MSAS – Assignment #2: Modeling

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1 Questions

Question 1

1) List the stages of dynamic investigation and their meaning. 2) When going from the *real system* to the *physical model* a number of assumptions are made; report the most important ones along with their mathematical implications. 3) For each of the assumptions below, shortly state what sort of simplification may result: i) The gravity torque on a pendulum is taken proportional to the pendulum angle θ; ii) Only wind forces and gravity are assumed in studying the motion of an aircraft; iii) A temperature sensor is assumed to report the temperature exactly; iv) The pressure in a hydraulic actuator is assumed uniform throughout the chamber. 4) List the *effort* and *flow* variables for the domains treated and discuss their similarity.

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1) The dynamic investigation defines the steps to follow in order to analyze a real system, and it's made of few main stages:

- Modeling and simulation: Whenever a complex real system must be analyzed, i.e. a system which interacts with the real word, it's possible to define a simple physical model of it, which necessarily implies a lot of approximations. Then, the set of equations describing it, must be written down, so to determine its mathematical model as well, out of which the system response is obtained.
- Synthesis: At this point, the behaviour of the system can suggest how to adjust the model, by making design decisions, in order to let the system behave as desired.

2) Passing from the real system to the physical model, results, as said, in some approximations, each of what has a meaning behind its choice and produces some effects either on the physical model and at mathematical level. The most important ones are then listed below:

- Neglecting small effects: it simplifies the model, which will only take into account the very relevant effects which are meaningful in terms of response. Avoiding these effects, will lead to a decrease in the number of variables and equations.
- Independent environment: it's based on the assumption that the system under study, does not modify the environment. Also in this case, the numbers of variables and equations decreases, but it must be applied on a relative basis.
- Lumping: This approximation leads to a completely different structure of the mathematical model. In fact, by considering lumped physical elements instead of distributed ones, as it is in the real world, the equations to solve are no more partial differential equations, but ordinary differential equations, which are clearly easier to solve.
- Linearity: It's possible to represent a non-linear system by linear differential equations, which are simpler, more general and have a clearer physical meaning. Also, allows to exploit a set of tools developed for linear systems such as transfer functions and so on.
- Constant parameters: With constant parameters, the system will be linear time invariant in case it's not linear.

- Neglecting uncertainty and noise: If they were considered, this would lead to stochastic differential equation. To recover them, instead of producing one system response, we reproduce a thousand responses, by changing the initial conditions or the parameters of the model. This allows to recover the stochasticity of the model, but using a deterministic approach made of ODEs, which is for example the Monte Carlo analysis, where the changing parameters are samples taken from a statistical distribution.

3) Here follows the simplifications for some assumptions:

1. The gravity torque on a pendulum can be assumed to be proportional to the pendulum angle θ, only when the angle is small. In fact when the angular displacement amplitude of the pendulum is large enough that the small angle approximation no longer holds,the equation of motion has a nonlinear form.

2. When the aircraft motion is under analysis, for sure it's necessary to consider wind forces and gravity because they provide the most relevant contribution to the body, but it's needed also to consider the environment in which the aircraft is moving, and since it flies inside the atmosphere of the earth, effects such as solar radiation pressure, gravity gradient or earth magnetic field would add a meaningless contribution to the overall forces and for this reason can be neglected.

3. Assuming that a temperature sensor is reporting the exact temperature implies that the sensitivity of the instrument is not being considered. As consequence, eventual random or systematic errors are set to zero, and the statistical analysis of the measurement is neglected, i.e. the noise is not taken into account.

4. Assuming the pressure uniform inside an idraulic component, means that some pressure losses are not considered and this may imply the presence of constant parameters such as viscosity.

4) An *effort variable* is something that drives the system to change, while the *flow variable* is something that changes as consequence of the effort variable.
By looking at the types of system treated in this course, for each of them, it's possible to identify what these variables correspond to.

System	**EFFORT VARIABLE**	**FLOW VARIABLE**
MECHANICAL	Force	Velocity
ELECTRICAL	Voltage	Current
FLUID	Pressure	Volume flow rate
THERMAL	Temperature	Heat flow

By modelling different kind of dynamic systems results in the same or similar differential equations. This is possible because they are all characterized by *Resistance*, *Capacitance* and *Inductance*. These systems handle energy in a similar way as well, and they are all bound thanks to the behaviour due to the effort and flow variables. In fact:

- Effort= RESISTANCE x Flow
- Flow = CAPACITANCE x $\frac{dEffort}{dt}$
- Effort = INDUCTANCE x $\frac{dFlow}{dt}$

Question 2

1) Derive from scratch the mathematical model for RC and RL circuits and express the system response in closed form. 2) Consider a real DC motor and a) sketch its physical model (list the assumptions made); b) derive its mathematical model; c) show how the motor constant depends on the physical parameters.

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1) RC and RL, are first order circuits, since inside them, only one dyamical element is present, whether its a capacitor or a resistor, and then, their behaviour is described by a first order differential equation. These two circuits, have dual properties.

-An *RC circuit* is composed by a resistor and a capacitor connected in series, where the first controls capacitor's charging and discharging, while the second is capable of storing energy. By supposing that the capacitor is charged at an initial instant t_0=0, and that the voltage $v_c(0)$ $\neq$0 is known, the voltage over time v_c is derived. By applying II Kirchoff's law to the loop, it's possible to write:

$$Ri(t) + v_c(t) = 0 \tag{1}$$

By substituting in Eq.1, the capacitor characteristic equation, namely i= C$\frac{dv_c}{dt}$, the following relation is obtained

$$RC\frac{dv_c}{dt} + v_c(t) = 0 \rightarrow \frac{dv_c}{dt} + \frac{1}{\tau}v_c(t) = 0 \tag{2}$$

where τ= RC, that assumes the name of **time constant**.

-An *RL circuit* is composed by a resistor and an inductor connected in series. By supposing that at an initial time t_0=0, the inductor is crossed by current and that at the initial instant it is known, so $i_L(0) \neq 0$, the current over time i_L(t) is derived. After applying Kirchoff's I law, it's possible to write:

$$\frac{v(t)}{R} + i_L(t) = 0 \tag{3}$$

By substituting in Eq.2, the inductor characteristic equation, namely v= L$\frac{di_L}{dt}$, the following relation is obtained

$$\frac{L}{R}\frac{di_L}{dt} + i_L(t) = 0 \rightarrow \frac{di_L}{dt} + \frac{1}{\tau}i_L(t) = 0 \tag{4}$$

where again, a **time constant** is found, now equal to $\tau = \frac{L}{R}$.

These circuits, are described by the **same differential equation**, which, in standard form, is expressed as shown in Eq.5.

$$\frac{dx(t)}{dt} + \frac{1}{\tau}x(t) = 0 \tag{5}$$

Eq.5 is a first order linear differential equation, with constant coefficients and omogeneous, x(t) unkown. Once x(0) is fixed, the solution is unique, and it's exponential as Eq.6 shows.

$$x(t) = Ke^{\alpha t} \tag{6}$$

To determine K and α constants, it's possible to substitute Eq.6 in Eq.5 to obtain Eq.7.

$$Ke^{\alpha t} = (\alpha + \frac{1}{\tau}) \tag{7}$$

so to get

$$x(t) = Ke^{-\frac{t}{\tau}} \tag{8}$$

Since at t= 0, x(0) = K, the solution is:

$$x(t) = x(0)e^{-\frac{t}{\tau}} \tag{9}$$

So, by going back to the circuits, the solutions are the following.

-RC solution : $v_c(\text{t}) = v_c(0)\ e^{-\frac{t}{RC}}$
-RL solution : $i_L(\text{t}) = i_L(0)\ e^{-\frac{tR}{L}}$

2) A direct current (DC) motor is a fairly simple electric motor that uses electricity and a magnetic field to produce torque, which causes it to turn. At its most simple, it requires two magnets of opposite polarity and an electric coil, which acts as an electromagnet. The repellent and attractive electromagnetic forces of the magnets provide the torque that causes the motor to turn. A DC motor requires at least one electromagnet, which switches the current flow as the motor turns, changing its polarity to keep it running. The other magnet or magnets can either be permanent magnets or other electromagnets.

a. The aim at this step, is to study the dynamic response of a DC motor when the voltage applied to its rotor windings is varying over time, while a variable load is applied to the output drive. So, the *input* to the system is voltage/current and the *output* is the angular velocity/torque. A reasonable sketch of the physical model of this system, is shown in Fig.1.

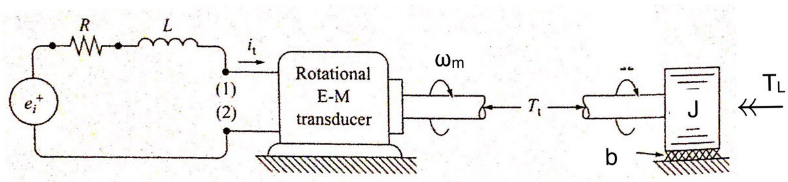

Figure 1: DC motor physical model.

The assumptions made are that the armature coil, is represented as a resistor and an inductance plus an additional voltage drop proportional to the angular velocity ω_m. Also it's considered that the DC motor drives a shaft with negligible small flexibility, the mechanical load is an inertia (disk) J, the bearings have linear friction b, and the disturbance load is T_L.

b. The mathematical model of this system, is derived by taking into account the combination of the electrical and mechanical elements. By applying the Kirchoff's laws and force equilibrium, the following system of equations is derived:

$$\begin{cases} e_1 - e_2 = Ri \\ L\frac{di}{dt} = e_2 - e_m \end{cases} \tag{10}$$

By substituting T_m = Ki and e_m = Kω_m in Eq.10, the final mathematical model for this system is obtained, and reported in Eq.11.

$$\begin{cases} L\frac{di}{dt} + Ri + K\omega_m = e_1 \\ J\dot{\omega_m} + b\omega_m - Ki = -T_L \end{cases} \tag{11}$$

The states of the system are the current in the equivalent circuit and the angular velocity of the driven shaft, so the state vector is made of x_1 and x_2:

$$\begin{cases} x_1 = i \\ x_2 = \omega_m \end{cases} \tag{12}$$

c. The motor constant K collects a series of physical properties characteristic of each motor, which are the number turns of the filament winding N, the length and radius (half width)of the coil l and r, and the magnetic field B. In fact K = 2NlBr.

Question 3

1) Write down the Fourier law and show how it is specialized in the case of conduction through a thin plate; discuss the concept of thermal resistance. 2) Report the equation for thermal radiation in case of a) black body and b) real body and discuss them.

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1) Fourier's law is one of the fundamental laws ruling heat transfer's phenomena, in which the matter of interest is the rapidity with which energy is transmitted. In particular, it regards conduction processes, which verify at molecular level between particles from a warm body to a colder one. It assumes different forms according to the geometry of the body, but in case of a thin plate it becomes

$$\dot{q} = -kA\frac{dT}{dx}[W] \tag{13}$$

where
k: thermal conductivity
A : cross-sectional area

The negative sign, is the translation of the physical meaning of the heat transmission from bodies with higher temperature to bodies with a lower one. Since it's possible to write the transmitted heat in terms similar to the electric field, as shown in Eq.2

$$\dot{q} = \frac{T_1 - T_2}{R}[W] \tag{14}$$

therefore,the thermal resistance of the material where the heat exchange happens, is introduced in Eq.3

$$R = \frac{L}{kA}\frac{[K]}{[W]} \tag{15}$$

Similar to how electrical resistance resists the flow of current in ohms, thermal resistance resists the flow of heat in Kelvins per watt, or in degrees Celsius per watt.The rate of heat transfer through a layer corresponds to the electric current, the thermal resistance corresponds to electrical resistance, and the temperature difference corresponds to voltage difference across the layer. The temperature difference is the potential or driving function for the heat flow, resulting in the Fourier equation being written in a form similar to Ohm's Law of Electrical Circuit Theory. We can use thermal resistance to estimate how hot a particular part might get under various loads based on how easily the heat is able to be transferred from one place to another. The R-value depends on the type of insulation, its thickness, and its density. An area and a temperature difference is required to solve for heat transferred.

2) Thermal radiation, is related to the propagation of energy as electromagnetic radiations, originated by charged particles. It happens though vacuum, it transmit heat faster since it's related to light speed, and it doesn't cross the matter of a body.

Case **a**: The black body, is a simplification of a real body, since it's a perfect emitter, since it emits uniformly towards every direction at the same intensity and at each wavelength and temperature, emits the maximum radiation possible with respect to a real body, and a perfect absorber of radiation, since it's able to soak the whole radiation impacting on it. The heat transmitted by a black body, takes the mathematical form of Eq.4

$$\dot{q} = A\sigma T^4[W] \tag{16}$$

where T: temperature of the body
A: surface area

σ: Stefan-Boltzmann constant, $\sigma = 5.67810[W/Cm^2]$

Case **b**: For a real body, the radiation is partly absorbed, so that it gains energy, partly transmitted and partly reflected. Also, it takes into account factors that are ideally neglected in the case of black bodies.
First, it has worse emission properties, thus the emissivity factor F_e ($0 \leq F_e \leq 1$) has to be accounted. Secondly, the radiation is always exchanged between two bodies at different temperatures and thus the net heat transfer between a body at higher temperature T_H and another one at lower temperature T_L has to be considered. Finally, the viewing factor F_v is added as a parameter of the lost radiation that is not impinging the other body. All these considerations,lead to

$$\dot{q} = F_e F_v \sigma A(T_H^4 - T_L^4)[W] \tag{17}$$

2 Exercises

Exercise 1

A miniaturized reaction wheel can be modeled as a couple of massive disks, connected with a flexible shaft (Figure 35). The first disk is driven by a rigid shaft, linked to an electric motor. The motor provides a given torque, while the rigid shaft is subjected to viscous friction, due to motor internal mechanisms. At $t_0 = 0$, the motor provides the torque $T(t_0) = T_0$.
1) Write down the mathematical model from first principles. 2) Using the data given in the figure caption, and guessing a value for the flexible shaft stiffness k and the viscous friction coefficient b, compute the system response from t_0 to $t_f = 10$ s. 3) Two accelerometers placed on the two disks recorded samples at 100 Hz, which were saved in the file `samples.txt`; the samples are affected by measurement noise. Determine the values of k and b that allow retracing the experimental data, so avoiding parametric errors.

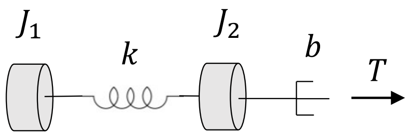

Figure 2: Physical model ($J_1 = 0.2\,\mathrm{kg{\cdot}m}$; $J_2 = 0.1\,\mathrm{kg{\cdot}m}$; $T_0 = 0.1\,\mathrm{Nm}$).

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1) Since the physical model of the reaction wheel under analysis is already provided, the first step of the dynamic investigation of this model, is already completed, so the following step is to determine a mathematical model from first principles, and then proceed with answering the other requests. The approach to follow, contemplate both linear and non-linear case,but in this analysis, the damper is considered as non linear.

Non-linear approximation approach: The aim of this analysis, is to obtain the response of the system, in terms of rotation, angular velocities and accelerations of both masses of the system, over time. Their motion, is due to a torque applied by an electric motor, connected to J_1 and J_2 through shafts. The effects to consider, are then the torque T, the flexible shaft stiffness, the rigid shaft viscous friction, and the inertia of the two masses, which are represented by resistance torques in opposition to the torque applied externally upon the system.
The set of equations needed for this kind of analysis, is made of two equations of motion, derived by considering the free-body diagram of each mass, and the actions of them, which is represented in Fig.3. The final system of equation obtained from the manipulation of the

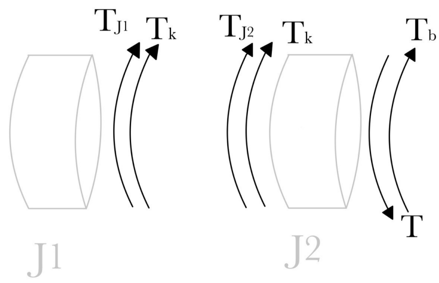

Figure 3: Free body diagram of the inertia of the systems.

equations, is shown in Eq.18

$$\begin{cases} \dot{\omega_1} = \frac{K}{J_1}\theta_2 - \frac{K}{J_1}\theta_1 \\ \dot{\omega_2} = \frac{T}{J_2} - \frac{K}{J_1}\theta_2 + \frac{K}{J_1}\theta_1 - sign(\omega_2)\frac{b}{J_2}{\omega_2}^2 \end{cases} \tag{18}$$

A system of first order linear differential equations, can be written in a state space representation, as expressed in Eq.19.

$$\begin{cases} x_1 = \omega_1 \\ x_2 = \omega_2 \\ x_3 = \theta_1 \\ x_4 = \theta_2 \end{cases} \tag{19}$$

The system then, has been integrated through ode45 MatLab solver, starting from the initial guessed values for the stiffness of the spring and the viscous friction parameters, respectively $k_{guess} = 1$ and $b_{guess} = 1$. The response of the system, is shown in Fig.4.

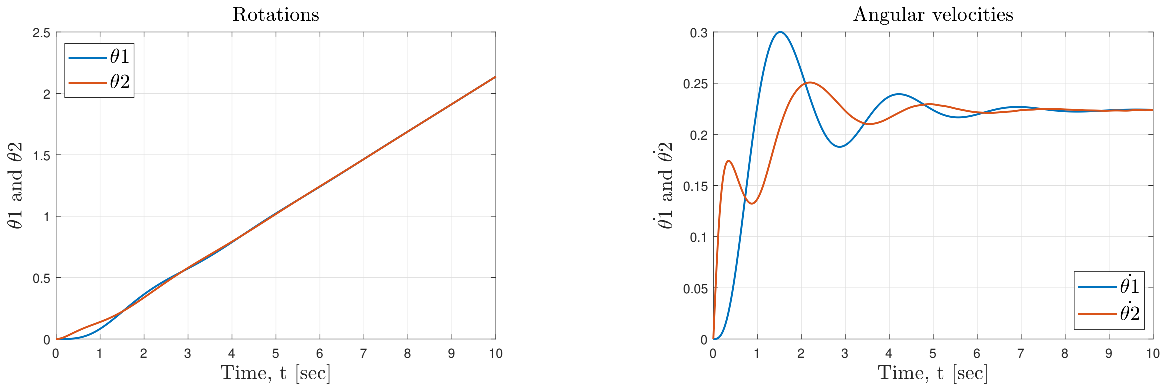

Figure 4: Responses of the systems with k and b guessed values.

Then, with the values obtained from the integration, the accelerations are computed and plotted together with the samples recorded at 100 Hz by the accelerometers, in order to verify the correctness of our dynamics, and the comparison, is shown in Fig.5, regarding J_1 and in Fig.6 for what concerns J_2. It's clear that the modelled data, aren't matching the sampled values, so an optimization to find the real values of k and b has been implemented.

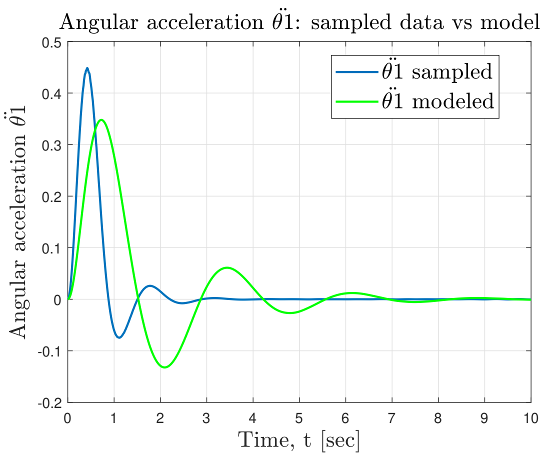

Figure 5: Responses of the systems over time, with k and b guessed values.

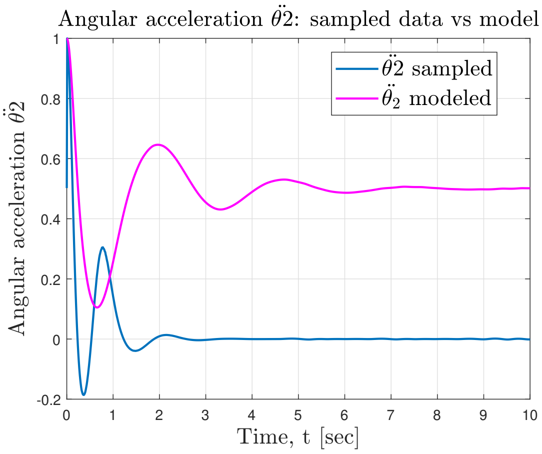

Figure 6: Responses of the systems with k and b guessed values.

The main goal now, is to find the values for these parameters, for which the difference between the accelerations computed at each instant though the model, and the sampled data, is as small as possible, and after computing the new k and b, to compute again the response of the system, which must match the real behaviour.
To do so, the *fminunc* MatLab tool is exploited.
It minimizes an objective function, which in this case is defined as the norm of the sum of the absolute errors e_1 and e_2, between the samples and the data processed by the model, at each instant.
Each error is made of the accelerations referred to both masses, so that $\mathbf{e_1} = ||\ddot{\boldsymbol{\theta}}_\mathbf{1}(t) - \mathbf{a_1}(t)||$ $[rad/s^2]$ and $\mathbf{e_2} = ||\ddot{\boldsymbol{\theta}}_\mathbf{2}(t) - \mathbf{a_2}(t)||$ $[rad/s^2]$. Fig.7 and Fig.8 are showing that, with the new values found after the optimization, namely k = 3.142 and b = 2.713, an optimal matching is obtained, which suggest that the model is correctly formulated. Clearly, even though it's fair result, it's not perfect as expected, since a margin of error is occurring. So Fig.9 shows the absolute error treated in the previous lines of this document, as function of time, referring to each mass. Indeed, it maintains a reasonable order of magnitude.

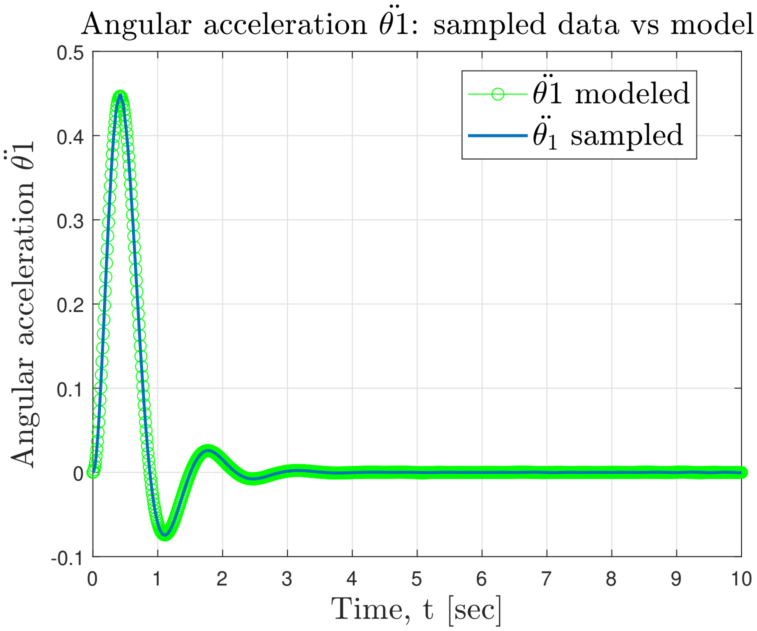

Figure 7: Responses of the systems over time, with k and b correct values.

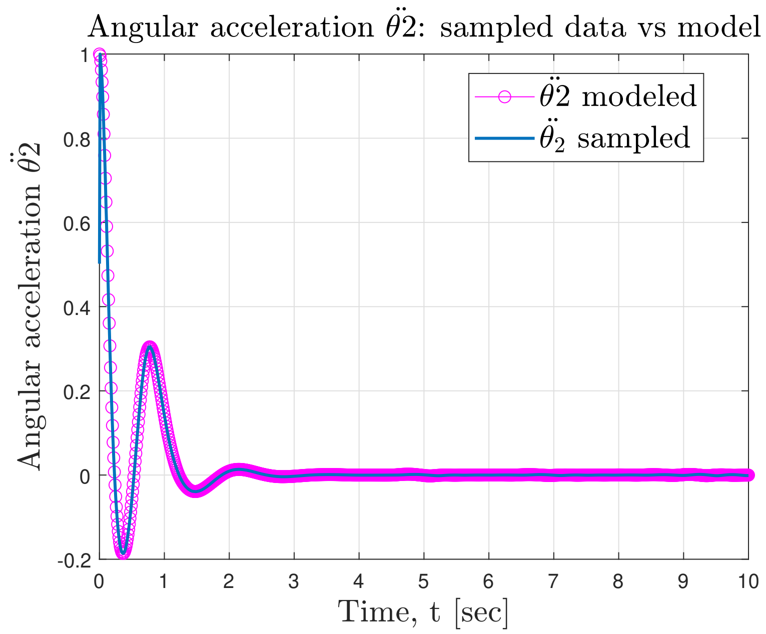

Figure 8: Responses of the systems with k and b correct values.

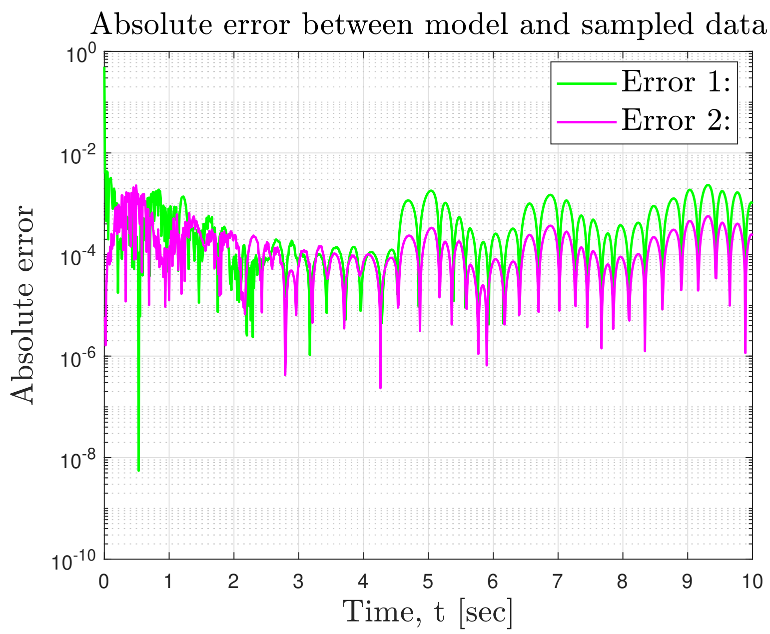

Figure 9: Responses of the systems with k and b correct values.

Exercise 2

Consider the ideal physical model network shown in Figure 10. The resistance R_2 varies its value with the value of the current, i.e., $R_2 = R_{2k} i$.The switch has been open for a long time. The capacitor is charged and has a voltage drop between its ends equal to 1 V. Then, at $t = 0$, the switch is closed. 1) Plot the subsequent time history of the voltage V_C across the capacitor.

2) Assume a voltage source characterized by $v(t) = \sin(2\pi f t)\arctan(t)$ having the positive terminal downward inserted in place of the switch. What is in this case the voltage history across the capacitor?

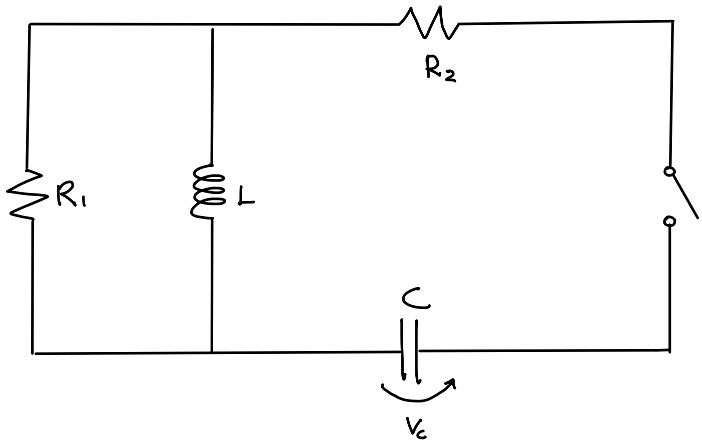

Figure 10: Circuit physical model ($R_1 = 100\,\Omega$; $R_{2k} = 10\,\Omega/\text{A}$; $L = 10\,\text{H}$; $C = 1\,\text{mF}$; $f = 5\,\text{Hz}$.)

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1) The physical model of the system provided by the text, shows an open RCL network. In fact, the circuit contains resistors, an inductor and a capacitor. The capacitor is charged and there is no current generator at this stage. This means that, once the switch is opened, the current will flow inside the circuit until the capacitor will be discharged, because some resistances are present. Then, the transient intercurring between the first instant of time, establishing the closure of the circuit, and the last, in which the current stops flowing, is studied. The fundamental circuit laws are applied to the system under analysis in order to formulate its mathematical model.
So, a brief recap of Kirchoff's I and II law is presented: Eq.20 (Kirchoff's junction rule) states that the algebraic sum of currents in a network of conductors, meeting at a point in zero; Eq.21 (Kirchoff's loop rule) states that the directed sum of the potential differences (voltages) around any closed loop is zero.

$$\sum_{k=1}^{n} i_k = 0 \tag{20}$$

$$\sum_{k=1}^{n} V_k = 0 \tag{21}$$

To apply them, and arbitrary direction is set for the current flowing inside each closed loop, as shown in Fig.11.

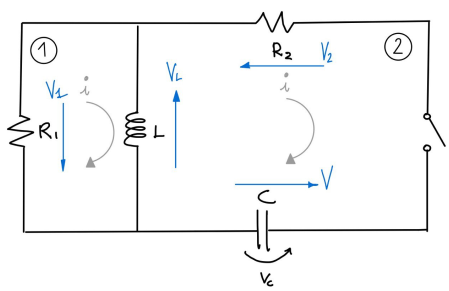

Figure 11: Circuit conventions chosen.

Recalling the constitutive equations of the elements of the circuit, namely $i_c = C\frac{dV_c}{dt}$ for what concerns the capacitor, $V_L = L\frac{di_L}{dt}$ for the inductor and $V_R = i_R R$ for the resistor, the basic system of equation is shown in Eq.22, coming out from the Kirchoff's laws, is manipulated in order to obtain Eq.23, containing the differential equation to integrate in MatLab to get the system response.

$$\begin{cases} i_R = i_C + i_L \\ V_1 = -V_L \\ V_L = V_2 + V_C \end{cases} \tag{22}$$

$$\begin{cases} \dot{x_1} = \frac{dV_C}{dt} \\ \dot{x_2} = \frac{d^2V_C}{dt^2} = \frac{\dot{V} + \frac{R_1 V}{L} + \frac{R_1 R_{2k} C^2 \dot{V}^2}{L}}{-R_1 C - 2R_{2k} C^2 \dot{V}} \end{cases} \tag{23}$$

The state vector is

$$\begin{cases} x_1 = V_C \\ x_2 = \frac{dV_C}{dt} \end{cases} \tag{24}$$

Therefore, the response of the system is evaluated analyzing the history of discharge of the voltage across the capacitor, starting from a value $V_0 = 1$ V, and it's shown in Fig.12.

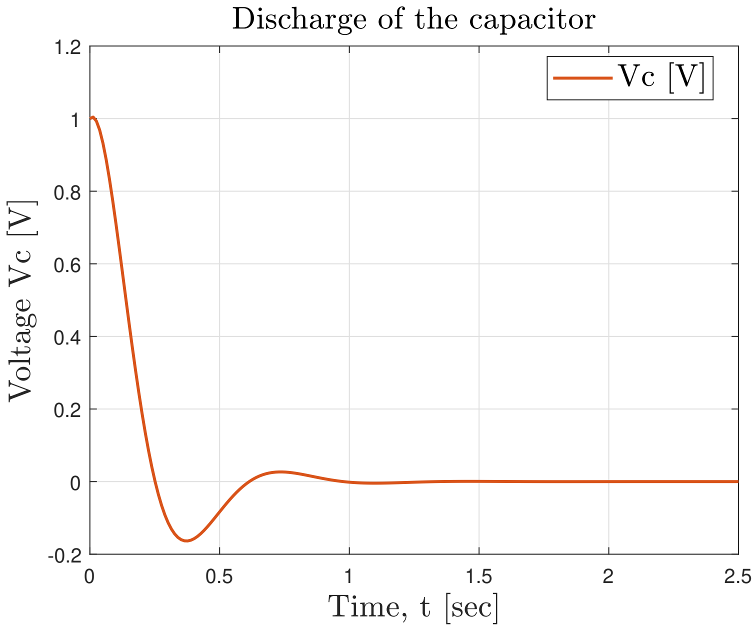

Figure 12: Transient analysis in presence of a discharge of a capacitor: switch case .

It's clear that the capacitor discharges, the voltage drops to zero.

2) Once a voltage source is introduced into the circuit, the conventions remain the one in Fig.11, but the system of equations coming from the Kirchoff's laws becomes:

$$\begin{cases} i_R = i_C + i_L \\ V_1 = -V_L \\ V_L = V_2 + V_C + v(t) \end{cases} \tag{25}$$

The additional term due to the presence of the generator, is found in the differential system of equations, which becomes Eq.26

$$\begin{cases} \dot{x_1} = \frac{dV_C}{dt} \\ \dot{x_2} = \frac{d^2V_C}{dt^2} = \frac{\dot{V} + \frac{R_1 V}{L} + \frac{R_1 R_{2k} C^2 \dot{V}^2}{L} + \frac{dv(t)}{dt}}{-R_1 C - 2R_{2k} C^2 \dot{V}} \end{cases} \tag{26}$$

The state vector is still depicted by Eq.24.
The updated response of the system, is pictured by Fig.13, where the voltage is now oscillating around zero, as the current is flowing on time thanks to the presence of the current generator.

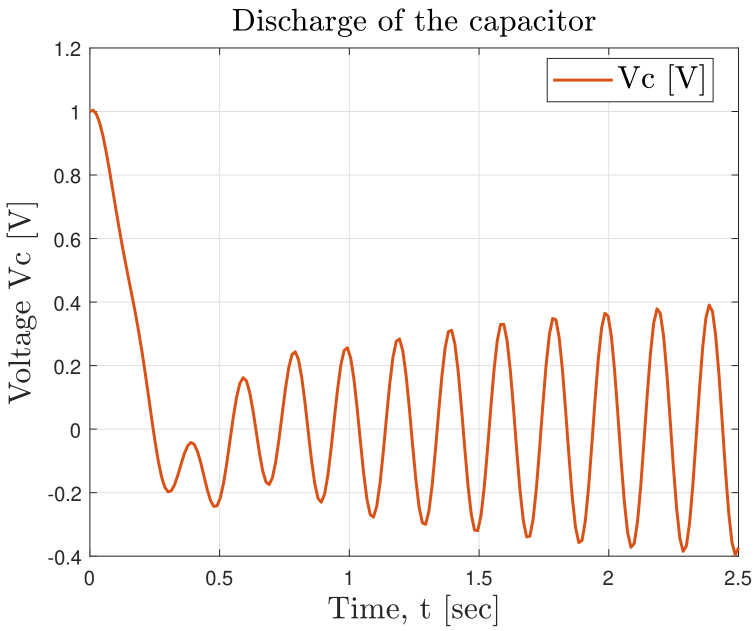

Figure 13: Transient analysis in presence of a discharge of a capacitor: generator case .

Exercise 3

The rocket engine in Figure 28a is fired in laboratory conditions. With reference to Figure 28a, the nozzle is made up of an inner lining (k_1), an inner layer having specific heat c_2 and high conductivity k_2, an insulating layer having specific heat c_4 and low conductivity k_4, and an outer coating (k_5). The interface between the conductor and the insulator layers has thermal conductivity k_3. 1) Select the materials of which the nozzle is made of[1], and therefore determine the values of k_i ($i = 1, \dots, 5$), c_2, and c_4. Assign also the values of ℓ_i (i=1,...,5), L, and A in Figure 28a. 2) Derive a physical model and the associated mathematical model using one node per each of the five layers and considering that only the conductor and insulator layers have thermal capacitance. The inner wall temperature, T_i, as well as the outer wall temperature, T_o, are assigned. 3) Using the mathematical model at point 2), carry out a dynamic simulation to show the temperature profiles across the different sections. At initial time, $T_i(t_0) = T_o(t) = 20$ C°. When the rocket is fired, $T_i(t) = 1000$ C°, $t \in [t_1, t_f]$, following a ramp profile in $[t_0, t_1]$. Integrate the system using $t_1 = 1$ s and $t_f = 60$ s. 4) Repeat the simulation in point 3) using a mathematical model implementing two nodes for the conductor and insulator layers.

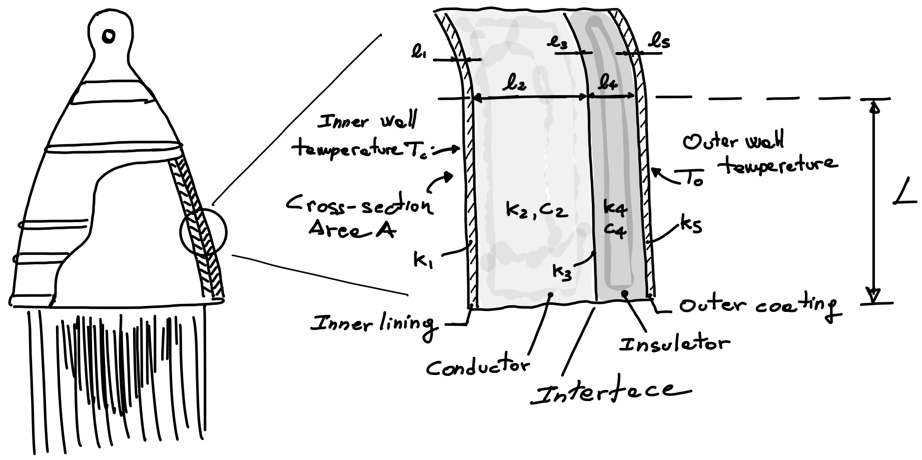

Figure 14: Real thermal system.

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1) Before starting the analysis of the heat transfer processes that happens through the layers of the nozzle treated, it's necessary to model the materials of which it must be made of, as well as its dimensions, to fully satisfy its role. In fact, in rocket designs, material selection for the components or structure of a launch vehicle is of paramount importance. In the selection of materials for the rocket, it is desirable to use a material that has high-to-weight ratio, good mechanical properties and ease of fabrication. Tab. below, gives an overview of the choices made for each of the five layers under analysis:

Layer	**Material**	**Length[m]**	**k[W/mk]**	**c[J/kgK]**	ρ**[kg/m^3]**
Inner lining	Silicon Nitride	0.004	30	-	-
Conductor	Molybdenum	0.05	138	250	10220
Interface	-	0.001	1	-	-
Insulator	Zirconium	0.01	1.7	420	5680
Outer Coating	Carbon composite	0.004	10	-	-

[1]The interface layer is not made of a physically existing material, though it produces a thermal resistance. For this layer, the value of the thermal resistance R_3 can be direcly assumed, so avoiding to choose k_3 and ℓ_3.

2) To build the physical model of this system, it's important to keep in mind the analogy between thermal and electrical systems. In fact, each material, with its characteristics, opposes a resistance to heat passage, which can be either conductive, or convective in case of fluids, as in electrical circuits it happens towards the current. Therefore, in this case, to each layer, corresponds one node, to which the resistances converge. For what concerns the second and fourth layer, a capacitance is considered as well, and it's sketched as a capacitor connected to the respective node. The physical model is shown in Fig.15.

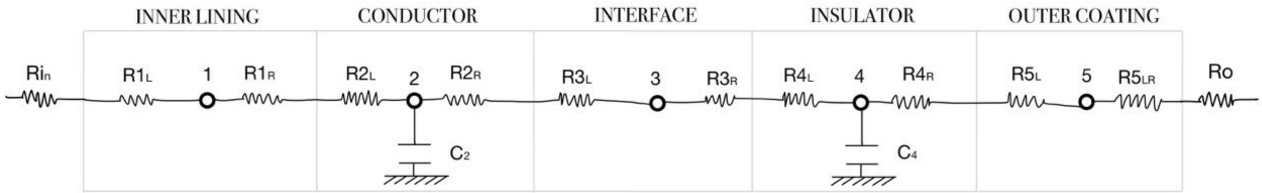


Figure 15: Physical model: One node for each layer case.

The temperatures inside the layers, considering the ability to store energy for the capacitive elements, will vary as function of time, as well as function of the position, since the heat is flowing from the left to the right side. The mathematical model here implemented, has the aim to track the temperature behaviour, and it's the outcome of an energy balance applied to each node.
Firstly, non capacitive elements are analyzed, and since the energy variation within time is null, namely $\frac{dU}{dt} = 0$, the heat flowing into the node at each instant, must be equal to the heat flowing out the node $\dot{q}_{in} = \dot{q}_{out}$, where the general expression for the heat flow between two points, can be expressed as shown in Eq.27

$$\dot{q}_{ab} = \frac{T_a - T_b}{R_{ab}} \tag{27}$$

In this system, only series resistors are present, so the equivalent resistance between nodes, is made of the sum of the resistances in the middle. The materials with which the nozzle is made, are characterized by conductive resistance, $R_{cond} = \frac{l}{kA}$, where 'l' is the length of the material considered, while the fluid which laps the external layers of the object, can be described by convective resistance, namely $R_{conv} = \frac{1}{hA}$. Therefore, the system of algebraic equations becomes:

$$\begin{cases} \dot{q}_{(1-i)} = \dot{q}_{(1-2)} \\ \\ \dot{q}_{(2-3)} = \dot{q}_{(3-4)} \\ \\ \dot{q}_{(4-5)} = \dot{q}_{(5-o)} \end{cases} \leftrightarrow \begin{cases} T_1 = \frac{T_i(R_{1R}+R_{2L})+T_2(t)(R_{in}+R_{1L})}{R_{in}\rightarrow R_{2L}} \\ \\ T_3 = \frac{T_i(R_{3R}+R_o)+T_o(t)(R_{in}+R_{3L})}{R_{in}\rightarrow R_o} \\ \\ T_5 = \frac{T_o(R_{3R}+R_{5L})+T_3(t)(R_{5R}+R_{Ro})}{R_{3R}\rightarrow R_o} \end{cases} \tag{28}$$

Now, moving on the capacitive materials, the energy variation within time can be expressed as $\frac{dU}{dt} = \dot{q}_{left} + \dot{q}_{right}$, and the two differential equations governing the temperature variation inside layer 2 and 4 are

$$\begin{cases} \frac{dT_2}{dt} = \frac{2}{\rho_1 l_2 c_2 A}\left(\frac{T_{in}-T_2(t)}{R_{in}\rightarrow R_{2L}} - \frac{T_2(t)-T_4(t)}{R_{2R}\rightarrow R_{4L}}\right) \\ \\ \frac{dT_4}{dt} = \frac{2}{\rho_1 l_4 c_4 A}\left(\frac{T_2(t)-T_4(t)}{R_{2R}\rightarrow R_{4L}} - \frac{T_4(t)-T_o}{R_{4R}\rightarrow R_o}\right) \end{cases} \tag{29}$$

that constitute the derivative of the state, namely

$$\begin{cases} x_2 = T_2 \\ x_4 = T_4 \end{cases} \tag{30}$$

All the equations have been written in terms of the inner and outer temperatures.
3) The system, has been integrated for 60 s, considering that from $t_0 = 0$ to t = 1, the inner temperature, follows an increment of it's value with a ramp profile, such that

$$T_i = T_i0 + (T_{[t=1s]} - T_i0)t \tag{31}$$

The resulting temperature profiles across the different layers are shown in Fig. 16 As expected, the first nodes reach higher temperatures as not only they are closer to the heat source, but mostly because the second layer is made of a conductor material. Then the temperature of the other nodes is smaller and smaller because of the insulation layer.

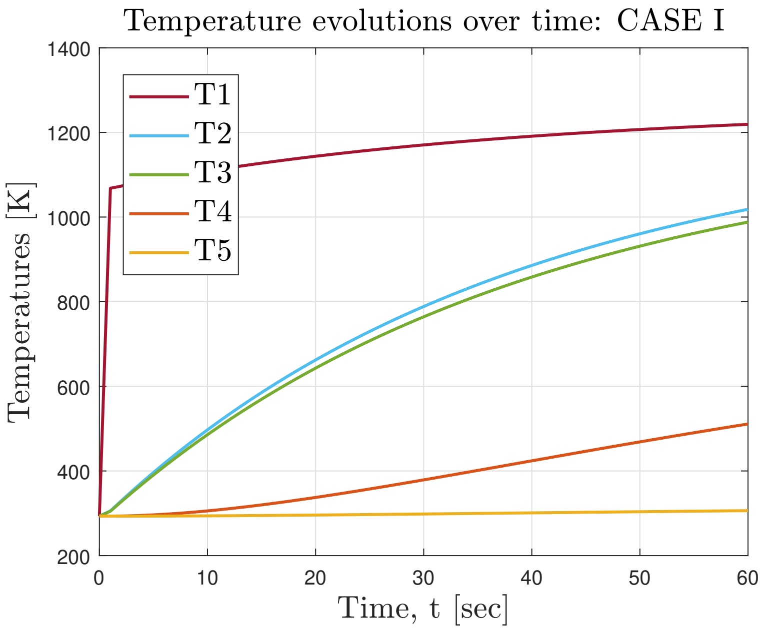


Figure 16: Temperature profiles: CASE I.

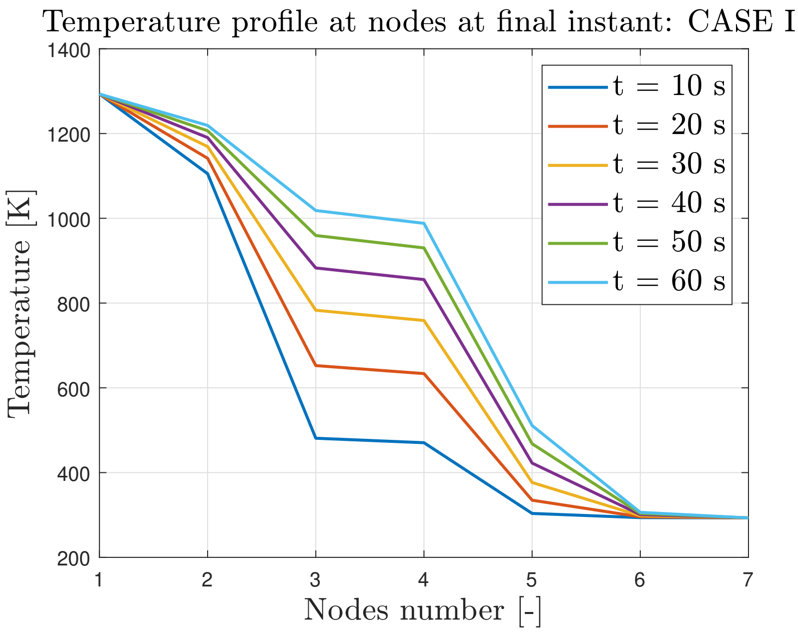


Figure 17: Temperature variation at interfaces between materials for different time instants.

It's possible to visualize that as time passes, the temperature at each node,intended as cross point between different materials, is increasing, and that it's higher in the nodes closer to the inner part of the nozzle. This behaviour is shown in Fig.17, and the temperatures are evaluated every 10 s.

4) At this point, to make the results of the integration more accurate, for the two capacitive elements, two nodes are considered, so the resultant system of equation undergoes some alterations. First of all, let's see what the new physical model looks like, by looking at Fig.18. Again, the system of equations describing the behaviour of this thermal system, comes from

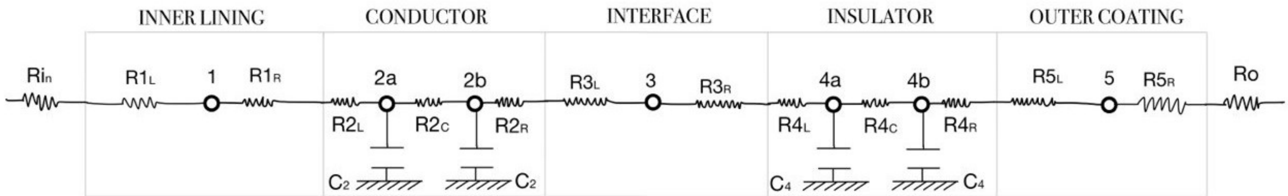


Figure 18: Physical model: Two nodes for layer 2 and 4 case.

energy balance considerations upon each node, in particular,for the non capacitive elements, the net balance of energy is zero, namely $\frac{dU}{dt} = 0$. Three algebraic equations are derived for these nodes and they are reported in Eq. 32 as follows.

$$\begin{cases} \dot{q}_{(1-i)} = \dot{q}_{(1-2a)} \\ \\ \dot{q}_{(2b-3)} = \dot{q}_{(3-4a)} \\ \\ \dot{q}_{(4b-5)} = \dot{q}_{(5-o)} \end{cases} \leftrightarrow \begin{cases} T_1 = \frac{T_i(R_{1R}+R_{2L})+T_{2a}(t)(R_{in})}{R_{in}\rightarrow R_{2L}} \\ \\ T_3 = \frac{T_{2b}(t)(R_{3R}+R_{4L})+T_{4a}(t)(R_{2R}+R_{3L})}{R_{2R}\rightarrow R_{4L}} \\ \\ T_5 = \frac{T_o(R_{4L}+R_{5L})+T_{4b}(t)(R_o)}{R_{4L}\rightarrow R_o} \end{cases} \tag{32}$$

Then, for each node inside layers 2 and 4, a differential system of equations is derived, considering they are still able to store energy, so $\frac{dU}{dt} = \dot{q}_{left} + \dot{q}_{right}$, therefore Eq.33 is showing what is integrated through MatLab *ode45* to have the temperature response in the case II, in which the equations become 4.

$$\begin{cases} \frac{dT_{2a}}{dt} = \frac{1}{\rho_2 l_2 c_2 A}\left(\frac{T_{i1}-T_{2a}(t)}{R_{in}\rightarrow R_{2L}} - \frac{T_{2a}(t)-T_{2b}(t)}{R_{2c}}\right) \\ \\ \frac{dT_{2b}}{dt} = \frac{1}{\rho_2 l_2 c_2 A}\left(\frac{T_{2a}(t)-T_{2b}(t)}{R_{2C}} - \frac{T_{2b}(t)-T_{4a}(t)}{R_{2R}\rightarrow R_{4L})}\right. \\ \\ \frac{dT_{4a}}{dt} = \frac{1}{\rho_4 l_4 c_4 A}\left(\frac{T_{2b}(t)-T_{4a}(t)}{R_{2R}\rightarrow R_{4L}} - \frac{T_{4a}(t)-T_{4b}(t)}{R_{4C}}\right) \\ \\ \frac{dT_{4b}}{dt} = \frac{1}{\rho_4 l_4 c_4 A}\left(\frac{T_{4a}(t)-T_{4b}(t)}{R_{4C}} - \frac{T_{4b}(t)-T_{5o}}{R_{4R}\rightarrow R_o}\right) \end{cases} \tag{33}$$

The state vector is instead reported in Eq.34.

$$\begin{cases} x_1 = T_{2a} \\ x_2 = T_{2b} \\ x_3 = T_{4a} \\ x_4 = T_{4b} \end{cases} \tag{34}$$

After integrating the differential equations and solving the algebraic ones, the behaviour of the temperatures is obtained and shown in Fig.19, while Fig.20 shows the temperatures in each node at particular instants, with the same considerations of case I, but with a solution which is more accurate.

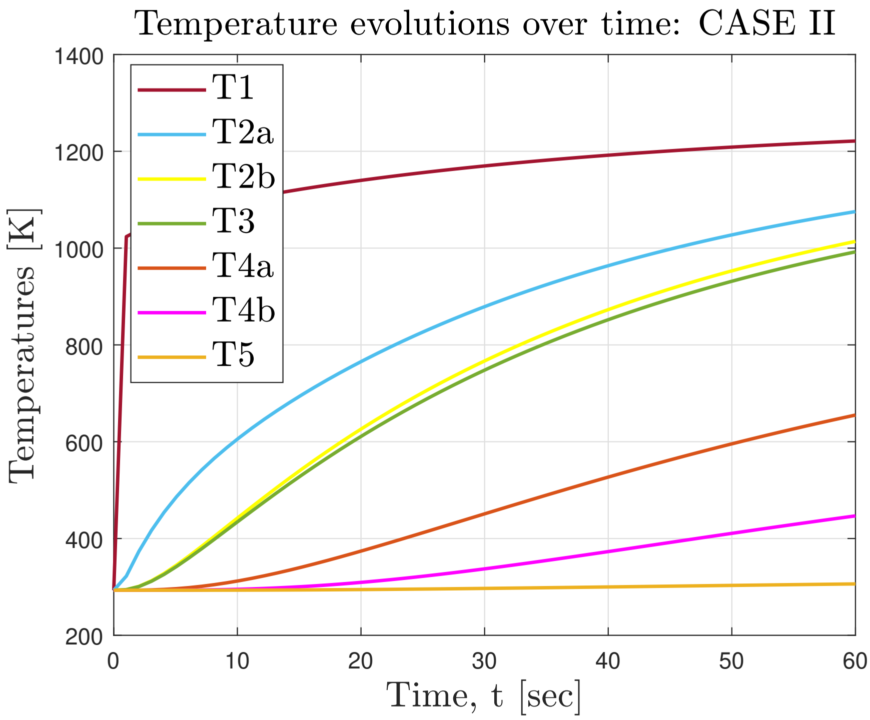


Figure 19: Temperatures profile: CASE II.

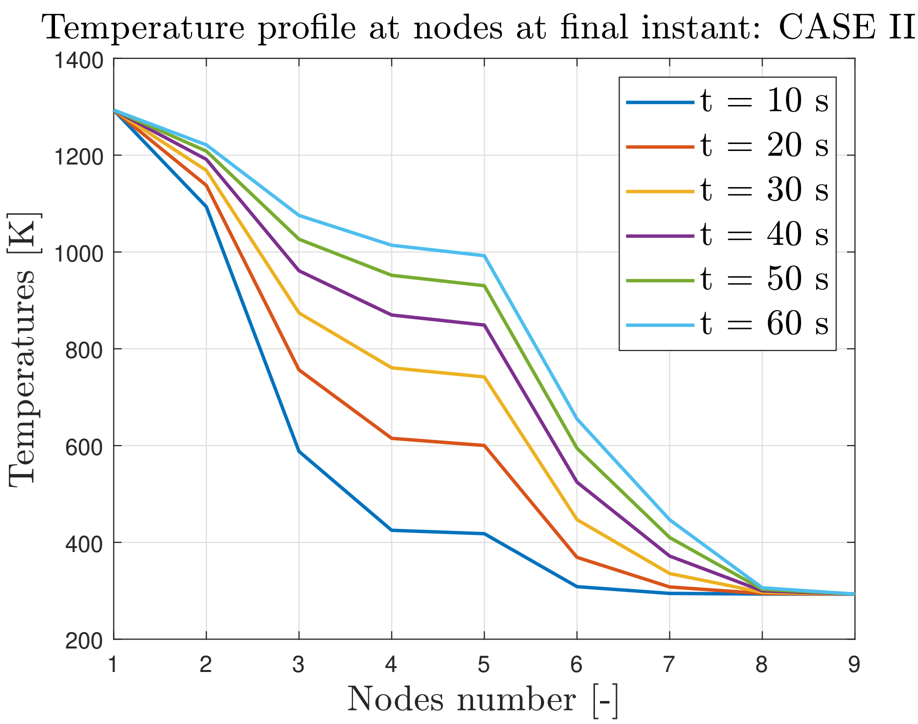


Figure 20: Temperature variation at interfaces between materials for different time instants.

Exercise 4

The electro-mechanical system in Figure 21 is constituted by an electric circuit that drives a mechanical part. A voltage source $v(t) = v_0 \ \cos(\omega t) \ e^{-\beta t}$ is activated at $t = 0$. Assume a coefficient K_m such that $T(t) = K_m \ i(t)$ and $e(t) = K_m \ \Omega_1$, where T is the torque acting on the mechanical part, i the current in the circuit, and Ω_1 the angular velocity of the first disk. Considering $K_m = 20$; $R = 200$; $v_0 = 2$ V; $\omega = 5$ Hz; $\beta = 0.2$ Hz; $L = 2$ mH; $J_1 = 0.5$ kg m^2; $J_2 = 0.3$ kg m^2; $b = 0.1$ kg m^2 s^{-1}; $k = 0.5$ Nm, it is asked to:

1) Derive the mathematical model of the system;
2) Determine the system eigenvalues;
3) Select and motivate the most appropriate integration scheme;
4) Show the system response until $t = 30$ s;
5) Setup, discuss, and run a procedure to find the values of K_m and R that allow matching the second disk angular velocity profile sampled at 10 Hz given in the file `Profile.txt`

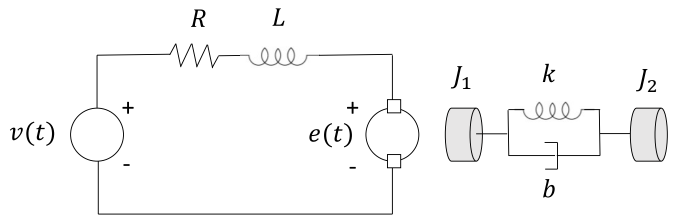


Figure 21: Electro-mechanical system.

The physical model of an electro-mechanical system is provided,so found that its purpose is to convert the electrical signal given by the voltage source acting in the circuit into mechanical motion, the first step is to derive the mathematical model of it. 1) Therefore, it's necessary to couple the electrical part to the mechanical part, so firstly the electrical circuit is analyzed. After setting arbitrarily the direction for the current as positive clockwise, the II Kirchoff's law is applied to the one closed loop. (The law quoted, is recalled in Exercise 2 section, Eq.21). According to this choice, the equation governing this circuit becomes Eq.35, which is witten on the basis of constitutive equations of the circuit, again, recalled in Exercise 2.

$$v(t) = i(t)R - L\frac{di}{dt} - e(t) = 0 \tag{35}$$

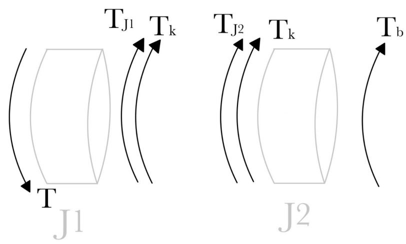


Figure 22: Free body diagram of the inertia of the systems.

With some substitutions based on the data provided by the text and after analyzing the free body diagram of each inertia, shown in Fig. 22, the system of equations to be integrated, is derived in Eq.36.

$$\begin{cases} \dot{i} = \frac{v(t)}{L} - \frac{iR}{L} - \frac{Km}{L}\omega_1 \\ \dot{\omega}_1 = \frac{Km}{J_1}i(t) - \frac{k}{J_1}\theta_1 + \frac{k}{J_1}\theta_2 - \frac{b}{J_1}\omega_1 + \frac{b}{J_1}\omega_2 \\ \dot{\omega}_2 = -\frac{k}{J_2}\theta_2 + \frac{k}{J_2}\theta_1 - \frac{b}{J_2}\omega_2 + \frac{b}{J2}\omega_1 \end{cases} \tag{36}$$

The system is the result of a linear approximation applied to the damper, and provides the differential equation describing how the current in the circuit and velocities of the masses are changing over time.
2) Thanks to the linear approximation, a state-space mathematical model can be built.
The state-space equation is:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}(t) + \mathbf{B} \tag{37}$$

$$\dot{x} = \begin{bmatrix} \dot{i} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\omega}_3 \\ \dot{\omega}_4 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & -\frac{Km}{L} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{Km}{J_1} & -\frac{k}{J_1} & \frac{k}{J_1} & -\frac{b}{J_1} & \frac{b}{J_1} \\ 0 & \frac{k}{J_2} & -\frac{k}{J_2} & \frac{b}{J_2} & -\frac{b}{J_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} \frac{v(t)}{L} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{38}$$

And the state variables are

$$\begin{cases} x_1 = i \\ x_2 = \theta_1 \\ x_3 = \theta_2 \\ x_4 = \omega_1 \\ x_5 = \omega_2 \end{cases} \tag{39}$$

3) To choose properly the integrator to this problem, the eigenvalues of A are computed. In fact, among them, $\lambda = -9.9995 \cdot 10^4$ is found, and its huge value suggests that thee problem is stiff, so the system is solver though MatLab *ode15s* stiff differential equations solver.
4) The response is shown in Fig.23 until 30 s, reporting the variation of angular positions and velocities thought time,for each mass.

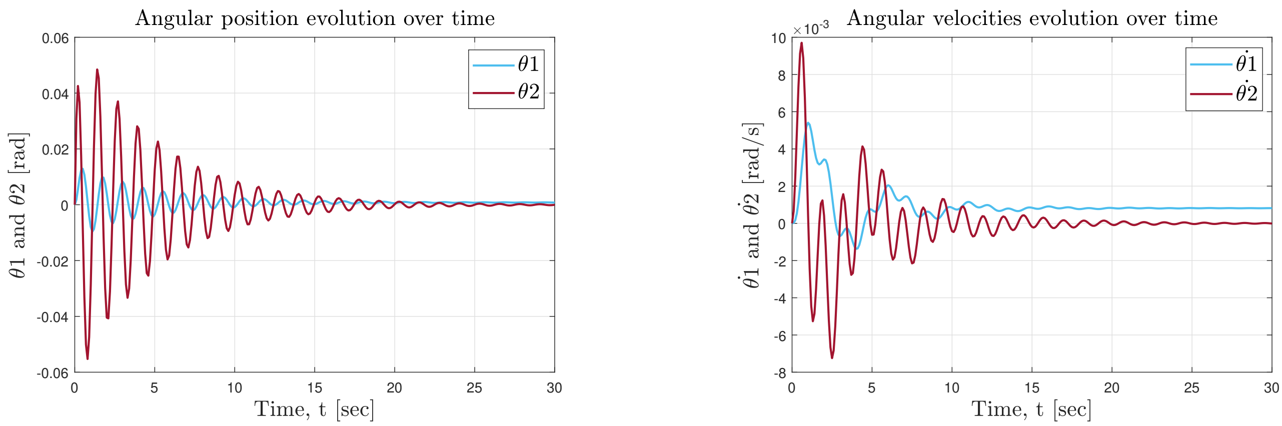


Figure 23: Responses of the system through time.

5) The result obtained until this point are processed using the values of the constant Km and R, provided by the text. Now, considering a set of angular velocities recorded at 10 Hz for the second mass, by accosting the sampled data to the angular velocities modeled as shown in Fig.24, it's possible to realize that the model must be optimized, in order to compute

those values of Km and R, such that the samples and the model can almost perfectly match. Similarly to Exercise 1, it's necessary to minimize the error between the velocities computed at

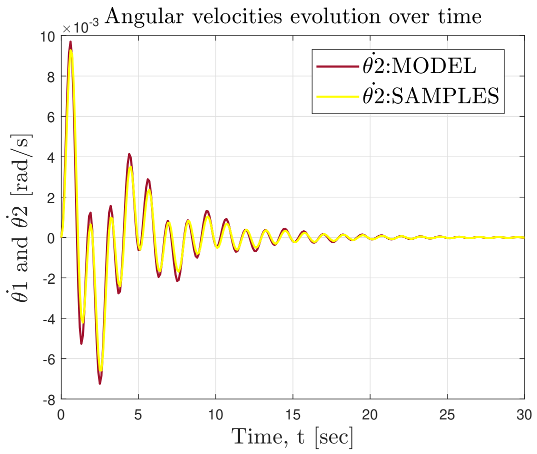


Figure 24: Comparison between modeled angular velocities and samples.

each instant and the sampled data. So the MatLab tool *fminunc* is applied to this case, and it works minimizing an objective function, which in this case is made of the norm of the absolute error between the velocities, namely $\mathbf{e} = ||\dot{\theta}_2(t) - \omega_2(t)||$ [rad/s], in this case only regarding the motion of J_2. Fig.25, shows that after the optimization, the states perfectly match, with a

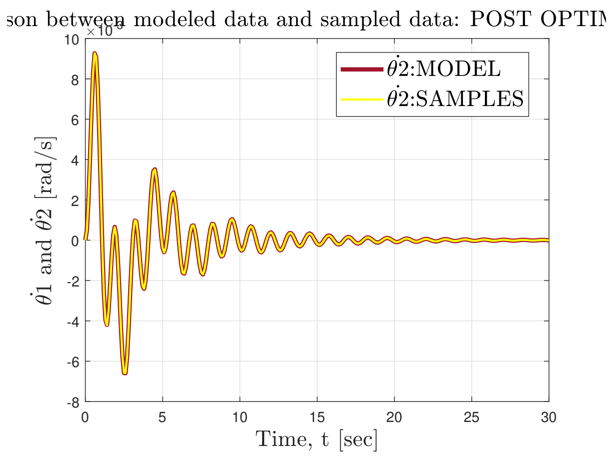


Figure 25: Comparison between modeled $\ddot{\theta}_2$ and samples:POST OPTIMIZATION .

margin of error shown in Fig.26. The correct values derived are Km = 16.70 and R = 227.74. Indeed, the response in terms of current is shown as well in Fig.27.

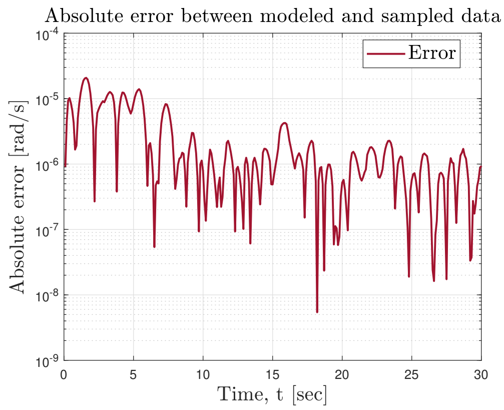


Figure 26: Error of optimization .

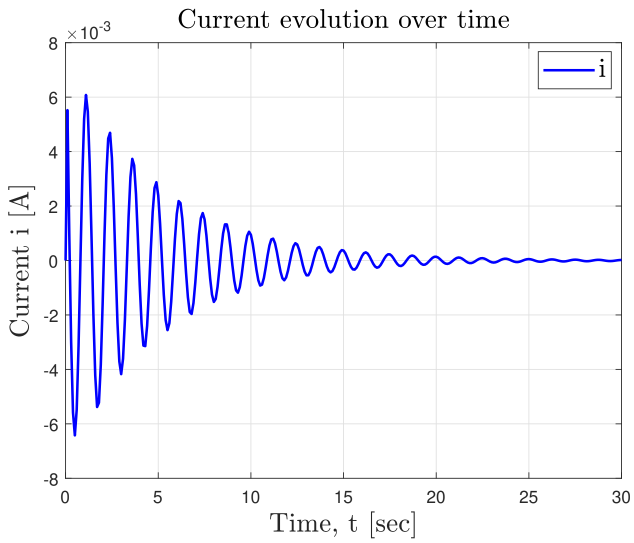


Figure 27: Responses of the system through time.

Exercise 5

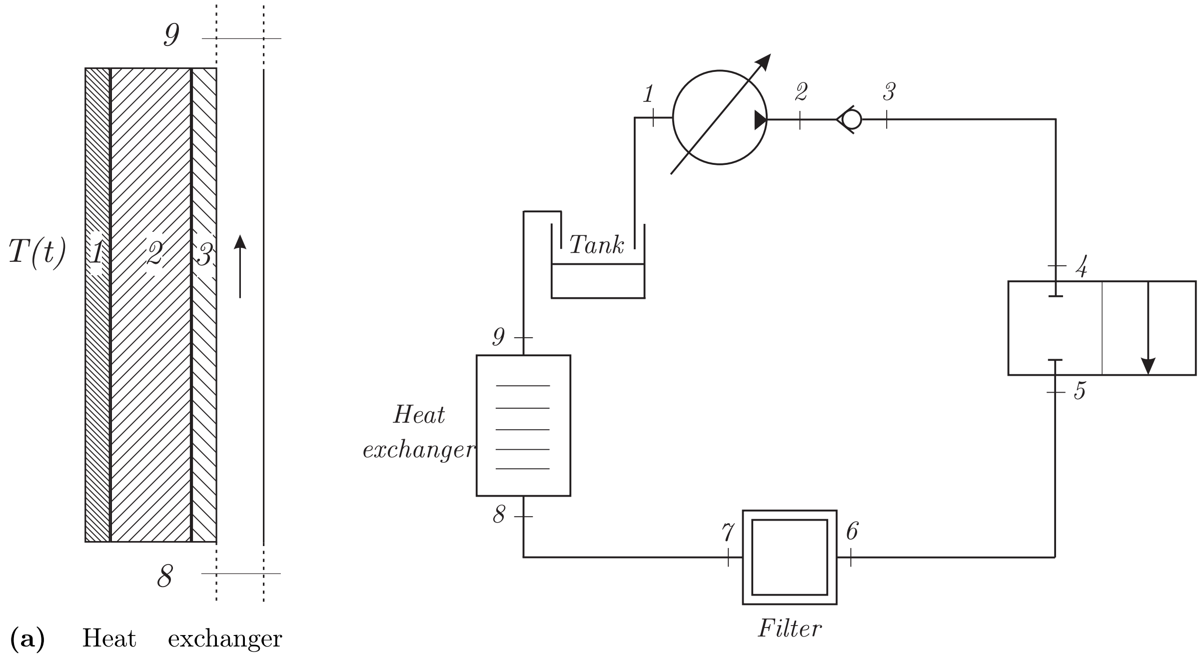
The hydraulic system in Figure 28b is made of a tank, a pump, a check valve, a distribution valve, a filter and an heat exchanger, plus the lines. The heat exchanger is used to cool down an external system, having a temperature profile $T(t) = T_0 + k_T \cos(\omega t)$, and its wall is made up of three layers with different thermal properties, as depicted in Figure 28a. The pressure drop inside the heat exchanger can be modelled as a simplified Rayleigh flow, such that $P_{out} = e^{\dot{Q}/\kappa} P_{in}$. Assuming:

- Fluid: Incompressible fluid, $\rho = 1000$ kg/m^3, specific heat $c_w = 4186$ J/(kg $\cdot$ K)
- Lines: Coefficient of pressure drop across the check valve $k_{cv} = 2$, diameter of the lines $D = 20$ mm;
  - Branch T–1: Length $L_{T1} = 0.5$ m, friction factor $f_{T1} = 0.032$;
  - Branch 3–4: Length $L_{34} = 1.5$ m, friction factor $f_{34} = 0.032$;
  - Branch 5–6: Length $L_{56} = 2.7$ m, friction factor $f_{56} = 0.040$;
  - Branch 7–8: Length $L_{78} = 2.5$ m, friction factor $f_{78} = 0.028$;
  - Branch 9–T: Length $L_{9T} = 1$ m, friction factor $f_{9T} = 0.032$.
- Tank: Adiabatic tank with constant pressure $P_T = 0.1$ MPa;
- Pump:

  Pistons:
  - Number: $N = 9$;
  - Diameter: $D_p = 0.7$ cm
  - Distance between the shaft of the shaft: $d_p = 1.5$ cm
  - Nominal pressure: 5 atm

  Pilot piston:
  - Diameter: $D_k = 1$ cm
  - Control lever length: $l_c = 10$ cm
  - Maximum angle of the control plate: $\theta_{\max} = 20$ deg
  - Rotation speed: $n = 4000$ rpm
  - Equivalent mass: $m_k = 2$ kg
  - Pre-loaded force: $F_0 = 5$ N
  - Friction coefficient: $r_k = 1$ Ns/m
  - Diameter of the pipe: $d_k = 1$ mm
  - Pilot pipe head loss: $k_p = 2.5$
- Distributor: Coefficient of pressure drop across the distributor $k_d = 15$, diameter $d_o = 10$ mm. At $t_0 = 0$ s the valve is half open; it is fully open after $\Delta t = 2$ s.
- Cooler: Diameter of the pipe $D = 20$ mm; Heat flux $\dot{Q}_c = 100$ W.
- Filter: Coefficient of pressure drop across the filter $k_f = 35$, leaking coefficient $k_l = 2.5\%$[2];

[2] $Q_{leak} = k_l Q_{in}$



(a) Heat exchanger model close-up.

(b) Hydraulic system physical model

Figure 28: Thermo-hydraulic system. Assume any other missing data.

- Heat exchanger: Diameter of the pipe $D = 20$ mm, length of the pipe inside the exchanger: $L_e = 0.5$ m; planar exchanger with exchange area $A_e = 1000\ \text{cm}^2$; First layer thermal properties: $k_1 = 395\ \text{W}/(\text{m}\cdot\text{K})$, $\ell_1 = 1$ cm, second layer thermal properties: $k_2 = 310\ \text{W}/(\text{m}\cdot\text{K})$, $\rho_2 = 8620\ \text{kg}/\text{m}^3$, $c_2 = 100\ \text{J}/(\text{kg}\cdot\text{K})$, $\ell_2 = 2.5$ cm, third layer thermal properties: $k_3 = 125\ \text{W}/(\text{m}\cdot\text{K})$, $\ell_3 = 1$ cm, heat transfer coefficient with the fluid $h = 20\ \text{W}/(\text{m}^2\cdot\text{K})$;
- $T_0 = 350\text{K}$, $k_T = 20$, $\omega = 5\ \text{s}^{-1}$, $\kappa = 1000$ W;
- The temperatures are propagated instantaneously along the pipes;
- The heat exchanger layers have an initial temperature of 320 K.

It is asked to:

1) Derive a lumped-approach physical model for the heat exchanger;
2) Derive the mathematical model of the whole system;
3) Select and motivate the most appropriate integration scheme;
4) Show the system response until $t = 25$ s;
5) Discuss at least one possible way to modify the system in order to keep the fluid temperature as close as possible to 20 °C.

The hydraulic system depicted above, is characterized by pressure drops and heat exchanges. The heat is exchanged by the fluid, which flows inside the pipes of the whole system, and the material with which the thermal exchanger is made, in order to cool down an external system from which the heat must be then removed, and transferred to the fluid. Therefore, it's necessary to derive a lumped-approach physical model for the heat exchanger. Similarly to Exercise 3, to each layer, a certain number of nodes is assigned, and the energy balance in computed with respect to them. For simplicity, one node per layer is assumed. According to the provided properties of the materials, the physical model derived for the heat exchanger, is shown in Fig.29.

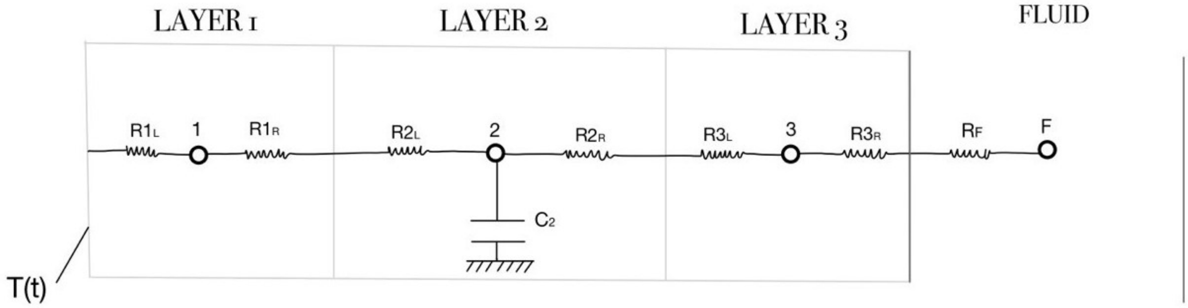


Figure 29: Heat exchanger physical model .

2) To describe the response of the system, is now necessary to detect which are the variables that change over time and why, so to write a mathematical model describing it, and integrate it over time. Starting from the heat exchanger itself, the variables of interests are the temperture of the layer 2, which has capacity capabilities, and for which the energy balance is given by $\frac{dU}{dt} = \dot{q}_{left} + \dot{q}_{right}$, and the temperature of the fluid, which is assumed to absorb the whole heat coming from the layers. From this point, it's necessary to analyze the behaviour of the pump, since its mathematical model, provides the remaining two fundamental equations of motion to integrate over time. In fact, the pump serves as a pressure regulation mechanism. It's made by 9 pistons connected to a plate which can be inclined of a certain maximum angle. The lower part of this plate, is connected as well to a pilot piston, that regulates the pressure which outgoes the pump, and then modifies it in the whole circuit. it's important to track the motion of the pilot piston, since it serves to track all the pressures as well. The mathematical model for the pilot piston, is given by a mass-damper-spring motion model. From these considerations, four differential equations are obtained and shown in Eq.40

$$\begin{cases} \frac{dT_2}{dt} = \frac{1}{\rho c_2 l_2 k_2}\left(\frac{T(t)-T_2(t)}{R_{1R \to} R_{2L}} - \frac{T_2(t)-T_{fluid}(t)}{R_{2R \to} R_{fluid}}\right) \\ \\ \frac{dT_{fluid}}{dt} = \frac{1}{c_{fluid}}\left(\frac{T_2(t)-T_{fluid}(t)}{R_{2R \to} R_{fluid}}\right) \\ \\ \dot{x}_k = v_k \\ \\ \dot{v}_k = \frac{1}{m_k}(P_k A_k - F_0 - h x_k - r v_k) \end{cases} \tag{40}$$

The four states are:

$$\begin{cases} x_1 = T_2 \\ x_2 = T_{fluid} \\ x_3 = x_k \\ x_4 = v_k \end{cases} \tag{41}$$

3)From the integration of these four differential equation that form a stiff system integrated with the *ode15s* MatLab solver choice, due to the stiffness of the system, the variation over time of the state vector made of x_1, x_2, x_3 and x_4 is obtained, and the behaviour of each of them is shown in Fig.30, Fig.31, Fig,32, so that the request 4) is satisfied.

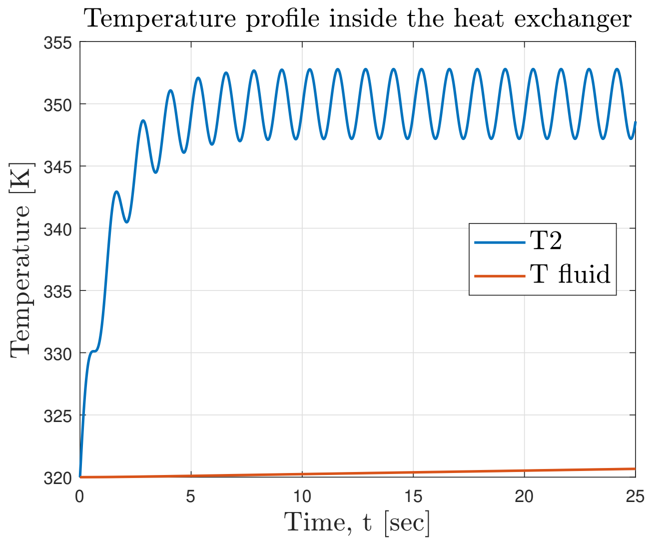


Figure 30: Temperature profiles .

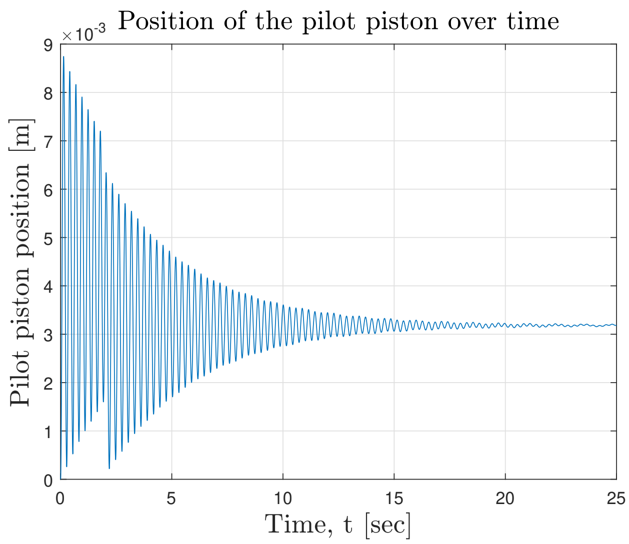


Figure 31: Pilot piston position tracking over time .

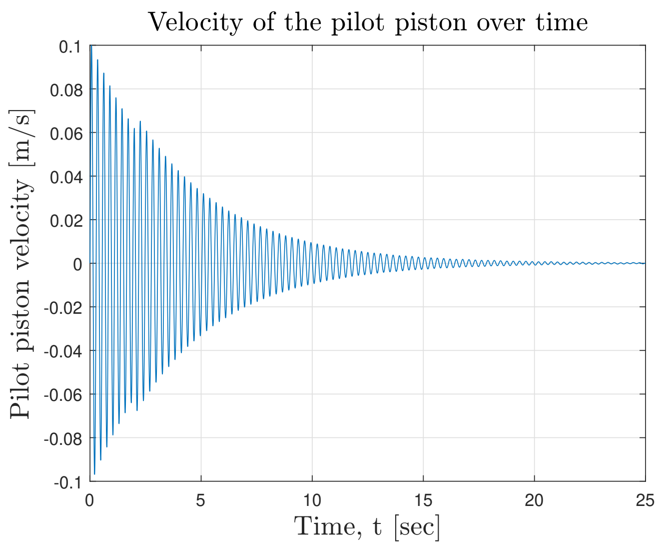


Figure 32: Pilot piston velocity tracking over time .

Unfortunately, the response of the system in terms of position and velocity, oscillates much more then expected, due to integration errors which couldn't be solved for the moment.

Lastly, a mathematical model for the computation of the pressures inside the hydraulic system is carried out. The following system of equation for the pressures, is derived taking into account that between the points of interest, there are pressure drops of different nature. The most interesting, a part from the pressure drops due to the friction of the pipes, are in *Section 1-2* in which the outgoing pressure in related to the pressure inside the pilot piston P_k, *Section 4-5*, in which it's necessary to consider the progressive opening of the valve in the first two seconds, *Section 6-7* in which the flow rate undergoes a change in it's value due to the leakages, so from Q the values passes to Q_2, and *Section 8-9*, the site of heat exchanger, where the temperature increment, causes an increment of pressure as well, so that $P_9 > P_8$. Finally, the system of equations describing the pressure variation inside the system, is shown in Eq.42.

$$\begin{cases} P_9 = P_T + \frac{1}{2}\rho_{fluid} f_{9T} \frac{L_{9T}}{D} \frac{Q2}{A}^2 \\ P_8 = \frac{P_9}{e^{\frac{\dot{q}}{k}}} \\ P_7 = P_8 + \frac{1}{2}\rho_{fluid} f_{78} \frac{L_{78}}{D} \frac{Q2}{A}^2 \\ P_6 = P_7 + \frac{1}{2}\rho_{fluid} k_{filt} (\frac{Q2}{A})^2 \\ P_5 = P_6 + \frac{1}{2}\rho_{fluid} f_{56} \frac{L_{56}}{D} \frac{Q}{A}^2 \\ P_4 = P_5 + \frac{1}{2}\rho_{fluid} k_{distr} (\frac{Q}{A(\alpha)})^2 \\ P_3 = P_4 + \frac{1}{2}\rho_{fluid} f_{34} \frac{L_{34}}{D} (\frac{Q}{A})^2 \\ P_2 = P_3 + \frac{1}{2}\rho_{fluid} k_{cv} (\frac{Q}{A})^2 \\ P_k = P_2 - \frac{1}{2}\rho_{fluid} k_{pil} (\frac{v_k A_k}{A_p})^2 \\ P_1 = P_T - \frac{1}{2}\rho_{fluid} f_{1T} \frac{L_{1T}}{D} (\frac{Q}{A})^2 \end{cases} \tag{42}$$

All the pressures are computed backward, by starting from the tank, along the pipes to the left, ending with the pump, such that it's possible to close the loop, considering the heat coming from the heat exchanger as input to the system.
It's noteworthy to say that the pressure regulation mechanism, rules the flow rate of the system as well, which is subjected to variations caused by the leakages.
Therefore, the flow rate dictated by the pump, is a function of the stroke as Eq.43 shows, and after the filter, it decreases until it becomes Q2.

$$\begin{cases} Q = nN_p A_p s(x_k) \\ Q2 = Q - Q_{leak} \\ Q_{leak} = k_{leak} Q \end{cases} \tag{43}$$

Additional notes, are reserved to the distributor. In fact, by looking at it's physical model in Fig.33, it can be noticed that the passage area across which the fluid flows, is function of the angle α. Since at the beginning of the integration the valve is half open, $\alpha = \pi$, and this value passes to $\alpha = 2\pi$ in the first 2 s, after which the area of the valve becomes the one used for all the pipes. The variation of this angle, will allow to obtain the area to consider for the computation of P_4, which equation is Eq.44.

$$A_(\alpha) = \frac{r_0^2}{2}(\alpha - sin\alpha) \tag{44}$$

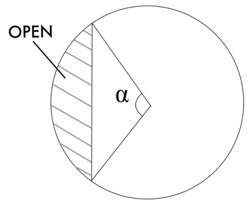


Figure 33: Pressure profiles over time .

Finally, the behaviour of the pressure is shown in Fig.34 with a zoom in Fig.35, to distinguish better the different pressures of the system.

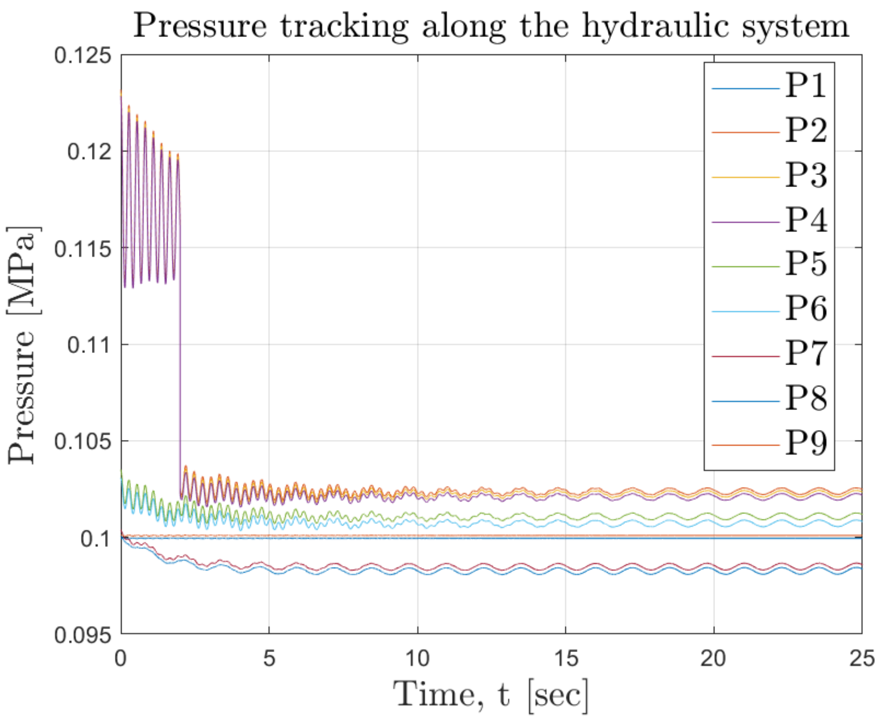


Figure 34: Pressure profiles over time .

5) In order to keep the fluid temperature as close as possible to 20 degrees, one solution that could fit, is the insertion of a cooler after the heat exchanger. in which the temperature is rising.
The heat exchanged, would take into account the temperature at which the fluid comes into the cooler and the temperature it's needed to reach, such that

$$Q = \dot{m}c_p\Delta T \tag{45}$$

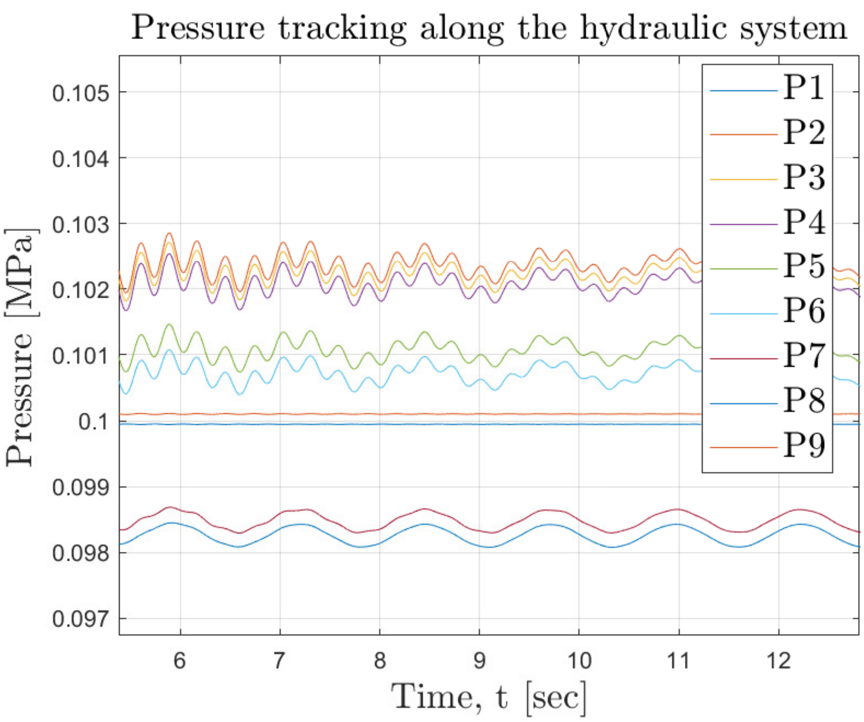


Figure 35: Pressure profiles over time: ZOOM IN .