AP Physics C: Electricity and Magnetism Notes

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Contents

1   Circuit

Equipment:

- D-cell battery/cells: with positive and negative terminals.

- Battery holder.

- Ammeter (machine labeled "<u>A</u>").

- Multimeter (can be ammeter, voltmeter or ohmmeter): has "test leads" or "probes."

- Single pole single throw switch and single pole double throw switch.

- Wires: alligator-alligator wires (alligator clips on both ends), alligator-banana wires, banana-banana wires.

- Sockets (light bulbs screw into the hole).

- Round bulbs and long bulbs.

There are electrons everywhere in the circuit (all metal solids have delocalized electrons). The battery serves as a "pump" to move electrons around through the circuit. We can use the voltmeter in the multimeter to measure the "pump strength."

We can find the potential difference across the battery $\Delta V_{\text{battery}}$ and across the two light bulbs ΔV_{B_1} and ΔV_{B_2}.

$$\Delta V_{\text{battery}} = 4.34\,\text{V}$$

$$\Delta V_{\text{B}_1} = 1.84\,\text{V}$$

$$\Delta V_{\text{B}_2} = 1.83\,\text{V}$$

Since energy is conserved the potential difference of the bulbs should be the same as that of the battery, but it is not. We need to take ΔV of the wires and switch into account as well.

Potential difference (ΔV) is measured in volts (V), and is related to energy. In Ye Olde Times, it is denoted V or emf (electromotive force), and ΔV_{bat} is denoted ε.

Flow rate/current (I) measures the number of electrons that pass a certain point per unit of time, and is measured in amperes (A).

Resistance (R) is how difficult it is for electrons to flow through and is measured in ohms (Ω).

The current is equal across the circuit.

Ohm's Law

$$\Delta V = IR$$

where:

$$\Delta V = \text{potential difference (V)}$$
$$I = \text{current (A)}$$
$$R = \text{resistance } (\Omega)$$

Resistors convert electrical energy to thermal energy. Light bulbs are a special case of resistor that convert electrical energy to thermal and light energy.

Resistors in series

When resistors are connected in series, current is equal across any part of the circuit:

$$I_{\text{bat}} = I_1 = I_2 = I_3$$

Assuming no potential is lost anywhere:

$$\Delta V_{\text{bat}} = \Delta V_1 + \Delta V_2 + \Delta V_3$$
$$R_{\text{total}} = R_1 + R_2 + R_3$$

Resistance

$$R = \frac{\rho l}{A}$$

where ρ is the resistivity of the material ($\Omega\,\text{m}$), l is length (m) and A is cross-sectional area (m^2).

A **junction/node** is a point where three or more wires are connected.

A **branch** connects two nodes.

To connect a parallel circuit, multiple junctions need to be made.

Resistors in parallel

The currents are different at different parts of the circuit:

$$I_{\text{bat}} = I_1 + I_2 + I_3$$

$$\Delta V_{\text{bat}} = \Delta V_1 = \Delta V_2 = \Delta V_3$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Similar to springs in parallel.

A shortcut: if the total equivalent resistance of a parallel circuit is R_T and the current entering the circuit is I_T, the current through branch i is. Since the circuit is parallel R_T is less than the individual R_i's.

$$I_i = I_T \cdot \frac{\frac{1}{R_i}}{\sum_k \frac{1}{R_k}} = I_T \cdot \frac{\frac{1}{R_i}}{\frac{1}{R_T}} = I_T \cdot \frac{R_T}{R_i}$$

From $I = \Delta V / R$ and $\Delta V_{\text{bat}} = \Delta V_1 = \Delta V_2 = \Delta V_3$:

$$I_{\text{bat}} = I_1 + I_2 + I_3 \implies \frac{\Delta V_{\text{bat}}}{R_{\text{total}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} + \frac{\Delta V_3}{R_3} \implies \boxed{\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Electron Sea Model

In metals, atoms form metallic bonds: each metal atom releases its valence electrons, which then form a sea of delocalized electrons. These electrons are then attracted to multiple atoms. These bonds are very strong and lead to high melting points for metals.

Delocalized electrons can move in a direction caused by an electric force.

Electrons flow from the negative terminal to the positive terminal. Conventional current is the direction in which positive charges flow, and is opposite to the direction electrons flow. The direction electrons flow is marked e^- and conventional current is marked I. The "long" end of the battery symbol is positive.

Ohmmeters are connected when there are no charges flowing; voltmeter and ammeter need the circuit to be active. Ammeter is connected in a circuit; voltmeter and ohmmeter measure across a circuit component (with a multimeter). In reality, these devices will modify the circuit when they are attached. An ammeter actually has a very small resistance, but we assume it has <u>zero resistance</u>. Also assume that a voltmeter has <u>infinite resistance</u>.

If two components are on different paths to the battery, they have the same potential difference (parallel).

A **combination circuit** is one which is not purely series or purely parallel. We can analyze such a circuit by calculating the total/equivalent resistance.

> **Power**
>
> $$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$
>
> where:
>
> $$P = \text{power (W)}$$
> $$I = \text{current (A)}$$
> $$\Delta V = \text{potential difference (V)}$$
> $$R = \text{resistance } (\Omega)$$
>
> The sum of power in each resistors equals the power in the battery.

Resistors dissipate energy by converting electrical energy to thermal energy at a rate equal to $P = I\Delta V = \ldots$

> **Light bulb ratings**
>
> Suppose a light bulb is rated at p W and v V. p relates to the brightness of the light bulb. From the formula $P = (\Delta V)^2/R$, we can determine the resistance of the light bulb: $R = v^2/p$.
>
> Light bulb assumptions:
>
> - The brightness of a light bulb is directly proportional to the power.
>
> - The resistance of a light bulb is constant.

> **Electric bills**
>
> The house is billed by energy used in kWh, which is actually a unit of energy.
>
> $$1\,\text{kWh} = 3.6 \times 10^6\,\text{J}$$

In a short circuit, a path with small/virtually zero resistance forms, adding a parallel path. The overall/equivalent resistance drops even though ΔV_{bat} is constant, increasing the current and therefore the power. A circuit breaker/fuse will trip/melt and open the circuit when current is higher than a certain limit.

1.1 Internal Resistance

In an ideal battery, there is no resistance: $R = 0$. In practice, a battery contains chemical reactions to generate a potential difference/current, and as the products of the reaction increase, the internal resistance increase.

In a circuit diagram, a resistor is placed to either side of the symbol for a battery. Sometimes, a dotted line includes these components. The potential difference of the battery and its internal resistance is ΔV_{T} (V).

> **Potential difference and internal resistance of battery**
>
> $$\Delta V_T = \varepsilon - Ir$$
>
> where:
>
> $$\Delta V_T = \text{terminal potential difference (includes internal resistance) (V)}$$
> $$\varepsilon = \text{potential difference of ideal battery (V)}$$
> $$I = \text{current through battery (A)}$$
> $$r = \text{internal resistance } (\Omega)$$

When batteries are connected in series, the ideal potential difference (ε) increases, but internal resistance also increases. The actual increase in terminal potential difference (ΔV_T) might be quite low, and a lot of thermal energy is produced.

If batteries are connected in parallel, there is no increase in potential difference. But the current in the branches where batteries are connected is reduced as batteries are added in parallel, which reduces internal resistance and ΔV due to internal resistance Ir. The battery also lasts longer.

Potential difference opposing each other: positive terminals are adjacent to each other. The battery with a lower potential difference is being "charged," until the batteries have the same potential difference.

> **Kirchhoff's Laws**
>
> 1. Junction/node law (law of conservation of charges): at any junction (place where 3 or more wires are connected together), the sum of all currents flowing into the junction equals the sum of all currents flowing out of the junction. Charges are conserved.
>
> 2. Loop law: around <u>any</u> loop in the circuit, the sum of all ΔV changes must equal zero. In conventional current: the battery produces a positive ΔV, resistors and light bulbs produce a negative ΔV. The sum of all components equals 0. Opposite of using flow of electrons as direction.

> The **ground** is where potential is zero volts. It is an ideal infinite source or sink of charge that can absorb unlimited charge without changing its potential of zero. It has both positive and negative charges.

1.2   Ideal Assumptions

- An ideal wire has <u>zero</u> resistance.

- An ideal ammeter has <u>zero</u> resistance.

- An ideal voltmeter has <u>infinite</u> resistance.

- An ideal battery has <u>zero</u> internal resistance.

- An ideal resistor is ohmic: it follows Ohm's law in that potential difference (ΔV) is directly proportional to current (I), with the slope being resistance R.

  A light bulb is <u>not ohmic</u>: its temperature will increase, increasing its resistance.

1.3 Charge

Current is the flow of charged particles, the charge (q, Q) being measured in coulombs (C). One coulomb is the amount of charge delivered by a current of one ampere per second.

$$\left[A = \frac{C}{s}\right] \iff [C = A \cdot s]$$

An electron has a charge of -1.6×10^{-19} C, and the charge of a proton has the same magnitude but opposite sign: $+1.6 \times 10^{-19}$ C.

Charge and current

$$I = \frac{Q}{t} = \frac{dq}{dt}$$

where:

$$I = \text{current (A)}$$
$$Q = \text{charge (C)}$$
$$t = \text{time (s)}$$

This is helpful in non-steady state circuits where current is not constant and changes over time.

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2 Electrostatics

Coulomb's law

$$F = \frac{kq_1 q_2}{r^2} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

where:

$$F = \text{electrostatic force (N)}$$
$$k = \text{Coulomb constant}$$
$$= 8.99 \cdot 10^9 \, \text{Nm}^2/\text{C}^2$$
$$\varepsilon_0 = \text{permittivity of free space } (\text{C}^2/\text{Nm}^2)$$
$$= 8.85 \cdot 10^{-12} \, \text{C}^2/\text{Nm}^2$$
$$q_1, q_2 = \text{charge (C)}$$
$$r = \text{distance between particles (m)}$$

There are positive and negative charges. The magnitude of the force is:

$$\|F_E\| = \frac{k \, |q_1| \, |q_2|}{r^2}$$

from which the direction of the force can be figured out by logic: opposite charges attract.

A **Van de Graaff generator** generates electrostatic energy by using a belt to transport electrons to a spherical dome. Electrons try to spread out as much as possible and are position on the dome's surface.

For two electrons spaced 1 millimeter apart, we can calculate $F_E = k\,|q_1|\,|q_2|\,/r^2$ and $F_G = G m_1 m_2 / r^2$. It turns out that F_E is about $4 \cdot 10^{42}$ times greater than F_G. So the gravitational force can often be ignored because it is so weak compared to electric force.

Electric force is a vector quantity, whose direction can be indicated with $+/-$ sign or with $\hat{i}, \hat{j}, \hat{k}$.

> **Electrostatics vs circuits**
>
> In electrostatics, charges are not moving (hence "static"). In a circuit, which for our purposes is direct current (DC), charges are flowing through the circuit and are not stationary.

2.1 Atomic Structure

Matter is made of atoms, ions and molecules, which are in turn made of protons (with positive charge), electrons (with negative charge) and neutrons (with no charge).

The mass of an electron is <u>significantly lower</u> than the mass of a proton or neutron:
$$(m_e = 9.11 \cdot 10^{-31} \text{ kg}) \ll m_p = m_n = 1.67 \times 10^{-27} \text{ kg}$$

Charge is quantized: the magnitude of charge on a proton or electron is the elementary charge:
$$|Q_e| = |Q_p| = 1.6 \cdot 10^{-19} \text{ C}$$

There is <u>no</u> unit of charge smaller than the elementary charge, and all possible charges is a multiple of it:
$$|Q| = n\left(1.6 \cdot 10^{-19}\right) \text{ C} \qquad n \in \mathbb{Z}^+$$

> An object being "charged" indicates the sign of the <u>net</u> charge, which is either positive or negative.
>
> - Positively charged: the object is deficient of electrons (has <u>less</u> electrons than protons).
>
> - Negatively charged: the object is in excess of electrons (has <u>more</u> electrons than protons).

The mass of excess electrons in a negatively charged object or the mass of electrons removed in a positively charged object is negligible, because the mass of an electron is so low.

> Charges on a metal/conductive sphere (solid or hollow) will always be on the surface of the sphere, because charges tend to maximize distance between each other. Thus, the <u>net charge is always on the surface</u>.

> In an insulator, there is no free flow of charges. Charges are stationary on the surface of the object.

2.2 Charging

Charging by friction. Different objects (insulators) have a different tendency to gain or lose net charge when rubbed with another object. Some will gain electrons while some will lose electrons.

- Glass and human hair will lose electrons and become more positive.

- Nylon and wood will gain electrons and become more negative.

Charging by conduction. This can be used for both insulators and conductors. It involves the movement of charges to/from a neutral conductor from/to a charged object by contact.

Charging by induction.

1. Start with a charged object ("rod") and a neutral conductor ("sphere").

2. When the rod is brought <u>near</u> (not necessarily contacting) the sphere, free electrons will rush towards/away from the rod. The sphere becomes <u>polarized.</u>

3. Connect the sphere to ground.

4. Electrons from ground will rush towards the sphere, and displace the positively charged particles.

<p align="center"><u>OR</u></p>

   Electrons from the sphere will rush towards ground, leaving only positively charged particles.

5. When the ground wire is removed from the sphere and the rod is moved away, the sphere remains charged.

This can be used to charge the sphere with either positive charge or negative charge. It would depend on the charge of the rod we start with.

- Negatively charged rod → positively charged sphere.

- Positively charged rod → negatively charged sphere.

2.3 Field Model

The field model is related to <u>non-contact forces</u>, which are gravity, electric force and magnetic force. The field model describes the alteration of space.