

AP PHYSICS C: ELECTRICITY AND MAGNETISM NOTES

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1 Circuit

Equipment:

- D-cell battery/cells: with positive and negative terminals.
- Battery holder.
- Ammeter (machine labeled “A”).
- Multimeter (can be ammeter, voltmeter or ohmmeter): has “test leads” or “probes.”
- Single pole single throw switch and single pole double throw switch.
- Wires: alligator-alligator wires (alligator clips on both ends), alligator-banana wires, banana-banana wires.
- Sockets (light bulbs screw into the hole).
- Round bulbs and long bulbs.

There are electrons everywhere in the circuit (all metal solids have delocalized electrons). The battery serves as a “pump” to move electrons around through the circuit. We can use the voltmeter in the multimeter to measure the “pump strength.”

We can find the potential difference across the battery $\Delta V_{\text{battery}}$ and across the two light bulbs ΔV_{B_1} and ΔV_{B_2} .

$$\Delta V_{\text{battery}} = 4.34 \text{ V}$$

$$\Delta V_{B_1} = 1.84 \text{ V}$$

$$\Delta V_{B_2} = 1.83 \text{ V}$$

Since energy is conserved the potential difference of the bulbs should be the same as that of the battery, but it is not. We need to take ΔV of the wires and switch into account as well.

Potential difference (ΔV) is measured in volts (V), and is related to energy. In Ye Olde Times, it is denoted V or emf (electromotive force), and ΔV_{bat} is denoted ε .

Flow rate/current (I) measures the number of electrons that pass a certain point per unit of time, and is measured in amperes (A).

Resistance (R) is how difficult it is for electrons to flow through and is measured in ohms (Ω).

The current is equal across the circuit.

Ohm's Law

$$\Delta V = IR$$

where:

ΔV = potential difference (V)

I = current (A)

R = resistance (Ω)

Resistors convert electrical energy to thermal energy. Light bulbs are a special case of resistor that convert electrical energy to thermal and light energy.

Resistors in series

When resistors are connected in series, current is equal across any part of the circuit:

$$I_{\text{bat}} = I_1 = I_2 = I_3$$

Assuming no potential is lost anywhere:

$$\Delta V_{\text{bat}} = \Delta V_1 + \Delta V_2 + \Delta V_3$$

$$R_{\text{total}} = R_1 + R_2 + R_3$$

Resistance

$$R = \frac{\rho l}{A}$$

where ρ is the resistivity of the material ($\Omega \text{ m}$), l is length (m) and A is cross-sectional area (m^2).

A **junction/node** is a point where three or more wires are connected.

A **branch** connects two nodes.

To connect a parallel circuit, multiple junctions need to be made.

Resistors in parallel

The currents are different at different parts of the circuit:

$$\begin{aligned}I_{\text{bat}} &= I_1 + I_2 + I_3 \\ \Delta V_{\text{bat}} &= \Delta V_1 = \Delta V_2 = \Delta V_3 \\ \frac{1}{R_{\text{total}}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\end{aligned}$$

Similar to springs in parallel.

A shortcut: if the total equivalent resistance of a parallel circuit is R_T and the current entering the circuit is I_T , the current through branch i is. Since the circuit is parallel R_T is less than the individual R_i 's.

$$I_i = I_T \cdot \frac{\frac{1}{R_i}}{\sum_k \frac{1}{R_k}} = I_T \cdot \frac{\frac{1}{R_i}}{\frac{1}{R_T}} = I_T \cdot \frac{R_T}{R_i}$$

From $I = \Delta V/R$ and $\Delta V_{\text{bat}} = \Delta V_1 = \Delta V_2 = \Delta V_3$:

$$I_{\text{bat}} = I_1 + I_2 + I_3 \implies \frac{\Delta V_{\text{bat}}}{R_{\text{total}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} + \frac{\Delta V_3}{R_3} \implies \boxed{\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Electron Sea Model

In metals, atoms form metallic bonds: each metal atom releases its valence electrons, which then form a sea of delocalized electrons. These electrons are then attracted to multiple atoms. These bonds are very strong and lead to high melting points for metals.

Delocalized electrons can move in a direction caused by an electric force.

Electrons flow from the negative terminal to the positive terminal. Conventional current is the direction in which positive charges flow, and is opposite to the direction electrons flow. The direction electrons flow is marked e^- and conventional current is marked I . The “long” end of the battery symbol is positive.

Ohmmeters are connected when there are no charges flowing; voltmeter and ammeter need the circuit to be active. Ammeter is connected in a circuit; voltmeter and ohmmeter measure across a circuit component (with a multimeter). In reality, these devices will modify the circuit when they are attached. An ammeter actually has a very small resistance, but we assume it has zero resistance. Also assume that a voltmeter has infinite resistance.

If two components are on different paths to the battery, they have the same potential difference (parallel).

A **combination circuit** is one which is not purely series or purely parallel. We can analyze such a circuit by calculating the total/equivalent resistance.

Power

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

where:

P = power (W)

I = current (A)

ΔV = potential difference (V)

R = resistance (Ω)

The sum of power in each resistors equals the power in the battery.

Resistors dissipate energy by converting electrical energy to thermal energy at a rate equal to $P = I\Delta V = \dots$

Light bulb ratings

Suppose a light bulb is rated at p W and v V. p relates to the brightness of the light bulb. From the formula $P = (\Delta V)^2/R$, we can determine the resistance of the light bulb: $R = v^2/p$.

Light bulb assumptions:

- The brightness of a light bulb is directly proportional to the power.
- The resistance of a light bulb is constant.

Electric bills

The house is billed by energy used in kWh, which is actually a unit of energy.

$$1 \text{ kWh} = 3.6 \cdot 10^6 \text{ J}$$

In a short circuit, a path with small/virtually zero resistance forms, adding a parallel path. The overall/equivalent resistance drops even though ΔV_{bat} is constant, increasing the current and therefore the power. A circuit breaker/fuse will trip/melt and open the circuit when current is higher than a certain limit.

1.1 Internal Resistance

In an ideal battery, there is no resistance: $R = 0$. In practice, a battery contains chemical reactions to generate a potential difference/current, and as the products of the reaction increase, the internal resistance increase.

In a circuit diagram, a resistor is placed to either side of the symbol for a battery. Sometimes, a dotted line includes these components. The potential difference of the battery and its internal resistance is ΔV_T (V).

Potential difference and internal resistance of battery

$$\Delta V_T = \varepsilon - Ir$$

where:

ΔV_T = terminal potential difference (includes internal resistance) (V)

ε = potential difference of ideal battery (V)

I = current through battery (A)

r = internal resistance (Ω)

When batteries are connected in series, the ideal potential difference (ε) increases, but internal resistance also increases. The actual increase in terminal potential difference (ΔV_T) might be quite low, and a lot of thermal energy is produced.

If batteries are connected in parallel, there is no increase in potential difference. But the current in the branches where batteries are connected is reduced as batteries are added in parallel, which reduces internal resistance and ΔV due to internal resistance Ir . The battery also lasts longer.

Potential difference opposing each other: positive terminals are adjacent to each other. The battery with a lower potential difference is being “charged,” until the batteries have the same potential difference.

Kirchhoff's Laws

1. Junction/node law (law of conservation of charges): at any junction (place where 3 or more wires are connected together), the sum of all currents flowing into the junction equals the sum of all currents flowing out of the junction. Charges are conserved.
2. Loop law: around any loop in the circuit, the sum of all ΔV changes must equal zero. In conventional current: the battery produces a positive ΔV , resistors and light bulbs produce a negative ΔV . The sum of all components equals 0. Opposite of using flow of electrons as direction.

The **ground** is where potential is zero volts. It is an ideal infinite source or sink of charge that can absorb unlimited charge without changing its potential of zero. It has both positive and negative charges.

1.2 Ideal Assumptions

- An ideal wire has zero resistance.
- An ideal ammeter has zero resistance.
- An ideal voltmeter has infinite resistance.
- An ideal battery has zero internal resistance.
- An ideal resistor is ohmic: it follows Ohm's law in that potential difference (ΔV) is directly proportional to current (I), with the slope being resistance R .

A light bulb is not ohmic: its temperature will increase, increasing its resistance.

1.3 Charge

Current is the flow of charged particles, the charge (q, Q) being measured in coulombs (C). One coulomb is the amount of charge delivered by a current of one ampere per second.

$$\left[A = \frac{C}{s} \right] \iff [C = A \cdot s]$$

An electron has a charge of $-1.6 \cdot 10^{-19}$ C, and the charge of a proton has the same magnitude but opposite sign: $+1.6 \cdot 10^{-19}$ C.

Charge and current

$$I = \frac{Q}{t} = \frac{dq}{dt}$$

where:

I = current (A)

Q = charge (C)

t = time (s)

This is helpful in non-steady state circuits where current is not constant and changes over time.

2 Electrostatics

Coulomb's law

$$F_E = \frac{kq_1q_2}{r^2} \quad k = \frac{1}{4\pi\epsilon_0}$$

where:

F = electrostatic force (N)

k = Coulomb constant

$$= 8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$$

ϵ_0 = permittivity of free space (C^2/Nm^2)

$$= 8.85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$$

q_1, q_2 = charge (C)

r = distance between particles (m)

There are positive and negative charges. The magnitude of the force is:

$$\|F_E\| = \frac{k|q_1||q_2|}{r^2}$$

from which the direction of the force can be figured out by logic: opposite charges attract.

A **Van de Graaff generator** generates electrostatic energy by using a belt to transport electrons to a spherical dome. Electrons try to spread out as much as possible and are position on the dome's surface.

Constants (on equation sheet)

The mass of an electron is:

$$m_e = 9.11 \cdot 10^{-31} \text{ kg}$$

The elementary charge is the smallest possible charge, and is equal to the magnitude of the charge of a proton or electron.

$$Q_e = 1.60 \cdot 10^{-19} \text{ C}$$

The gravitational constant from Mechanics is:

$$G = 6.67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$$

SI prefixes: 10^{-3} milli (m), 10^{-6} micro (μ), 10^{-9} nano (n), 10^{-12} pico (p).

For two electrons spaced 1 millimeter apart, we can calculate $F_E = k |q_1| |q_2| / r^2$ and $F_G = G m_1 m_2 / r^2$. It turns out that F_E is about $4 \cdot 10^{42}$ times greater than F_G . So the gravitational force can often be ignored because it is so weak compared to electric force.

Electric force is a vector quantity, whose direction can be indicated with $+/-$ sign or with $\hat{i}, \hat{j}, \hat{k}$.

Electrostatics vs circuits

In electrostatics, charges are not moving (hence “static”). In a circuit, which for our purposes is direct current (DC), charges are flowing through the circuit and are not stationary.

2.1 Atomic Structure

Matter is made of atoms, ions and molecules, which are in turn made of protons (with positive charge), electrons (with negative charge) and neutrons (with no charge).

The mass of an electron is significantly lower than the mass of a proton or neutron:

$$(m_e = 9.11 \cdot 10^{-31} \text{ kg}) \ll m_p = m_n = 1.67 \cdot 10^{-27} \text{ kg}$$

Charge is quantized: the magnitude of charge on a proton or electron is the elementary charge:

$$|Q_e| = |Q_p| = 1.6 \cdot 10^{-19} \text{ C}$$

There is no unit of charge smaller than the elementary charge, and all possible charges is a multiple of it:

$$|Q| = n (1.6 \cdot 10^{-19}) \text{ C} \quad n \in \mathbb{Z}^+$$

An object being “charged” indicates the sign of the net charge, which is either positive or negative.

- Positively charged: the object is deficient of electrons (has less electrons than protons).
- Negatively charged: the object is in excess of electrons (has more electrons than protons).

The mass of excess electrons in a negatively charged object or the mass of electrons removed in a positively charged object is negligible, because the mass of an electron is so low.

Charges on a metal/conductive sphere (solid or hollow) will always be on the surface of the sphere, because charges tend to maximize distance between each other. Thus, the net charge is always on the surface.

In an insulator, there is no free flow of charges. Charges are stationary on the surface of the object.

2.2 Polarization

Polarization is a separation of positive and negative charges within an object. This is done by shifting the statistical distribution of electrons to one side of an object (as explained by quantum mechanics, electron orbitals, etc.). It does not create charge, and will still have the same net charge as before.

A charged object can induce polarization in nearby neutral objects. Once the neutral object is polarized, like charges repel and are now further apart, meaning opposite charges are now closer together. Therefore, the two objects will attract.

In conductors, electrons move more readily. Polarization results in a side with excess electrons (negative) and a side with a deficit in electrons (positive).

In insulators, electrons are not free to move, and polarization occurs by changing the shape of electron clouds/orbitals. The electron cloud becomes asymmetrically shaped, and the “center of electrons/charge” is not at the atomic nucleus. The individual atoms become polarized.

Polar molecules (have a dipole moment) can orient themselves in an electric field due to attraction.

Polarization explains a number of real-world occurrences, such as:

- Balloons stick to a wall because the balloon is negative charged (has high affinity for electrons), and the wall molecules polarize.
- Water streams bend because water molecules reorient toward a charged object.
- Small pieces of paper jump toward a balloon because the negatively charged balloon surface polarizes it.

2.3 Charging

Charging by friction. Different objects (insulators) have a different tendency to gain or lose net charge when rubbed with another object. Some will gain electrons while some will lose electrons.

- Glass and human hair will lose electrons and become more positive.
- Nylon and wood will gain electrons and become more negative.

Charging by conduction. This can be used for both insulators and conductors. It involves the movement of charges to/from a neutral conductor from/to a charged object by contact.

Charging by induction.

1. Start with a charged object (“rod”) and a neutral conductor (“sphere”).

2. When the rod is brought near (not necessarily contacting) the sphere, free electrons will rush towards/away from the rod. The sphere becomes polarized.
3. Connect the sphere to ground.
4. Electrons from ground will rush towards the sphere, and displace the positively charged particles.

OR

Electrons from the sphere will rush towards ground, leaving only positively charged particles.

5. When the ground wire is removed from the sphere and the rod is moved away, the sphere remains charged.

This can be used to charge the sphere with either positive charge or negative charge. It would depend on the charge of the rod we start with.

- Negatively charged rod → positively charged sphere.
- Positively charged rod → negatively charged sphere.

Electroscopes. An electroscope is a device used to detect charge. It consists of a large model knob connected via a stem to two metal leaves. If a charged object is brought near the knob, electrons will move towards/away from the leaves, causing them to move upwards (since the leaves become charged and repel each other).

2.4 Field Model

The field model is related to non-contact forces, which are gravity, electric force and magnetic force. The field model describes the alteration of space.

Gravity

In a gravitational field, all masses are attracted to the center (of mass M) by a force given by the law of universal gravitation:

$$F_G = \frac{GMm}{r^2}$$

The gravitational field strength g is:

$$g = \frac{GM}{r^2}$$

and has units $\text{m/s}^2 = \text{N/kg}$. g is a vector quantity depending on the distance vector r and the location of the test mass. The force exerted on a mass m is:

$$F = gm \implies \Sigma F = ma$$

Field lines all point towards the mass M . The field is radially symmetric and the field lines point radially inward. On the Earth's surface, field lines point down and is uniform at $g = 9.81 \text{ m/s}^2$.

Electricity

In an electric field, charges are either attracted to or repelled from the central charge (Q), given by:

$$F_E = \frac{kQq}{r^2}$$

The electric field strength E is:

$$E = \frac{kQ}{r^2}$$

and has units N/C . E is a vector quantity depending on the distance vector r and the location of the test charge. The force exerted on a charge q is:

$$\Sigma F = Eq$$

Field lines are more complex because there are both attractive and repulsive force. The direction of field lines are the direction of F_E on a positive charge. When close to Q , the field lines are approximately straight and are uniform.

2.5 Electric Fields

A **capacitor** is made of two plates with opposite charge. If the distance between the plates is small, it can be thought of as being uniform.

A field is **uniform** if the electric force on a test charge is the same (in both direction and magnitude) at all points in space. A field is **nonuniform** if it is not.

Properties of electric fields:

- Electric field is always a vector quantity around the source charge (Q). At any location, the direction of the electric field is tangent to field lines. Field lines can be curved.
- Field lines are perpendicular to conducting surfaces.
- Field lines point in the direction of a force on a positive
- The density/spacing of field lines indicate the relative strength of the field. Denser lines/lower distance between lines indicate a stronger electric field. If the strength of the electric field $E = kQ/r^2$ doubles, the number of lines doubles.
- Field lines do not cross each other, as this would indicate two different electric field values/forces at the same point (the electric field is not a function).
- Electric field strength E is a vector quantity.

Electric field strength

The strength of an electric field created by a point charge with charge Q from a certain distance r is:

$$\|E\| = \frac{k|Q|}{r^2}$$

The units for electric field strength E is either newton per coulomb (N/C) or volt per meter (V/m). This is not dependent on the strength of any test charge. The force acting on a test charge of charge q is:

$$\|F_E\| = E|q| = \frac{k|Q||q|}{r^2}$$

In a uniform gravitational field, change in potential energy is:

$$\Delta U_g = mg\Delta h$$

where g is the gravitational field strength, which is 9.81 m/s^2 on the surface of the Earth.

Gravitational potential

Gravitational potential is gravitational potential energy per unit mass:

$$V = \frac{U_g}{m} = \frac{mgh}{m} = gh$$

and is measured m^2/s^2 or J/kg.

This concept can be similarly used for electric potential.

Electric potential and potential energy, uniform field

In a uniform electric field (between two charged plates) of strength E , the electric potential energy U_E of a charge q at a distance x to the plate of opposite charge from q is:

$$\Delta U_E = qEx$$

Electric potential is potential energy per unit charge:

$$V = \frac{U_E}{q} = \frac{qEx}{q} = Ex = \frac{kQ}{x}$$

Change in potential energy associated with a change in potential is:

$$|\Delta U_E| = \Delta |q \underbrace{Ex}_V| = \Delta |qV| = |q| |\Delta V|$$

Equipotential lines/surfaces are lines/surfaces on which electric potential is constant. They are described in terms of the electric potential on all points on the line/surface measured in volts (V). They behave similarly to contours.

- Potential is highest when close to a charge, and decreases as distance r, x increases.
- Positive point charges have positive potential and negative point charges have negative potential. For both positive and negative charges, electric potential at infinite distance is 0 V.
- Positive charges move from high to low potential values.
- Electric field lines/surfaces are perpendicular to equipotential lines.
- Field strength is inversely proportional to the spacing between field lines.
- Electric field is a vector quantity, and electric potential is a scalar quantity.

If a point P is affected by multiple charges, the electric potential at P is the sum of the electric potential created by each charge:

$$V_i = \frac{k|Q_i|}{x_i} \quad V_P = \sum_i V_i$$

Electric potential is measured in volts (V). From this equation:

$$\left[V = \frac{\text{N/C}}{\text{m}} = \frac{\text{Jm}}{\text{Cm}} = \frac{\text{J}}{\text{C}} \right]$$

If the potential difference ΔV is known, change in electric potential energy ΔU_E can be found. Kinetic energy K can also be found: since energy is conserved, $\Delta K = -\Delta U_E$.