

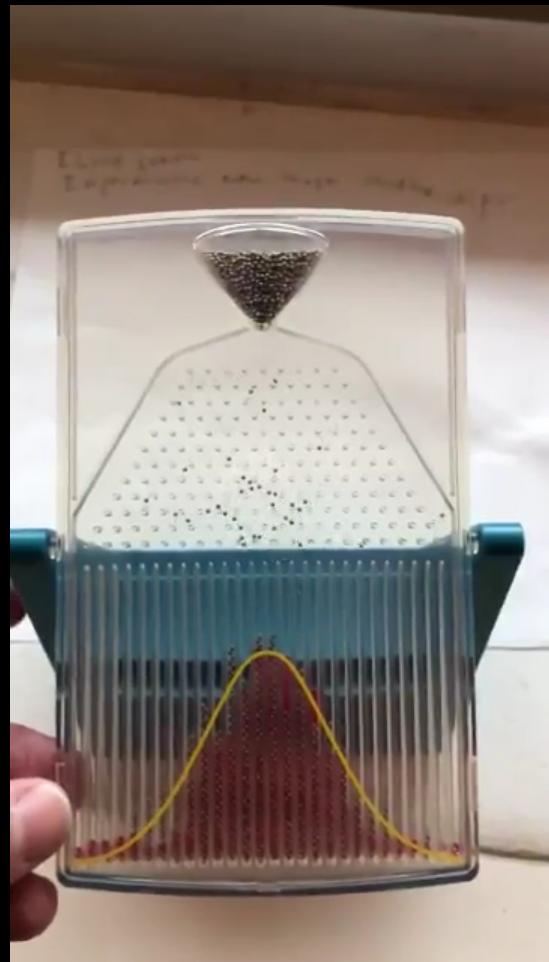
# Likelihood-free inference in Physical Sciences

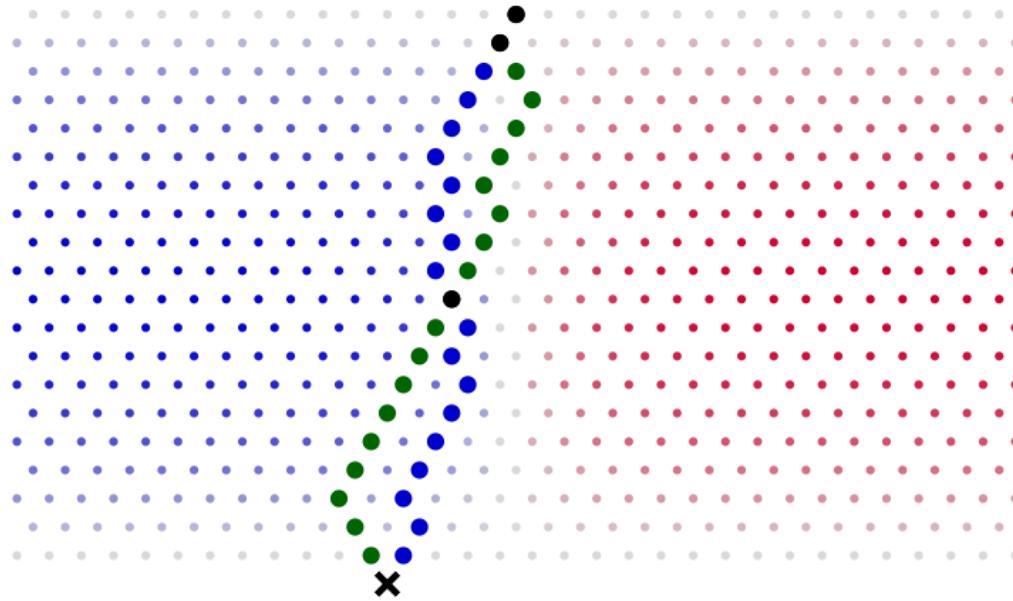
Artificial Intelligence and Physics

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March 22, 2019

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[@glouppe](https://twitter.com/glouppe)







The probability of ending in bin  $\textcolor{teal}{x}$  corresponds to the total probability of all the paths  $\textcolor{teal}{z}$  from start to  $\textcolor{teal}{x}$ .

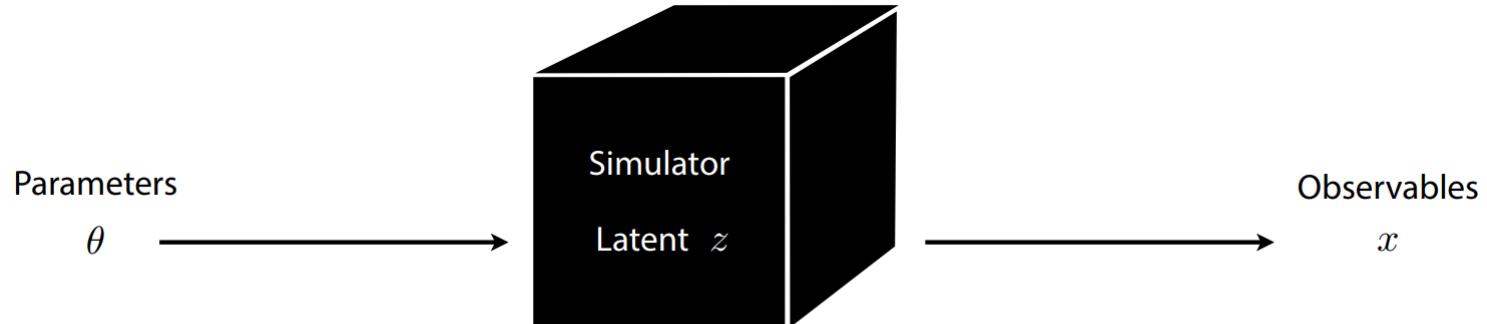
$$p(x|\theta) = \int p(x, z|\theta) dz = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$

But what if we shift or remove some of the pins?

The Galton board is a **metaphor** of simulation-based science:

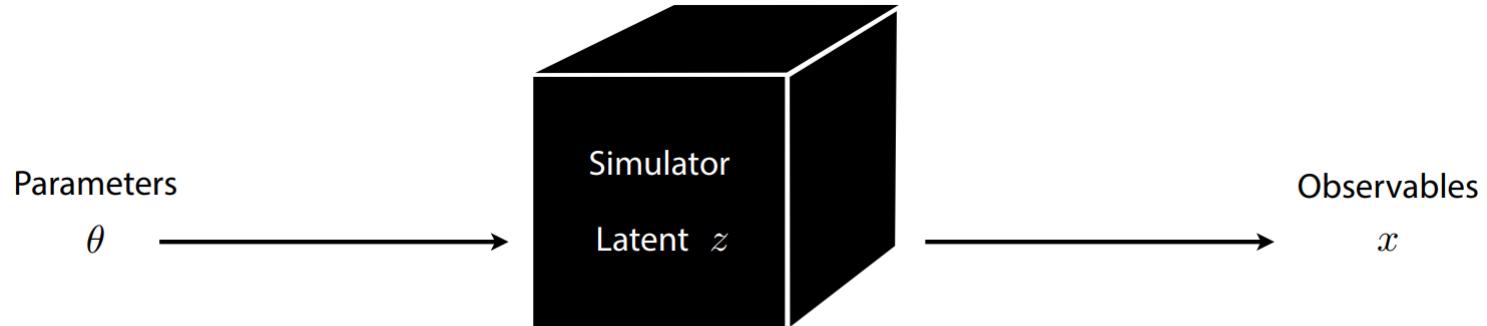
Galton board device	→	Computer simulation
Parameters $\theta$	→	Model parameters $\theta$
Buckets $x$	→	Observables $x$
Random paths $z$	→	Latent variables $z$ (stochastic execution traces through simulator)

Inference in this context requires **likelihood-free algorithms**.



Prediction:

- Well-understood mechanistic model
- Simulator can generate samples



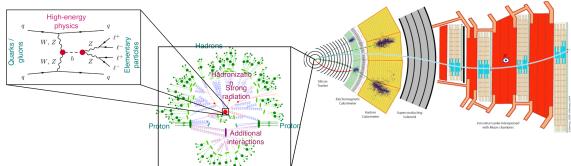
Prediction:

- Well-understood mechanistic model
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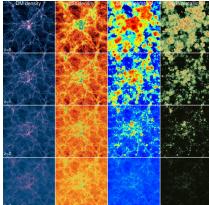
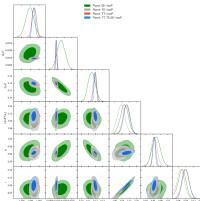
Inference:

- Likelihood function  $p(x|\theta)$  is intractable
- Inference based on estimator  $\hat{p}(x|\theta)$

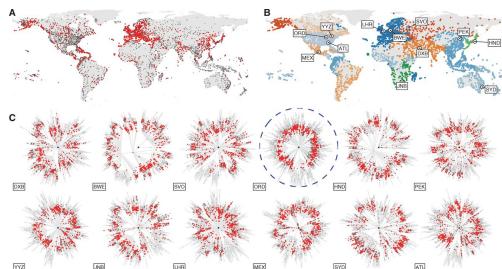
# Applications



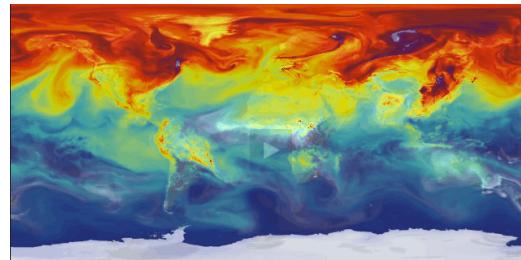
Particle physics



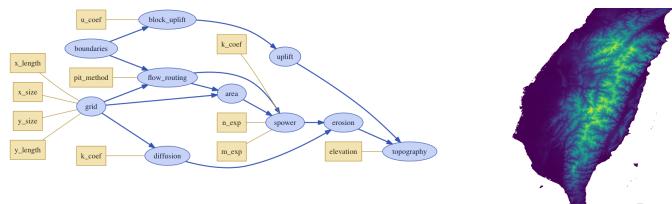
Cosmology



Epidemiology



Climatology

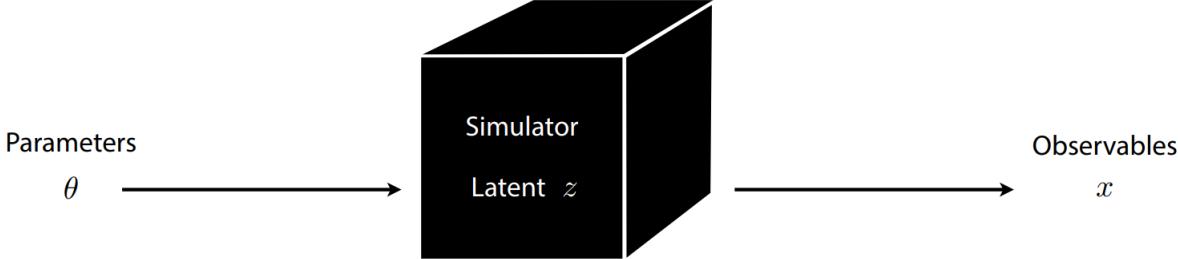


Computational topography

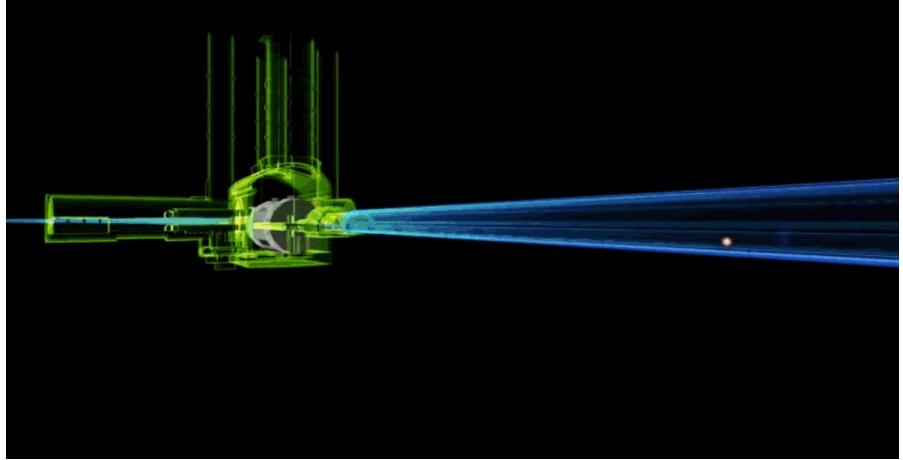


Astronomy

# Particle physics



$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_\nu f^{abc} \partial_\nu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \\
 M^2 W_\mu^+ W_\mu^- & - \frac{1}{2}g_\mu Z_\mu^0 \partial_\mu Z_\mu^0 - \frac{1}{2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - ig_{sw}(\partial_\mu Z_\mu^0)W_\nu^+ W_\nu^- \\
 W_\mu^+ W_\nu^- & - Z_\mu^0(W_\mu^+ \partial_\mu W_\nu^- - W_\mu^- \partial_\mu W_\nu^+) + Z_\mu^0(W_\mu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\mu W_\mu^+) - \\
 ig_{sw}(\partial_\mu A_\nu(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)) & - A_\nu(W_\mu^+ \partial_\mu W_\nu^- - W_\mu^- \partial_\mu W_\mu^+) + A_\nu(W_\mu^+ \partial_\mu W_\mu^- - \\
 W_\mu^- \partial_\mu W_\mu^+) - Z_\mu^0(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) - A_\mu(W_\mu^+ \partial_\mu W_\nu^- - W_\mu^- \partial_\mu W_\mu^+) + A_\mu(W_\mu^+ \partial_\mu W_\mu^- - \\
 W_\mu^- \partial_\mu W_\nu^+) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ + \frac{1}{2}g^2 W_\mu^+ W_\nu^+ W_\mu^- + g^2 c_w^2(Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - \\
 Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + g^2 s_w^2(A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\nu^+ W_\mu^-) + g^2 s_w c_w(A_\mu Z_\mu^0(W_\mu^+ W_\nu^- - \\
 Z_\mu^0 W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\mu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
 \beta_h \left( \frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M}{g^2} \alpha_h - \\
 g \alpha_h M(H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-) - \\
 \frac{1}{8}g^2 \alpha_h(H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\nu^0 H - \\
 \frac{1}{2}ig(W_\mu^+(\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\nu^-(\phi^0 \partial_\mu \phi_+ - \phi^+ \partial_\mu \phi^0)) + \\
 \frac{1}{2}g(W_+(\partial_\mu \phi^- - \phi^- \partial_\mu H) + W_-(H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g_{sw}^{\perp}(Z_\mu^0(H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
 M(\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{g^2}{c_w^2} M Z_\mu^0(W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu(W_\mu^+ \phi^- - \\
 W_\mu^- \phi^+) - ig \frac{1-2s_w^2}{2c_w^2} Z_\mu^0(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8}g^2 \frac{M}{c_w^2} Z_\mu^0 Z_\nu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 \frac{1}{2}g^2 s_w^2 Z_\mu^0 \phi^0(W_\mu^+ \phi^- - W_\mu^- \phi^+) - \frac{1}{2}g^2 \frac{s_w^2}{c_w^2} Z_\mu^0 H(W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0(W_\mu^+ \phi^- + \\
 W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu H(W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{2w}{(2s_w^2 - 1)} A_\mu \phi^+ \phi^- - \\
 g^2 s_w^2 A_\mu \phi^+ \phi^- + \frac{1}{2}ig s_w (q_\mu^\lambda \gamma^\mu q_\nu^\lambda) g_\mu^\nu - \bar{e}^\lambda (\gamma^\mu + m_\lambda^2) \bar{e}^\lambda - \bar{p}^\lambda (\gamma \partial + m_\lambda^2) p^\lambda - \bar{u}_j^\lambda (\gamma \partial + \\
 m_\lambda^2 u_j^\lambda - d_j^\lambda (\gamma \partial + m_\lambda^2) u_j^\lambda) + ig s_w A_\mu(-(e^{\lambda \gamma^\mu} e^\lambda) + \frac{1}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda))) + \\
 \frac{ig}{4c_w} Z_\mu^0((\bar{p}^\lambda \gamma^\mu(1 + \gamma^5) p^\lambda) + (\bar{e}^{\lambda \gamma^\mu}(4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu(\frac{1}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
 (i_j^\lambda \gamma^\mu(1 - \frac{1}{3}s_w^2 + \gamma^5) u_j^\lambda) + \frac{ig}{2\sqrt{2}} W_\mu^+(i(p^\lambda \gamma^\mu(1 + \gamma^5) l^\mu p^\lambda e^\lambda) + (i_j^\lambda \gamma^\mu(1 + \gamma^5) C_{\lambda k} d_j^\lambda) + \\
 \frac{ig}{2\sqrt{2}} W_\mu^-(i(\bar{e}^{\lambda \gamma^\mu} l^\mu \lambda_k(1 - \gamma^5) \nu^\lambda) + (\bar{d}_j^\lambda C_{\lambda k}^\dagger \mu(1 + \gamma^5) u_j^\lambda)) + \\
 \frac{ig}{2M\sqrt{2}} \phi^+ (m_\lambda^2 (\bar{p}^\lambda U^{\dagger \mu} \lambda_k(1 - \gamma^5) \nu^\lambda) - m_\lambda^2 (\bar{e}^{\lambda \gamma^\mu} U^{\dagger \mu} \lambda_k(1 - \gamma^5) \nu^\lambda) - \frac{im_\lambda^2}{2} H(\bar{p}^\lambda \nu^\lambda) - \\
 \frac{ig}{2M\sqrt{2}} \phi^- (m_\lambda^2 (\bar{e}^{\lambda \gamma^\mu} U^{\dagger \mu} \lambda_k(1 - \gamma^5) \nu^\lambda) - m_\lambda^2 (\bar{e}^{\lambda \gamma^\mu} U^{\dagger \mu} \lambda_k(1 - \gamma^5) \nu^\lambda) - \frac{im_\lambda^2}{2} H(\bar{p}^\lambda \nu^\lambda) - \\
 \frac{g}{2} M^2 H(\bar{e}^{\lambda \gamma^\mu} \lambda_k(1 - \gamma^5) \nu^\lambda) + \frac{ig}{2} M^2 \phi^0(\bar{p}^\lambda \nu^\lambda \gamma^\mu \lambda_k) - \frac{ig}{2} M^2 \phi^0(e^{\lambda \gamma^\mu} \nu^\lambda) - \frac{1}{4} \bar{p}_\lambda M_{\lambda k}^R(1 - \gamma_5) \bar{\nu}_k - \\
 \frac{1}{4} \bar{p}_\lambda M_{\lambda k}^R(1 - \gamma_5) \bar{\nu}_k + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^2 (\bar{u}_j^\lambda C_{\lambda k}(1 - \gamma^5) d_j^\lambda) + m_u^2 (\bar{u}_j^\lambda C_{\lambda k}(1 + \gamma^5) d_j^\lambda) + \\
 \frac{ig}{2M\sqrt{2}} \phi^- (m_d^2 (\bar{d}_j^\lambda C_{\lambda k}^\dagger(1 + \gamma^5) u_j^\lambda) - m_u^2 (\bar{d}_j^\lambda C_{\lambda k}^\dagger(1 - \gamma^5) u_j^\lambda) - \frac{im_\lambda^2}{2} H(\bar{u}_j^\lambda u_j^\lambda) - \\
 \frac{g}{2} M^2 H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} M^2 \phi^0(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{ig}{2} M^2 \phi^0(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda) + G^\mu \partial^\nu G^\nu + g_s f^{abc} \partial_\mu G^\mu G^\nu g_\nu^a + \\
 X^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2) - \bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + Y \partial^2 Y + ig c_w W_\mu^+(\partial_\mu \bar{X}^0 X^- - \\
 \partial_\mu X^+ X^0) + ig s_w W_\mu^+(\partial_\mu \bar{X}^- Y - \partial_\mu X^+ Y) + ig c_w W_\mu^-(\partial_\mu \bar{X}^+ X^- - \\
 \partial_\mu X^0 X^+) + ig s_w W_\mu^-(\partial_\mu \bar{X}^- Y - \partial_\mu X^+ Y) + ig c_w Z_\mu^0(\partial_\mu \bar{X}^+ X^- - \\
 \partial_\mu X^- X^+) + ig s_w A_\mu(\partial_\mu \bar{X}^+ X^- + \frac{1-2s_w^2}{2c_w} i g M (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
 \frac{1}{2c_w} i g M (\bar{X}^0 X^- \phi^+ - X^0 X^+ \phi^-) + ig M s_w (\bar{X}^0 X^- \phi^+ - X^0 X^+ \phi^-) + \\
 \frac{1}{2}ig M (\bar{X}^+ X^0 \phi^0 - X^- X^0 \phi^0) .
 \end{aligned}$$



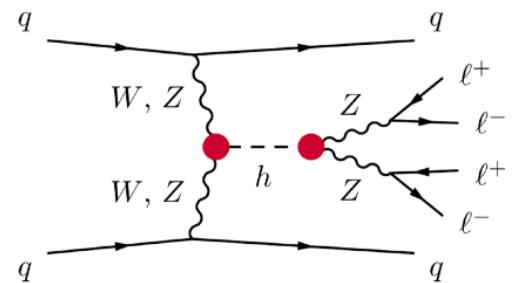
Latent variables

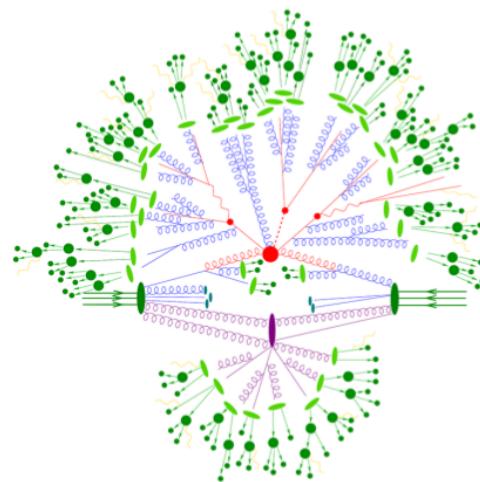
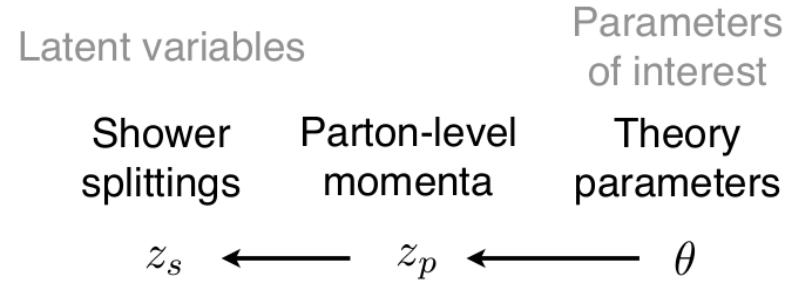
Parameters  
of interest

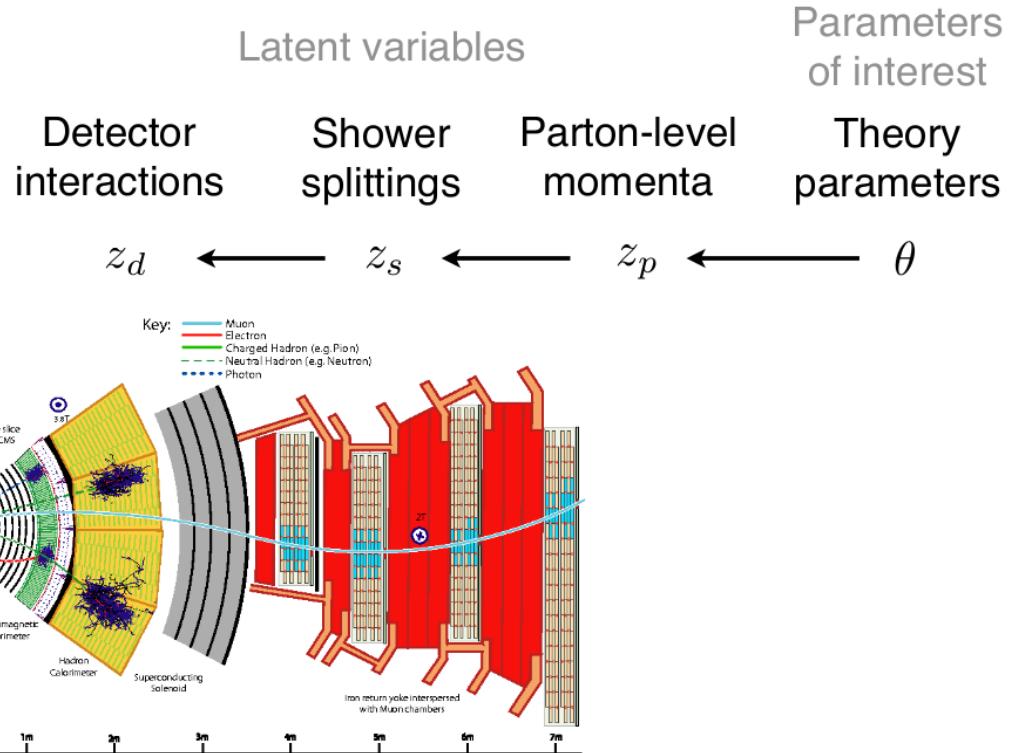
Parton-level  
momenta

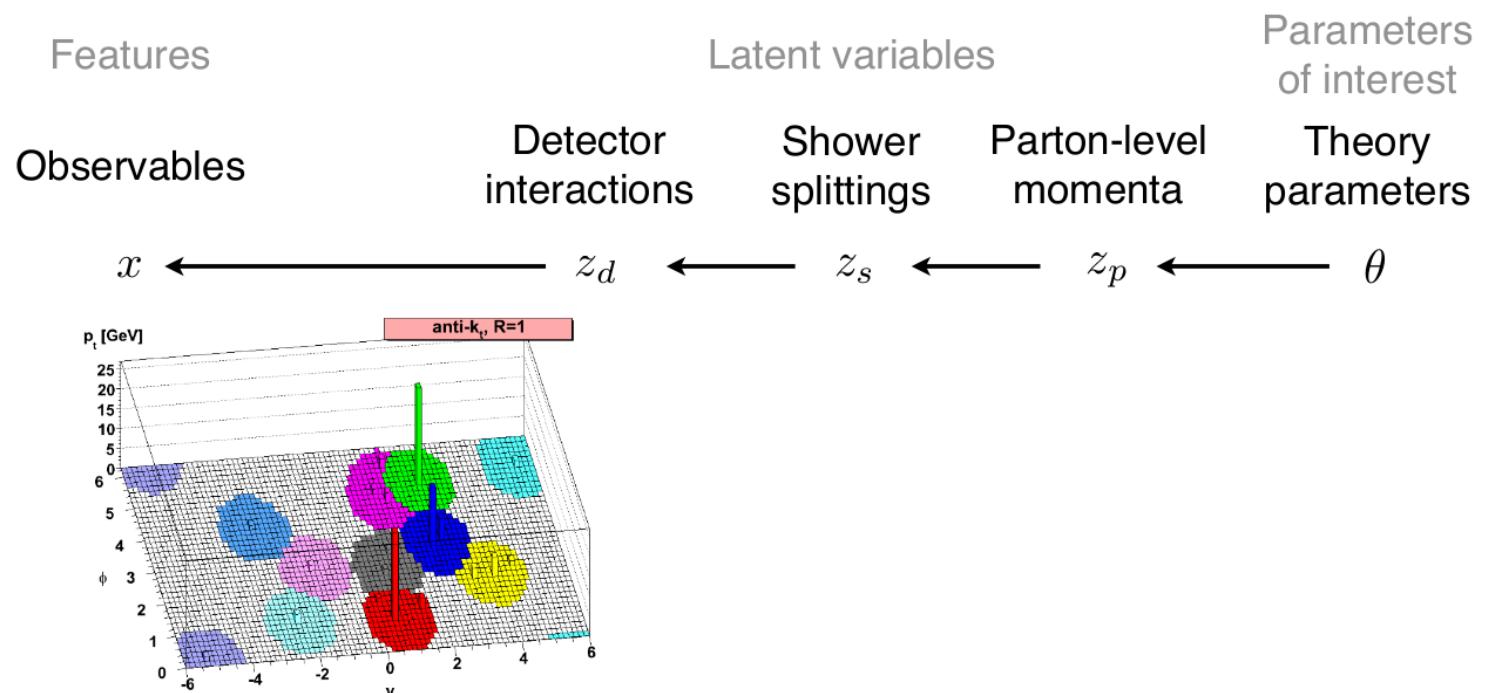
Theory  
parameters

$$z_p \leftarrow \theta$$









[Image source: M. Cacciari,  
G. Salam, G. Soyez 0802.1189]

$$p(x|\theta) = \underbrace{\iiint}_{\text{intractable}} p(z_p|\theta)p(z_s|z_p)p(z_d|z_s)p(x|z_d)dz_p dz_s dz_d$$

# Likelihood ratio

The likelihood ratio

$$r(x|\theta_0, \theta_1) = \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

is the quantity that is **central** to many **statistical inference** procedures.

## Examples

- Frequentist hypothesis testing
- Supervised learning
- Bayesian posterior sampling with MCMC
- Bayesian posterior inference through Variational Inference
- Generative adversarial networks
- Empirical Bayes with Adversarial Variational Optimization

The likelihood  $p(x|\theta)$  is actually rarely needed.

*When solving a problem of interest, do not solve a more general problem as an intermediate step. – Vladimir Vapnik*



$$\frac{p_{\mathbf{x}}(\mathbf{x}|\theta_0)}{p_{\mathbf{x}}(\mathbf{x}|\theta_1)} = r(\mathbf{x}; \theta_0, \theta_1)$$
A blue curved arrow points from the term  $p_{\mathbf{x}}(\mathbf{x}|\theta_0)$  to the term  $r(\mathbf{x}; \theta_0, \theta_1)$ . A red curved arrow points from the term  $p_{\mathbf{x}}(\mathbf{x}|\theta_1)$  to the same term  $r(\mathbf{x}; \theta_0, \theta_1)$ .

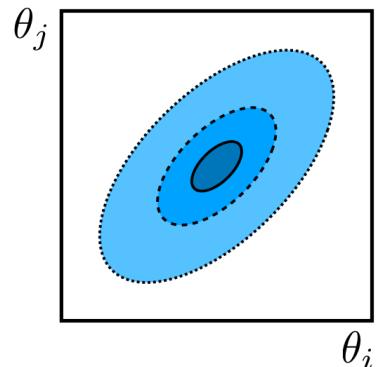
Direct likelihood ratio estimation is simpler than density estimation.  
(This is fortunate, we are in the likelihood-free scenario!)

# The frequentist physicist's way

The Neyman-Pearson lemma states that the likelihood ratio

$$r(x|\theta_0, \theta_1) = \frac{p(x|\theta_0)}{p(x|\theta_1)}$$

is the **most powerful test statistic** to discriminate between a null hypothesis  $\theta_0$  and an alternative  $\theta_1$ .



IX. *On the Problem of the most Efficient Tests of Statistical Hypotheses.*

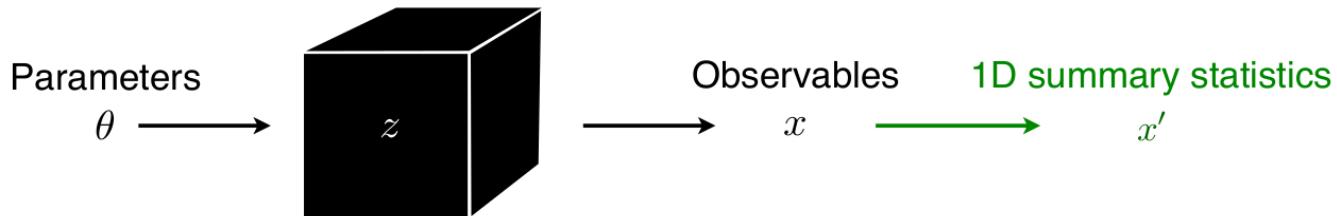
*By J. NEYMAN, Nencki Institute, Soc. Sci. Lit. Varsoviensis, and Lecturer at the Central College of Agriculture, Warsaw, and E. S. PEARSON, Department of Applied Statistics, University College, London.*

(Communicated by K. PEARSON, F.R.S.)

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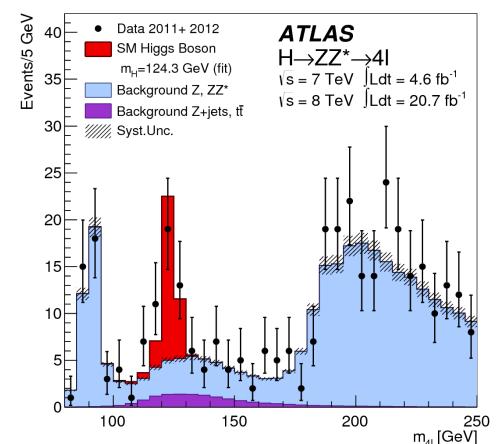
Define a projection function  $s : \mathcal{X} \rightarrow \mathbb{R}$  mapping observables  $x$  to a summary statistics  $x' = s(x)$ .

Then, approximate the likelihood  $p(x|\theta)$  as

$$p(x|\theta) \approx \hat{p}(x|\theta) = p(x'|\theta).$$

From this it comes

$$\frac{p(x|\theta_0)}{p(x|\theta_1)} \approx \frac{\hat{p}(x|\theta_0)}{\hat{p}(x|\theta_1)} = \hat{r}(x|\theta_0, \theta_1).$$

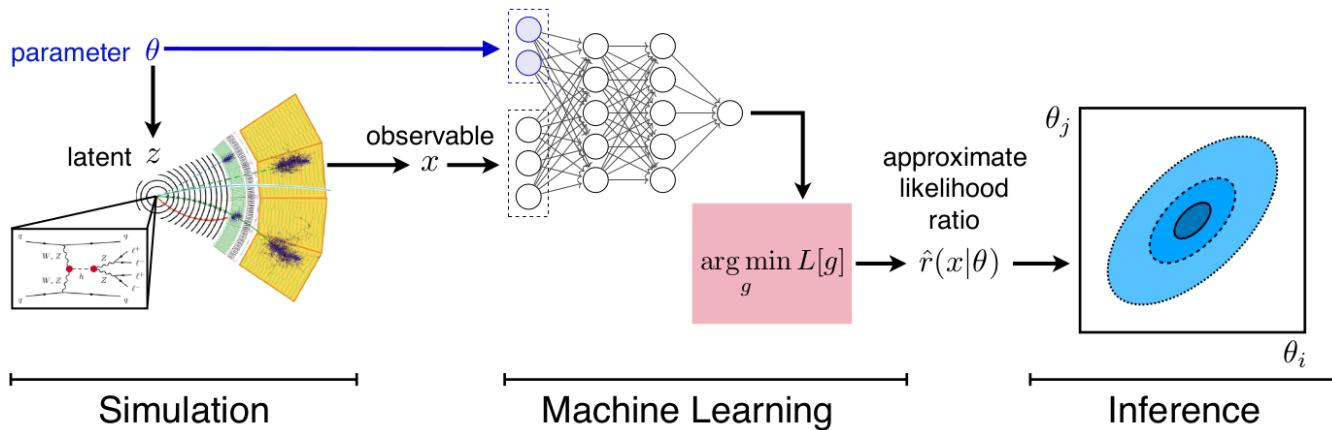


# CARL

Supervised learning provides a way to **automatically** construct  $s$ :

- Let us consider a binary classifier  $\hat{s}$  (e.g., a neural network) trained to distinguish  $x \sim p(x|\theta_0)$  from  $x \sim p(x|\theta_1)$ .
- $\hat{s}$  is trained by minimizing the cross-entropy loss

$$L_{XE}[\hat{s}] = -\mathbb{E}_{p(x|\theta)\pi(\theta)}[1(\theta = \theta_0) \log \hat{s}(x) + 1(\theta = \theta_1) \log(1 - \hat{s}(x))]$$



The solution  $\hat{s}$  found after training approximates the optimal classifier

$$\hat{s}(x) \approx s^*(x) = \frac{p(x|\theta_1)}{p(x|\theta_0) + p(x|\theta_1)}.$$

Therefore,

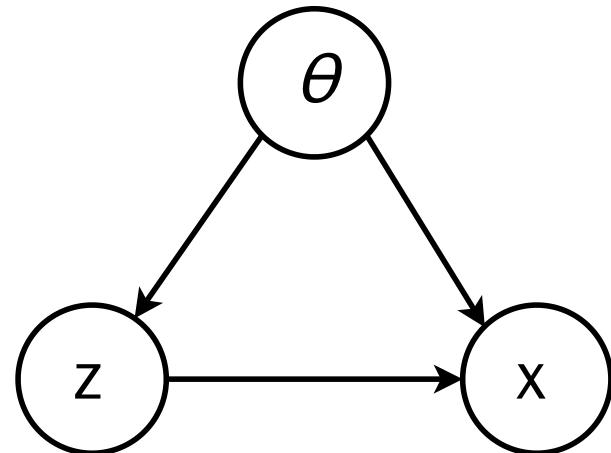
$$r(x|\theta_0, \theta_1) \approx \hat{r}(x|\theta_0, \theta_1) = \frac{1 - \hat{s}(x)}{\hat{s}(x)}$$

That is, **supervised classification is equivalent to likelihood ratio estimation.**

# Bayesian inference

For a given model  $p(x, z, \theta)$ ,  
Bayesian inference usually consists in  
computing the posterior

$$p(\theta|x) = \frac{p(x|\theta)p(\theta)}{p(x)}.$$



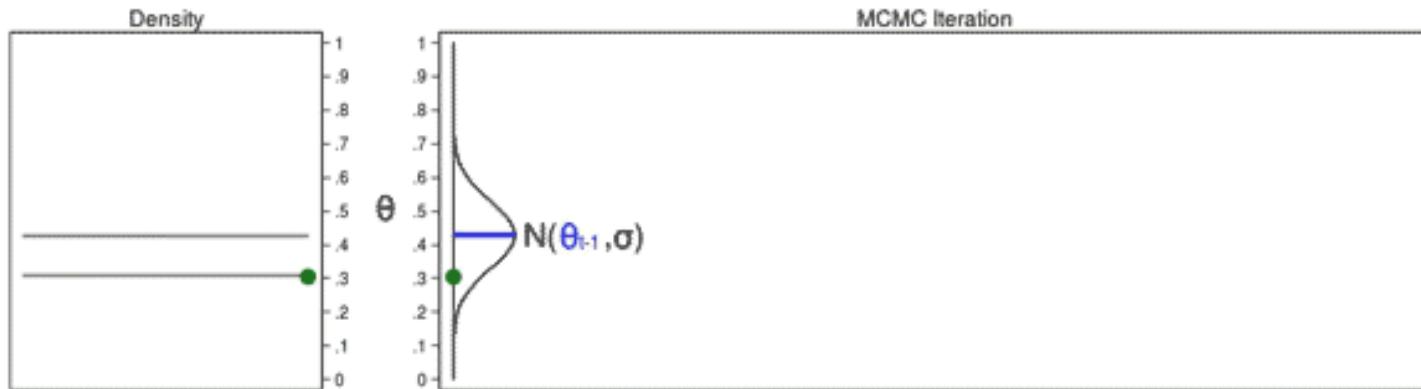
For most cases, this is intractable since it requires evaluating the evidence

$$p(x) = \int p(x|\theta)p(\theta)d\theta.$$

In the likelihood-free scenario, this is even less tractable since we cannot even evaluate the likelihood

$$p(x|\theta) = \int p(x, z|\theta)dz.$$

## Posterior sampling



$$\text{Step 1: } r(\theta_{\text{new}}, \theta_{t-1}) = \frac{\text{Posterior}(\theta_{\text{new}})}{\text{Posterior}(\theta_{t-1})} = \frac{\text{Beta}(1,1,0.306) \times \text{Binomial}(10,4,0.306)}{\text{Beta}(1,1,0.429) \times \text{Binomial}(10,4,0.429)} = 0.834$$

$$\text{Step 2: Acceptance probability } \alpha(\theta_{\text{new}}, \theta_{t-1}) = \min\{r(\theta_{\text{new}}, \theta_{t-1}), 1\} = \min\{0.834, 1\} = 0.834$$

Step 3: Draw  $u \sim \text{Uniform}(0,1) = 0.617$

Step 4: If  $u < \alpha(\theta_{\text{new}}, \theta_{t-1}) \rightarrow \text{If } 0.617 < 0.834 \quad \text{Then } \theta_t = \theta_{\text{new}} = 0.306$   
Otherwise  $\theta_t = \theta_{t-1} = 0.429$

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# Likelihood-free MCMC with Approximate Likelihood Ratios

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**Algorithm 2** Likelihood-free Metropolis-Hastings

*Inputs:*

Initial parameter  $\theta_0$ .  
Transition distribution  $q(\theta)$ .  
Parameterized classifier  $s(x, \theta)$ .  
Observations  $\mathcal{O}$ .

*Outputs:*

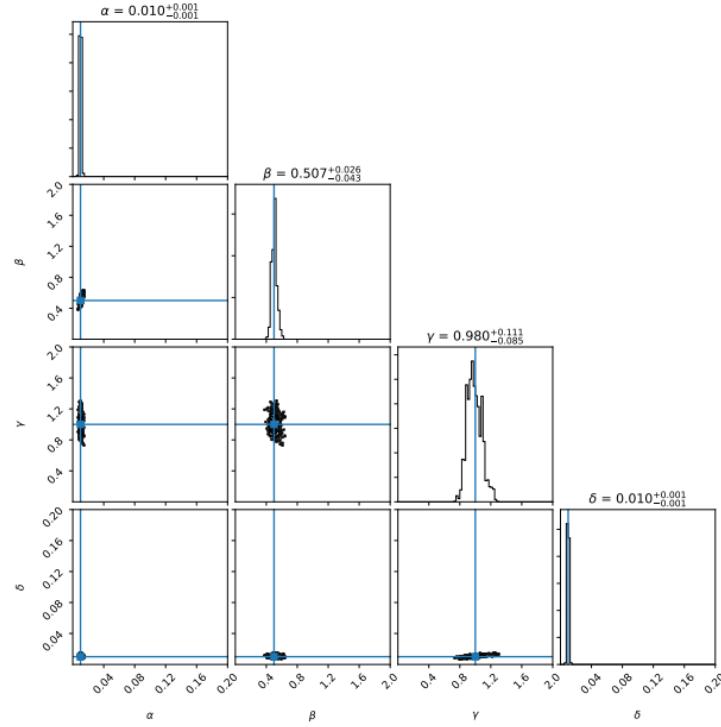
Markov chain  $\theta_{0:n}$

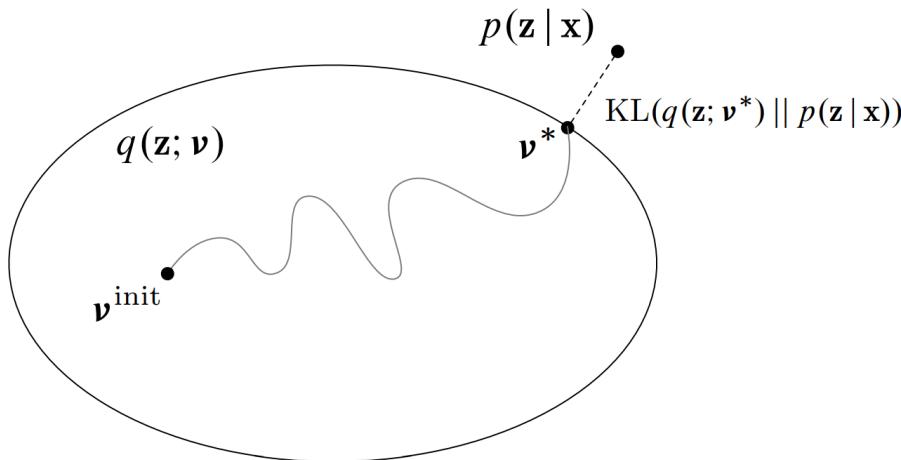
*Hyperparameters:*

Steps  $n$ .

```
1:  $t \leftarrow 0$ 
2:  $\theta_t \leftarrow \theta_0$ 
3: for  $t < n$  do
4:    $\theta' \sim q(\theta | \theta_t)$ 
5:    $\lambda \leftarrow \sum_{x \in \mathcal{O}} \log \hat{r}_e(x, \theta') - \sum_{x \in \mathcal{O}} \log \hat{r}_e(x, \theta_t)$ 
6:    $\rho \leftarrow \min \left\{ \exp(\lambda) \frac{q(\theta_t | \theta')}{q(\theta' | \theta_t)}, 1 \right\}$ 
7:    $\theta_{t+1} \leftarrow \begin{cases} \theta' & \text{with probability } \rho \\ \theta_t & \text{with probability } 1 - \rho \end{cases}$ 
8:    $t \leftarrow t + 1$ 
9: end for
10: return  $\theta_{0:n}$ 
```

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## Likelihood-free Variational inference

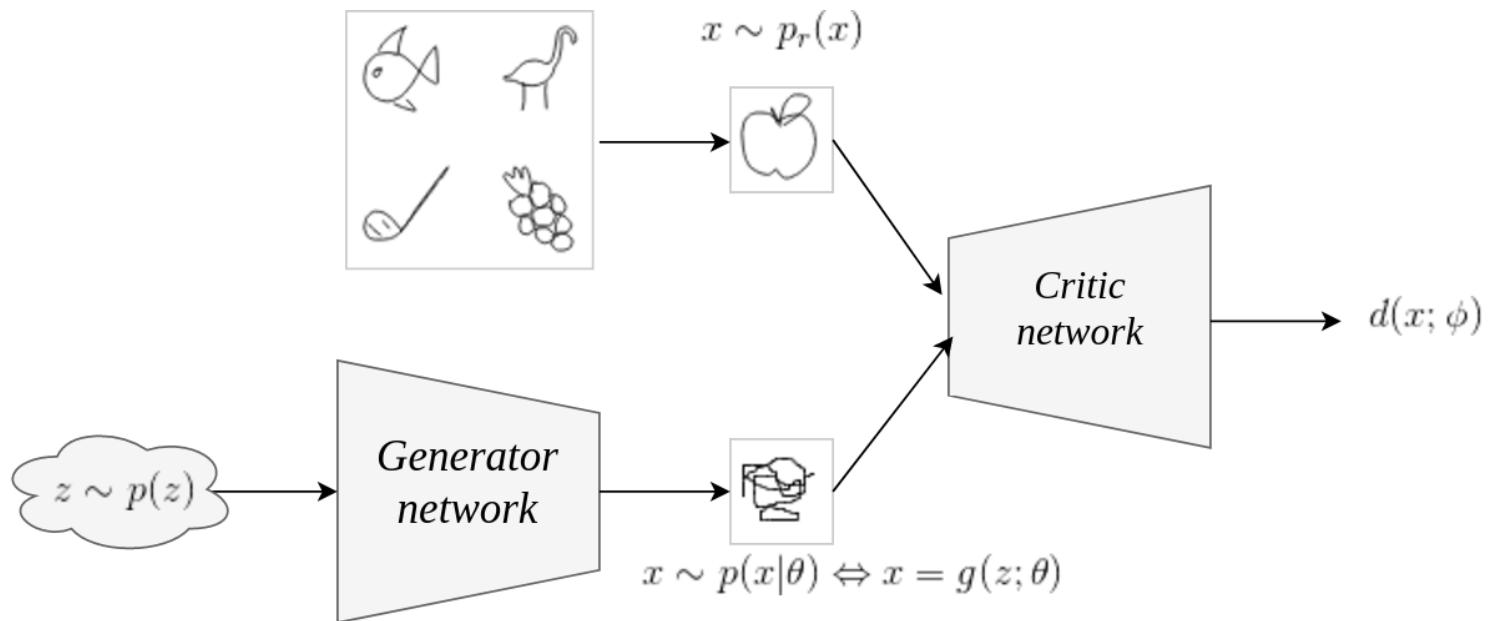
Let  $q(\mathbf{x}_n)$  be the empirical distribution on the observations  $\mathbf{x}$  and consider using it in a “variational joint”  $q(\mathbf{x}_n, \mathbf{z}_n | \boldsymbol{\beta}) = q(\mathbf{x}_n)q(\mathbf{z}_n | \mathbf{x}_n, \boldsymbol{\beta})$ . Now subtract the log empirical  $\log q(\mathbf{x}_n)$  from the ELBO above. The ELBO reduces to

$$\mathcal{L} \propto \mathbb{E}_{q(\boldsymbol{\beta})} [\log p(\boldsymbol{\beta}) - \log q(\boldsymbol{\beta})] + \sum_{n=1}^N \mathbb{E}_{q(\boldsymbol{\beta})q(\mathbf{z}_n | \mathbf{x}_n, \boldsymbol{\beta})} \left[ \log \frac{p(\mathbf{x}_n, \mathbf{z}_n | \boldsymbol{\beta})}{q(\mathbf{x}_n, \mathbf{z}_n | \boldsymbol{\beta})} \right]. \quad (4)$$

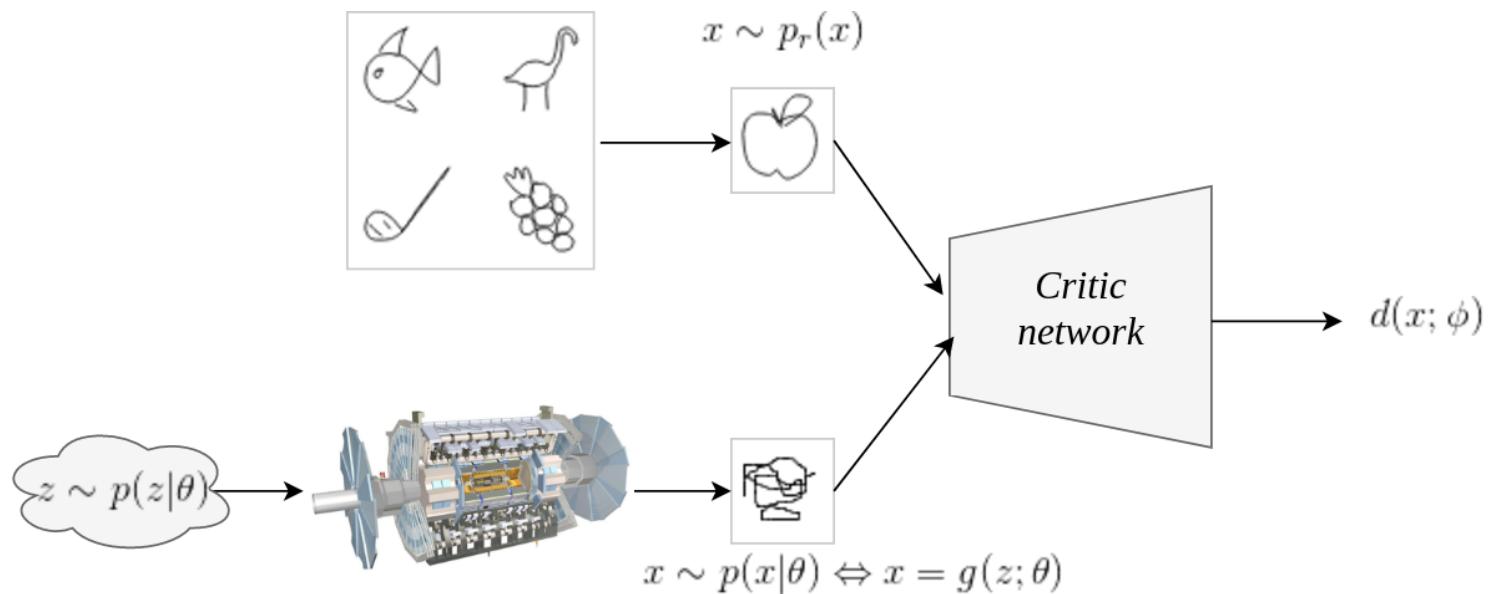
(Here the proportionality symbol means equality up to additive constants.) Thus the ELBO is a function of the ratio of two intractable densities. If we can form an estimator of this ratio, we can proceed with optimizing the ELBO.

We apply techniques for ratio estimation [51]. It is a key idea in GANs [37, 56], and similar ideas have reappeared in statistics and physics [21, 8].

# Generative adversarial networks



# Adversarial Variational Optimization



Replace  $g$  with an actual scientific simulator!

## Key insights

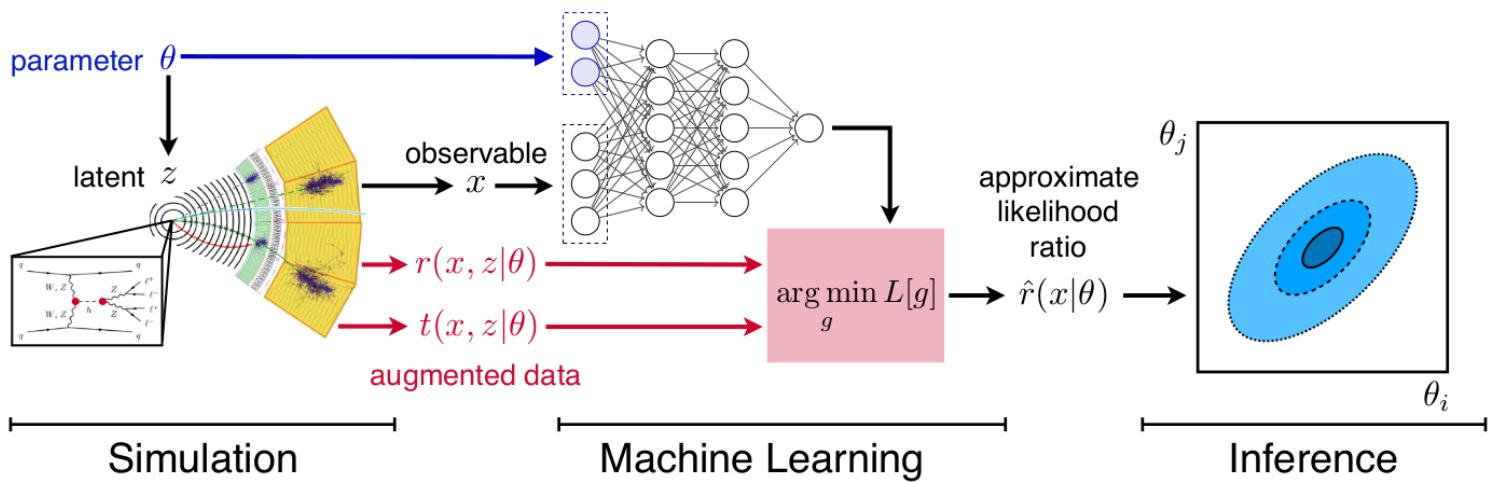
- Replace the generative network with a non-differentiable forward simulator  $g(z; \theta)$ .
- Let the neural network critic figure out how to adjust the simulator parameters.
- Combine with variational optimization to bypass the non-differentiability by optimizing upper bounds of the adversarial objectives

$$U_d(\phi) = \mathbb{E}_{\theta \sim q(\theta; \psi)} [\mathcal{L}_d(\phi)]$$
$$U_g(\psi) = \mathbb{E}_{\theta \sim q(\theta; \psi)} [\mathcal{L}_g(\theta)]$$

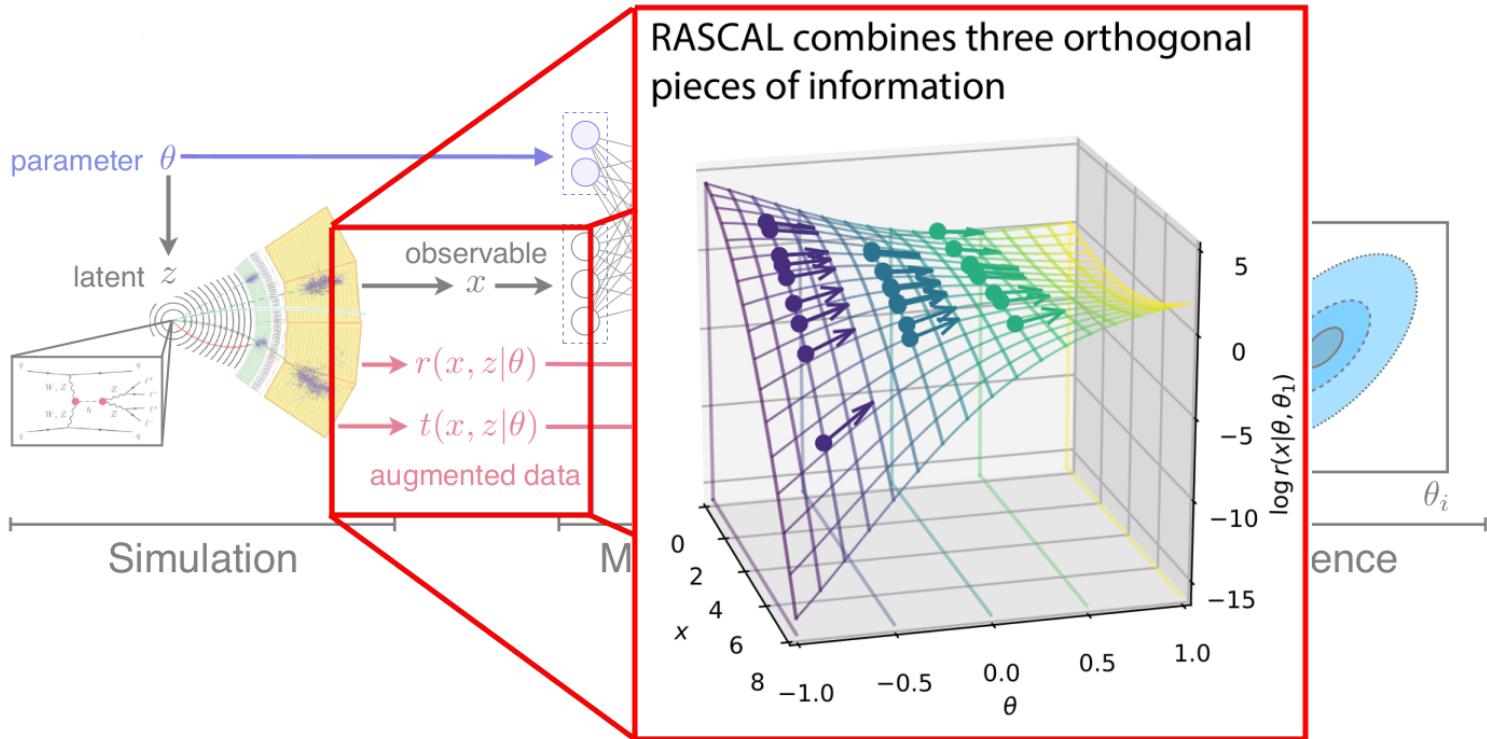
respectively over  $\phi$  and  $\psi$ .

- Effectively, this amounts to empirical Bayes guided by the likelihood ratios estimated from the critic.

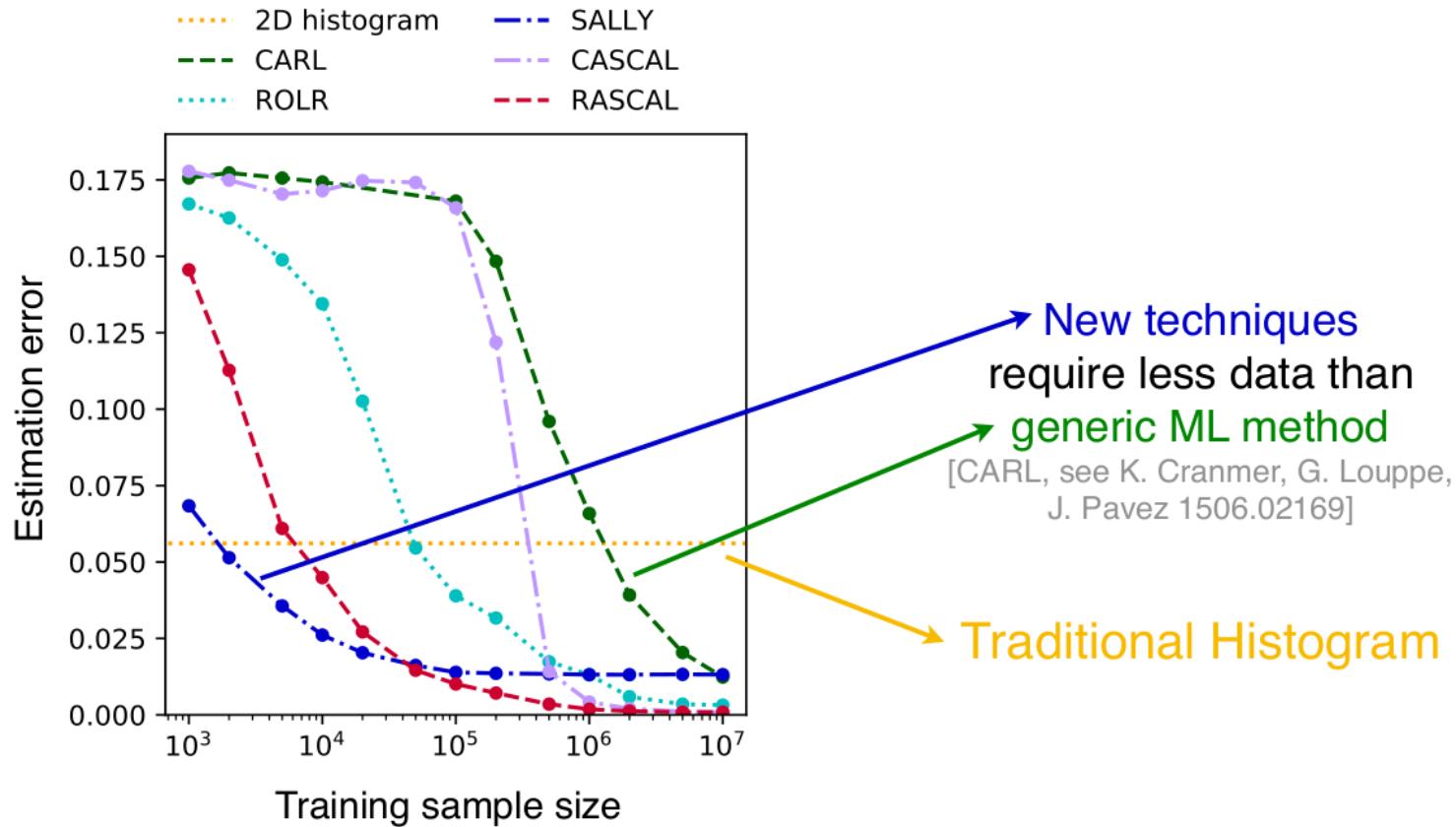
# Mining gold



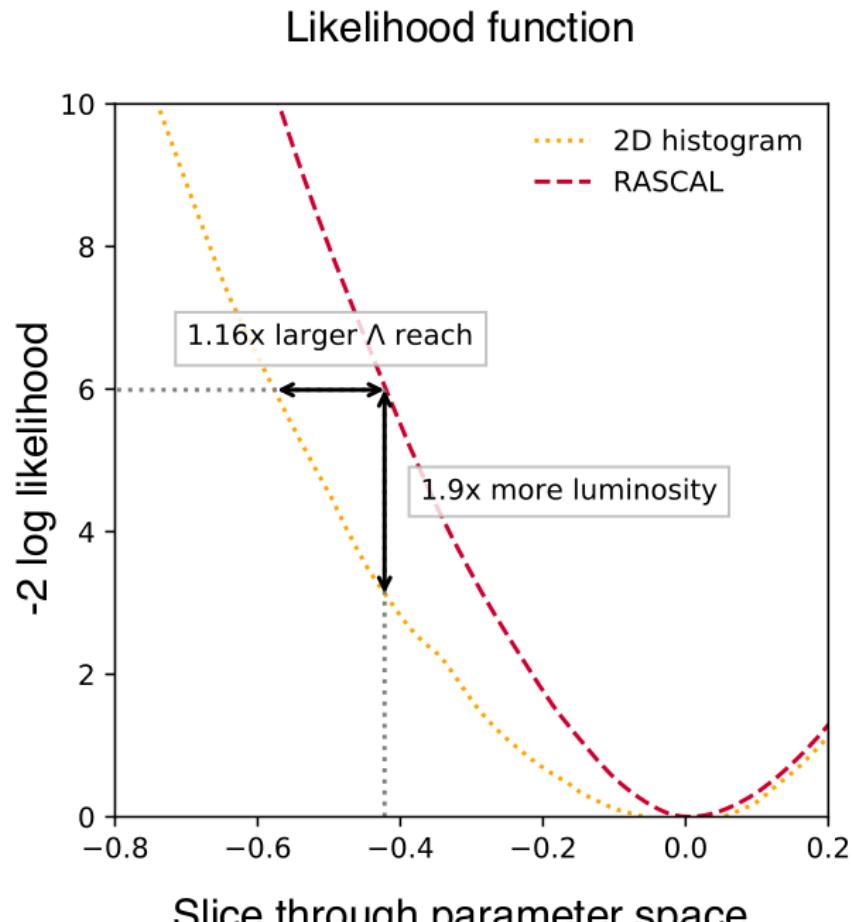
# Mining gold



## Increased data efficiency



## Better sensitivity



36 events, assuming SM

# Summary

- Much of modern science is based on "likelihood-free" simulations.
- The likelihood-ratio is central to many statistical inference procedures.
- Supervised learning enables likelihood-ratio estimation.
- Better likelihood-ratio estimates can be achieved by mining simulators.

# Collaborators



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The end.

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