideal Spring

Formulas like $F_s = -kx$ and $U_s = kx^2/2$ depend on a perfect, ideal spring, which requires some assumptions.

- The mass of the spring itself is negligible; the mass of the system is exactly the mass of the object attachde to the spring.
- Force exerted is $F_s = -kx$: force-displacement graph is linear and passes through the origin. From this, formula for kinetic energy follows: $U_s = -\int F_s \, \mathrm{d}x = -\int -kx \, \mathrm{d}x = kx^2/2$.
- No internal friction and losses. If there are, *damping* occurs. This is drawn in Figure 2.5. Even though the period does not change, each successive crest and trough decreases in magnitude.

Figure 2.6 shows the energy position graph of an ideal spring. By conservation of energy, total energy $\mathrm{TE} = U_s + K$.

2.3 Non-Ideal Springs

Real springs are non-ideal, obviously.