DESIGN OF A HIGH PRESSURE TURBINE STAGE

Gašper Lotrič – 10951450

Input data and design choices

Inlet total pressure $P_{tot,in}$	40 bar
Inlet total temperature $T_{tot,in}$	1700
Total to total expansion ratio $ß_{tt}$	2
Cooling air flow rate	0.1 of mass flow rate

Choices based on **Rolls-Royce Trent 1000 engine**

Mass flow rate $\dot{m}$	1200 kg/s
Bypass ratio	10:1
Rotational speed ω	25000 rpm
Fuel	Jet A-1 ($C_{12}H_{23}$)

Other assumptions

- Axial inlet and outlet: α_0=α_2=0°
- Constant mean diameter
- Constant blade height
- Perfect gas
- Free vortex design
- Cooling air is introduced in station 0

Combustion Model

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Combustion Model and Gas Properties

Air flow into engine core

$$\dot{m}_{core} = \frac{\dot{m}}{1+BPR}\ (1 - coolingAir) = 98.18\ \text{kg/s}$$

Fuel properties

$$M_{fuel} = 167.3 * 10^{-3}\ \text{kg/mol}$$
$$LHV_{fuel} = 43^{*}\ 10^{6}\ \text{J/kg}$$

Jet fuel A-1 And air mixture

$$C_{12}H_{23} + (1+\epsilon)\beta_{O_2,stoich}\left(O_2 + \frac{X_{O_2}}{X_{N_2}}N_2 + \frac{X_{O_2}}{X_{Ar}}Ar\right) \rightarrow 12CO_2 + 11.5H_2O + excess\ air$$

Fuel mass flow with $\epsilon = 3$

$$\alpha_{stoich} = \frac{\beta_{O_2,stoich}}{X_{O_2}} \cdot \frac{M_{fuel}}{M_{air}} = 14.64$$

$$\alpha = \alpha\ stoich\ \cdot (1+\varepsilon) = 58.54$$

$$\dot{m}_{fuel} = \frac{\dot{m}_{air}}{\alpha} = 1.68\ \text{kg/s}$$

Flue gas Composition

Gas	Mass fraction	Molar fraction
CO_2	0.0528	0.0348
H_2O	0.0207	0.0333
O_2	0.1709	0.1547
N_2	0.7430	0.7681
Ar	0.0126	0.0092

Combustion Model and Gas Properties

Gas Property calculation using NASA polynomials for specific gases

$$\frac{C_{p,i}}{R}(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3 + a_5 T^4 \longrightarrow c_{p,i} = \frac{C_{p,i}}{R}\frac{R}{M_i}$$

Specific gas constants

$$R_i = \frac{R}{M_i}$$

Where $R = 8.314\ \mathrm{J/molK}$

Specific heat at constant volume and gamma ratio

$$c_{v,i} = c_{p,i} - R_i$$

$$\gamma_i = \frac{c_{p,i}}{c_{v,i}}$$

$$T_{ref} = 298\mathrm{K}$$

Mixture properties

$$c_{p,mix} = \sum_i w_i c_{p,i} \qquad M_{mix} = \sum_i x_i M_i$$

$$c_{v,mix} = \sum_i w_i c_{v,i} \qquad R_{mix} = \frac{R}{M_{mix}}$$

Where w_i is the mass fraction of species i
And x_i is the mole fraction of species i

Combustion Model and Gas Properties

Resulting calculated gas properties at 1700 K

Gas	cp [kJ/kg K]	cv [kJ/kg K]	gamma	R [kJ/kg K]
CO_2	1.347078	1.15816	1.163124	0.188923
H_2O	2.733913	2.27238	1.203105	0.461533
O_2	1.158606	0.89878	1.289091	0.259828
N_2	1.290456	0.99365	1.298707	0.296809
Ar	0.520307	0.31218	1.666667	0.208123
Mixture	1.291107	1.00402	1.285937	0.287084

Combustion Model and Gas Properties

Dynamic viscosity calculation

Sutherland's law for temperature-dependent viscosity

$$\mu(T) = \mu_0 \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + S}{T + S}$$

Wilke's mixing rule for Dynamic viscosity

$$\mu_{mix} = \sum_{i=1}^{n} \frac{x_i \, \mu_i}{\sum_{j=1}^{n} x_j \phi_{ij}}$$

interaction terms $\phi_{i,j}$ are calculated as

$$\phi_{ij} = \frac{\left[1 + \sqrt{\frac{\mu_i}{\mu_j}} \left(\frac{M_j}{M_i}\right)^{1/4}\right]^2}{\sqrt{8\left(1 + \frac{M_i}{M_j}\right)}}$$

Where

- μ_0 is reference viscosity at T_0
- S is Sutherland's constant

With this, the calculated viscosity for the flue gas mixture at 1700 K is

$$\mu_{fg} = 5.66 \, 10^{-5} \ \mathrm{Pa\,s}$$

Turbine Meanline

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Turbine Meanline Design

The working fluid is the combustion products and the cooling air

Perfect gas Hypothesis

Adiabatic system through the stator stage – Total temperature remains constrained

Fixed expansion ratio -> repeating stage system can't be used because flow angles and velocities can change stage to stage

Turbine Meanline Design – Input parameters

$$m_{total} = \frac{\dot{m}_{core}}{1 + coolingAir} + \dot{m}_{fuel}$$

Chord lengths of rotor and stator are assumed $c_{stator} = c_{rotor} = 0.04\ \mathrm{m}$

Purely axial inlet flow $\alpha_0 = 0$

In the high pressure turbines of airplane engines the angular speed reaches up to $\omega = 25000\ \mathrm{rpm}$

Adiabatic through the stator $h_0 = h_1$

No swirl at the rotor outlet – purely axial outlet flow $\alpha_2 = 0$

Constant blade height in turbine stage $b_0 = b_1 = b_2$

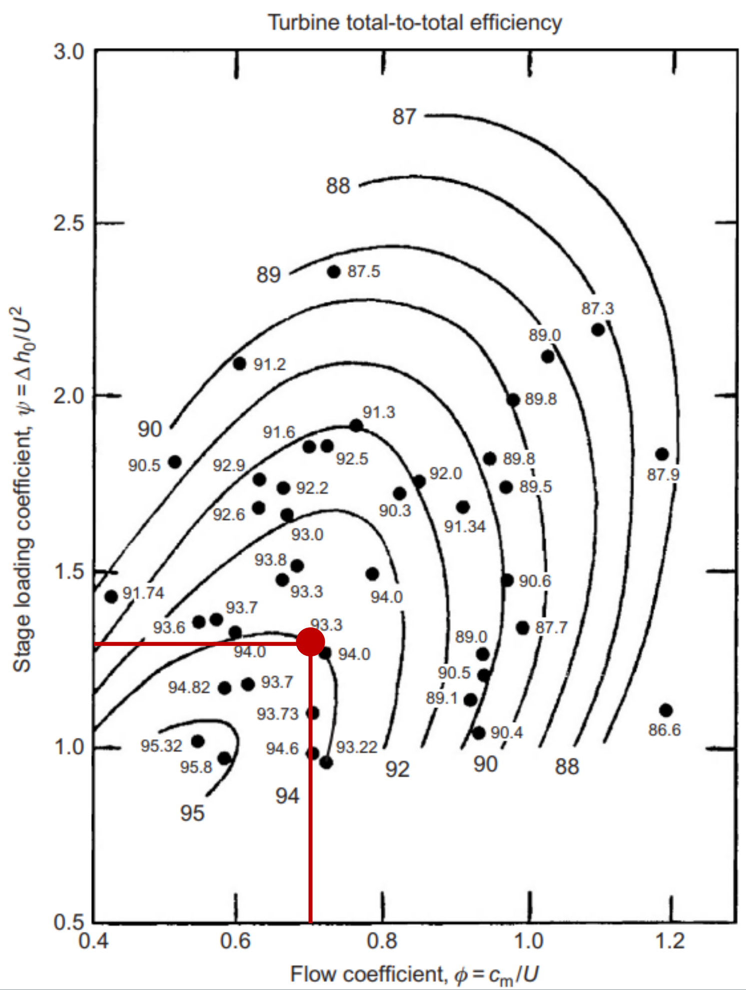
Turbine Meanline Design – Input parameters

Smith chart for nondimensional parameters

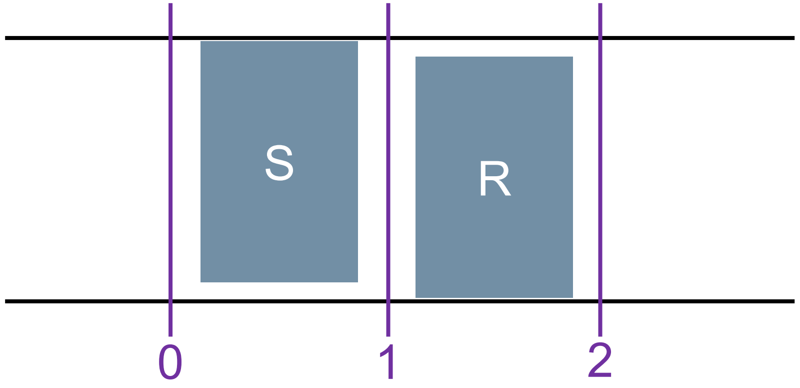
Flow coefficient $\phi = \frac{c_m}{U} = 0.7$

Stage loading coefficient $\psi = \frac{\Delta h_0}{U^2} = 1.3$

Total to total efficiency $\eta_{TT} = 0.933$



Turbine Meanline Design – Workflow



Input Data — $c_p, \gamma, R_{mix}, \dot{m}, \phi, \psi, \eta_{TT}$

↓

Station 1 — Rotor inlet

↓

Loss Loop @ Station 1 — Iteratively solves for P_1, ρ_1, b_1

↓

Station 0 — Stator inlet – computes P_0, V_0, T_0

↓

Station 2 — Losses + geometry via residual optimization

Turbine Meanline Design – Station 1

Input Data

$c_p, \gamma, R_{mix}, \dot{m}, \phi, \psi, \eta_{TT}, P_{0t}, T_{0t}, \omega$

$$L_{euler} = \eta_{TT}\, c_p T_{0t} \left[1 - \left(\frac{P_{2t}}{P_{0t}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$U_{mean} = \sqrt{\frac{L_{euler}}{\psi}}$$

$$r_{mean} = \frac{U_{mean}}{\omega}$$

$$\alpha_1 = \arctan\left(\frac{\psi}{\phi}\right)$$

$$V_{1,ax} = \phi U_{mean}$$

$$V_{1,tan} = V_{1,ax} \tan(\alpha_1)$$

$$V_1 = \sqrt{V_{1,ax}^2 + V_{1,tan}^2}$$

$$W_{1,ax} = V_{1,ax}$$

$$W_{1,tan} = V_{1,tan} - U_{mean}$$

$$W_1 = \sqrt{W_{1,ax}^2 + W_{1,tan}^2}$$

$$\beta_1 = \arctan\left(\frac{W_{1,tan}}{W_{1,ax}}\right)$$

$$T_{1t} = T_{0t}$$

$$T_1 = T_{1t} - \frac{V_1^2}{2c_p}$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$M_{1,rel} = \frac{W_1}{\sqrt{\gamma R T_1}}$$

$$T_{1t,rel} = T_1\left(1 + \frac{\gamma - 1}{2} M_{1,rel}^2\right)$$

Loss Loop @ Station 1

Turbine Meanline Design – Loss Loop @ Station 1

Station 1

$$b_1^i = b_1^{i+1}$$

$$Re = \frac{\rho_1^i V_1 c}{\mu}$$

$$Y_{sec} = \frac{c}{b}\left[0.0334\frac{\cos\alpha_1}{\cos\alpha_{mean}} + B\left(\frac{k}{c}\right)^{0.78}\right]\left(\frac{c_l}{s/c}\right)^2\frac{\cos^2\alpha_{out}}{\cos^3\alpha_{out}}$$

$$Y_p = Y_{p,ref}\left(\frac{Re_{ref}}{Re}\right)^{0.2}\left[1 + 7\left(\frac{t}{s} - \frac{t}{s}\bigg|_{ref}\right)\right]$$

$$Y_{tot} = Y_p + Y_{sec}$$

$$\frac{P_{t1}}{P_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{t1} = \frac{P_{t0}}{Y_{tot}\left(1 - \frac{P_1}{P_{1t}}\right) + 1}$$

$$P = \frac{P_{t1}}{P_{t1}/P_1}$$

$$\rho_1 = \frac{P_1}{RT_1}$$

$$b_1 = \frac{\dot{m}}{\rho_1 V_{1,ax}\, 2\pi r_{mean}}$$

$$error = \frac{b_1^{i+1} - b_1^i}{b_1^i}$$

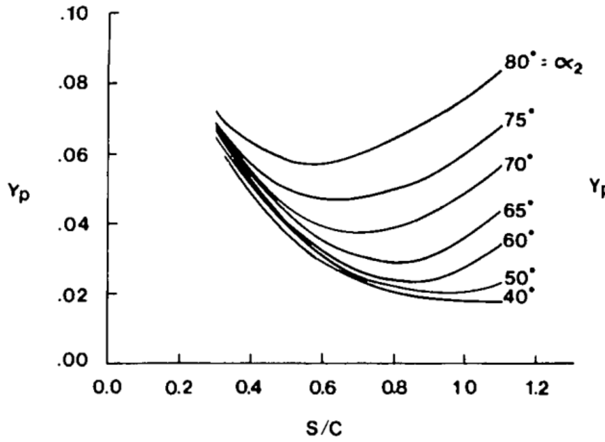
$$|error| < 10^{-6}$$

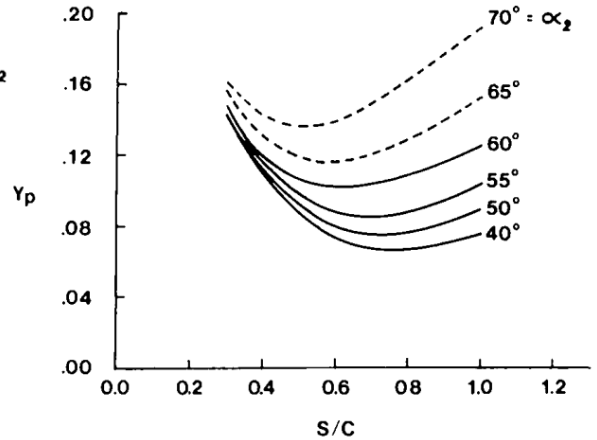
no

yes

Station 0

Turbine Meanline Design – Ainley-Mathieson

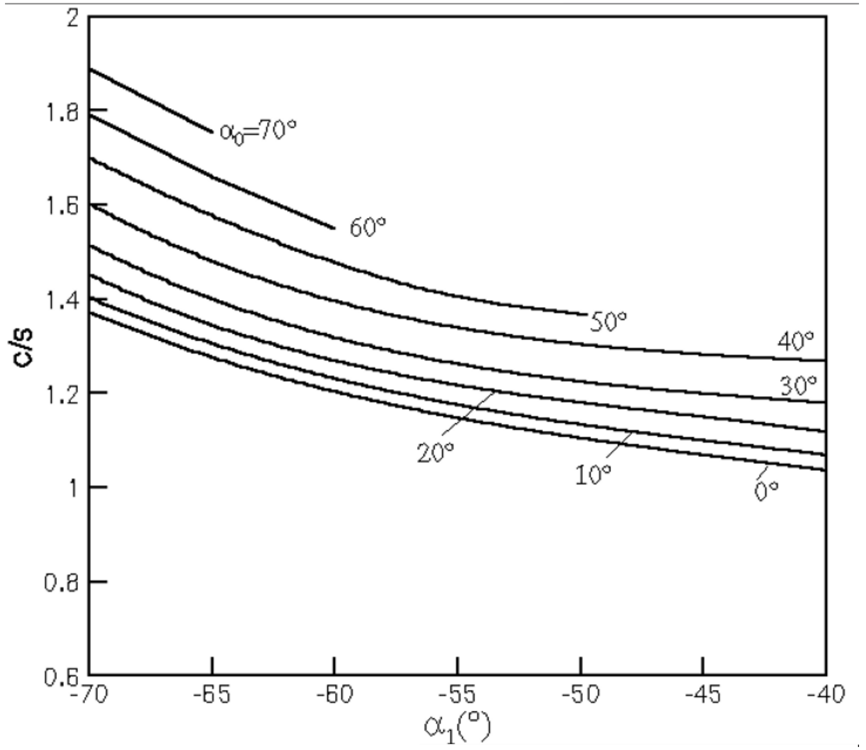




Solidity is chosen to minimise the profile losses in each of the two cases

Combined loss is calculated as

$$Y_p = \left[Y_{p,ai} + \frac{\beta_1}{\beta_2}\left(Y_{p,sb} - Y_{p,ai}\right)\right]\left(\frac{t_{max}/c}{t_{max,ref}/c}\right)^{\frac{\beta_1}{\beta_2}}$$



Using the β_1 and β_2 angles, solidity is obtained, and from there pitch s is calculated

Turbine Meanline Design – Ainley-Mathieson

Reynolds Correction

$$X_{Re} = \begin{cases} \left(\dfrac{Re}{2 \cdot 10^5}\right)^{-0.4} & ; Re \leq 2 \cdot 10^5 \\ 1\,; \quad 2 \cdot 10^5 \leq Re \leq 10^6 & \\ \left(\dfrac{Re}{10^6}\right)^{0.2} & ; Re \geq 10^6 \end{cases}$$

Trailing Edge Correction

$$X_{TE} = 1 + 7\left(\frac{t}{s} - \frac{t}{s}\bigg|_{ref}\right)$$

$$Y_{p,corrected} = Y_p \underbrace{\left(\frac{Re_{ref}}{Re}\right)^{0.2}}_{\text{Reynolds correction}} \underbrace{\left[1 + 7\left(\frac{t}{s} - \frac{t}{s}\bigg|_{ref}\right)\right]}_{\text{Trailing edge thickness correction}}$$

Turbine Meanline Design – Dunham-Came

$$Y_{sec} + Y_{cl} = \frac{c}{b} \cdot \left[0.0334 \frac{\cos\alpha_1}{\cos\alpha_0} + B \left(\frac{k}{c}\right)^{0.78} \right] \cdot \left(\frac{c_l}{s/c}\right)^2 \frac{\cos^2\alpha_1}{\cos^3\alpha_{mean}}$$

$B = 0.37$ (shrouded)
$B = 0.47$ (unshrouded)
$k = 0.5\ mm$

- Where $\alpha_{mean} = \arctan\left(\frac{\tan\alpha_0 + \tan\alpha_1}{2}\right)$
- Coefficient of lift $c_l = 2\left(\frac{s}{c}\right)|\tan\alpha_0 - \tan\alpha_1| \cos\alpha_{mean}$

Total stator losses become:

$$Y_{tot,stator} = Y_{p,corrected} + Y_{sec} + Y_{cl}$$

Turbine Meanline Design – Station 1 Results

Results after Station 1 iterations

$L_{is} = 313.51$ kJ/kg
$L_{euler} = 292.50$ kJ/kg
$U_{mean} = 474.34$ m/s
$r_{mean} = 0.1812$
$\omega_{rad} = 2617.99$

$V_{1,ax} = 332.04$m/s
$V_{1,tan} = 616.65$ m/s
$V_1 = 700.36$ m/s
$\alpha_1 = 61.699°$

$W_{1,ax} = 332.04$ m/s
$W_{1,tan} = 142.30$ m/s
$W_1 = 361.25$ m/s
$\beta_1 = 23.199°$

$T_{1t} = 1700$ K
$T_1 = 1510.04$ K
$T_{1t,rel} = 1560.58$ K

$M_1 = 0.938$
$M_{1,rel} = 0.484$

Turbine Meanline Design – Station 0 Loop

Station 1

ρ_0^i

$$b_0 = b_1$$

$$V_0 = \frac{\dot{m}}{\rho_0^i 2\pi r_m b_0}$$

$$T_0^{i+1} = T_{t0} - \frac{V_0^2}{2c_p}$$

$$M_0 = \frac{V_0}{\sqrt{\gamma R T_0^{i+1}}}$$

$$P_0 = P_{0t}\left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{-\frac{\gamma}{\gamma - 1}}$$

$$\rho_0^{i+1} = \frac{P_0}{R T_0^{i+1}}$$

ρ_0^{i+1}

T_0^{i+1}

$$error = \frac{\rho_0^{i+1} - \rho_0^i}{\rho_0^i}$$

$$|error| < 10^{-6}$$

no: $\rho_0^i = \rho_0^{i+1}$

yes: Station 2

Turbine Meanline Design – Station 0 Results

Results after Station 0 iterations

$$V_0 = 222.64 \text{ m/s}$$
$$T_0 = 1680.80 \text{ K}$$
$$M_0 = 0.2826$$
$$P_0 = 38.01 \text{ bar}$$
$$\rho_0 = 7.877 \text{ kg/m}^3$$
$$T_{0t} = 1700 \text{ K}$$
$$P_{0t} = 40 \text{ bar}$$

Turbine Meanline Design – Station 2 Assumptions

outlet flow angle $\alpha_2 = 0°$. There is no tangential component at the absolute frame of reference.

T_{2t} to match with the Euler Work assumed by the non-dimensional parameters.

Moreover, euler work is fixed by the non dimensional coefficients at the begining of the meanline design.

Hub Radius: 0.120 m

Trailing edge thicknes of the rotor: 1 mm

Minimum throat opening for rotor: 8 mm

Number of seals in rotor: 3

Tip clearance: $k = 0.5\ mm$

The maximum thickness of the blades: 10 mm

Turbine Meanline Design – Station 2

Station 0

V_2, b_2

The goal of the loop is to optimise V_2
The final result has to satisfy $P_{2t} = 20\ \mathrm{bar}$ and keep geometric continuity from station 1

$$V_{2,ax} = V_2$$
$$V_{2,tan} = V_{2,ax}\tan(\alpha_0)$$
$$V_2 = \sqrt{V_{2,ax}^2 + V_{2,tan}^2}$$
$$W_{2,ax} = V_{2,ax}$$
$$W_{2,tan} = V_{2,tan} - U_{mean}$$
$$W_2 = \sqrt{W_{2,ax}^2 + W_{2,tan}^2}$$
$$\beta_2 = \arctan\left(\frac{W_{2,tan}}{W_{2,ax}}\right)$$

$$T_{2t} = T_{1t} - \frac{L_{euler}}{c_p}$$
$$T_2 = T_{2t} - \frac{W_2^2}{2c_p}$$
$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}}$$
$$M_{2,rel} = \frac{W_2}{\sqrt{\gamma R T_2}}$$
$$T_{2t,rel} = T_2\left(1 + \frac{\gamma-1}{2}M_{2,rel}^2\right)$$

$$P_{2t} = \frac{P_{1t,rel}}{\left(\dfrac{1+\frac{\gamma-1}{2}M_{2,rel}^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}} + Y_{total,rotor}\left[\left(\dfrac{1+\frac{\gamma-1}{2}M_{2,rel}^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\frac{\gamma}{\gamma-1}} - \left(1+\frac{\gamma-1}{2}M_2^2\right)^{-\frac{\gamma}{\gamma-1}}\right]}$$

$$P_2 = \frac{P_{2t}}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}}$$
$$P_{2t,rel} = P_2\left(1+\frac{\gamma-1}{2}M_{2,rel}^2\right)^{\frac{\gamma}{\gamma-1}}$$
$$P_{2,rel} = P_{2t,rel}\left(\frac{T_{2,rel}}{T_{2t,rel}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\rho_2 = \frac{P_2}{RT_2}$$
$$b_2 = \frac{\dot{m}}{2\pi r_{mean} V_{2,ax}\rho_2}$$

$$error_P = \frac{|P_{2t}^{i+1} - P_{2t}^{i}|}{P_{2t}^{i}} \qquad error_b = \frac{|b_2^{i+1} - b_2^{i}|}{b_2^{i}}$$

$$Y_{total,rotor} = Y_{profile} + Y_{secondary} + Y_{TET} + Y_{TC} + Y_{shock}$$

$$residual = 100 \cdot error_P^6 + 10 \cdot error_b^2$$

$V_{2,opt}$

Radial Eq.

Turbine Meanline Design – Station 2 Losses

Trailing Edge Thickness Losses

$$Y_{TET} = \frac{\left[1 - \frac{\gamma - 1}{2} M_{2,rel}^2 \left(\frac{1}{1 - DTEEC} - 1\right)\right]^{-\frac{\gamma}{\gamma - 1}} - 1}{1 - \left(1 + \frac{\gamma - 1}{2} M_{2,rel}^2\right)^{-\frac{\gamma}{\gamma - 1}}}$$

DTEEC is interpolated from stator/rotor TEEC curves based on $\frac{t}{s}$

Tip Clearance Losses

$$Y_{TC} = 0.37 \cdot \frac{c}{b} \cdot \left(\frac{\delta}{c}\right)^{0.78} \cdot C_l^2 \cdot \frac{\cos^2 \beta_2}{\cos^3 \beta_{mean}}$$

δ tip gap height

$$\beta_{mean} = \arctan\left(\frac{\tan \beta_1 + \tan \beta_2}{2}\right)$$

Shock Loss

Interpolate hub mach number from radial position

$$M_{1,rel,hub} = f\left(\frac{r_H}{r_T}\right) \cdot M_{1,rel}$$

Calculate shock loss

$$\Delta \mathrm{P}_{q1,hub} = 0.75 \cdot \left(M_{1,rel,hub} - 0.4\right)^{1.75}$$

Scale by radius

$$\Delta \mathrm{P}_{q1,shock} = \frac{r_H}{r_T} \cdot \Delta P_{q1,hub}$$

Dimensionless pressure loss coefficient

$$Y_{shock} = \Delta P_{q1,shock} \cdot \frac{P_{1,rel}}{P_{2,rel}} \cdot \frac{1 - \left(1 + \frac{\gamma - 1}{2} M_{1,rel}^2\right)^{-\frac{\gamma}{\gamma - 1}}}{1 - \left(1 + \frac{\gamma - 1}{2} M_{2,rel}^2\right)^{-\frac{\gamma}{\gamma - 1}}}$$

Turbine Meanline Design – Station 2 Results

Results after Station 2 iterations

$V_{2,opt} = 448.53$
$T_{2t} = 1473.45$
$V_{2,ax} = 448.53$
$V_{2,tan} = 0$
$b_2 = 0.05548$
$Y_{total,rotor} = 0.12594$
$Y_{sec,rotor} = 0.05770$
$Y_{TC} = 0.01511$
$\rho_2 = 3.9010$

Turbine Meanline Design – Result Summary

omega (RPM)	25000	V1 (m/s)	700.36
L_is (kJ/kg)	313.51	V1ax (m/s)	332.04
L_euler (kJ/kg)	292.50	V1tang (m/s)	616.65
U_mean (m/s)	474.34	T1 (K)	1510.04
r_mean (m)	0.1812	T1t (K)	1700
b1 (m)	0.0555	P1 (bar)	22.90
V0 (m/s)	222.65	rho1 (kg/m^3)	5.2817
T0 (K)	1680.80	Y_total_stator	0.06128
P0 (bar)	38.01	beta1 (deg)	23.1986
rho0 (kg/m^3)	7.8768		

alfa0 (deg)	0	Y_TC	0.01511
V2opt (m/s)	448.53	T0t (K)	1700
V2ax (m/s)	448.53	P0t (bar)	40
V2tang (m/s)	0	P2t (bar)	20
b2 (m)	0.0558	Expansion Ratio	2
T2t (K)	1473.45	Mass Flow Total (kg/s)	110.77
rho2 (kg/m^3)	3.910	Work Coeff	1.3
Y_total_rotor	0.126	Flow Coeff	0.7
Y_secondary_rotor	0.0577	Efficiency_TT	0.933
Y_Secondary_calculated	0.0422		

2D Blade Generation – Procedure

Inputs

- Axial chord c_a
- Discretized percentage of chord $x_{\%}$
- Local thickness percentages $t_{\%}$
- Target trailing edge thickness t_{te}
- Maximum thickness t_{max}

Outputs

- $error$ — axial chord matching error
- x_{camber}, y_{camber} — camber line
- x_{ss}, y_{ss} — suction side
- x_{ps}, y_{ps} — pressure side
- t_{interp} — thickness distribution along camber

[1] Thickness Distribution over actual chord
├─ $t_{max,half} = \frac{t_{max}}{2}$
├─ $t_{TE_{half}} = \frac{t_{te}}{2}$
└─ interpolate final half-thickness

[2] Camber Line Generation (Circular Arc)
├─ $R = \frac{c_a}{2\sin\left(\frac{|\Delta\theta|}{2}\right)}$, stator: $\Delta\theta = \alpha_0 - \alpha_1$ - rotor: $\Delta\theta = \beta_1 - \beta_2$
├─ camber angle sweep in interval $[90 - \theta_{in}, 90 - \theta_{out}]$
├─ $x_{camber} = R \cdot \cos(\theta)$
├─ $y_{camber} = R \cdot \sin(\theta) - R$
└─ Normalize camber and rescale so it matches chordlength

[3] Interpolate Thickness Along Camber Line
└─ $t_{interp} = interpolate(x_{camber}, x_{\%}, t_{max})$

[4] Calculate Surface Normals
├─ $\frac{\mathrm{d}y}{\mathrm{d}x} = \nabla(y_{camber}, x_{camber})$
├─ $\theta_{slope} = \arctan\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$
└─ surface normal $n_x = -\sin(\theta_{slope})$, $n_y = \cos(\theta_{slope})$

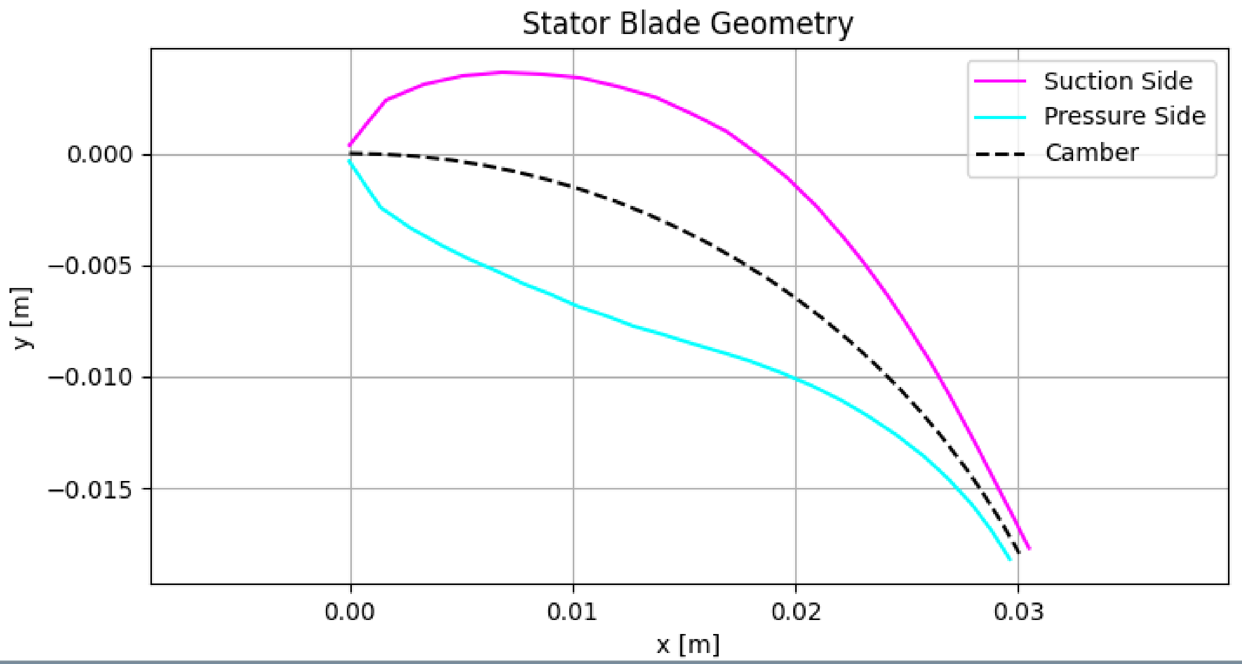
[5] Construct Blade Surfaces
├─ Suction Side: $x_{ss} = x_{camber} + t_{interp}\, n_x$, $y_{ss} = y_{camber} + t_{interp} n_y$
└─ Pressure Side: $x_{ps} = x_{camber} - t_{interp} n_x$, $y_{ps} = y_{camber} - t_{interp} n_y$

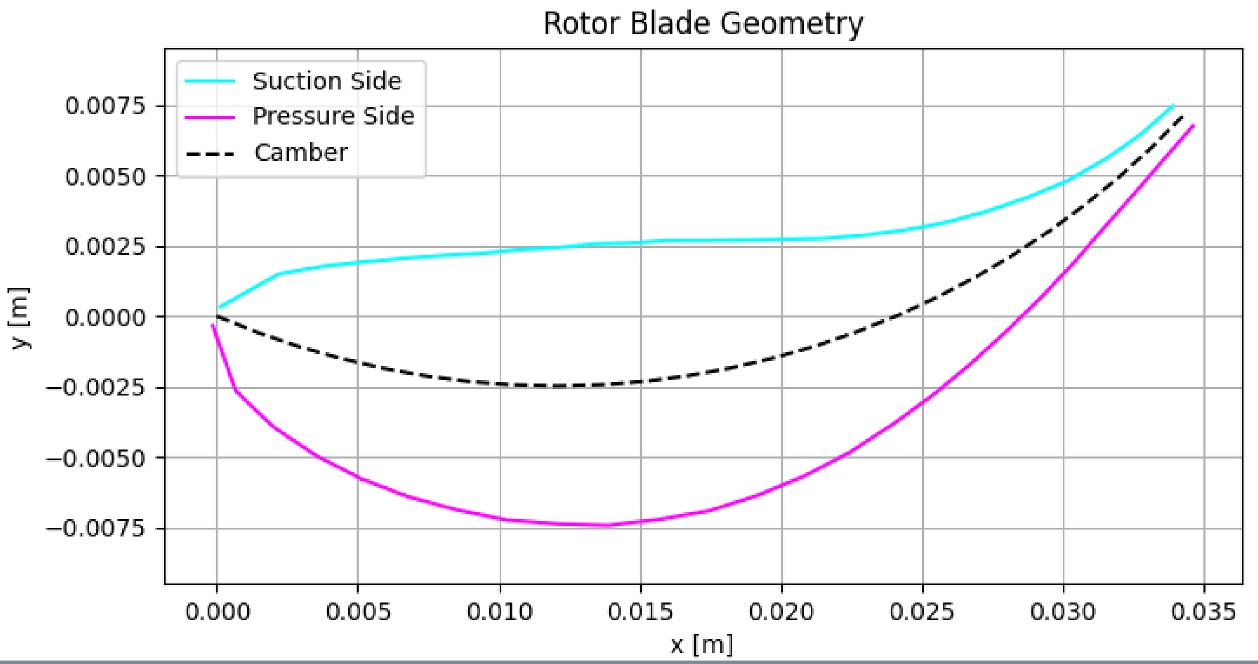
[6] Compute Chord Error
├─ $c_{actual} = \sqrt{(x_{end} - x_{start})^2 + (y_{end} - y_{start})^2}$
└─ $error = |c_{actual} - c_a|$

2D Blade Generation – NACA 65 series

Stator and rotor blades are generated from a NACA 65 blade.
The original blade is deflected, rotated, and translated

X%	0	1.25	2.5	5	7.5	10	15	20	30	40	50	60	70	80	90	95	98	100
Y%	0	1.124	1.571	2.222	2.709	3.111	3.746	4.218	4.824	5.057	4.87	4.151	3.038	1.847	0.749	0.354	0.2	0.15





TURBINE RADIAL EQUILIBRIUM

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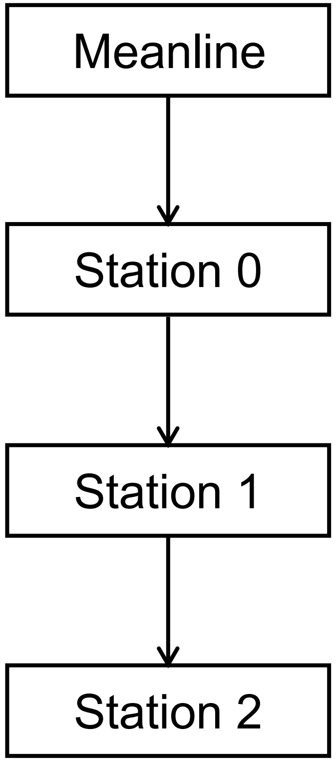
Turbine Radial Equilibrium – Flowchart



Uniform flow conditions for station 0

Mass-Averging to compare results with meanline values

$$x_{massAvg} = \frac{\int x(r)\dot{m}(r)dr}{\int \dot{m}(r)dr}$$

Turbine Radial Equilibrium – Station 0

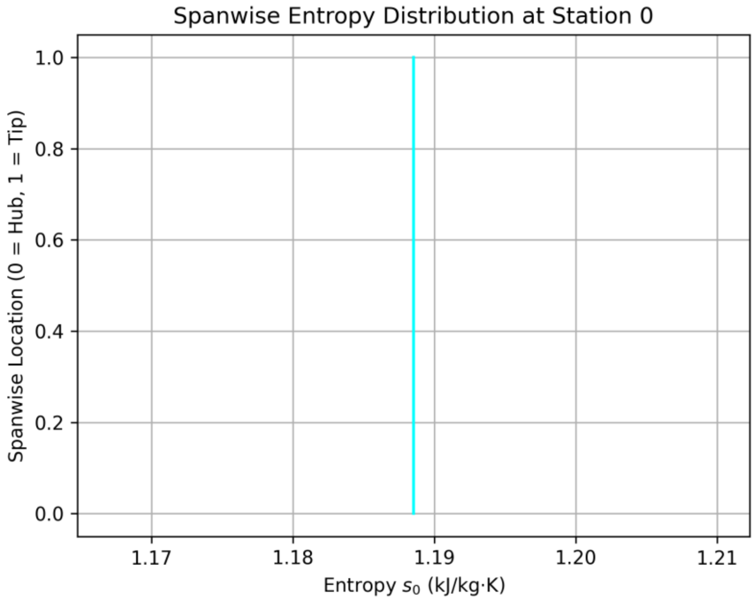
Meanline

$\downarrow b^i$

$$s_0 = c_p \ln\left(\frac{T_0}{T_{ref}}\right) - R \ln\left(\frac{P_0}{P_{ref}}\right)$$

$\downarrow$

Station 1



Uniform conditions at the inlet of the stator are assumed

Blade span is discretised, and calculations are carried on for each section to get spanwise dependency

Turbine Radial Equilibrium – Station 1

Station 0

$$V_{1,tan}(r) = V_{1,tan}^{midspan}\left(a\frac{r}{r_{mean}} + (1-a)\frac{r_{mean}}{r}\right) \qquad V_{1,ax}(r) = V_{1,ax}^{meanline} \quad \text{ISRE}$$

$a = 0.5$ Blended vortex – (0 – free vortex, 1 – forced vortex)

Assume constant $V_{1,ax}$

Initialize values that will be used in the loop calculation →

$$V_1 = \sqrt{V_{1,ax}^2 + V_{1,tan}^2} \qquad \rho_1 = \frac{P_1}{RT_1}$$

$$T_1 = T_{1t} - \frac{V_1^2}{2c_p} \qquad \dot{m}(r) = \rho_1(r)V_{1,ax}2\pi r b_1$$

$$P_1 = P_0\left(\frac{T_1}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$

Initialize loss distributions

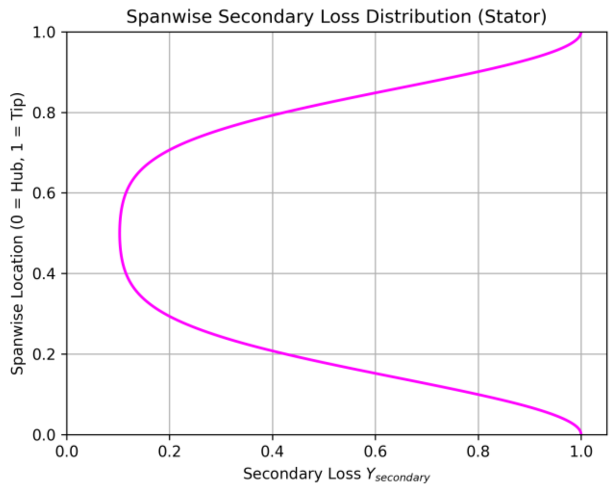
Shrouded blade assumed -> $Y_{tipClearance}(r) = 0$

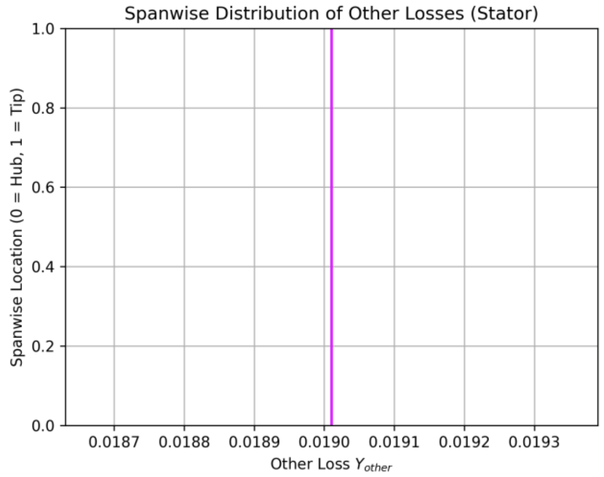
$Y_{secondary}(r)$ Gaussian profile from hub to tip

$Y_{other}(r)$ Scaled linear profile

Loop

Station 2





Turbine Radial Equilibrium – Loop @ Station 1

St.1 Init.

$$Y_{total} = Y_{other} + Y_{secondary}$$

$$P_{1t} = \frac{P_{0t} + P_1 Y_{total}}{1 + Y_{total}}$$

$$s_1 = s_0 - R \ln\left(\frac{P_{1t}}{P_{0t}}\right)$$

$$\frac{\partial h_t}{\partial r} = T\frac{\partial s}{\partial r} + V_{ax}\frac{\partial V_{ax}}{\partial r} + \frac{V_t}{r}\frac{\partial (rV_t)}{\partial r}$$ Non Isentropic Radial Equilibrium

$$\frac{\partial V_{ax}(r)}{\partial r} = -\frac{T(r)}{V_{ax}(r)}\frac{\partial s(r)}{\partial r}$$ scipy.integrate.solve_ivp

$$V_1 = \sqrt{V_{1,ax}^2 + V_{1,tan}^2}$$

Update values

$$T_1 = T_{1t} - \frac{V_1^2}{2c_p}$$

$$P_1 = \frac{P_{1t}}{\left(\frac{T_{1t}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\rho_1 = \frac{P_1}{RT_1}$$

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

$$W_{1,ax} = V_{1,ax}$$

$$W_{1,tan} = V_{1,tan} - U$$

$$W_1 = \sqrt{W_{1,ax}^2 + W_{1,tan}^2}$$

$$\beta_1 = \mathrm{arctg}\left(\frac{W_{1,tan}}{W_{1,ax}}\right)$$

$$M_{1,rel} = \frac{W_1}{\sqrt{\gamma R T_1}}$$

$$P_{1t,rel} = P_1\left(1 + \frac{\gamma - 1}{2}M_{1,rel}^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\alpha_2 = \mathrm{arctg}\left(\frac{V_{2,tan}}{V_{2,ax}}\right)$$

calculate $\dot{m} = \rho_1(r) V_{1,ax} 2\pi r b_1$

$$error = \left|\frac{\dot{m} - \dot{m}_{actual}}{\dot{m}_{actual}}\right|$$

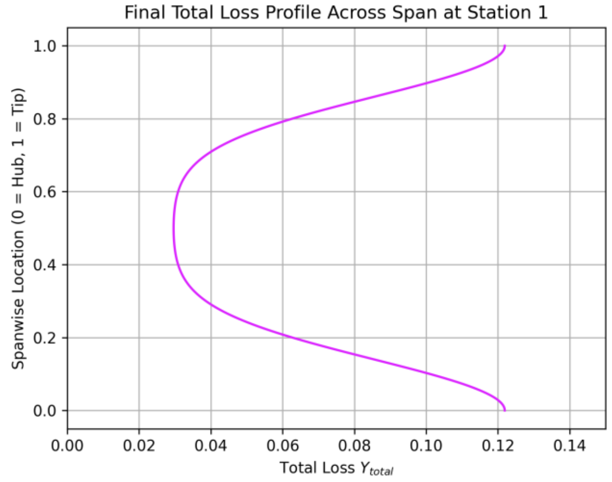
$|error| < 10^{-8}$

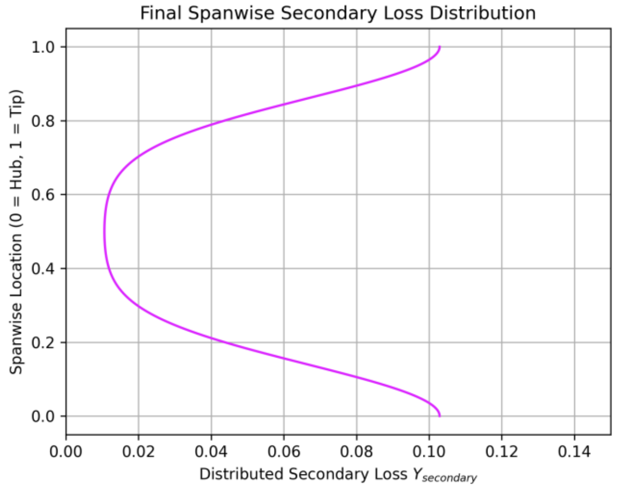
no

yes

Station 2

Turbine Radial Equilibrium – Station 1



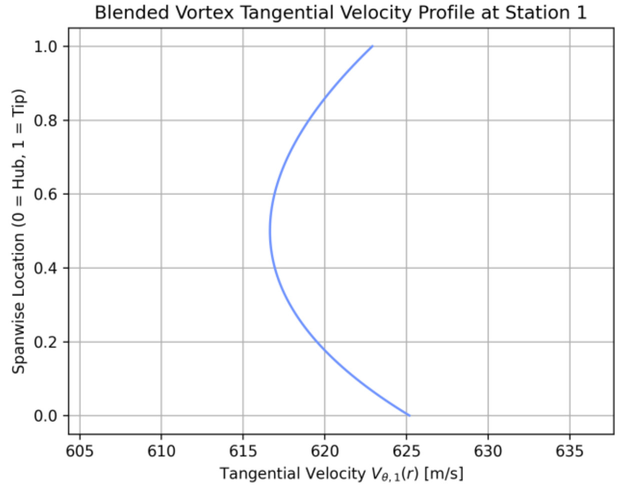


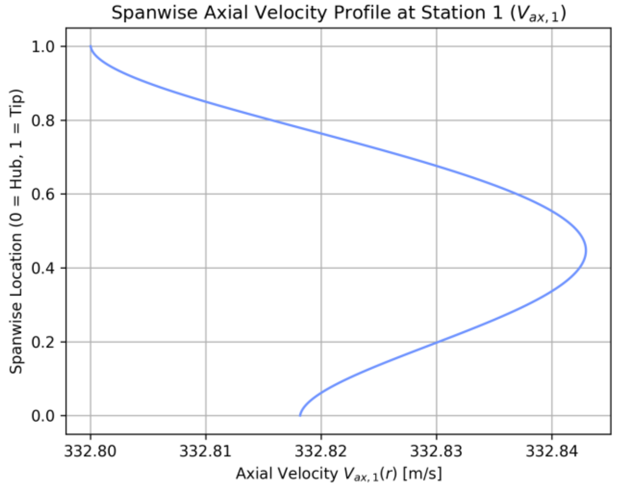
Mass averaged quantities

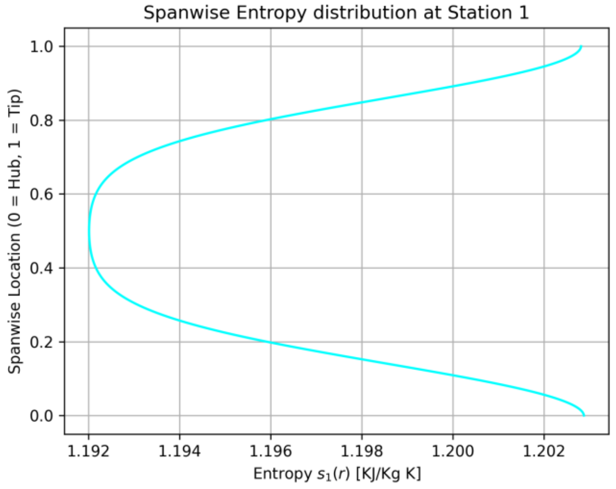
V_{1ax}	V_{1tan}	V_1	α_1	T_1	T_{1t}	P_1	s_1	$\dot{m}_{total}$
332.83	619.03	702.83	61.73	1508.7	1700	22.81	1.1957	110.77

Quantities from meanline calculations

V_{1ax}	V_{1tan}	V_1	α_1	T_1	T_{1t}	P_1	$\dot{m}_{total}$
332.00	616.65	700.36	61.69	1510.0	1700	22.90	110.77







Turbine Radial Equilibrium – Station 2

Station 1

$$V_{2,tan}(r) = V_{2,tan}^{midspan}\left(a\frac{r}{r_{mean}} + (1-a)\frac{r_{mean}}{r}\right) \qquad V_{2,ax}(r) = V_{2,ax}^{meanline} \text{ ISRE}$$

$a = 0.5$ Blended vortex – (0 – free vortex, 1 – forced vortex)

Assume constant $V_{2,ax}$

Initialize values that will be used in the loop calculation →

$$V_2 = \sqrt{V_{2,ax}^2 + V_{2,tan}^2} \qquad \rho_2 = \frac{P_2}{RT_2}$$

$$T_2 = T_{2t} - \frac{V_2^2}{2c_p} \qquad M_2 = \frac{V_2}{\sqrt{\gamma RT_2}}$$

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \qquad \dot{m}(r) = \rho_2(r)V_{2,ax}(r)2\pi r b_2$$

Initialise loss distributions:

$Y_{tipClearance}(r)$ Tip clearance loss (Gaussian)

$Y_{secondary}(r)$ Gaussian profile assumed

$Y_{other}(r)$ Scaled linear profile

↓

Loop

↓

Blade generation

Turbine Radial Equilibrium – Loop @ Station 2

St.2 Init.

$$Y_{total} = Y_{other} + Y_{secondary} + Y_{tipClearance}$$

$$P_{2t,rel} = \frac{P_{1t,rel} + P_2 Y_{total}}{1 + Y_{total}} \quad P_{2t} = P_{2t,rel}\left(\frac{T_{2t}}{T_{2t,rel}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$s_2 = s_1 + c_p \ln\left(\frac{T_{2t}}{T_{1t}}\right) - R\ln\left(\frac{P_{2t}}{P_{1t}}\right)$$

$$\frac{\partial h_t}{\partial r} = T\frac{\partial s}{\partial r} + V_{ax}\frac{\partial V_{ax}}{\partial r} + \frac{V_t}{r}\frac{\partial (rV_t)}{\partial r}$$ Non Isentropic Radial Equilibrium

$\Delta h_{euler} = U(V_{1,tan} - V_{2,tan})$ And gradient $\frac{\partial}{\partial r}\Delta h_{euler}$

$$\frac{\partial V_{ax}(r)}{\partial r} = \frac{1}{V_{ax}c_p}\frac{\partial \Delta h_{euler}}{\partial r} - \frac{T(r)}{V_{ax}(r)}\frac{\partial s(r)}{\partial r}$$ scipy.integrate.solve_ivp

Update values

$$V_2 = \sqrt{V_{2,ax}^2 + V_{2,tan}^2}$$

$$T_2 = T_{2t} - \frac{V_2^2}{2c_p}$$

$$P_2 = \frac{P_{2t}}{\left(\frac{T_{2t}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\rho_2 = \frac{P_2}{RT_2}$$

$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}}$$

$$W_{2,ax} = V_{2,ax}$$

$$W_{2,tan} = V_{2,tan} - U$$

$$W_2 = \sqrt{W_{2,ax}^2 + W_{2,tan}^2}$$

$$\beta_2 = \mathrm{arctg}\left(\frac{W_{2,tan}}{W_{2,ax}}\right)$$

$$\alpha_2 = \mathrm{arctg}\left(\frac{V_{2,tan}}{V_{2,ax}}\right)$$

calculate $\dot{m} = \rho_1(r)V_{1,ax}2\pi r b_1$

$$error = \left|\frac{\dot{m} - \dot{m}_{actual}}{\dot{m}_{actual}}\right|$$

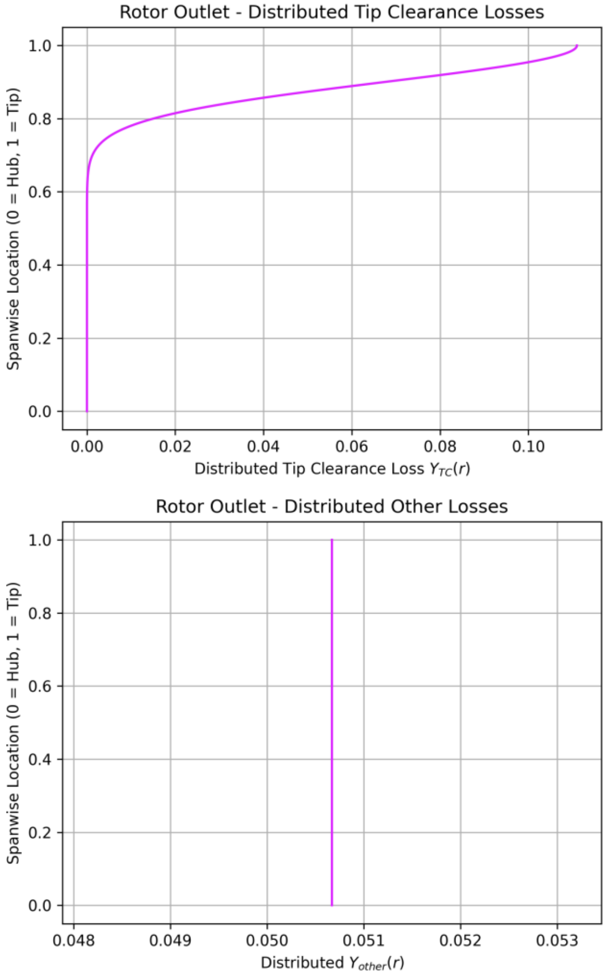
$|error| < 10^{-8}$

no

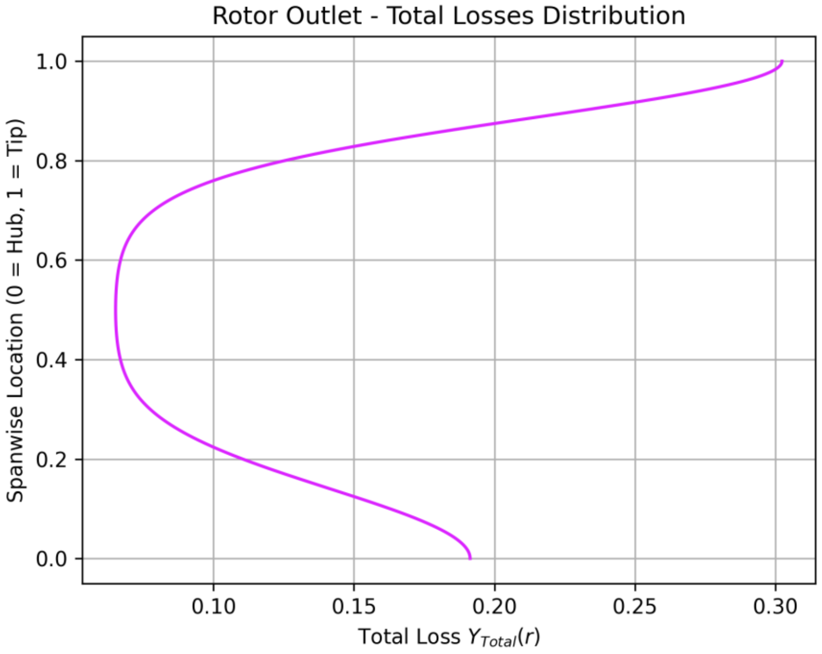
yes

Blade gen.

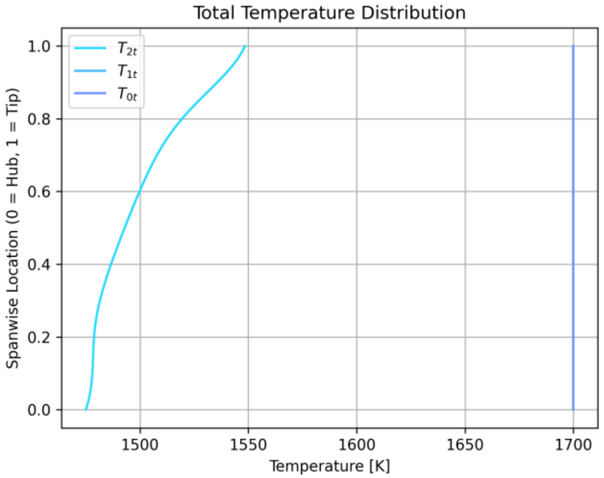
Turbine Radial Equilibrium – Losses Station 2

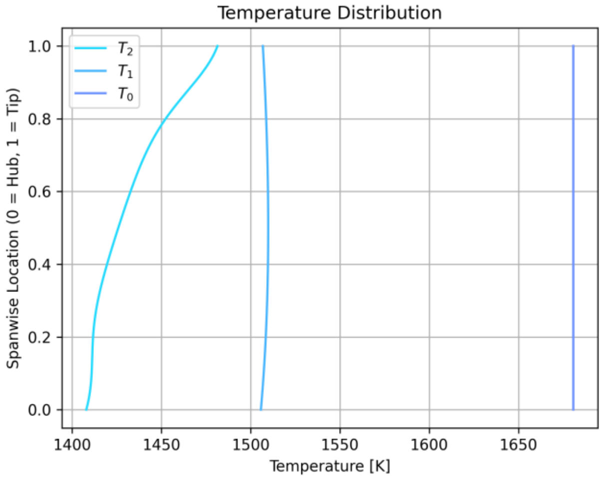


These are the distributions of separate losses from different effects, and the combined total losses



Turbine Radial Equilibrium – P&T Station 2



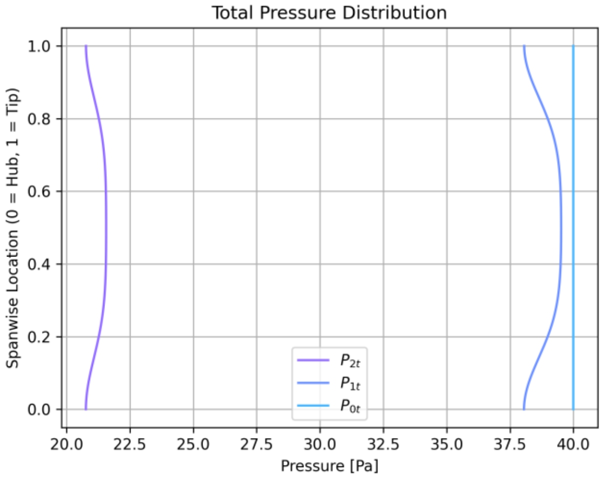


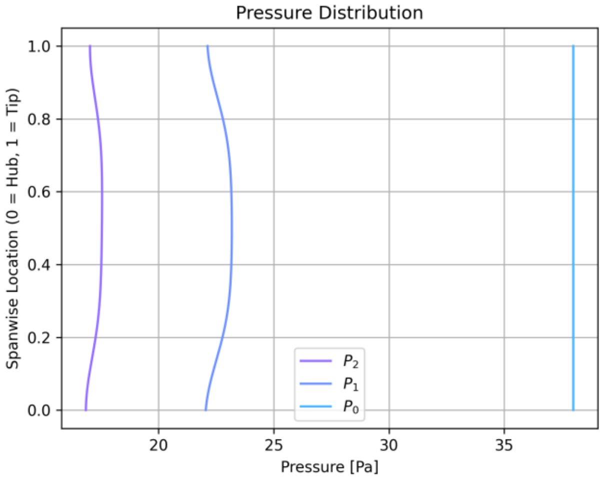
$V_{2,ax}$	$V_{2,tan}$	V_2	α_2	T_2	P_2	s_2	$\dot{m}_{total}$
416.19	0	416.1892	0	1433.8	17.33	1.2087	110.77

Thre's a bit of a discrepancy between the mass averaged values and the ones calculated during meanline calculations.

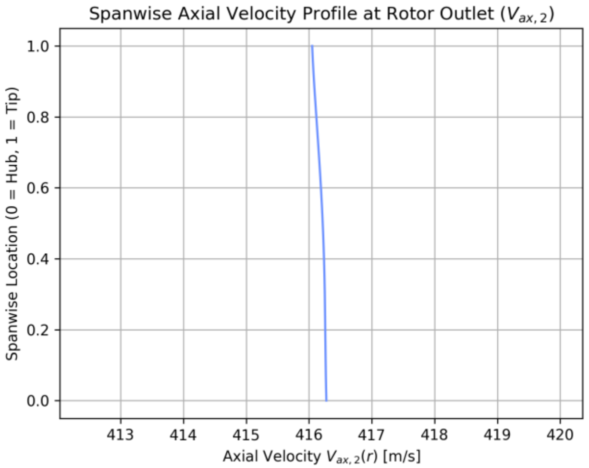
$V_{2,ax}$	$V_{2,tan}$	V_2	α_2	T_2	P_2	$\dot{m}_{total}$
448.53	0	448.53	0	1395.54	15.664	110.77

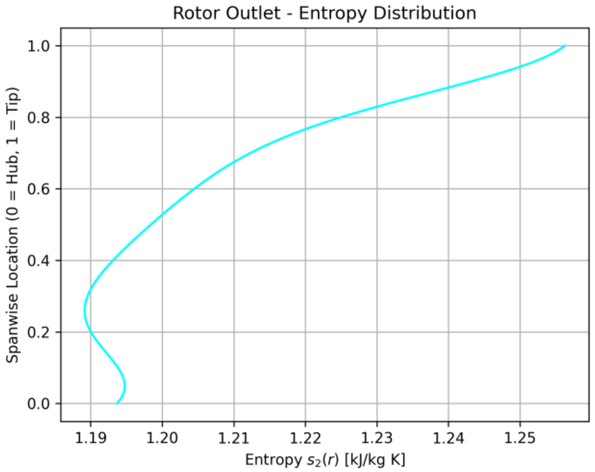
This probably occurs because of initial assumptions of loss distributions and temperature profiles.



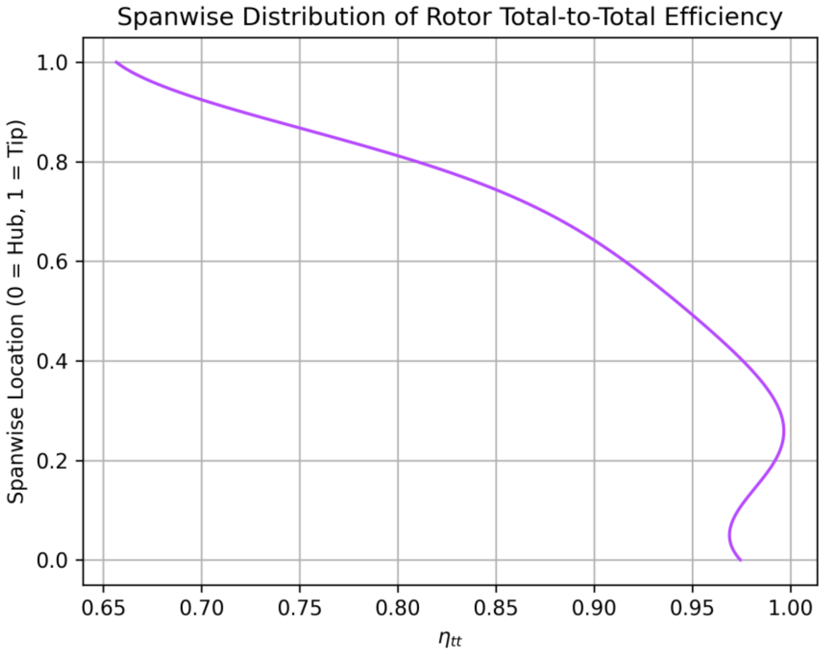


Turbine Radial Equilibrium – Results Station 2





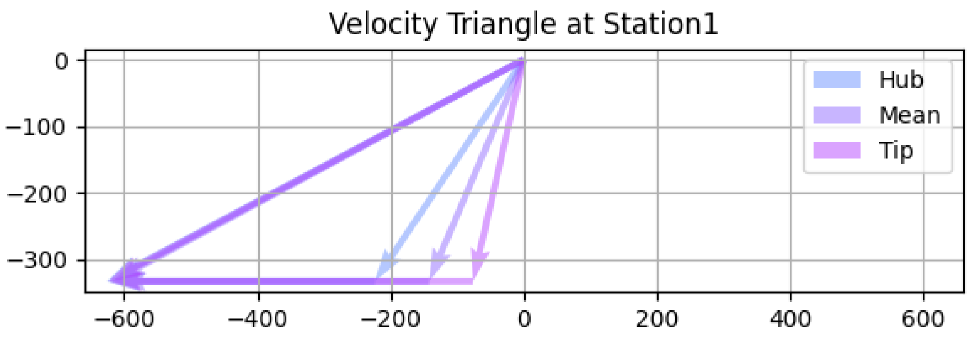
Mass averaged efficiency
$\eta_{TT} = 0.89599$

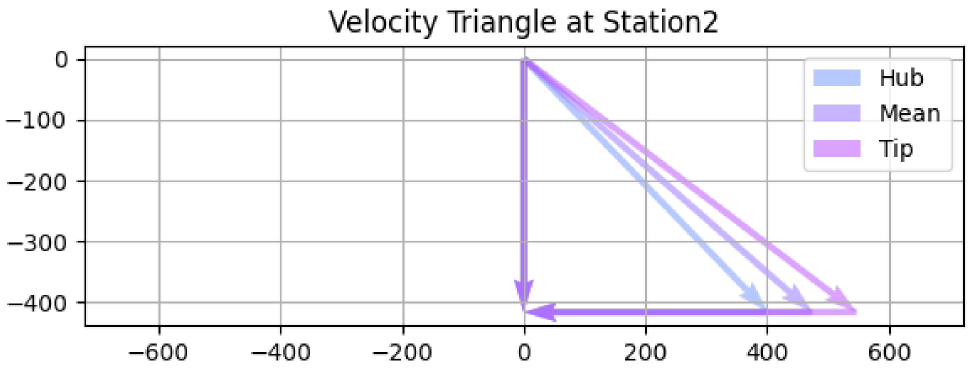


Again there's a difference from the value assumed in the begining

Turbine Radial Equilibrium – Velocity Triangles

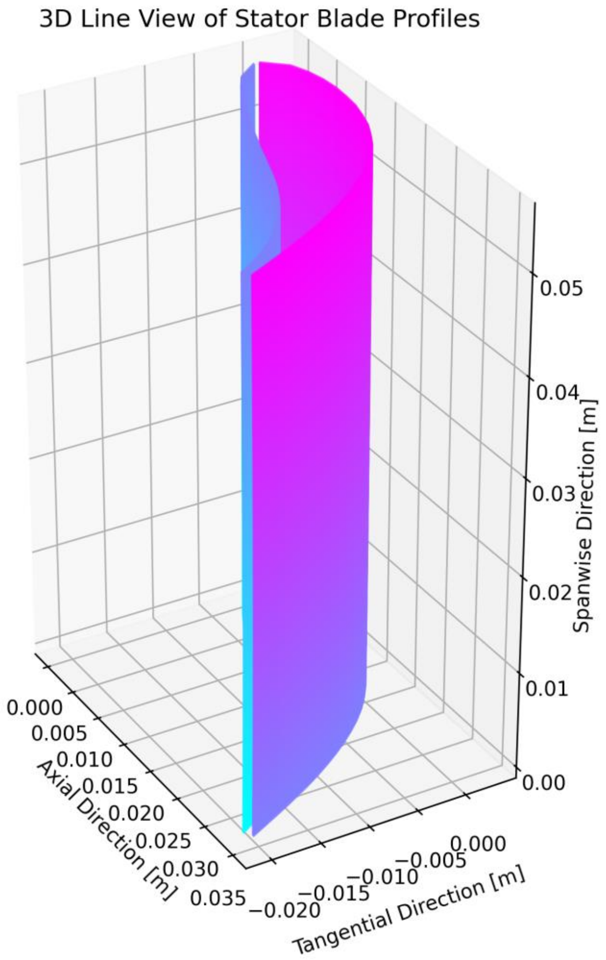
- **Hub:** Low blade speed → modest turning → reduced work and losses
- **Midspan:** Matches meanline design → balanced efficiency and loading
- **Tip:** High blade speed → sharp turning → high work, high losses

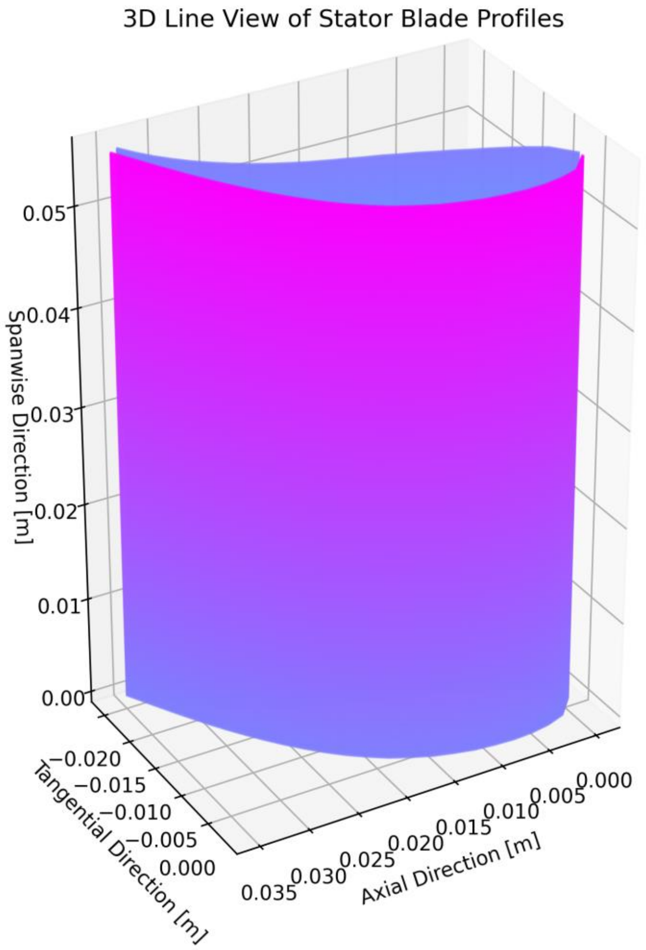




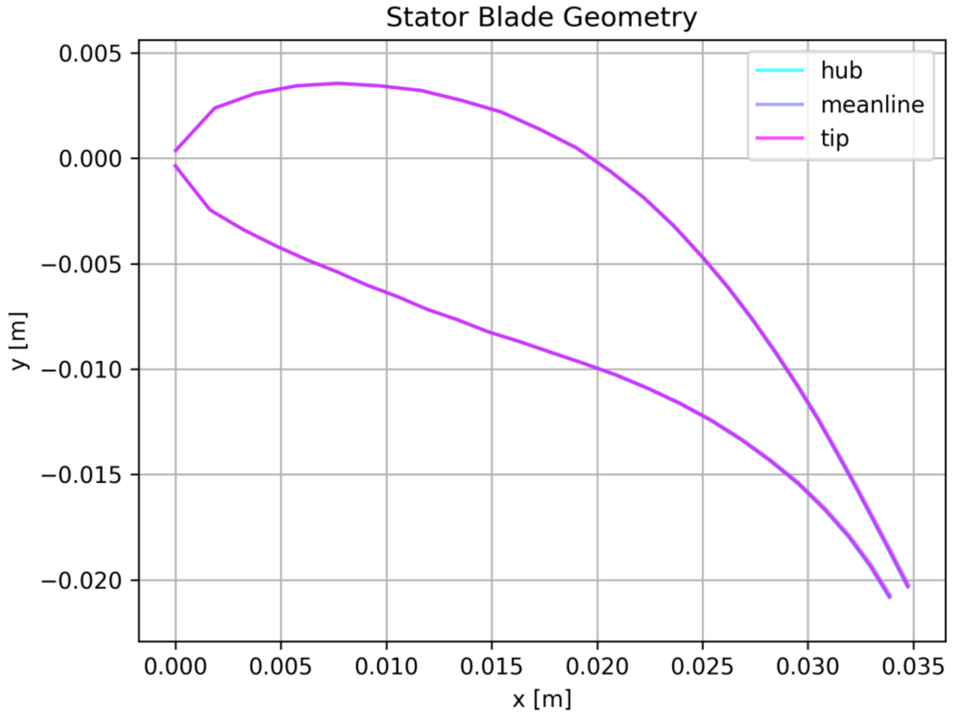
Design Implication: Blade must be twisted and tapered to align with spanwise flow angles

3D Blade Generation – Stator

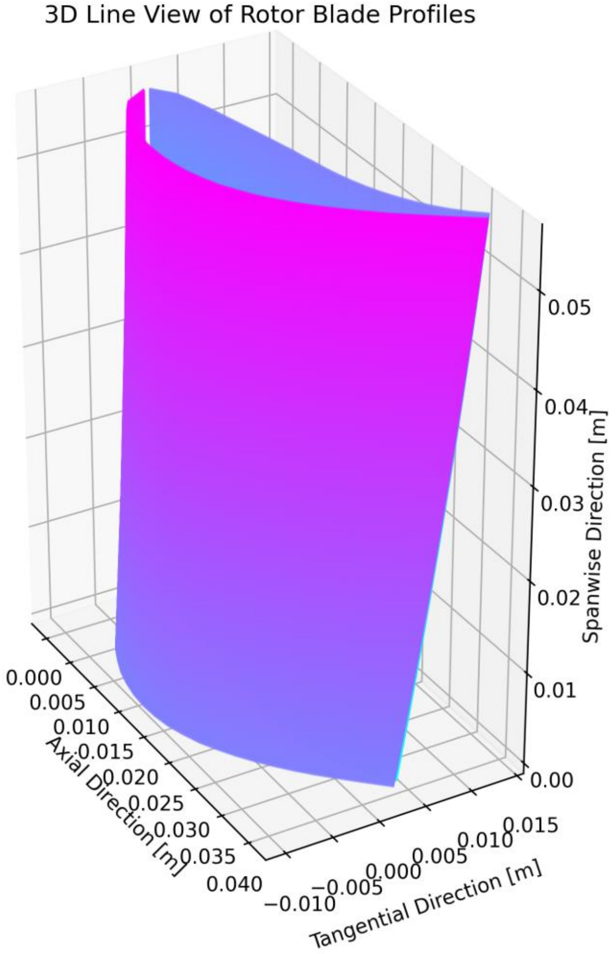


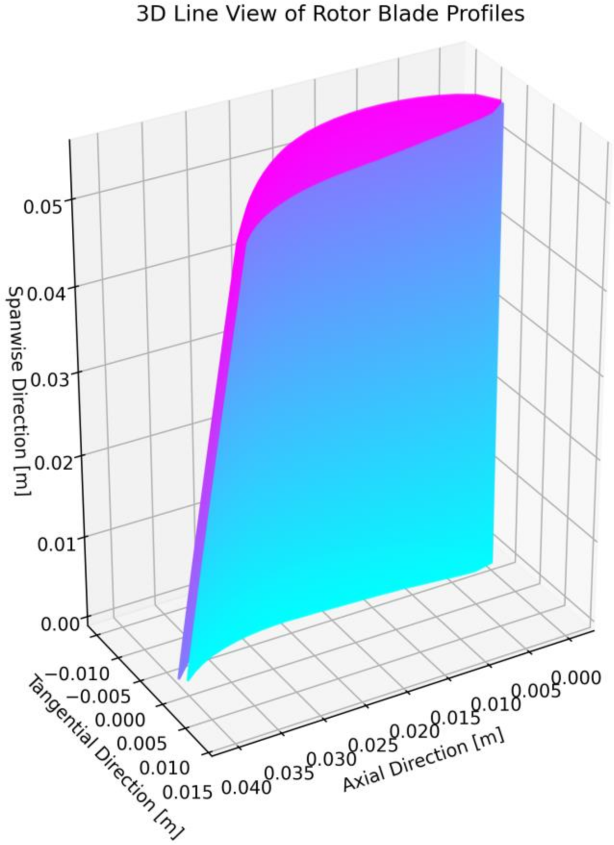


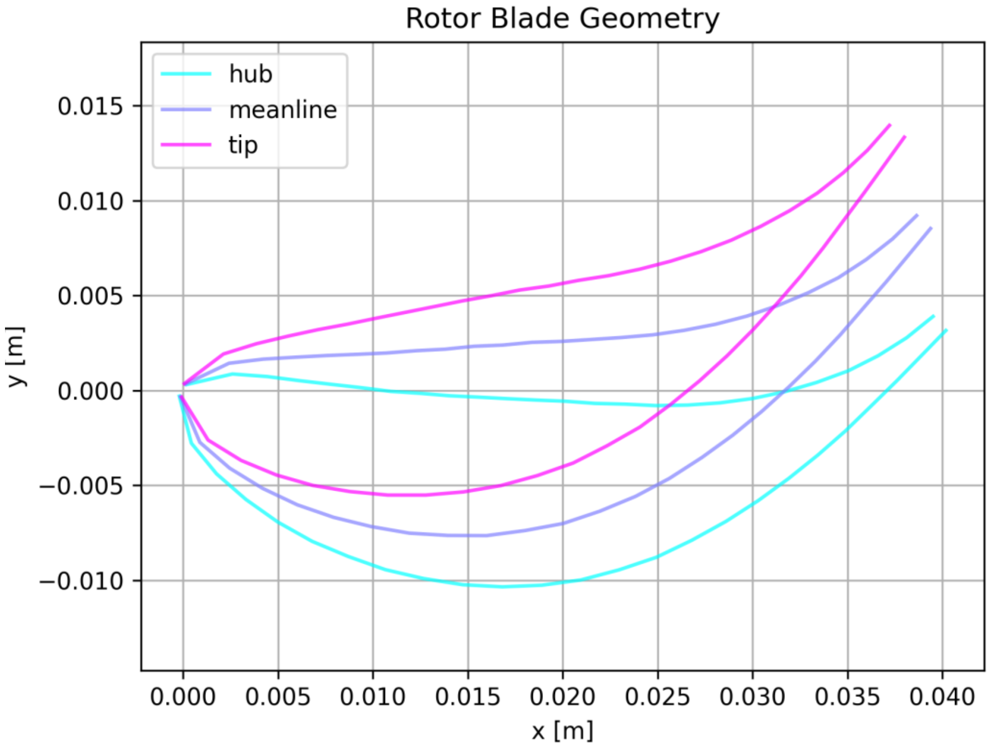
Following the same procedure as before, calculated at each blade height



3D Blade Generation – Rotor







COMPRESSOR

Gašper Lotrič, Turbomachinery A – Course Project

Compressor – Design Choices

Repeating stage assumption: $V = const.$
Through stages 0-1-2

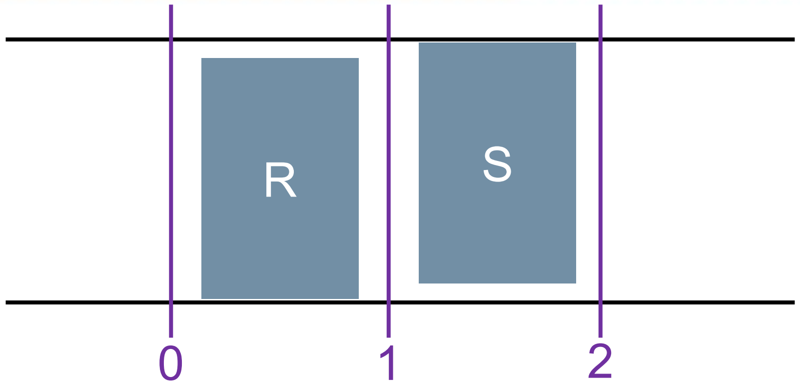
Rotational speed similar to the turbine stage

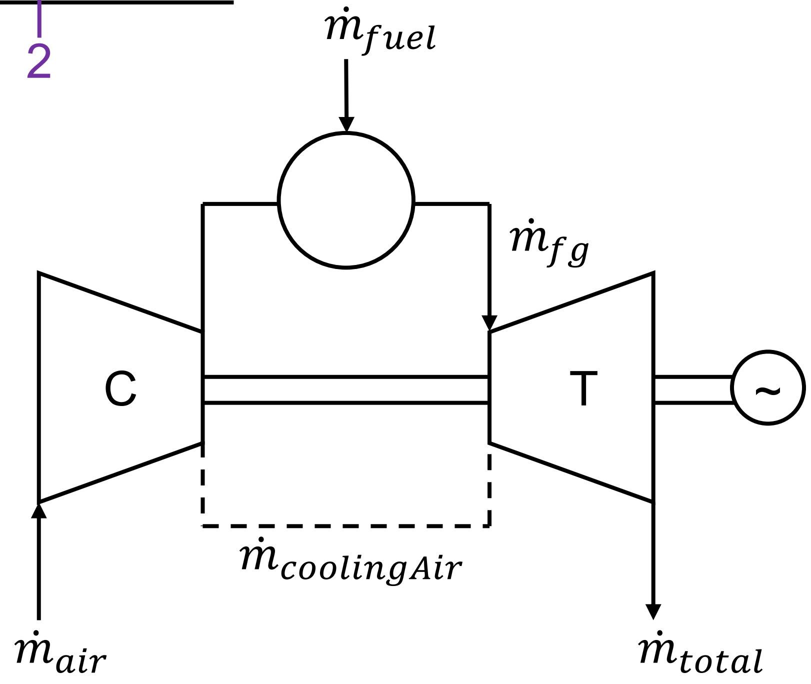
Constant axial velocity

Constant mean diameter

Perfect gas

Cooling air is discharged from the last stage





Compressor – Inlet and Outlet Conditions

Combustor energy balance

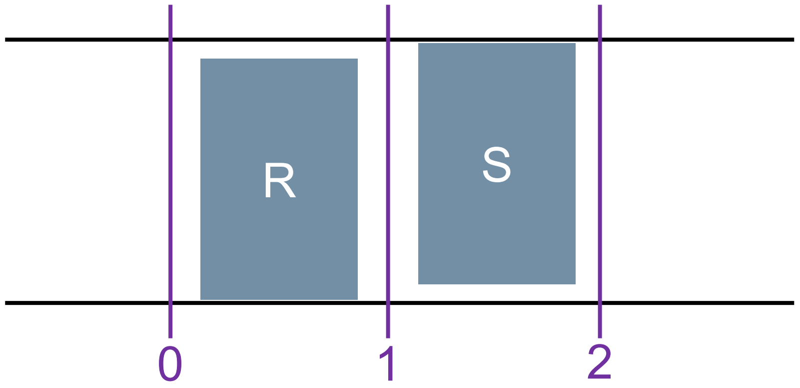
$$\dot{m}_{air}c_{p,air}\left(T_{t,out}-T_{ref}\right)+\dot{m}_{fuel}c_{p,fuel}\left(T_{fuel}-T_{ref}\right)+\dot{m}_{fuel}LHV\eta_{comb}=\dot{m}_{fg}c_{p,fg}(T_0-T_{ref})$$

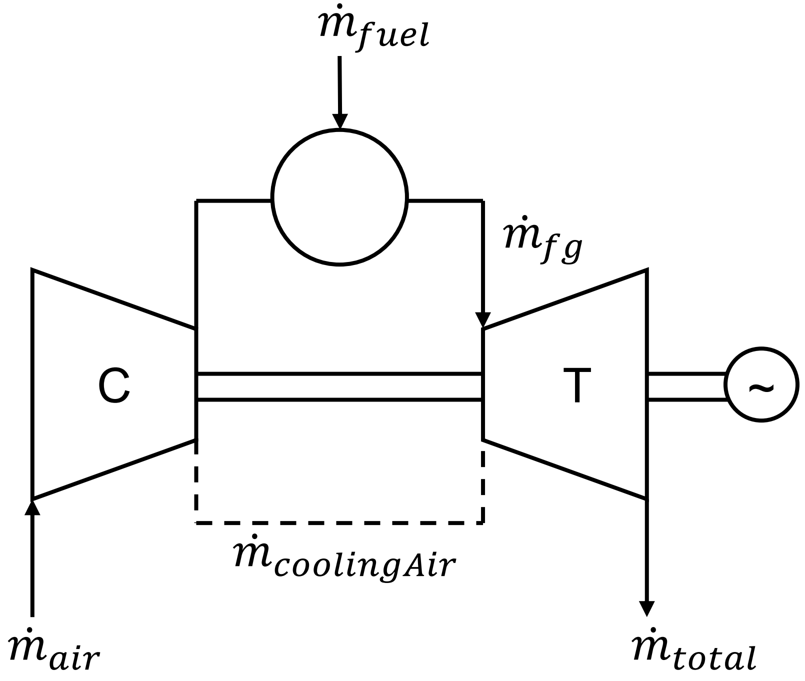
Mass balance

$$\dot{m}_{fg}=\frac{\dot{m}_{air}}{1-coolingAir}+\dot{m}_{fuel}$$

Euler power (TURBINE) – must be equal to the compressor power

$$P_{euler}=\dot{m}_{fg}c_p(T_{0t}-T_{2t})$$





Compressor – Combustor Balance

Compressor outlet Temperature from combustor balance

$$h_{out} = (\dot{m}_{air} + \dot{m}_{fuel})c_{p,mix}(T_{0t} - T_{ref}) - \dot{m}_{fuel}(LHV + c_{p,fuel}(T_{fuel} - T_{ref})$$

$$T_{t,out} = \frac{h_{out}}{c_{p,air}\dot{m}_{air}} + T_{ref}$$

$$T_{t,in} = T_{out} - \frac{P_{euler}}{c_{p,air}\dot{m}_{air}}$$

Isentropic outlet temperature

$$T_{t,out,is} = T_{in,comp} + \frac{T_{t,out} - T_{t,in}}{\eta_{TT}}$$

Pressure ratio

$$\frac{P_{out}}{P_{in}} = \left(\frac{T_{t,out,is}}{T_{t,in}}\right)^{\frac{\gamma}{\gamma-1}}$$

Compressor – Stages

Assume pressure ratio per stage from data for similar engines

$\pi_{stage} = 1.29$

Calculate the Number of Stages needed

$$N_{stages} = integer\left(\frac{\ln\left(\frac{P_{out}}{P_{in}}\right)}{\ln(\pi_{stage})}\right) = 6$$

Update pressure ratio per stage to match

$\pi_{stage}^{new} = 1.23$

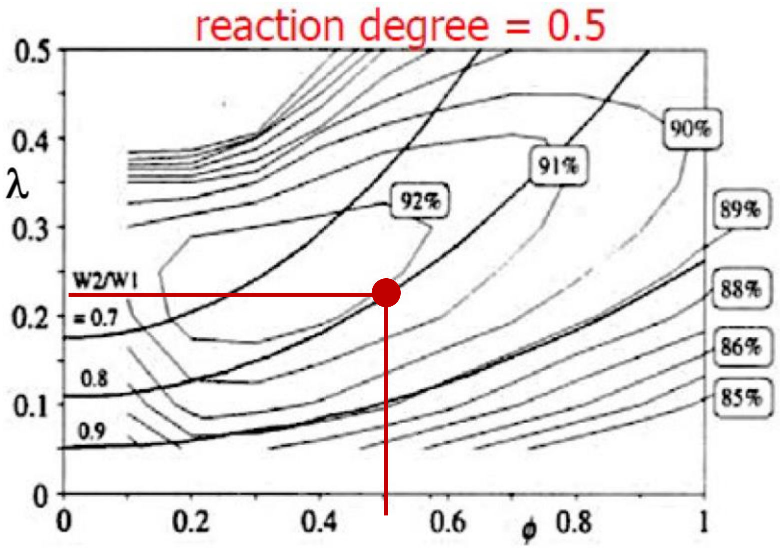
Compressor – Non dimensional Performance Charts

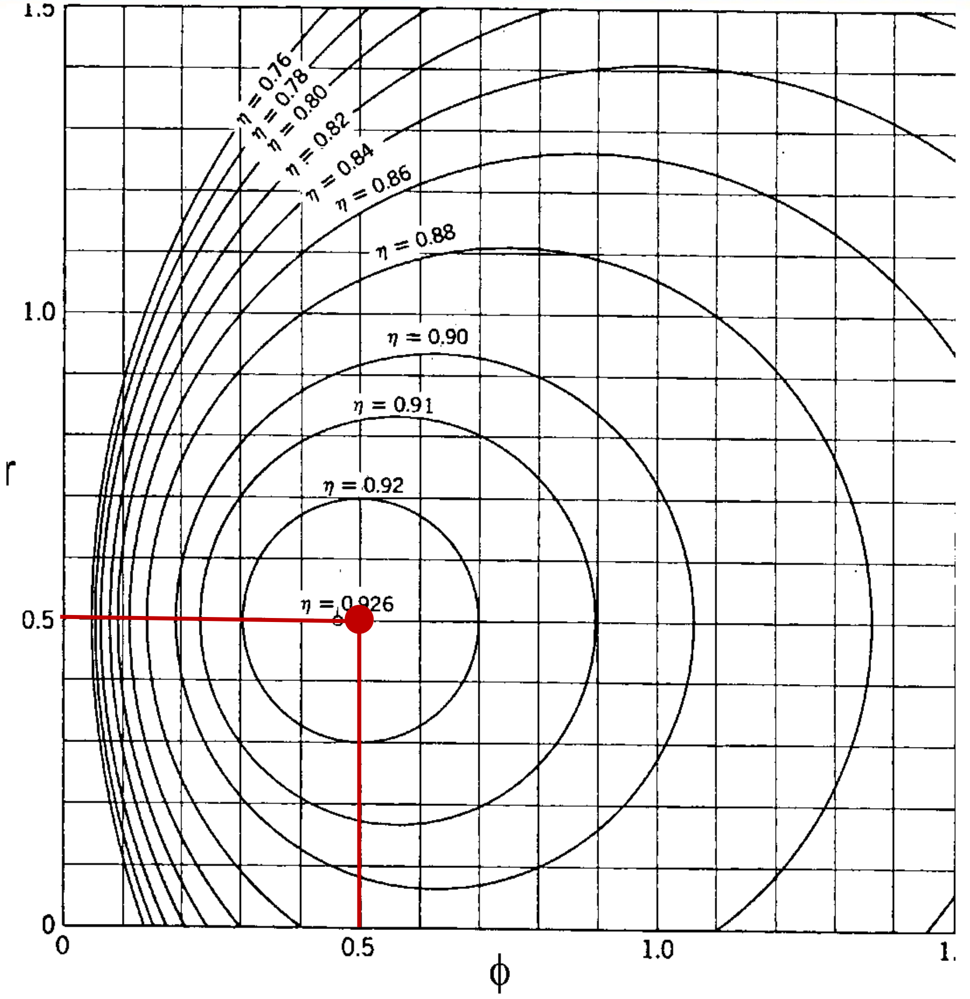
Vavra Chart

Blade loading chart $\chi = 0.5$

$$\lambda = \frac{L_{euler}}{U^2}$$

Flow coefficient: $\phi = 0.5$
Reaction degree: $\chi = 0.5$
Work coefficient: $\lambda = 0.25$
Efficiency: $\eta_{TT} = 0.92$





Compressor – Blade Geometry

Mean radius from non-dimensional coefficients

$$r_{mean} = \frac{1}{\omega}\sqrt{\frac{L_{euler}}{\lambda}}$$

$$U = r_{mean}\omega$$

Absolute velocity

$$V_{1,ax} = \phi U$$

$$V_{1,tan} = U\left(1 - \chi - \frac{\lambda}{2}\right)$$

Assuming constant mean radius and blade height (in stage)

$$\Delta T_{stage} = \frac{T_{t,out} - T_{t,in}}{N_{stages}}$$

Work done per stage

$$w_{stage} = c_p(T)\Delta \mathrm{T_{stage}}$$

For each stage

Total quantities

$$T_t^{i+1} = T_t^i + \Delta T_{stage}$$

$$P_t^{i+1} = P_t^i + \pi_{stage}$$

$$\Delta \mathrm{h_t^i} = \mathrm{w_{stage}}$$

Static quantities

$$T_s^i = T_t^i - \frac{V_{1,ax}^2}{2c_p(T)}$$

$$P_s^i = \frac{P_t^i}{\left(1 + \frac{\gamma - 1}{2} M_{ax}^2\right)^{\frac{\gamma}{\gamma-1}}}$$

$$M_{ax} = \frac{V_{1,ax}}{\sqrt{\gamma R T_s^i}}$$

$$\rho^i = \frac{P_s^i}{R T_s^i}$$

Blade Height

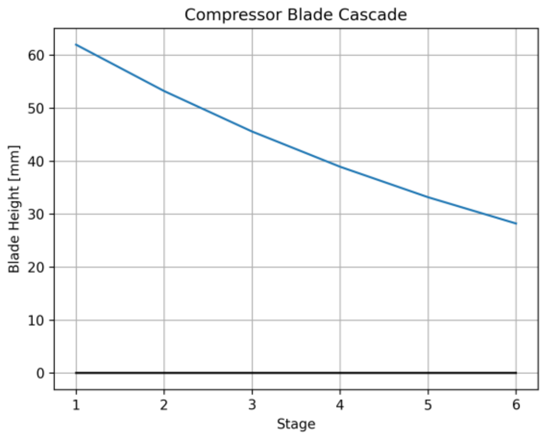
$$b_i = \frac{\dot{m}}{2\pi r_{mean}\rho_i V_{1,ax}}$$

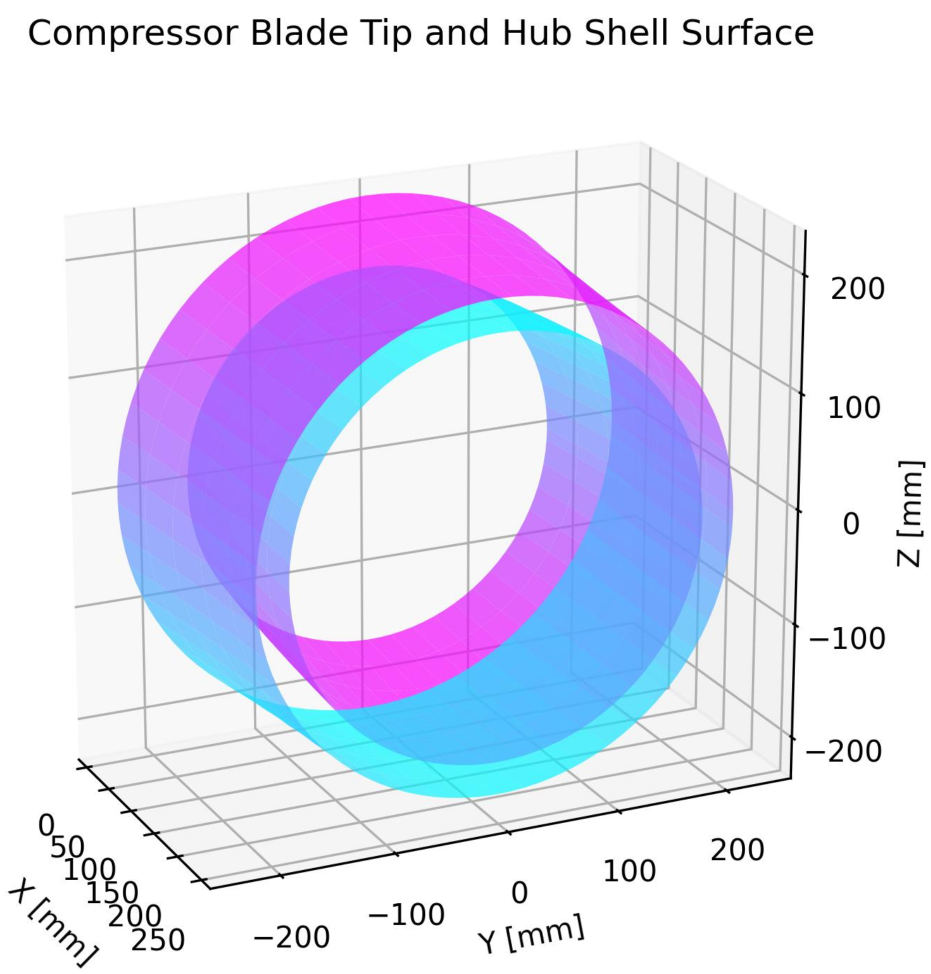
Compressor – Results

Stage	P_t [bar]	T_t [K]	Δh [kJ/kg]	T_s [K]	M_{ax}	P_s [bar]	ρ [kg/m^3]	b [m]
1	14.2	892	74.54	867.7	0.447	12.45	5.00	0.06199
2	17.5	948	75.13	924.1	0.433	15.44	5.82	0.05323
3	21.5	1005	75.71	980.4	0.421	19.13	6.80	0.04558
4	26.4	1061	76.26	1036.8	0.409	23.69	7.96	0.03894
5	32.5	1117	76.79	1093.2	0.398	29.30	9.34	0.03318
6	40	1174	77.30	1149.5	0.389	36.24	10.98	0.02822

Turbine and compressor power match to the 11th decimal

32400.022759904372
32400.022759904376





THANK YOU FOR LISTENING