



Data Structures

Chapter 5 Tree

1. introduction
2. Binary tree
 - Definition and Properties
 - Traversal
 - **Coding**
3. Binary search tree
4. Tree balancing

사람아
주께서 산한 것이 무엇임을
네게 보이셨나니

여호와께서 네게 구하시는 것이
오직 공의를 행하며 인자를 사랑하며
겸손히 네 하나님과 함께 행하는 것이 아니냐

미
가
6
장
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절

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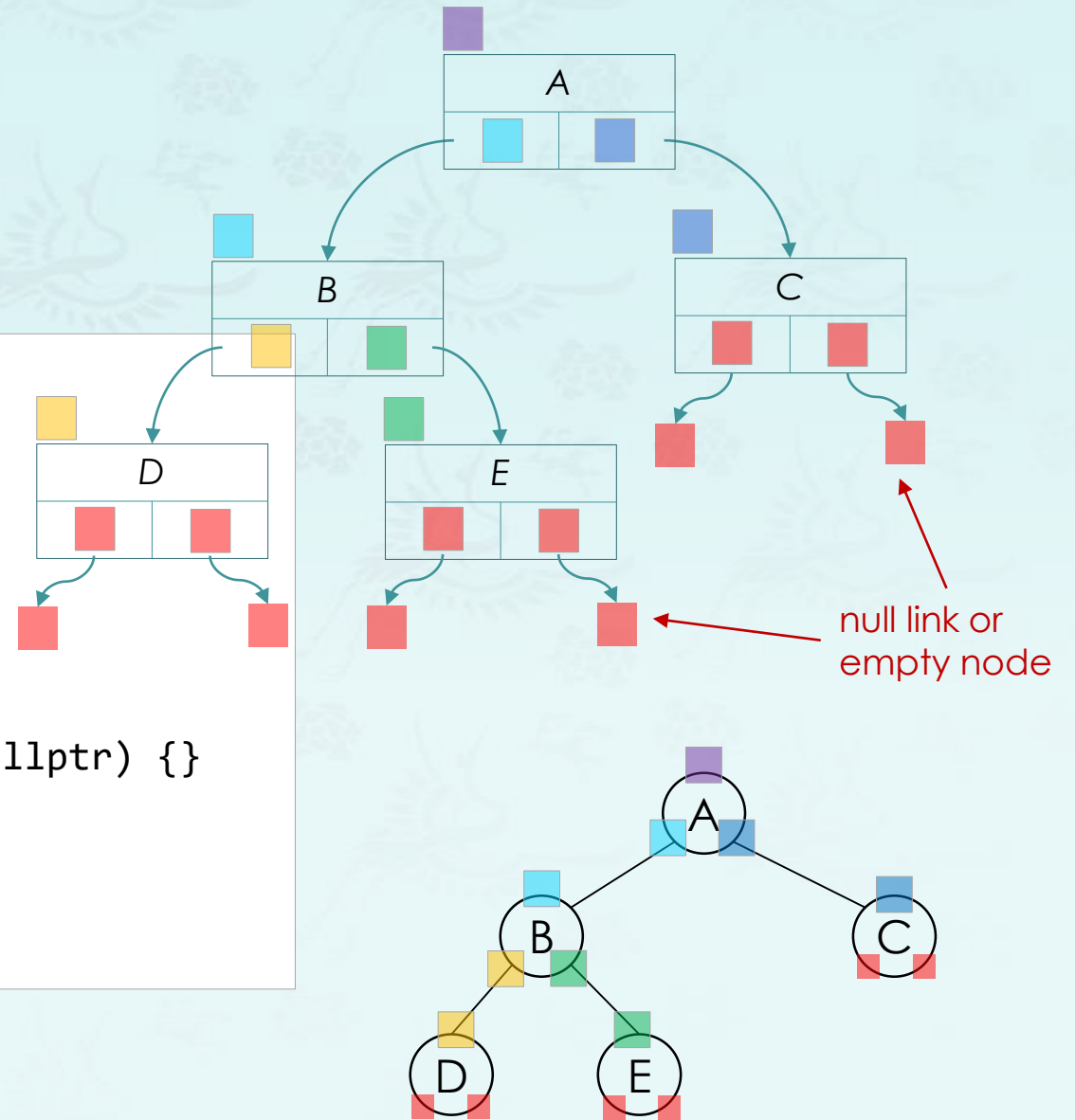
He has showed you, O man, what is good. And what does the LORD require of you?
To act justly and to love mercy and to walk humbly with your God. Micah 6:8

하나님이 우리를 구원하사 기록하신 소명으로 부르심은 우리의 행위대로 하심이 아니요 오직
자기의 뜻과 영원 전부터 그리스도 예수 안에서 우리에게 주신 은혜대로 하심이라 (딤후1:9)

Recursion & Tree Structure

```
struct TreeNode{  
    int      key;  
    TreeNode* left;  
    TreeNode* right;  
};  
using tree = TreeNode*;
```

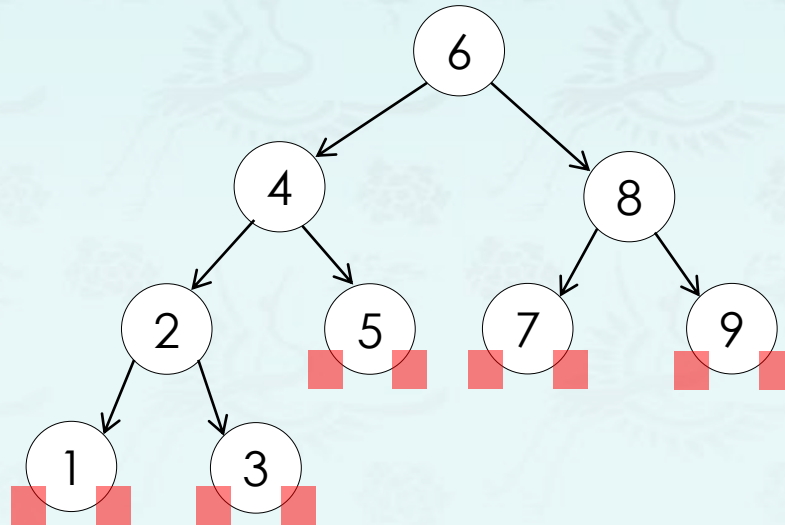
```
struct TreeNode{  
    int      key;  
    TreeNode* left;  
    TreeNode* right;  
  
    TreeNode(int k, TreeNode* l, TreeNode* r) {  
        key = k; left = l; right = r;  
    }  
    TreeNode(int k) : key(k), left(nullptr), right(nullptr) {}  
  
    ~TreeNode(){}  
};  
using tree = TreeNode*;
```



Operations: inorder()

```
// Given a binary tree, its node values in inorder are passed  
// back through the argument v which is passed by reference.  
void inorder(tree node, vector<int>& v) {  
    if (empty(node)) return;  
  
    inorder(node->left, v);  
    v.push_back(node->key);  
    inorder(node->right, v);  
}
```

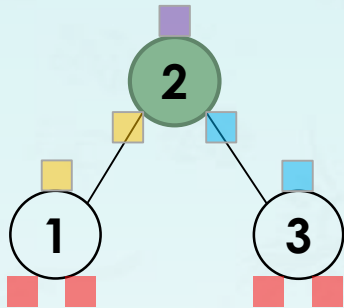
```
// pinorder() and its result  
vec.clear();  
inorder(root, vec);  
cout << "inorder: ";  
for (auto i : vec)  
    cout << i << " ";  
cout endl;
```



Binary tree traversals

■ **Example: Inorder traversal(LVR)**

- Step 1 – Recursively traverse left subtree.
- Step 2 – Visit root node. (print or save it.)
- Step 3 – Recursively traverse right subtree.

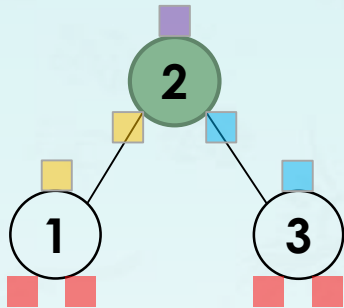


```
void inorder(tree root) {  
    if (root == nullptr) return;  
  
    inorder(root->left);    L  
    cout << root->key;      V  
    inorder(root->right);   R  
}
```


Binary tree traversals

■ **Example: Inorder traversal(LVR)**

- Step 1 – Recursively traverse left subtree.
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- Step 3 – Recursively traverse right subtree.



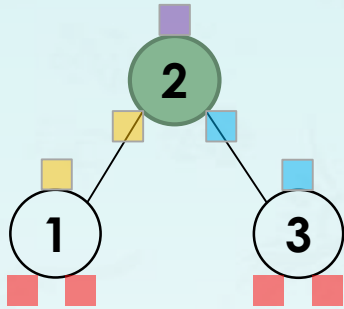
```
void inorder(tree root) {  
    if (root == nullptr) return;  
  
    inorder(root->left);    L  
    cout << root->key;      V  
    inorder(root->right);   R  
}
```

```
int main() {  
  
}
```

Binary tree traversals

■ Example: Inorder traversal(LVR)

- Step 1 – Recursively traverse left subtree.
- Step 2 – Visit root node. (print or save it.)
- Step 3 – Recursively traverse right subtree.



```
void inorder(tree root) {  
    if (root == nullptr) return;  
  
    inorder(root->left);    L  
    cout << root->key;    V  
    inorder(root->right);  R  
}
```

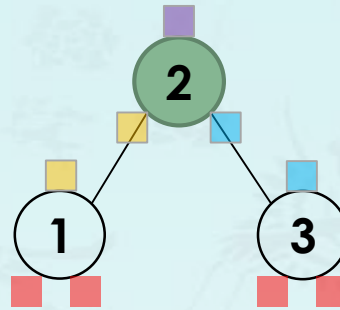
```
int main() {  
    tree l = new TreeNode(1);  
    tree r = new TreeNode(3);  
    tree root = new TreeNode(2, l, r);  
    inorder(root);  
}
```


Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {  
    if (root == nullptr) return;  
    push → inorder(root->left);  
    pop → cout << root->key;  
    push → inorder(root->right);  
    pop →  
}
```

push pop
system stack

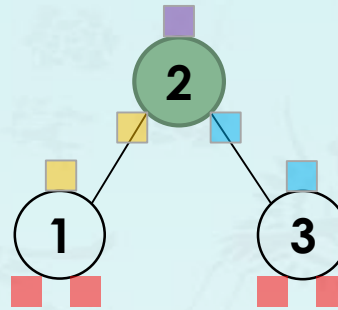
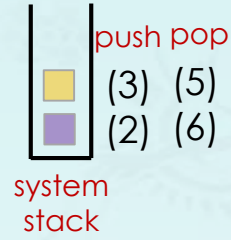


Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

push
pop
push
pop



- (1) The initial call `inorder(purple, key=2)` is made.
- (2) The purple calls `inorder()` with left (or orange).
The purple FC (the initial call) stops and stacked.
- (3) The orange calls `inorder()` with left (or pink)
The orange FC stops and stacked.
The pink FC returns immediately since it is null.
The pink FC is over 1st time. Then, stack pops. ■
- (4) The orange pops from stack & continues where it left.
■ `cout << root->key;` ➡ 1
- (5) The orange calls `inorder()` with right(or pink).
The orange FC stops and stacked. ← how many calls in stack?
The pink FC returns immediately since it is null.
The orange FC finishes its job here. Stock pops. ■
- (6) The purple pops from stack & continues where it left. ■ ← how many calls in stack?
■ `cout << root->key;` ➡ 2

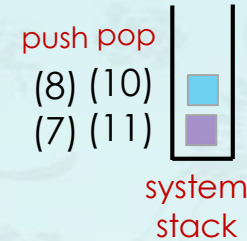
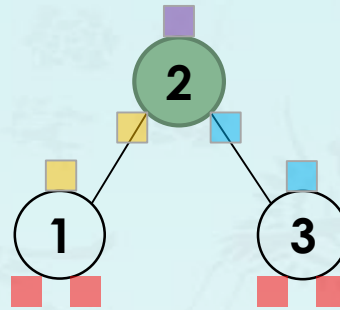
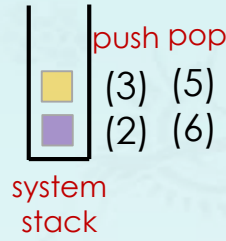
❖ FC stands for function call.

Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {
    if (root == nullptr) return;
```

```
    push → inorder(root->left);
    pop → cout << root->key;
    push → inorder(root->right);
    pop → }
```



(7) (8) (10)

```
void inorder(tree root) {
    if (root == nullptr) return;
```

```
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

- (1) The initial call `inorder(purple, key=2)` is made.
- (2) The purple calls `inorder(orange)` with left (or orange).
The purple FC (the initial call) stops and stacked.
- (3) The orange calls `inorder(pink)` with left (or pink).
The orange FC stops and stacked.
The pink FC returns immediately since it is null.
The pink FC is over 1st time. Then, stack pops.
- (4) The orange pops from stack & continues where it left.
orange cout << root->key; → 1
- (5) The orange calls `inorder(pink)` with right (or pink).
The orange FC stops and stacked.
The pink FC returns immediately since it is null.
The orange FC finishes its job here. Stock pops.
- (6) The purple pops from stack & continues where it left.
purple cout << root->key; → 2

- (7) The purple call `inorder(blue)` with right (or blue).
The purple FC stops and stacked.
- (8) The blue calls `inorder(pink)` with left (or pink).
The blue FC stops and stacked.
The pink FC returns immediately since it is null.
The pink FC is over. Then, stack pops.
- (9) The blue pops from stack & continues where it left.
blue cout << root->key; → 3
- (10) The blue calls `inorder(pink)` with right (or pink).
The blue FC stops and stacked.
The pink FC returns immediately since it is null.
The blue FC finishes its job here. Stock pops.
- (11) The purple pops from stack & continues where it left.
The purple finishes its job and returns to the caller(main).

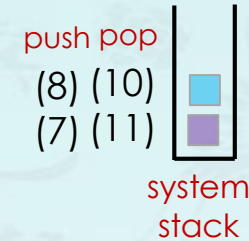
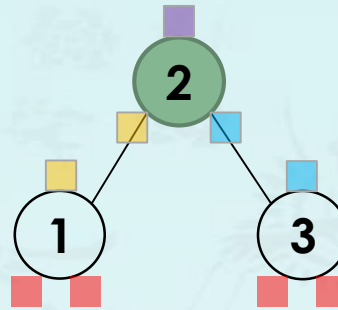
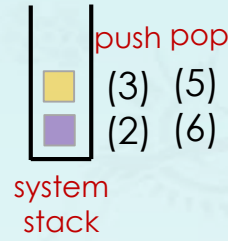
❖ FC stands for function call.

Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

push
pop
push
pop



(7) (8) (10)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

- (1) The initial call `inorder(purple, key=2)` is made.
- (2) The purple calls `inorder(orange)` with left (or orange). The purple FC (the initial call) stops and stacked.
- (3) The orange calls `inorder(pink)` with left (or pink). The orange FC stops and stacked. The pink FC starts.
- (4) The orange pops from stack & continues where it left. `cout << root->key;`
- (5) The orange calls `inorder(blue)` with right (or blue). The orange FC stops and stacked. The blue FC starts.

- **The final output:**
- **The number of times of `inorder()` invoked:**
- **The number of times of the first line return executed:**
- **The number of times of the hidden return executed:**
- **List root's keys passed as an argument and its count:**

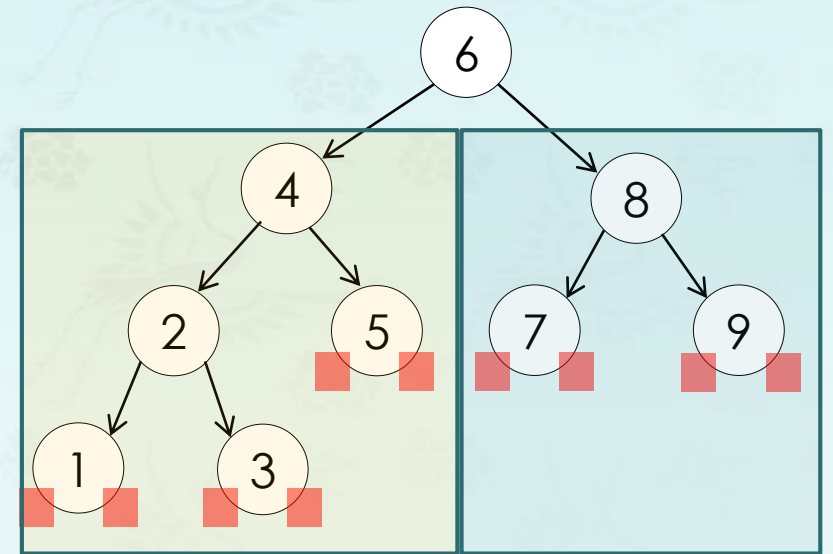
- (6) The purple pops from stack & continues where it left. `cout << root->key;` → 2

- (7) The purple call `inorder(blue)` with right (or blue). The purple FC stops and stacked.
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- (10) The blue calls `inorder(blue)` with right (or blue). The blue FC stops and stacked. The pink FC starts.
- (11) The purple pops from stack & continues where it left. The purple finishes its job and returns to the caller(main).

❖ FC stands for function call.

Operations: size()

```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    return size(node->left) + size(node->right) + 1;
}
```

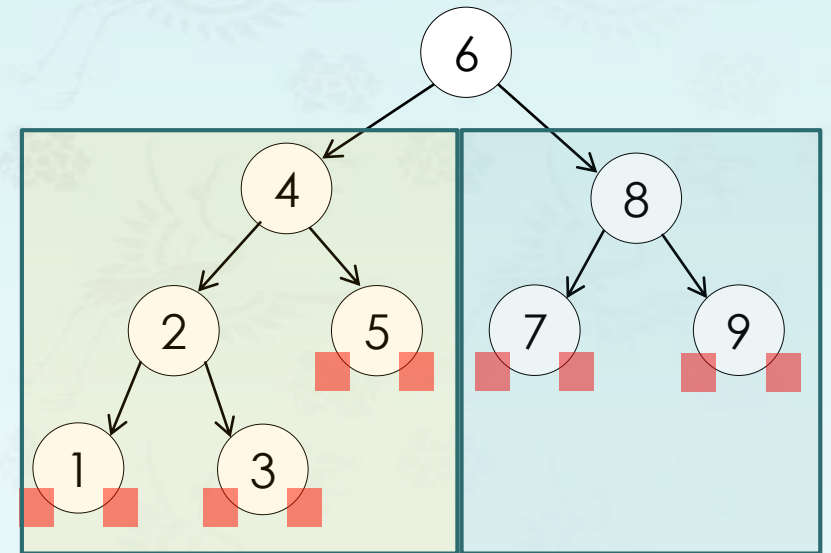


Operations: size()

```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    return size(node->left) + size(node->right) + 1;
}
```

- Q1. What is the total number of the function calls to complete with the tree and how many returns each?

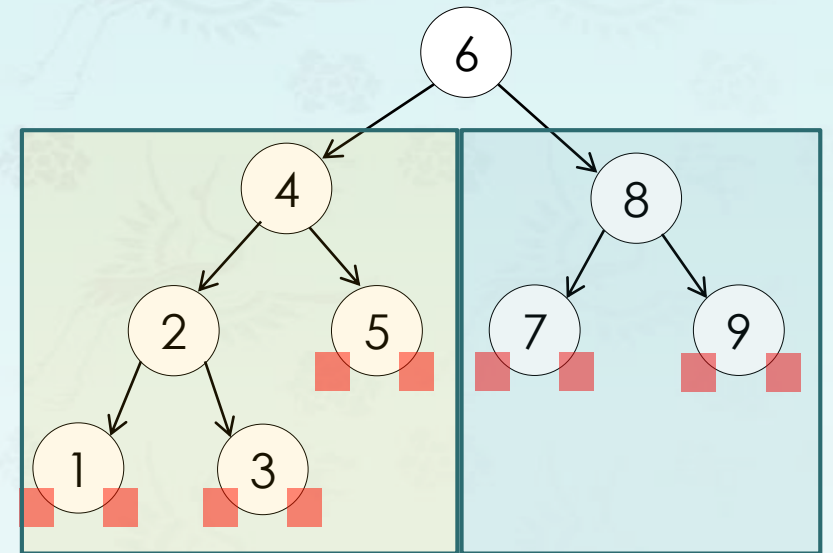
```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    int left = size(node->left);
    int right = size(node->right);
    return left + right + 1;
} // debug & trace friendly version
```



Operations: size()

```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    return size(node->left) + size(node->right) + 1;
}
```

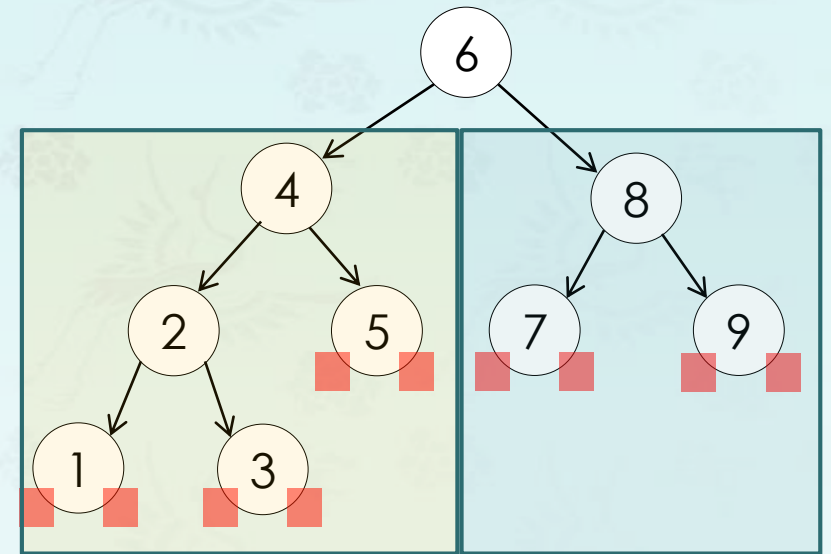
- Q1. What is the total number of the function calls to complete with the tree and how many returns each?
- Q2. Which node invokes the last function call?
- Q3. Which node finishes its size function call and returns size = 1 for the first time?



Operations: height()

```
// returns the max depth of a tree.  
// height = -1 for empty tree, 0 for root only tree  
int height(tree node) {  
    if (empty(node)) return -1;  
    int left  = height(node->left);  
    int right = height(node->right);  
    return max(left, right) + 1;  
}
```

- Q1. What is the total number of the function call to complete with the tree below?
- Q2. What is the return value of the 10th and 12th function call?
- Q3. What is the return value of the node 2?



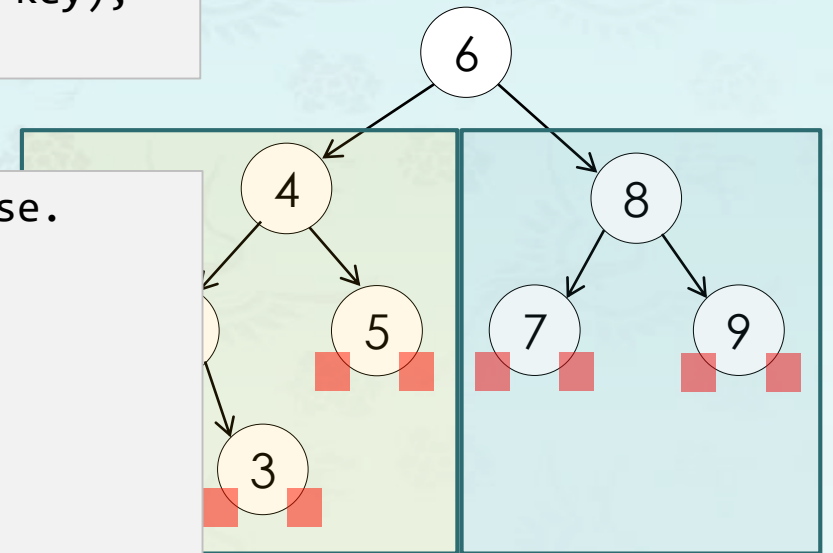
Operations: containsBT(), findBT()

```
// returns true if key is in a given binary tree, false otherwise.
bool containsBT(tree root, int key) {
    if (empty(root)) return false;
    if (key == root->key) return true;

    return containsBT(root->left, key) || containsBT(root->right, key);
}
```

```
// returns true if key is in a given binary tree, false otherwise.
bool containsBT(tree root, int key) {
    if (empty(root)) return false;
    if (key == root->key) return true;

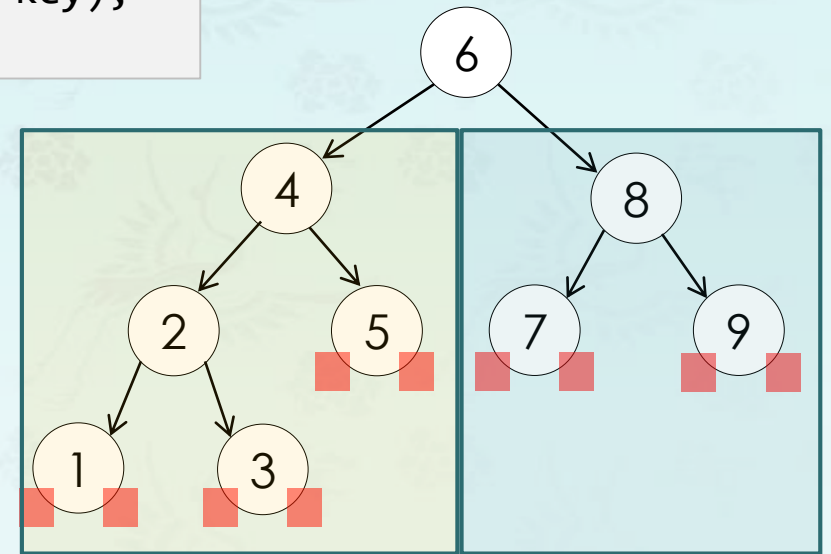
    if (containsBT(root->left, key)
        return true;
    if (containsBT(root->right, key)
        return true;
    return false;
} // debug & trace friendly version
```



Operations: containsBT(), findBT()

```
// returns true if key is in a given binary tree, false otherwise.  
bool containsBT(tree root, int key) {  
    if (empty(root)) return false;  
    if (key == root->key) return true;  
  
    return containsBT(root->left, key) || containsBT(root->right, key);  
}
```

- Q1: Which node invokes **containsBT(root->right, key)** for the first time?
- Q2: Which node will invoke **return false** for the first time?
- Q3: How many function calls are made to reach the node **key=5**?
- Q4: How many function calls still remains in the system stack to finish after key=5 is found and what are they?



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