Navier-Stokes equation is the basic equation of fluid dynamics. Given the boundary conditions and this equation, one can get the detail of a fluid field and make further prediction. However, this equation has not yet been solved. The currently most widely-used solution is called SIMPLE algorithm. This algorithm can be described as prediction-calculation-updating. It is an iterative method. To get a reasonable result, it may need dozens of iterations. This is computational-costly even for today's computer. For example, it takes months to simulate the flow field of a small area like a hand in supercomputers (using the most accurate DNS model). Even for less accurate model, such as LES and RANS, it may take hours or even days in PC.

$$\rho \frac{d\nu}{dt} = -\nabla p + \rho F + \mu \Delta \upsilon$$

Eq1： The Navier-Stokes equation. It is totally nonlinear between velocity *v* and pressure *P*.

To solve this problem, this project tries to use neural networks to learn the underlying N-S equation. The model is water flowing around a 2-dimensional cylinder (Kármán vortex street) as shown in Fig 1. This model has been widely researched and has experimental results. In our project, Kármán vortex street appears in laminar flow condition. It has a period which depends on the velocity of water and the diameter of cylinder. It means the fluid filed is almost the same after a period.

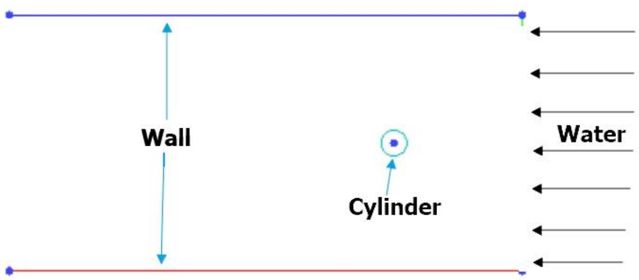


Fig 1: Model of this project. Water enters from the entrance and flows around the cylinder and gets out from the exit.

1. Input data set:

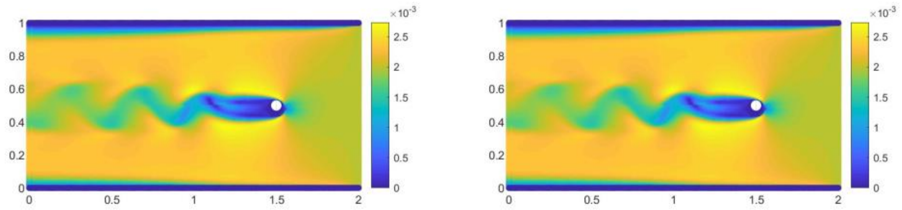
We have computational fluid dynamics (CFD) software FLUENT, which can simulate the fluid field relatively precisely. The output is the velocity of each point in the fluid field in a txt file.

1.501233109E+00	4.500319231E-01	0.000000000E+00	0.000000000E+00
1.498715598E+00	4.500332510E-01	0.000000000E+00	0.000000000E+00
1.503739594E+00	4.501566044E-01	0.000000000E+00	0.000000000E+00
1.496209246E+00	4.501605830E-01	0.000000000E+00	0.000000000E+00
1.506213280E+00	4.504037574E-01	0.000000000E+00	0.000000000E+00
1.493735826E+00	4.504103719E-01	0.000000000E+00	0.000000000E+00
1.508633600E+00	4.507664841E-01	0.000000000E+00	0.000000000E+00
1.491315898E+00	4.507757102E-01	0.000000000E+00	0.000000000E+00
1.510981914E+00	4.512348718E-01	0.000000000E+00	0.000000000E+00
1.488968100E+00	4.512466765E-01	0.000000000E+00	0.000000000E+00
1.513242098E+00	4.517964771E-01	0.000000000E+00	0.000000000E+00
1.486708548E+00	4.518108200E-01	0.000000000E+00	0.000000000E+00
1.515400984E+00	4.524369065E-01	0.000000000E+00	0.000000000E+00
1.484550403E+00	4.524537409E-01	0.000000000E+00	0.000000000E+00
1.517446911E+00	4.531454035E-01	0.000000000E+00	0.000000000E+00

Fig 3: Simulation results of the fluid field. The first, second and third columns are x, y and z coordinates of each point. Since it is a 2-dimensional model, the z coordinate

of all points are zero. The fourth column is velocity of this field.

Based on the txt file, we can get a visualized picture of the fluid field. Fig 3 is the colormap of two timesteps in this fluid field. The tails are the famous Kármán vortex street. If we get hundreds of these pictures in different timesteps, we can see how water actually flows. This is shown in the appendix video.



(a) T=0.3 (b) T=2.3

Fig 3: Simulation results of the fluid field. Different colors represent different velocities in the fluid field, which are shown in the color bar. This model has a period of 2 second. Note that the two pictures are almost the same.

Since we use CFD software to simulate fluid field, we can get almost unlimited data. This is an advantage of this method.

2.Objectives:

This project can be divided into three parts or stages.

1): Given the data set of a time span, such as 0-4s, predicting the fluid field of any time afterwards. In other words, this is learning the fluid field in time domain.

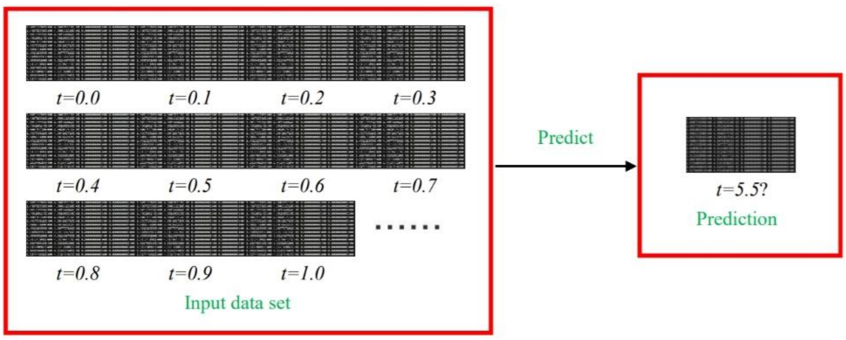


Fig 4: Objective 1: predicting fluid field at any time given previous data set.

2): Given the data set of a certain area in a timestep, predicting the fluid field out of this area. This is learning the fluid field in space domain.

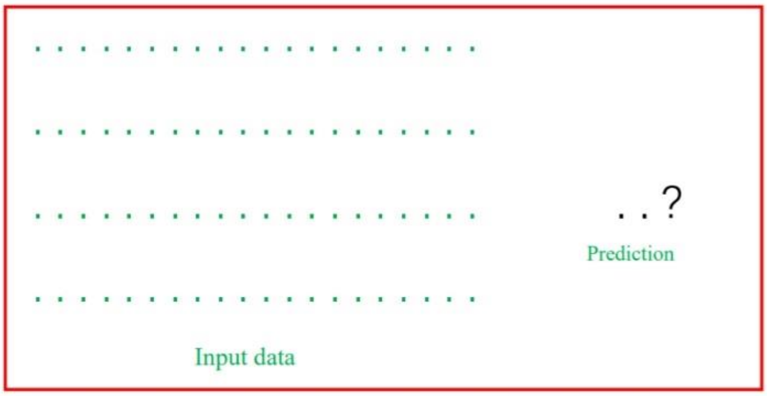


Fig 5: Objective 2: predicting fluid field at any position given adjacent data set.

3): The combination of stage 1 and stage 2. This is learning the fluid field both in time and space domain.

3.Methods:

CNN or GAN?

4.Difficulties:

1). We have txt data and graph data. Which data should be used as the input to neural networks?

2). Due to the extreme nonlinearity of N-S equation, pooling and dilation should be used to increase nonlinearity?

3). The data set is too large, it may take a long time to train.