Task 2:

let $A, B, C, D \in \mathbb{R}$ constants.

$$Au(x-\Delta x) = Au(x) - A\Delta x u'(x) + A\frac{\Delta x^2}{2}u''(x) - A\frac{\Delta x^3}{3!}u'''(x) + A\frac{\Delta x^4}{4!}u''''(x)$$

$$Bu(x) = Bu(x)$$

$$Cu(x+\Delta x) = Cu(x) + C\Delta x u'(x) + C\frac{\Delta x^2}{2}u''(x) + C\frac{\Delta x^3}{3!}u'''(x) + C\frac{\Delta x^4}{4!}u''''(x)$$

$$Du(x+2\Delta x) = Du(x) + D(2\Delta x)u'(x) + D\frac{4\Delta x^2}{2}u''(x) + D\frac{8\Delta x^3}{3!}u'''(x) + D\frac{16\Delta x^4}{4!}u''''(x)$$

we want $Au(x-\Delta x) + Bu(x) + Cu(x+\Delta x) + D(x+2\Delta x)$

such that the following cancel: ① $u(x)=0$, ② $u'(x)$ remains!
③ $u''(x)=0$
④ $u'''(x)=0$

① $A+B+C+D=0$
② $-A\Delta x + C\Delta x + 2D\Delta x = 1$
③ $A\frac{\Delta x^2}{2} + C\frac{\Delta x^2}{2} + D\frac{4\Delta x^2}{2} = 0$
④ $-A\frac{\Delta x^3}{3!} + C\frac{\Delta x^3}{3!} + D\frac{8\Delta x^3}{3!} = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 4 \\ -1 & 0 & 1 & 8 \end{bmatrix}\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 1/\Delta x \\ 0 \\ 0 \end{bmatrix}$$

Matlab $\rightarrow$ $$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \frac{1}{\Delta x}\begin{bmatrix} -1/3 \\ -1/2 \\ 1 \\ -1/6 \end{bmatrix}$$

$$\Rightarrow u'(x) = \frac{1}{6\Delta x}\left(-2u(x-\Delta x) - 3u(x) + 6u(x+\Delta x) - u(x+2\Delta x)\right)$$