Task 1:

$$\frac{du}{dt} = \lambda u \qquad u(0) = 1 \qquad \lambda \in \mathbb{C}$$

$$t \in [0, T]$$

$$u = e^{\lambda t} + C$$

but $u(0) = 1 \Rightarrow C = 0$

$$\boxed{u = e^{\lambda t}}$$

② (see figure 1)

③ EE: $u^{n+1} - u^n = \Delta t \lambda u^n$

$$\boxed{u^{n+1} = (1 + \lambda \Delta t) u^n} \rightarrow \boxed{G(z) = 1 + z} \quad \lim_{z \to \infty} G(z) \to \infty$$

IE: $$\boxed{u^{n+1} = \frac{1}{1 - \lambda \Delta t} u^n} \rightarrow \boxed{G(z) = \frac{1}{1 - z}} \quad \lim_{z \to \infty} G(z) \to 0$$

CN: $u^{n+1} - u^n = \frac{1}{2} \Delta t \left[\lambda u^{n+1} + \lambda u^n \right]$

$$u^{n+1} = u^n \left(\frac{1 + \lambda \Delta t \frac{1}{2}}{1 - \lambda \Delta t \frac{1}{2}} \right)$$

$$\boxed{u^{n+1} = \left(\frac{2 + \lambda \Delta t}{2 - \lambda \Delta t} \right) u^n} \rightarrow \boxed{G(z) = \frac{2 + z}{2 - z}} \quad \lim_{z \to \infty} \left(G(z) \right) = \frac{\frac{2}{z} + \frac{z}{z}}{\frac{2}{z} - \frac{z}{z}} \to 1$$

Analytic: $$\boxed{G(z) = e^z}$$

(see figure 2)