Task 2 cont...

b) leading error term:

$$\left(A\frac{\Delta x^4}{4!} + 0 + C\frac{\Delta x^4}{4!} + D\frac{16\Delta x^4}{4!}\right)u''''(x)$$

$$= \frac{\Delta x^4}{6\Delta x^2 \times 4!}\left(-2 + 6 - 16\right)u''''(x)$$

$$= \frac{-12\Delta x^3}{6 \times 4!} = \frac{-\Delta x^3}{12}$$

order 3.

Task 3

$$\boxed{k_0 = \lambda u^n}$$

$$\boxed{k_1 = \lambda\left(u^n + \frac{\Delta t}{2}(\lambda u^n)\right) = u^n\lambda\left(1 + \frac{\lambda\Delta t}{2}\right) = \underline{u^n\lambda\left(1 + \frac{z}{2}\right)}} \quad \overset{z}{\swarrow}$$

$$\boxed{k_2 = \lambda\left(u^n + \frac{\Delta t}{2}\left(u^n\lambda\left(1 + \frac{z}{2}\right)\right)\right)}$$

$$= \lambda u^n\left(1 + \frac{z}{2}\left(1 + \frac{z}{2}\right)\right) = \underline{\lambda u^n\left(1 + \frac{z}{2} + \frac{z^2}{4}\right)}$$

$$\boxed{k_3 = \lambda\left(u^n + \Delta t\left(\lambda u^n\left(1 + \frac{z}{2} + \frac{z^2}{4}\right)\right)\right)}$$

$$= \lambda u^n\left(1 + z + \frac{z^2}{2} + \frac{z^3}{4}\right)$$

~~u''''=u''=...~~

$$u^n = u^n + \frac{\Delta t}{6}\left[u^n\lambda\left(1 + \frac{z}{2}\right)\times 2 + \lambda u^n\left(1 + \frac{z}{2} + \frac{z^2}{4}\right)\times 2 + \cancel{2\lambda} \lambda u^n\left(1 + z + \frac{z^2}{2} + \frac{z^3}{4}\right) + \lambda u^n\right]$$

~~u^{n+1} = u^n(1+z+\frac{z^2}{2})~~

$$u^{n+1} = u^n\left[1 + \frac{z}{6}\left(1 + 2 + z + 2 + z + \frac{z^2}{2} + 1 + z + \frac{z^2}{2} + \frac{z^3}{4}\right)\right]$$

$$\boxed{u^{n+1} = u^n\left[1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24}\right]} \quad \text{see figure 3.}$$