Question 3 cont...

Note: $|X_1 + X_2| = \frac{1}{2\Delta x}\left|\left[2\Delta x f'(x) + \frac{2\Delta x^3}{3!}f'''(x) + \ldots\right]\right|$ as before

$$|X_1| + |X_2| = \frac{1}{2\Delta x}\left[\left|2f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2}f''(x) + \ldots\right| + \left|f(x) - \Delta x f'(x) + \frac{\Delta x^2}{2}f''(x) + \ldots\right|\right]$$

since Δx is small we only take leading terms

$$|X_1 + X_2| \simeq \frac{1}{2\Delta x}\left|2\Delta x f'(x)\right| = |f'(x)|$$

$$|X_1| + |X_2| \simeq \frac{1}{2\Delta x}\left[|f(x)| + |f(x)|\right] = \frac{|f(x)|}{\Delta x}$$

now $\mathcal{L}_p = \dfrac{\varepsilon\left[|X_1| + |X_2|\right]}{|X_1 + X_2|}$ (as $\varepsilon_1 = \varepsilon_2 = \varepsilon$)

$$\simeq \frac{\varepsilon\left[\frac{|f(x)|}{\Delta x}\right]}{|f'(x)|} = \frac{|f(x)|\varepsilon}{|f'(x)|\Delta x}$$