Q1:

example equations & solutions

$x_1 = 2$
$x_2 = 3$
$\vdots$
$x_n = $ etc.

$3x_1 + 2x_2 = 12$
$5x_1 + x_2 = 13$
$\vdots$ etc.
$ax_n + x_{n-1} = c$

$8x_1 + 3x_2 = 25$

$8(2) + 3(3) = 25$

solution remains the same
bc you're adding equations together

Q2:

1) There are more variables than equations ∴ there are ∞ solutions & the system is consistent

$x - 2y + 4z = 12$
$2x + y - z = 4$

$y = 4 - 2x + z$
$x - 8 + 4x - 2z + 4z = 12$

$5x + 2z = 20$
$x = 4 - \frac{2}{5}z$

$y = 4 - 8 + \frac{4}{5}z + z$
$y = -4 + \frac{9}{5}z$

$x(\alpha) = 4 + \frac{3}{2}\alpha$
$y(\alpha) = -4 + \frac{9}{5}\alpha$
$z(\alpha) = \alpha$

2) $2x - 4y + 8z = 24$

3) $2x - 4y + 8z = 10$

4) $x + y + z = \frac{3}{2}\alpha + \frac{9}{5}\alpha + \alpha$

$x + y = \frac{33}{10}z$

Q3:

5) never; homogeneous sets have constants that are all zero, so they always have a solution (zero vector)

Q4:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 1 & -1 & 4 \\ 1 & -1 & 1 & 0 \end{array}\right]$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & -5 & 2 \\ 1 & -1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & -2 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 5 & -2 \\ 0 & -2 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & -2 & -1 & -1 \end{bmatrix}$$

$R_3 - R_1$ $\quad -R_2$ $\quad R_1 - R_2$

$2R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 9 & -5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & -3 & 3 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -\frac{5}{9} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & \frac{4}{3} \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -\frac{5}{9} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & \frac{4}{3} \\ 0 & 1 & 0 & \frac{7}{9} \\ 0 & 0 & 1 & -\frac{5}{9} \end{bmatrix} \begin{array}{l} \parallel zy \\ \parallel xz \\ \parallel xy \end{array}$$

$R_3 / 9$ $\quad 3R_3 + R_1$ $\quad R_2 - 5R_3$

Q5: $\dim(\text{span}(\{v_1, v_2, v_3, v_4\})) = 4$

Q6:

I don't actually know how to do any of the math of this

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \\ -3 \end{bmatrix} - \text{proj}_{\vec{v_1}} \begin{bmatrix} -1 \\ 0 \\ 2 \\ -3 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 4 \end{bmatrix} - \text{proj}_{\vec{v_1}} \begin{bmatrix} 2 \\ -1 \\ 0 \\ 4 \end{bmatrix} - \text{proj}_{\vec{v_2}} \begin{bmatrix} 2 \\ -1 \\ 0 \\ 4 \end{bmatrix}$$

$$v_4 = \begin{bmatrix} 2 \\ 1 \\ 5 \\ 5 \end{bmatrix} - \text{proj}_{\vec{v_1}} \begin{bmatrix} 2 \\ 1 \\ 5 \\ 5 \end{bmatrix} - \text{proj}_{\vec{v_2}} \begin{bmatrix} 2 \\ 1 \\ 5 \\ 5 \end{bmatrix} - \text{proj}_{\vec{v_2}} \begin{bmatrix} 2 \\ 1 \\ 5 \\ 5 \end{bmatrix}$$

Q7:

$$\begin{bmatrix} 5 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix}$$

Q8:

A: speed 10, carry 5
B: speed 15, carry 3

distance: 100 units
weight: 25 units

$$5x + 3y = 25$$
$$10x + 15y = 100$$
$$-10x + 6y = 50$$
$$9y = 50$$
$$y = \frac{50}{9}$$
$$x = \frac{10}{3}$$

$$\frac{150}{45} = \frac{10}{3}$$