Question 3 cont...

③ $f(x+\Delta x) = f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2} f''(x) + \frac{\Delta x^3}{3!} f'''(x) + O(\Delta x^4)$

$f(x-\Delta x) = f(x) - \Delta x f'(x) + \frac{\Delta x^2}{2} f''(x) - \frac{\Delta x^3}{3!} f'''(x) + O(\Delta x^4)$

$$\frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = \frac{2\Delta x f'(x) + 2\frac{\Delta x^3}{3!} f'''(x) + \dots}{2\Delta x}$$

$$= f'(x) + \frac{\Delta x^2}{3!} f'''(x) + O(\Delta x^4)$$

$$\varepsilon_d = \left| \frac{f'(x) - f_n'(x)}{f'(x)} \right|$$

$$= \left| \frac{f'(x) - f'(x) - \frac{\Delta x^2}{3!} f'''(x)}{f'(x)} \right|$$

$$= \left| \frac{-\frac{\Delta x^2}{3!} f'''(x)}{f'(x)} \right| = \frac{\Delta x^2}{6} \frac{|f'''(x)|}{|f'(x)|}$$

this minus is included in X_2

$$f_n'(x) = \frac{f(x+\Delta x)}{2\Delta x} - \frac{f(x-\Delta x)}{2\Delta x}$$

$\downarrow$ X_1 $\qquad$ $\downarrow$ X_2

$$\varepsilon_p = \frac{|X_1|}{|X_1 + X_2|}\varepsilon_1 + \frac{|X_2|}{|X_1 + X_2|}\varepsilon_2$$

but each has the same machine accuracy.

$\Rightarrow \varepsilon_1 = \varepsilon_2 = \varepsilon$