Data Structures Chapter 7: Graph

- 1. Introduction
 - Terminology, Representation, ADT
- 2. Basic Operations
 - DFS, CC, BFS, Processing
- 3. Digraph and Applications
- 4. Minimum Spanning Tree(MST)



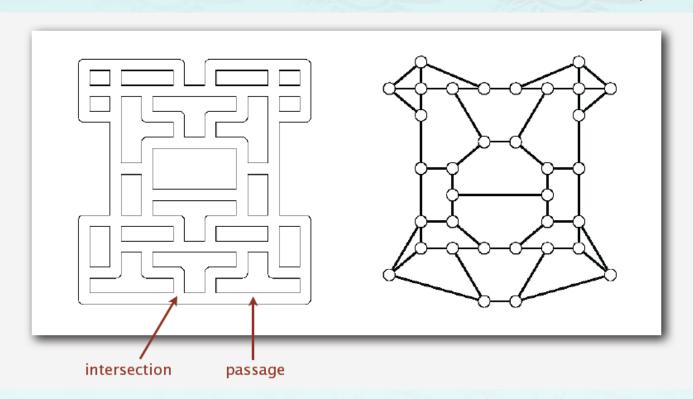
죄의 삯은 사망이요 하나님의 은사는 그리스도 예수 우리 주 안에 있는 영생이니라 (로마서 6:23)

모든 사람이 죄를 범하였으매 하나님의 영광에 이르지 못하더니 그리스도 예수 안에 있는 속량으로 말미암아 하나님의 은혜로 값없이 의롭다 하심을 얻은 자 되었느니라 (로마서 3:23-24)

Algorithm:

- Vertex = intersection
- Edge = passage

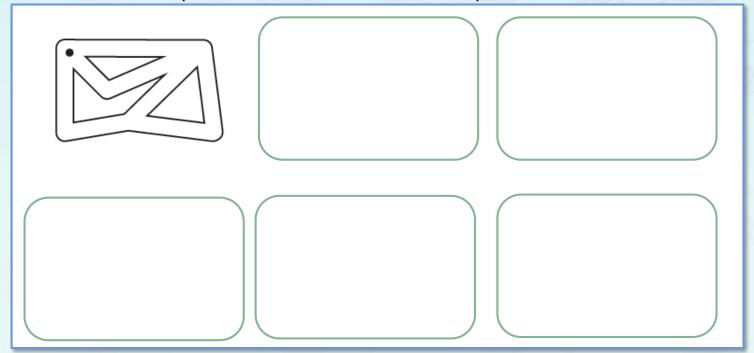
pacman



Maze Goal: Explore every intersection in the maze.

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options

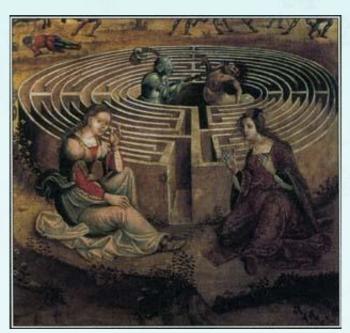


Maze Goal: Explore every intersection in the maze.

Good Visualization: https://www.cs.usfca.edu/~galles/visualization/DFS.html

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options



Theseus, a hero of Greek mythology, is best known for slaying a monster called the Minotaur. When Theseus entered the Labyrinth where the Minotaur lived, he took a ball of <u>yarn</u> to unwind and mark his route. Once he found the Minotaur and killed it, Theseus used the string to find his way out of the maze.

Read more: http://www.mythencyclopedia.com/Sp-Tl/Theseus.html#ixzz30wFO3ofe

Maze Goal: Explore every intersection in the maze.

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options



Shannon and his famous <u>electromechanical</u> mouse Theseus (named after <u>Theseus</u> from Greek mythology) which he tried to have solve the maze in one of the first experiments in <u>artificial intelligence</u>.

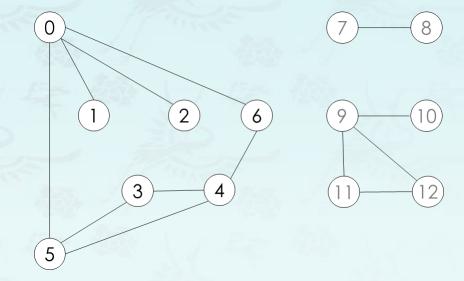
The Las Vegas connection: Shannon and his wife Betty also used to go on weekends to <u>Las Vegas</u> with <u>MIT</u> mathematician <u>Ed Thorp</u>, and made very successful forays in <u>blackjack</u> using <u>game theory</u>.

Maze Goal: Explore every intersection in the maze.

- Design pattern: Decouple graph data type
- Idea: Mimic maze exploration

DFS (to visit a vertex v)

- Mark v as visited.
- Recursively visit all unmarked vertices w adjacent to v.



Typical applications:

- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

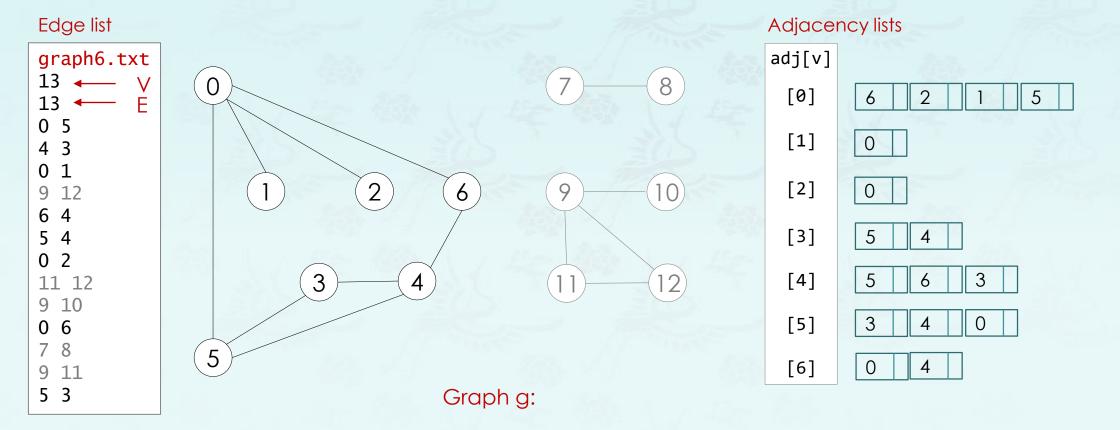
Challenge:

How to implement?

Goal: Systematically search through a graph from graph processing

- Create a graph object
- Pass the graph to a graph processing routine
- Query the graph-processing routine
 - path from v to w
 - distance from v to w
 - connected
 - bipartite
 - cyclic

- For each edge(v, w) in the list
- Insert front each vertex both (adj[v], w) and (adj[w], v) addEdgeFromTo(g, v, w); // from v to w.



Challenge: build adjacency lists?

Graph - ADT

```
// a structure to represent an adjacency list of vertices
struct Gnode {
  int    item;
    Gnode* next;
  Gnode (int i, Gnode *p = nullptr) {
    item = i;    next = p;
    }
    ~Gnode() {}
};
using gnode = Gnode *;
```

Graph ADT – graph.h

```
struct Graph {
 int V; // N vertices
 int E; // N edges
 gnode adj; // array of linked lists of vertices
 Graph(int v = 0) \{ // constructs a graph with v vertices \}
   V = V;
   E = 0;
   adj = new (nothrow) Gnode[v];
   assert(adj != nullptr);
   for (int i = 0; i < v; i++) { // initialize adj list as empty;
       adj[i].item = i;
                                 to begin with
                    unused:
                      but may store the degree of vertex i.
 ~Graph() {}
                      graph g = new Graph(v);
using graph = Graph *;
                      for (int i = 0; i < E; i++)
                        addEdge(g, from[i], to[i]);
```

Graph ADT – graph.cpp

```
// add an edge from v to w to an undirected graph
// A new vertex is added at the beginning of adj list of v.
void addEdgeFromTo(graph g, int v, int w) {
 gnode node = new Gnode(w);
 g->adj[v].next = node;
 g->E++;
   With a bug
// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
  addEdgeFromTo(g, v, w); // edge from v to w
 addEdgeFromTo(g, w, v);  // since undirected
```

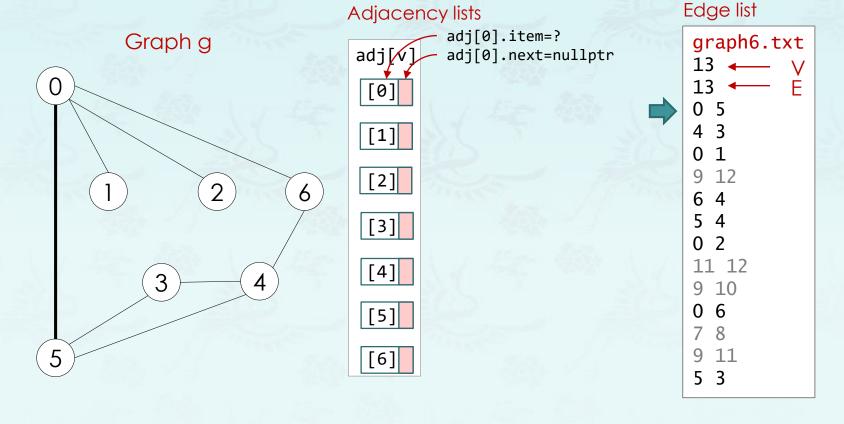
Graph ADT – graph.cpp

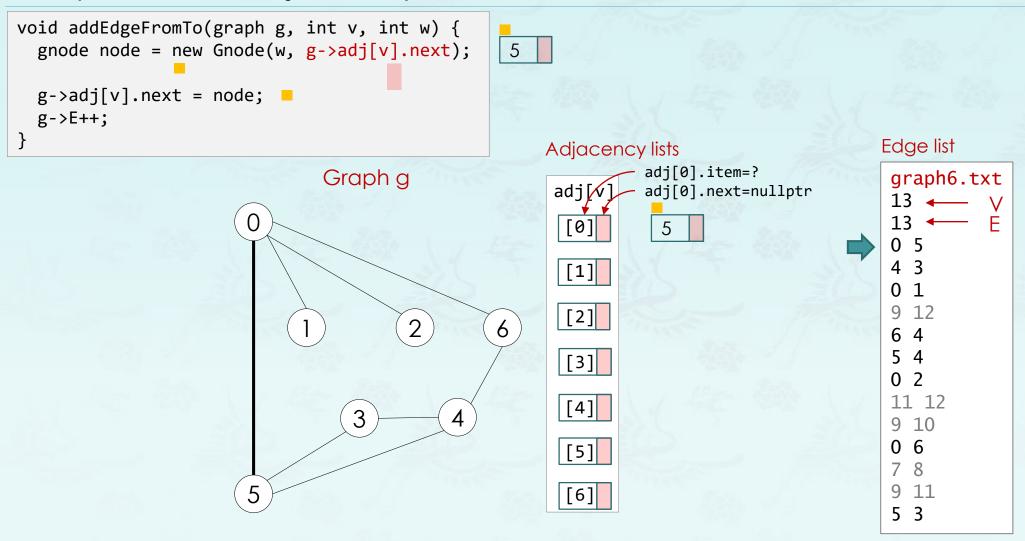
```
// add an edge from v to w to an undirected graph
// A new vertex is added at the beginning of adj list of v.
void addEdgeFromTo(graph g, int v, int w) {
                                                     instantiate a node w and
 gnode node = new Gnode(w);
                                                     make a link with vertex v.
 g->adj[v].next = node;
                                                     what is wrong?
 g->E++;
   With a bug
// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
  addEdgeFromTo(g, v, w); // edge from v to w
 addEdgeFromTo(g, w, v);  // since undirected
```

Graph ADT - graph.cpp

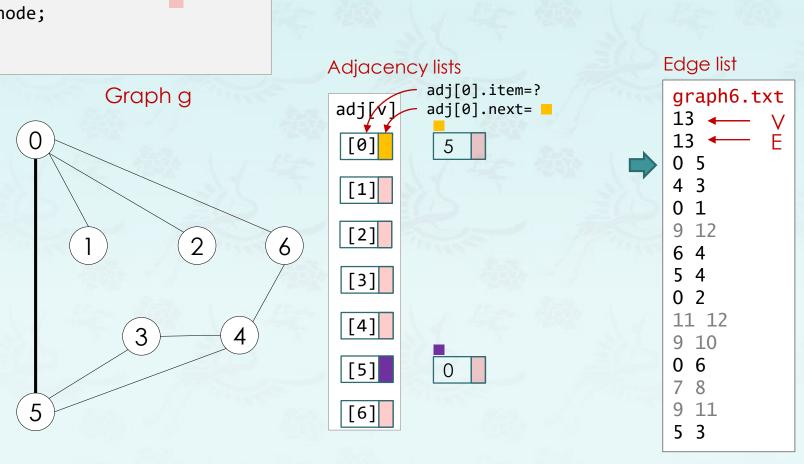
```
// add an edge from v to w to an undirected graph
// A new vertex is added at the beginning of adj list of v.
void addEdgeFromTo(graph g, int v, int w) {
                                                     instantiate a node w and
 gnode node = new Gnode(w, g->adj[v].next);
                                                     insert it at the front of adj[v]
 g->adj[v].next = node;
 g->E++;
// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
 addEdgeFromTo(g, v, w); // edge from v to w
 addEdgeFromTo(g, w, v);  // since undirected
```

```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```

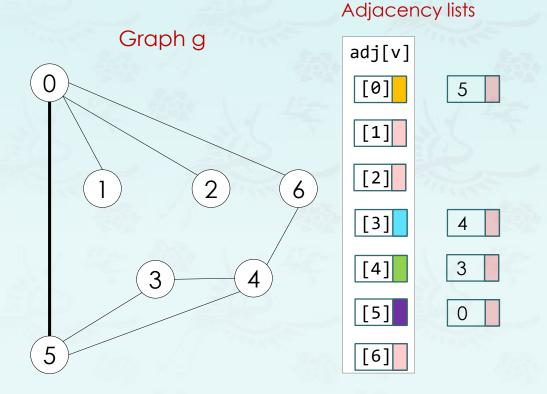


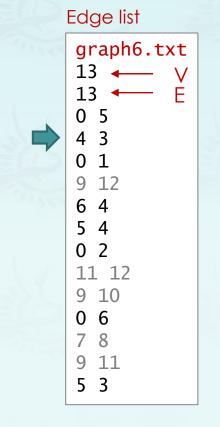


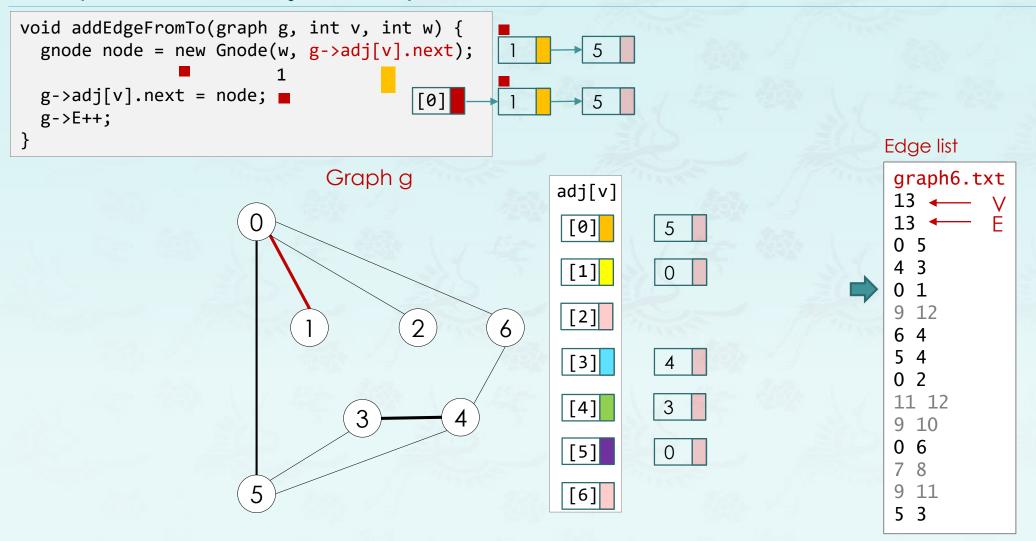
```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```



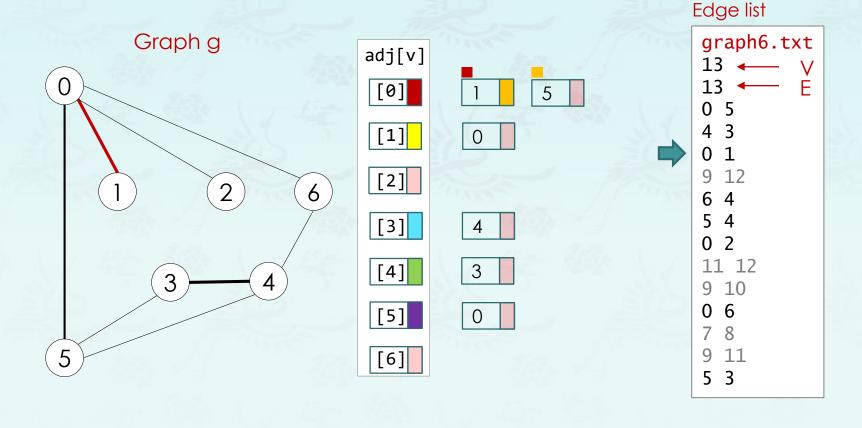
```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```



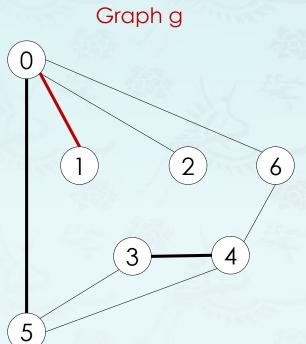




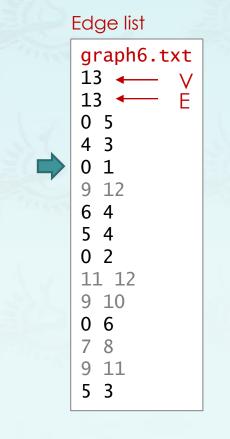
```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```



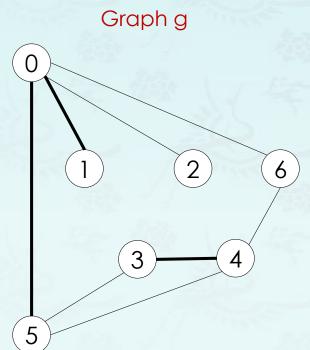
```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```

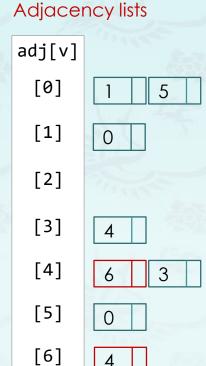


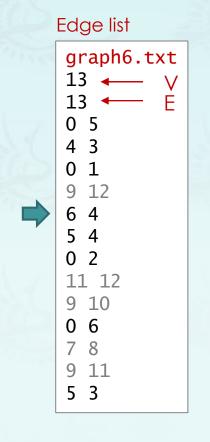
Adjacency lists adj[v] [0] [1] 0 [2] [3] 4 [4] 3 [5] 0 [6]



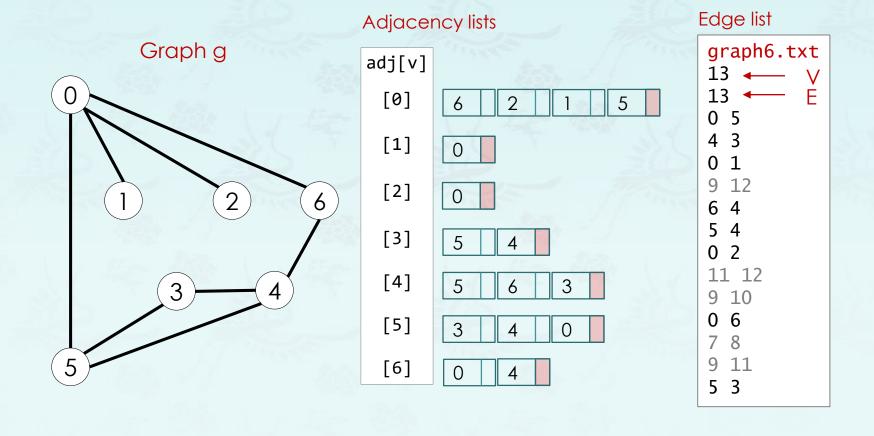
```
void addEdgeFromTo(graph g, int v, int w) {
  gnode node = new Gnode(w, g->adj[v].next);
  g->adj[v].next = node;
  g->E++;
}
```





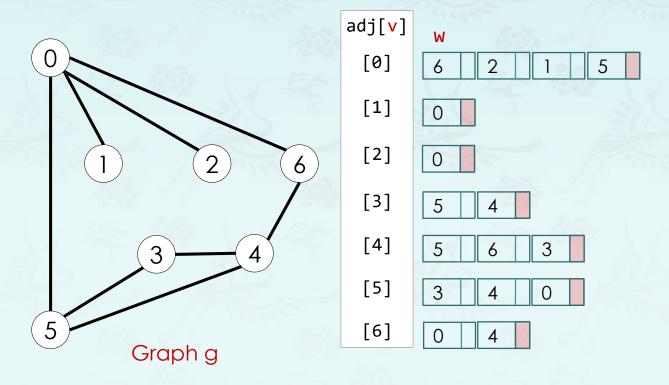


Warming-up: degree()

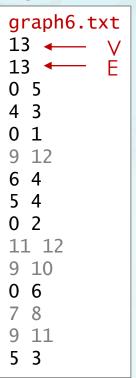


Warming-up: degree()

```
int degree(graph g, int v) {
  int deg = 0;
  for (gnode w = g->adj[v].next; w != nullptr; w = w->next)
    deg++;
  return deg;
}
```

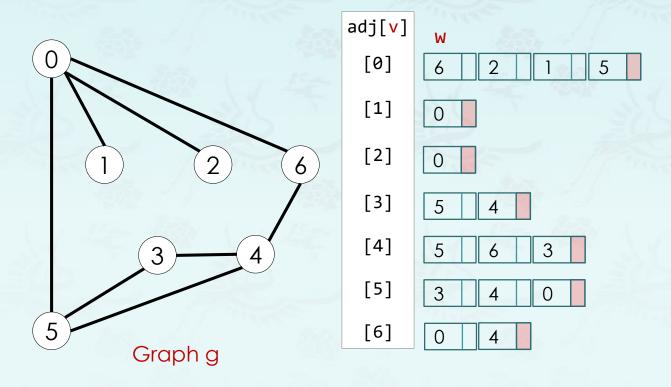


Edge list



Warming-up: degree()

```
int degree(graph g) {
  int deg = 0;
  for (int v = 0; v < V(g); v++)
   deg = max(degree(g, v), deg);
  return deg;
}</pre>
```

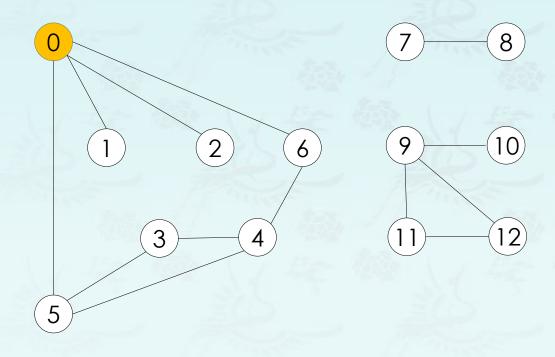


Edge list



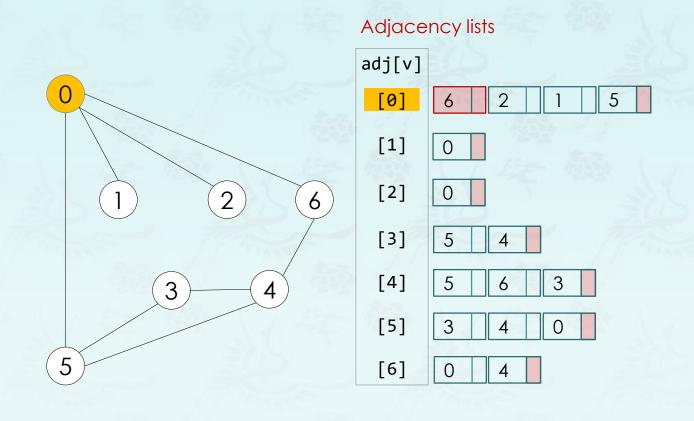
To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



V	marked[]	parent[v]
0	т	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-
10	9 F	-
1:	1 F	-
12	2 F	-

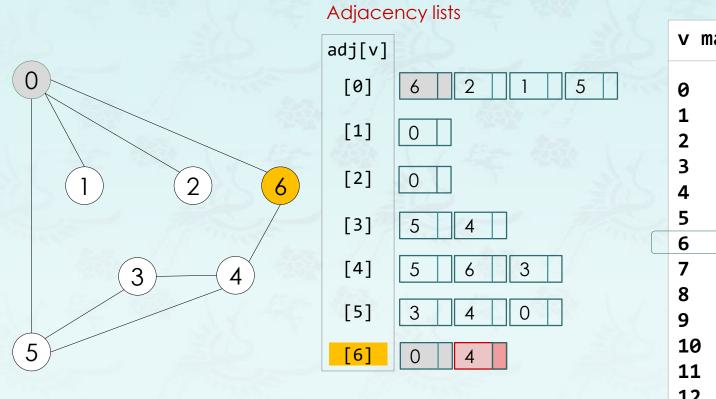
Which one first? visit 0:

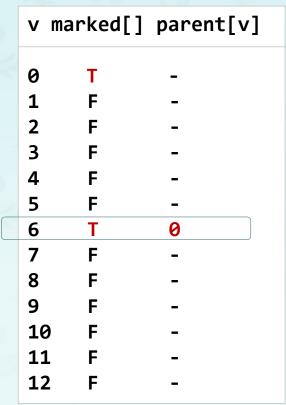


V	marked[]	<pre>parent[v]</pre>
_	_	
0	Т	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-
16) F	-
11	L F	-
12	2 F	-

visit 0: check 6, check 2, check 1, check 5

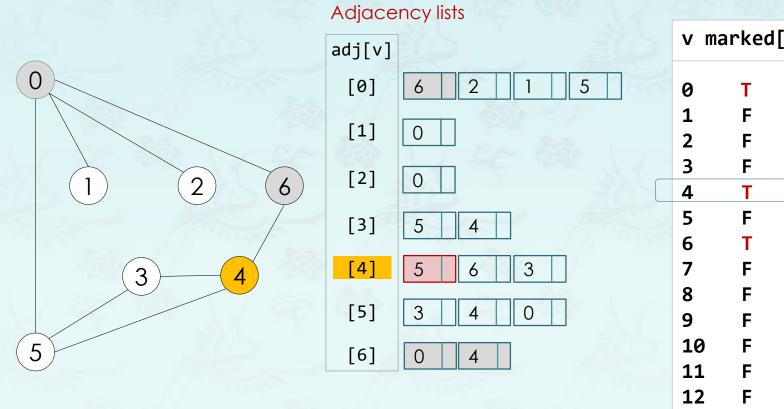
DFS 0





visit 6: check 0, check 4

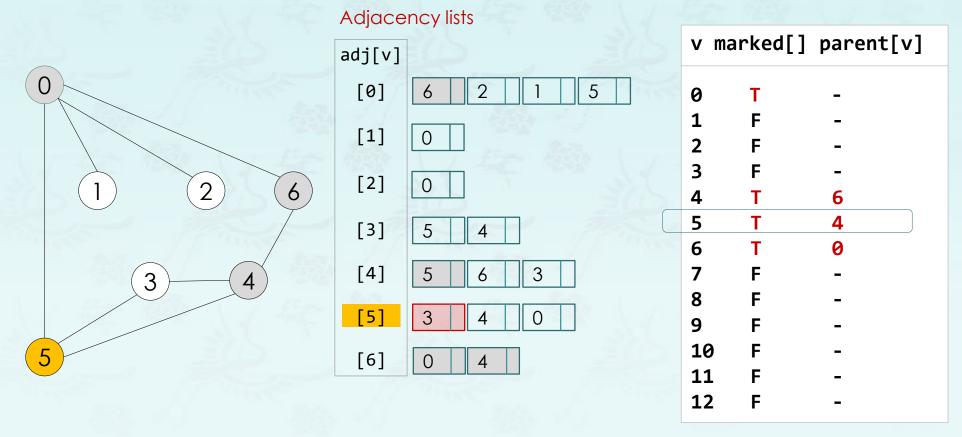
DFS 0 6



v marked[] parent[v] 6

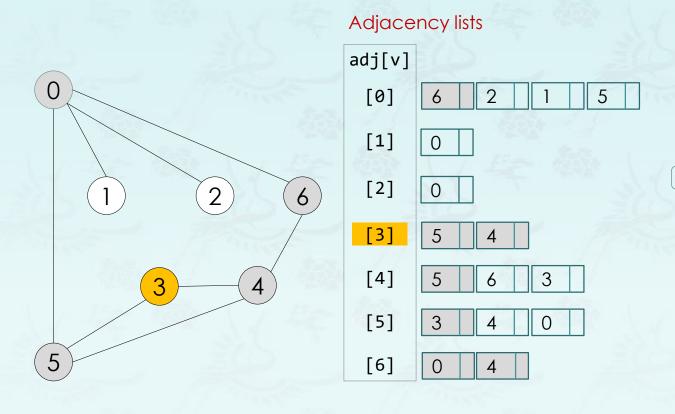
visit 4: check 5, check 6, check 3

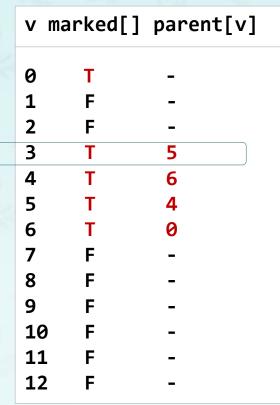
DFS 0 6 4



visit 5: check 3, check 4, check 0

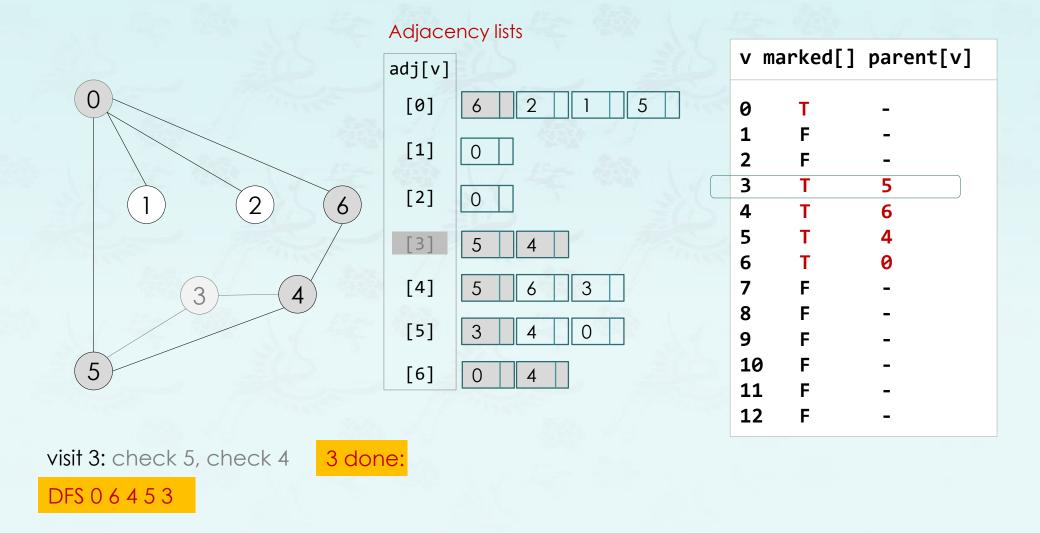
DFS 0 6 4 5

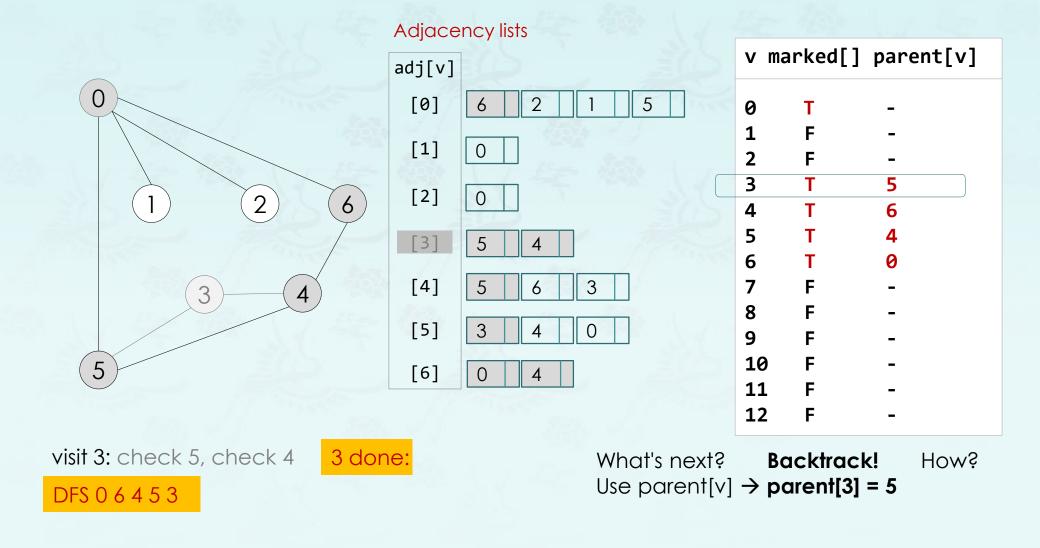




visit 3: check 5, check 4

DFS 0 6 4 5 3

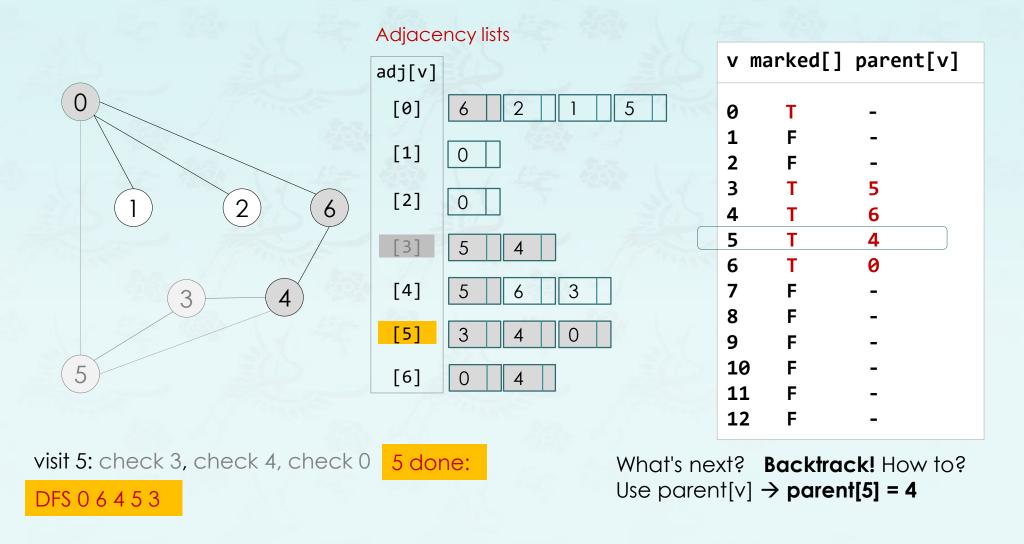


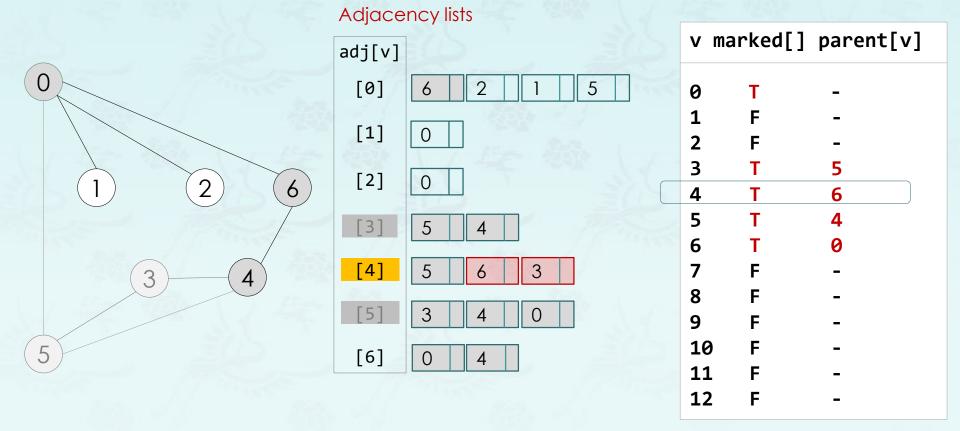




visit 5: check 3, check 4, check 0

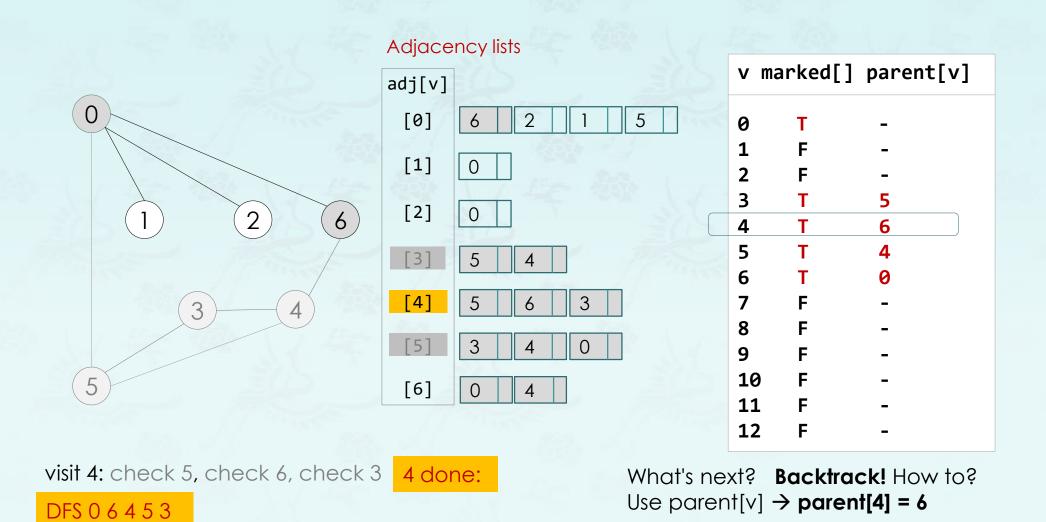
DFS 0 6 4 5 3

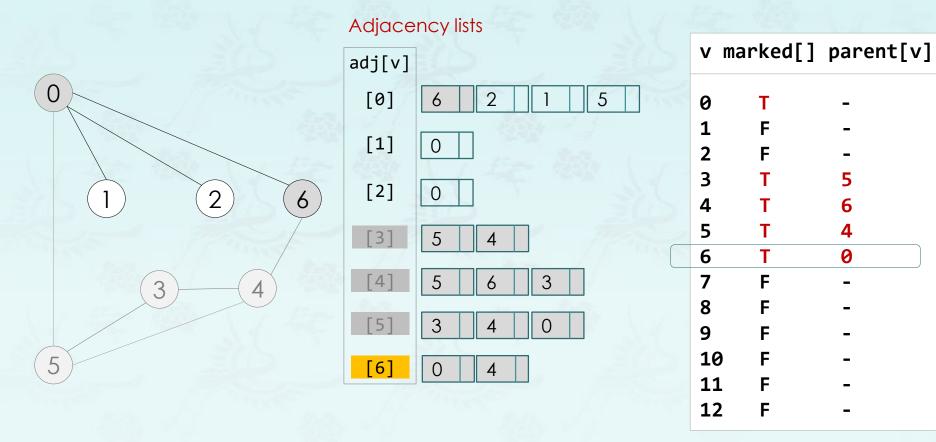




visit 4: check 5, check 6, check 3

DFS 0 6 4 5 3

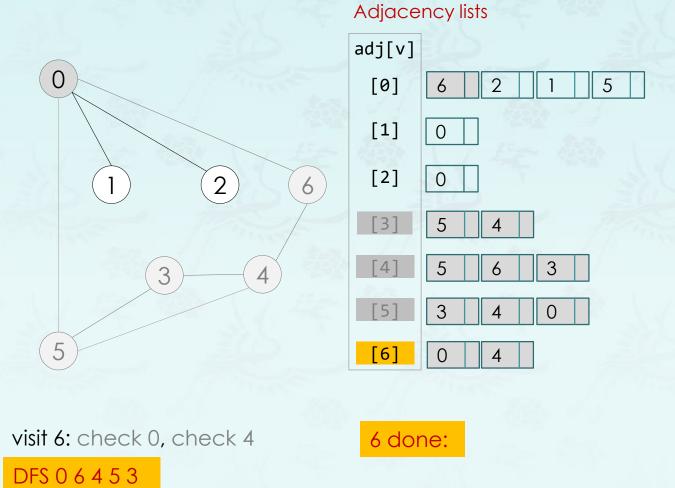




visit 6: check 0, check 4

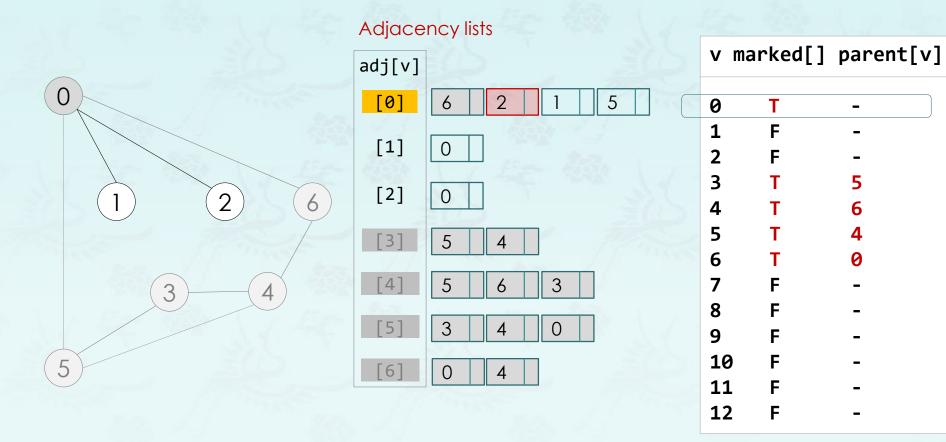
DFS 0 6 4 5 3

0



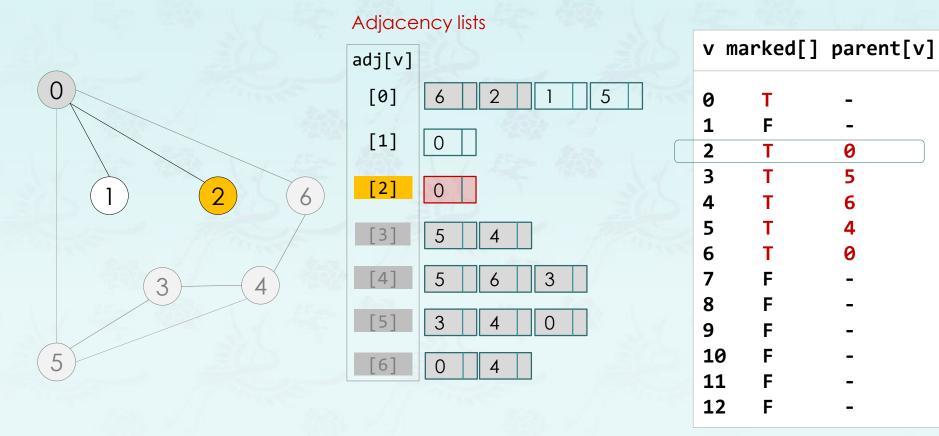
V	marked[]	<pre>parent[v]</pre>
0	Т	-
1	F	-
2	F	-
3	Т	5
4	T	6
5	Т	4
6	Т	0
7	F	-
8	F	-
9	F	-
16) F	-
11	L F	-
12	2 F	-

What's next? Backtrack! parent[6] = 0



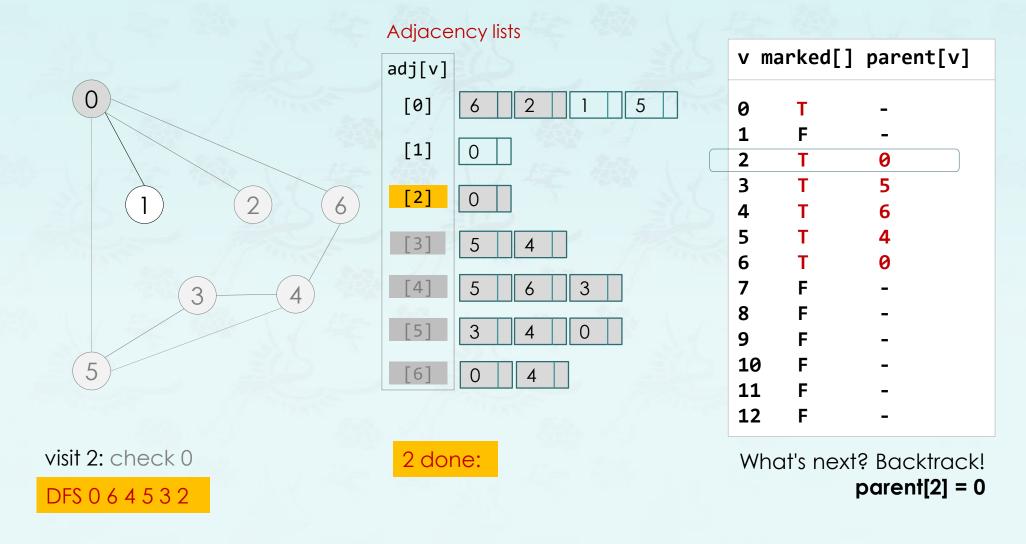
visit 0: check 6, check 2, check 1, check 5

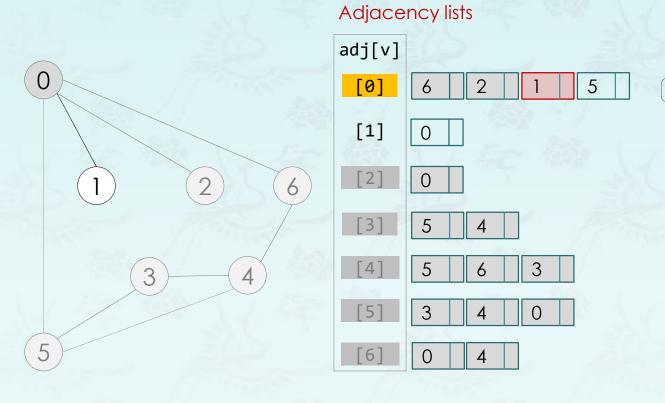
DFS 0 6 4 5 3



visit 2: check 0

DFS 0 6 4 5 3 2

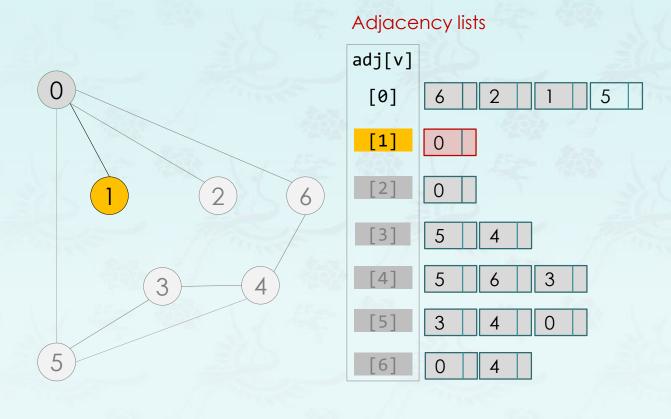




V	marked[]	parent[v]	
0	T	-	
1	F	-	
2	T	0	
3	T	5	
4	T	6	
5	T	4	
6	T	0	
7	F	_	
8	F	-	
9	F	_	
16) F	-	
11	L F	-	
12	2 F	-	

visit 0: check 6, check 2, check 1, check 5

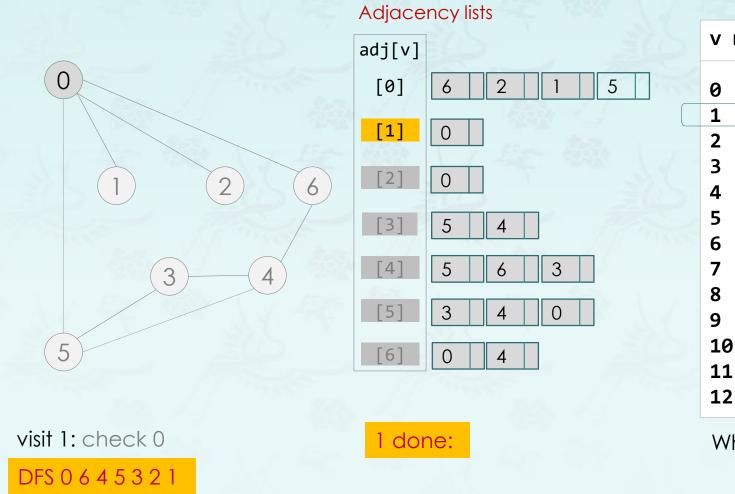
DFS 0 6 4 5 3 2





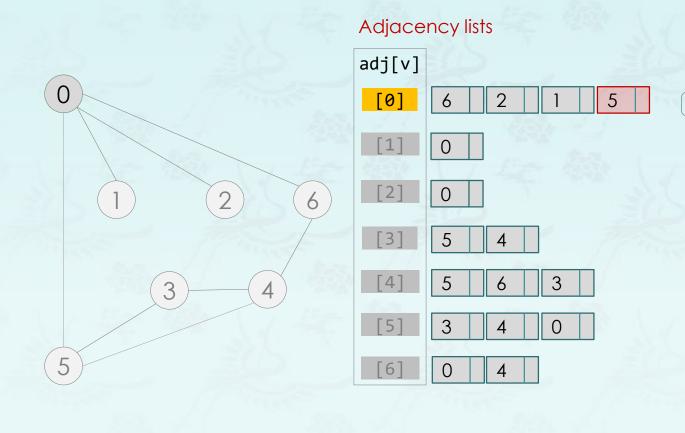
visit 1: check 0

DFS 0 6 4 5 3 2 1



V	marked[]	parent[v]
0	Т	-
1	Т	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	_
8	F	-
9	F	-
10	F	-
11	. F	-
12	. F	-

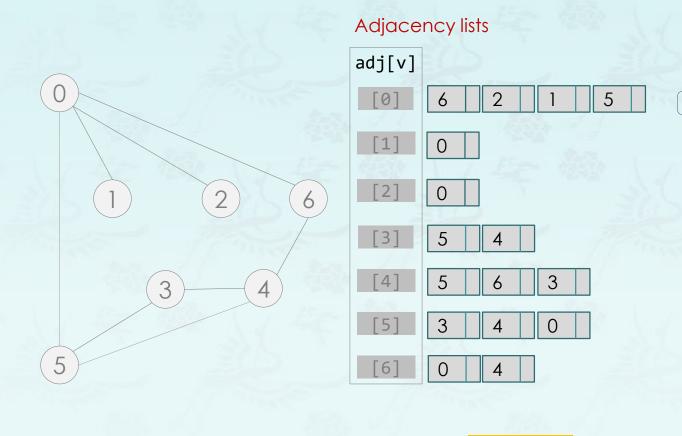
What's next? Backtrack! parent[1] = 0



V	marked[]	parent[v]	
0	T	_	
1	Т	0	
2	Т	0	
3	Т	5	
4	Т	6	
5	Т	4	
6	T	0	
7	F	_	
8	F	-	
9	F	-	
16) F	-	
11	. F	-	
12	2 F	-	

visit 1: check 6, check 2, check 1, check 5

DFS 0 6 4 5 3 2 1



V	marked[]	<pre>parent[v]</pre>	
0	Т	_	
1	Т	0	
2	Т	0	
3	T	5	
4	Т	6	
5	T	4	
6	T	0	
7	F	-	
8	F	-	
9	F	-	
16) F	-	
11	L F	-	
12	2 F	-	

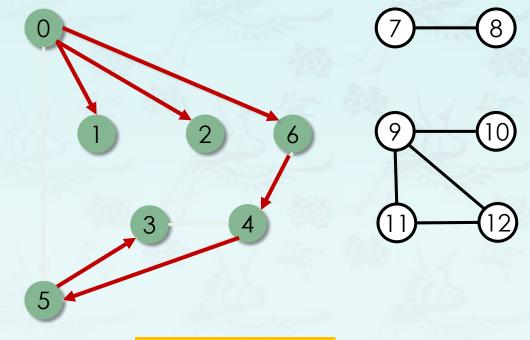
visit 1: check 6, check 2, check 1, check 5

0 done:

DFS 0 6 4 5 3 2 1

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



DFS Output:	DFS: 0 6 4 5 3 2 1
--------------------	--------------------

- found vertices reachable from 0
- build a data structure parent[v]

V	marked[]	<pre>parent[v]</pre>	
0	T	-	
1	Т	0	
2	Т	0	
3	T	5	
4	Т	6	
5	Т	4	
6	Т	0	
7	F	-	
8	F	-	
9	F	-	
16) F	-	
11	L F	-	
12	2 F	-	

Goal: Find all vertices connected to s (and a corresponding path).

Idea: Mimic maze exploration

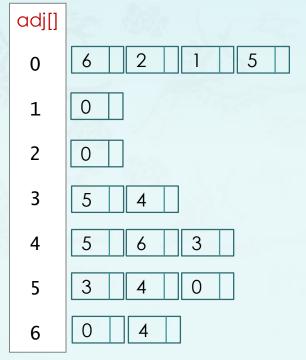
Algorithm:

- Use recursion (ball of string).
- Mark each visited vertex (and keep track of edge taken to visit it).
- Return (retrace steps) when no unvisited options.

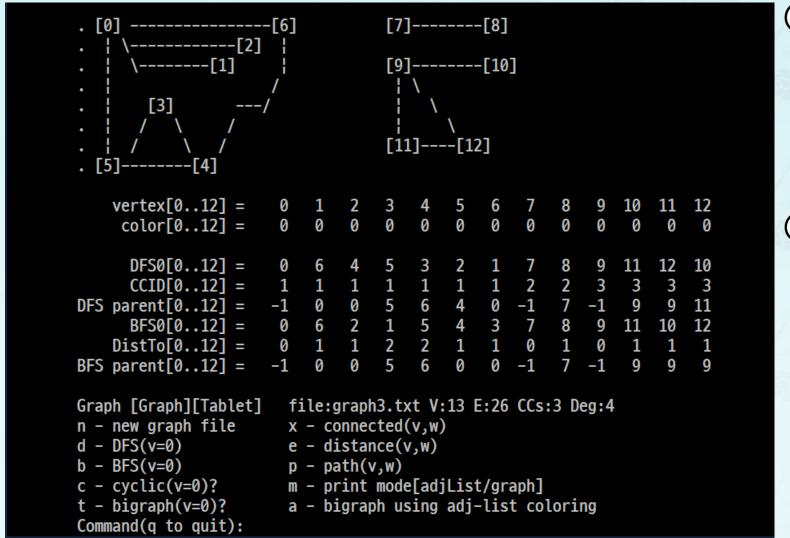
Data Structures:

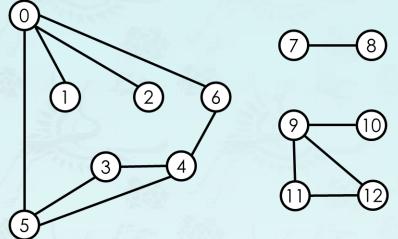
- Boolean[] marked to mark visited vetices.
- int[] parent to keep tree of paths.
 (parent[w] == v) means that edge v-w taken to visit w for first time

Adjacency lists



```
// runs DFS for all components and produces DFS0[], CCID[] & parentDFS[].
void DFS_CCs(graph g) {
  if (empty(g)) return;
  for (int i = 0; i < V(g); i++) {
    g->marked[i] = false;
    g->parentDFS[i] = -1;
    g \rightarrow CCID[i] = 0;
  queue<int> que;
                                    DFS() with a shortcoming
  DFS(g, 0, que);
  setDFS0(g, 0, que);
 g \rightarrow DFSv = \{\};
                                    // runs DFS for at vertex v recursively.
                                    // Only que, g->marked[v] and g->parentDFS[] are updated here.
                                    void DFS(graph g, int v, queue<int>& que) {
                                      g->marked[v] = true;  // visited
                                      que.push(v);
                                                                       // save the path
                                      for (gnode w = g->adj[v].next; w; w = w->next) {
                                        cout << "your code here (recursion) \n";</pre>
```





```
// returns a path from v to w using the DFS result or parentDFS[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
  if (empty(g)) return;
  for (int i = 0; i < V(g); i++) {
    g->marked[i] = false;
   g->parentDFS[i] = -1;
  queue<int> q;
  DFS(g, v, q); // DFS at v, starting vertex
  g->DFSv = q; // DFS result at v
  path = {};
  cout << "your code here\n"; // push v to w path to the stack path</pre>
```

DFS: Depth-First Search Properties

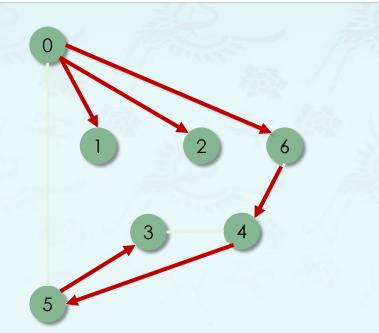
- Proposition: After DFS, can find vertices connected to sin constant time and can find a path to s (if one exists) in time proportional to its length.
- Proof: parent[] is parent-link representation of a tree rooted at s.

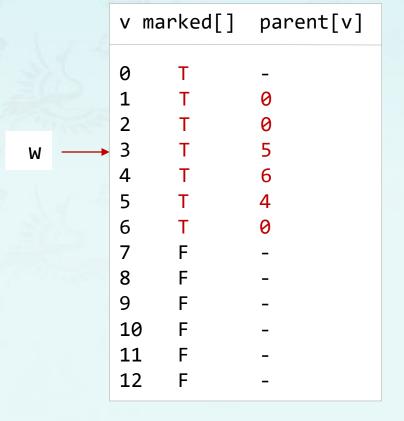
```
// returns a path from v to w using the DFS result or parentDFS[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
  if (empty(g)) return;
 for (int i = 0; i < V(g); i++) {
   g->marked[i] = false;
   g->parentDFS[i] = -1;
 queue<int> q;
 DFS(g, v, q); // DFS at v, starting vertex
 g->DFSv = q; // DFS result at v
 path = {};
 cout << "your code here\n"; // push v to w path to the stack path</pre>
```

V	marked[]	parent[v]
0	Т	-
1	Т	0
2	T	0
3	Т	5
4	Т	6
5	Т	4
6	T	0
7	F	-
8	F	-
9	F	-
16) F	-
1:	l F	-
12	2 F	-

DFS: Depth-First Search Properties

- Proposition: After DFS, can find vertices connected to sin constant time and can find a path to s (if one exists) in time proportional to its length.
- Proof: parent[] is parent-link representation of a tree rooted at s.
- What is the path from vertex 0 to vertex 3?
- In this case, what is in the stack when parent[] returns?
 void DFSpath(graph g, int v, int w, stack<int>& path)

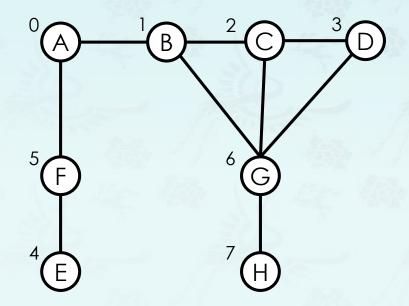




DFS - Exercise

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



aajacent list	adjacent li
A: B F	0: 1 5
B: G C A	1:620
C: DGB	2: 3 6 1
D: C G	3: 2 6
E: F	4: 5
F: E A	5: 40
G: HBCD	6:7123
H: G	7: 6

Graph g:

Hint: A B...?...F E

DFS/BFS - Exercise

- 1. Create an **antenna.txt** such that it generates the following adjacent list. Refer to some graph files provided with Pset to see how the graph file format works.
- 2. Find DFS/BFS and parent vertices, respectively. During BFS, additionally, get distTo[] as well. Record these results in antenna.txt
- 3. Show how you get these results as good as you can.
 Submit this part in a separate file as named like antenna.docx or antenna.hwp, etc.
- 4. Make sure that your antenna.txt can be processed by graphx.exe and produces the same adjacent list.

Data Structures Chapter 7: Graph

- 1. Introduction
 - Terminology, Representation, ADT
- 2. Basic Operations
 - DFS, CC, BFS, Processing
- 3. Digraph and Applications
- 4. Minimum Spanning Tree(MST)