Matrix Representation of Graphs

Group 3

Eva Rott
Michael Glatzhofer
Dominik Mocher
Julian Wolf

Institute for Information Systems and Computer Media (IICM),
Graz University of Technology
A-8010 Graz, Austria

15 Mai 2017

Abstract

By representing graphs in a matrix adjancy matrix it is possible to observe special patterns and reveal dependencies which might not be seen in the graph per se. By combining the benefits of both the matraix representation and the graph it self, a very powerfull approach of graph analysing may be achieved.

In this survey we present some powerfull techniques applyable to adjancy matrices to analyze graphs. furthermore, we present some tools utilizing these techniques.

Contents

List of Figures

List of Tables

stlistingnameList of Listings

Listings

Chapter 1

Basics

This chapter explains the different types of graphs and their corresponding matrices, which techniques are basically used on matrices and how to interpret the resulting patterns

1.1 Definitions

Basically, a graph in mathematics is an ordered pair $G = (V, E)$ containing a set of nodes V and a set of edges e. However, some literature refer to nodes as “vertices" (thus the V) or “points". Edges may be called “arc" or lines. On the other hand, in the case of an directed graph, edges may also be called arrows. Moreover:

- V is not allowed to be empty
- E is allowed to be empty
- The **order** of a graph is the number of vertices $|V|$
- The **size** of a graph is the number of edges $|E|$

In this paper, we define that every node in the graph has its distinctive unique id, which never changes. This holds for the reordering of the matrices too - when reordering rows and columns, the corresponding index stays with the column, otherwise the graph would be changed with this operation.

1.2 Types of Graphs

We distinguish between two types of edges (directed and undirected) and two types of cost calculations (weighted and unweighted) between the nodes, which leads to 4 different graphs

1.2.1 directed/undirected

As can be seen in figure **??** undirected edges may be traversed in any direction, whereas directed edges may just be traversed in one direction. In this paper we neither talk about the mathematical **quiver**, a directed graph which has multiple arrows pointing from node x to y, nor about a **multigraph**, which is a graph which contains multiple undirected edges connecting just two nodes.

1.2.2 weighted/unweighted

A graph without weights has uniform costs on all edges, which leads to equal costs by traversing the edges in any order. When adding weights or costs to the edges, this fact changes. The order or selection of traversing the edges does matter as it creates different costs.

TODO: some more stuff here

1.3 Use cases

Some use cases of the different graph types are (to name just a few examples):

- Navigation system (weighted directed)
  - Nodes: Cities/POIs
  - Edges: Routes directed (one way streets)
    - weights
      - length of street (find shortest way)
      - time to traverse the street (find fastest way)
- Subway map (undirected unweighted)
  - Nodes: stations
  - edges: connection between stations
- Relations of tweets (directed unweighted)
  - nodes: single tweet entry
  - edges: references to other tweets

1.4 Matrix representation of graphs

When representing graphs in a matrix, an adjacency matrix is used. Adjacency matrices are structured with every row and every column represents one node. This leads to a N x N square matrix, where N is the number of nodes.

These matrices show some patterns according to their corresponding graph but most times these patterns are not immediately visible. There are some techniques to reveal these patterns, all of them involving the reordering of the matrix.

1.4.1 Reordering

The main goal of reordering the matrix is to cluster the edges and thus reveal certain patterns. An example of this behaviour can be seen in figure **??**.

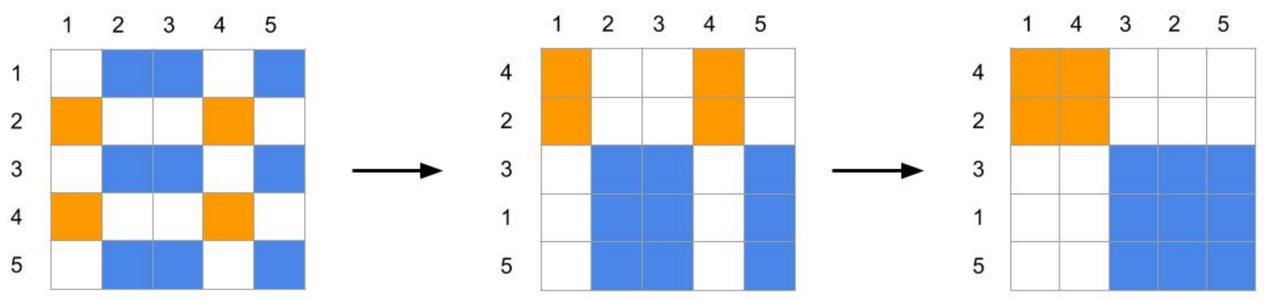
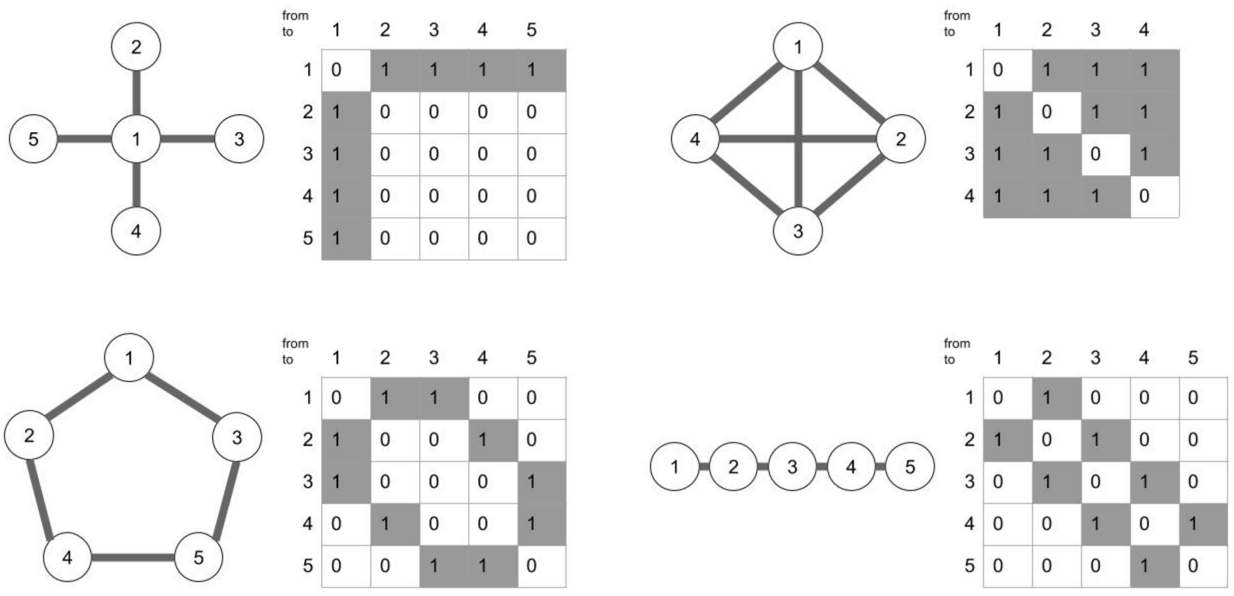


Figure 1.1: Reordering a matrix

When reordering the matrix, the indices of the single rows and columns stay with the rows, otherwise the graph would change by this workstep. In this example, at first the rows 1 and 4 get swapped and as a second step columns 2 and 4. In this way the full connection pattern of the two subgraphs may be observed.



from to	1	2	3	4	5
1	0	1	1	1	1
2	1	0	0	0	0
3	1	0	0	0	0
4	1	0	0	0	0
5	1	0	0	0	0

from to	1	2	3	4
1	0	1	1	1
2	1	0	1	1
3	1	1	0	1
4	1	1	1	0

from to	1	2	3	4	5
1	0	1	1	0	0
2	1	0	0	1	0
3	1	0	0	0	1
4	0	1	0	0	1
5	0	0	1	1	0

from to	1	2	3	4	5
1	0	1	0	0	0
2	1	0	1	0	0
3	0	1	0	1	0
4	0	0	1	0	1
5	0	0	0	1	0

Figure 1.2: 4 differnet patterns: star, full, circle and line

1.4.2 Patterns

There are 4 main patterns which may be revealed by reordering the matrix. These patterns may be combined in such a way, that for example a subgraph creates a circle, but one node if it is connected to every other node. This results in a combination of the star and the circle pattern. The four different patterns can be seen in the corresponding figures **??**.

Chapter 2

Scalability

Chapter 3

Cell types

Chapter 4

Matrix headers