Question 3 cont...

$$\mathcal{L}_g = \mathcal{L}_d + \mathcal{L}_p = \frac{\Delta x^2 |f'''(x)|}{6|f'(x)|} + \frac{|f(x)|\varepsilon}{|f'(x)|\Delta x}$$

Want to find min:

$$\frac{\partial \mathcal{L}_g}{\partial \Delta x} = \frac{\Delta x |f'''(x)|}{3|f'(x)|} - \frac{|f(x)|\varepsilon}{|f'(x)|\Delta x^2}$$

solve $\frac{\partial \mathcal{L}_g}{\partial \Delta x} = 0$

$$\Rightarrow \Delta x^3 = \frac{|f(x)|\varepsilon \times 3|f'(x)|}{|f'(x)| \times |f'''(x)|}$$

$$\Delta x^* = \sqrt[3]{\frac{|f(x)|}{|f'''(x)|}} \times 3\varepsilon$$

now $\frac{\partial^2 \mathcal{L}_g}{\partial \Delta x^2} = \frac{|f'''(x)|}{3|f'(x)|} + \frac{2|f(x)|\varepsilon}{|f'(x)|\Delta x^3} > 0 \quad \forall \Delta x, \varepsilon > 0$

$\Rightarrow$ min!