1. (a)

P_1		x = 1	x = 2	x = 3
y	1	F	F	F
	2	F	T	F
	3	F	F	F

P_2		x = 1	x = 2	x = 3
y	1	T	F	F
	2	T	F	F
	3	T	F	T

P_3		x = 1	x = 2	x = 3
y	1	T	F	F
	2	F	T	F
	3	F	F	T

P_4		x = 1	x = 2	x = 3
y	1	T	T	T
	2	T	T	T
	3	T	T	T

y	1	T	F	F
	2	F	T	F
	3	F	F	T

		1	2	3
y	1	T	T	T
	2	T	T	T
	3	T	T	T

(b)

	P_0	P_1	P_2	P_3	P_4
$\exists x \exists y\, P_i$	T	T	T	T	T
$\exists x \forall y\, P_i$	F	F	T	F	T
$\forall x \exists y\, P_i$	F	F	F	T	T
$\forall x \forall y\, P_i$	F	F	F	F	T

(c) P_2 from the above is a counterexample.

(d) P_3 is a counterexample. (from (b))

$\forall x \exists y\, P_i$	F	F	F	T	T
$\forall x \forall y\, P_i$	F	F	F	F	T

(c) P_2 from the above is a counterexample. (from (b))

(d) P_3 is a counterexample. (from (b))

(e) Let $P =$ "xy is even, $x, y \in \mathbb{Z}$". Then this is counterexample.

(f) P_3 is a counter example.

(oops, change the S_i's to X_i's)

2. (a) $S_1 \wedge S_2$ = true

(b) $S_4 \oplus S_5$ = true

(c) $(\neg S_1 \wedge S_2 \wedge S_3 \wedge S_4 \wedge S_5) \vee (\ldots) \vee \ldots$

$\vee (S_1 \wedge S_2 \wedge S_3 \wedge S_4 \wedge \neg S_5)$

(d) X_1 = true, X_2 = true, with

$(X_3, X_4 = T, X_5 = F)$ or $(X_3, X_5 = T, X_4 = F)$

$(X_3, X_4 = T, X_5 = F)$ or $(X_3, X_5 = T, X_4 = F)$

3. (a) All residues mod 3 of squares are $\{0^2, 1^2, 2^2\}$

$= \{0, 1\}$, so $n \equiv 2 \pmod 3$ cannot be a square.

(b) $(x-y)(x+y) = 10$. Let $a = x-y$, $b = x+y$,

Then $(a,b) = (\pm 1, \pm 10), (\pm 2, \pm 5), (\pm 5, \pm 2), (\pm 10, \pm 1)$,

but $2x = a+b \equiv 0 \pmod 2$, which is impossible by all combinations above.

4. (a) False, Assuming $n = 3k+1$ contradicts "$2n+1$ is NOT a multiple of 3."

(b) Incorrect proof. Cannot just plug in $n = n+1$, because we haven't established proof works for $n+1$.

Try instead: $n < 2^n \Rightarrow n+1 < 2^n + 1 < 2^{n+1}$

(c) Incorrect, base case is insufficient.

$\max(1, 0) = 1$ will imply $\max(0, -1) = 0$ from the proof, which he did not prove and is incorrect.

from the proof, which he did not prove and is incorrect.

5. (a) Base case: $n = 3$, $n^2 = 9 \geq 2n+1 = 7$

Assume it is true that $n = n_0$.

To prove $n = n_0 + 1$ works,

Plugging in, we must prove

$$(n_0+1)^2 \geq 2(n_0+1)+1$$

$$\Rightarrow n_0^2 + 2n_0 + 1 \geq 2n_0 + 1 + 2.$$ Since $2n_0 + 1 \geq 2 \ \forall n_0 \geq 3$,

Adding this to inductive hypothesis proves result.

5. (b) Base case $n=4$, $2^4 = 16 \geq 4^2 = 16$.

Assume true for $n = n_0$.

Then $n = n_0 + 1$, we must prove

$$2^{n_0+1} \geq (n_0+1)^2.$$

But $2^{n_0+1} = 2 \cdot 2^{n_0} \geq 2 \cdot (n_0)^2$, from inductive hypothesis,

and $2n_0^2 \geq (n_0+1)^2$

$\iff n_0^2 \geq 2n_0 + 1$, which is true from (a),

QED.

6. Yes. We use induction. $n=1$ stack case is trivial

Assume it can be done with $n=n_0$ stacks.

Take m to be the largest pizza, and

p_m to be the position, from bottom up.
If m is not the bottommost pizza,
Flip right beneath m, which will put
m on top. Then flip the bottom most
pizza, which will make $p_m=0$ (at the bottom)

By inductive hypothesis, we can then flip
the other pancakes into ascending order,
which finishes the proof.